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Guiding center orbit simulation of energetic particles in tokamak plasma

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Abstract: *A new relativistic guiding center orbit code (RGCO) has been developed and used to investigate the confinement mechanism of energetic electrons in QUEST tokamak. Benefiting from the orbit simulation in cylindrical coordinates, the particle tracking can be performed within the entire first wall, including the main plasma and the scrape-off layer (SOL), so it can provide a more realistic evaluation of the confinement and loss of the particle than previous results. Detailed analyses reveal that energetic electrons can be confined both inside and outside the last closed flux surface (LCFS) inside the first wall of QUEST tokamak, thus contributing to the non-inductive current driving. In future work, the confinement and loss mechanism of alpha particle will be researched through energetic electron in QUEST by physics experiment combined with the RGCO code simulation.*

Keywords: Alpha particle; Energetic particle; Guiding center orbit; Relativistic effect; Ripple field

2. INTRODUCTION

Generated by auxiliary heating or by fusion reactions, energetic particles (EP) perform an important role in plasma heating, current driving, and momentum inputting. The well confinement of EP is a key issue for the high performance and safety operation of both present-day tokamak and future fusion power plant. The loss of EP may cause serious damage to the plasma facing components (PFCs). Therefore, the confinement and loss mechanism of the EP should be systematically researched.

Particle orbit simulation is one of the powerful tools for the above study. As an effective method, the particle-in-cell (PIC) technique [1] follows the motion of a large number of particles in their self-consistent electromagnetic fields, which is suitable for solving the instability problems in plasma. However, the time consumption of the computation is unacceptable for physics problems with long time scales, such as the slowing down process of alpha particle. For this case, the test particle method is an attractive alternative that describes the single particle motion in the same way as the PIC method, but with an equilibrium electromagnetic field. A series of numerical codes based on the test particle method were developed, such as NUBEAM [2], ORBIT [3] and ASCOT [4]. These codes were mainly used to study the phase space dynamics of energetic ions. However, for energetic electrons, their energy is comparable to that of energetic ions, but their mass is only one thousandth of that of ions, thus having a strong relativistic effect. Therefore, it is urgent to develop an orbit simulation code with relativistic effects.

Under this background, the relativistic guiding center orbit code (RGCO) has been developed. Since the new code simulate particle orbits in cylindrical coordinates, it can provide a more realistic evaluation of the confinement and loss of the energetic electrons within the entire first wall. In general, the application of relativistic guiding center orbit simulation is a step forward in the study of energetic electron confinement in QUEST.

The remainder of this work is organized as follows. Descriptions of simulation models are presented in section 2. The confinement of energetic electrons in QUEST tokamak is discussed in section 3. Finally, a summary is given in section 4.

3. SIMULATION MODELS

In order to study the confinement and loss mechanism of EP in plasma, a guiding center orbit code with relativistic effect has been developed. In this section, the key model and features of this code are described.

3.1 Equilibrium and perturbations

There is no closed magnetic surface in the SOL, so the particles in SOL tend to attack the PFCs along the magnetic field line, causing the excessive localized heat load, which is one of the situations that future fusion power plant are trying to avoid. To consider the orbit calculation in SOL, the cylindrical coordinate is selected in the code.

The magnetic fields, both equilibrium and perturbation fields, must be calculated and contained in the code, for the purpose of performing particle orbits simulation. The equilibrium field can be calculated from the poloidal flux Ψ_p and poloidal current profile provided by the EFIT equilibrium [5], the Ψ_p is in 2D uniform (R, Z) mesh and $f = RB_\phi$ is defined in 1D uniform Ψ_p mesh. The poloidal view of the Ψ_p in QUEST shot 35442 at 2.5 s can be seen in Fig. 1. The radial and vertical components of the poloidal magnetic field can be calculated by the following formulas [6].

$$B_R(R,Z) = -\frac{1}{2\pi R}\frac{\partial \Psi_P(R,Z)}{\partial Z} \tag{1}$$

$$B_Z(R,Z) = \frac{1}{2\pi R}\frac{\partial \Psi_P(R,Z)}{\partial R} \tag{2}$$

In this way, the equilibrium magnetic field with divergence-free condition is obtained, which is the basis for the accurate orbit calculation.

$$B_\phi(R,Z) = B_{\phi 0}(R,Z) + B_{\phi 0}(R,Z) * \delta(R,Z) * \cos(\phi) \tag{3}$$

Where

$$\delta(R,Z)=(B_{\max}-B_{\min})/(B_{\max}+B_{\min}) \qquad (4)$$

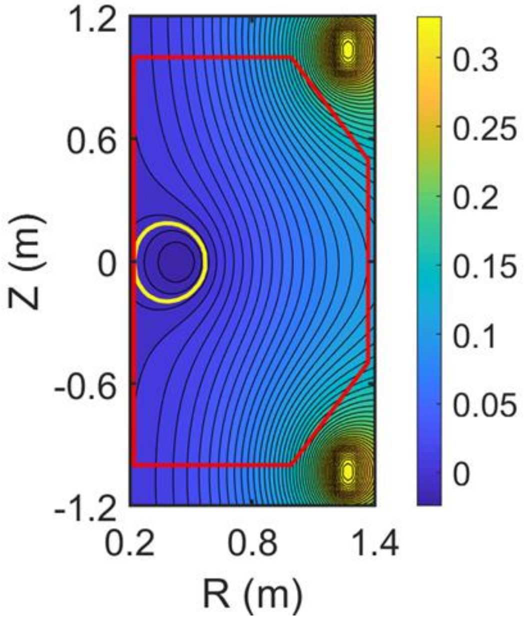

Fig. 1. The poloidal view of the Ψ_p in QUEST shot 35442 at 2.5 s. The red line is the first wall of the QUEST and the yellow circle is the LCFS.

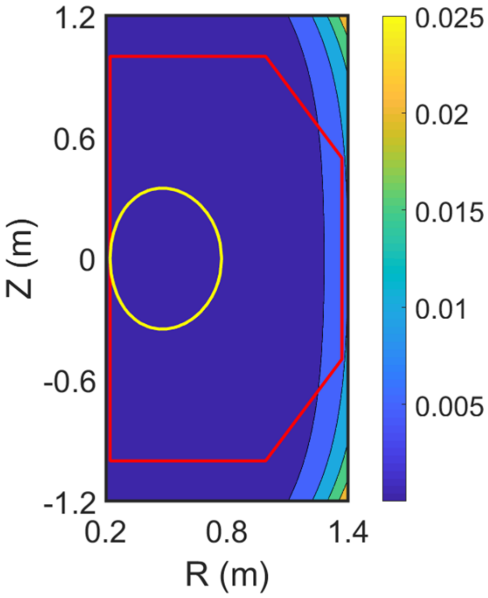

Fig. 2. The ripple of the QUEST tokamak. The red line is the first wall of the QUEST, and the yellow circle is the LCFS.

As one of the important perturbation magnetic field, ripple field, caused by the discrete toroidal field coils, is prone to cause the particle classical loss by the diffusion of banana orbits whose turning points lie within a certain zone close to top and bottom. Therefore, the ripple is an important engineering parameter that should be considered when designing a new tokamak. After considering the ripple field, the toroidal magnetic field used in the code is calculated by the following equation [7].

The first item on the right-hand side of equation 3 is the equilibrium magnetic field and the second item is the ripple field. δ is defined as ripple at one location in the tokamak. $B_{\max}=\max[B_\phi(R,\phi,Z)]$ is obtained on the R-Z plane where the TF coil is located ($\phi=0$). And $B_{\min}=\min[B_\phi(R,\phi,Z)]$ is obtained on the plane in the middle of the two R-Z planes where the two TF coils are located ($\phi=\pi/N_{coil}$). The magnitude of the magnetic field at any position in the tokamak can be calculated by the Biot-Savart Law. Combined with equation 4, the ripple of the tokamak can be calculated. Fig. 2 is the ripple of the QUEST tokamak. It can be seen that the ripple of QUEST inside the first wall is very small.

3.2 Guiding center motion of a single particle

The major component of the RGCO code is to follow particle orbits in 3D magnetic field. For the drift orbit model, the equations of guiding center motion under non-relativistic effect [8] can be written as

$$\frac{d\vec{X}}{dt}=\frac{\vec{B}^*}{B_{/\!/}^*}v_{/\!/}+\frac{\mu}{m\Omega B_{/\!/}^*}\vec{B}\times\nabla B+\frac{1}{BB_{/\!/}^*}\vec{E}\times\vec{B} \qquad (5)$$

$$\frac{dv_{/\!/}}{dt}=-\frac{\mu}{m}\frac{\vec{B}^*}{B_{/\!/}^*}\cdot\nabla B+\frac{Ze}{m}\frac{\vec{B}^*}{B_{/\!/}^*}\cdot\vec{E} \qquad (6)$$

where X is the guiding-center position, which can be divided into R, ϕ, Z three components in the cylindrical coordinate. $v_{/\!/}$ is the parallel (to the magnetic field) velocity of the particle, m and Ze are the mass and charge of the particle, respectively. $\mu=mv_\perp^2/2B$ is the magnetic moment, with $v_\perp$ being the perpendicular (to the magnetic field) velocity of the particle. $\Omega=BZe/m$ is the cyclotron angular frequency. $\vec{B}^*$ and $B_{/\!/}^*$ are defined by

$$\vec{B}^*=\vec{B}+B\frac{v_{/\!/}}{\Omega}\nabla\times\vec{b} \qquad (7)$$

$$B_{/\!/}^*=\vec{b}\cdot\vec{B}^*=B\left(1+\frac{v_{/\!/}}{\Omega}\vec{b}\cdot\nabla\times\vec{b}\right) \qquad (8)$$

where $\vec{b}=\vec{B}/B$. In right-hand side of equation 5, the first item, the second item and the third item are the curvature drift, the ∇B drift, and the $E\times B$ drift of the particle, respectively.

However, for the energetic particles, especially energetic electron, which can be accelerated to near the light speed by the auxiliary heating system in the tokamak, such as lower hybrid wave (LHW) and electron cyclotron resonance heating (ECRH). With such high energy, these electrons sometimes could lose confinement and then attack the PFCs, leading to excessive local heat load, which is a threaten to the steady state operation of the future fusion power plant. Therefore, the confinement of the energetic particle is an important subject to be researched. The guiding center motion of the particle under relativistic effect [9] can be described as follow.

$$\frac{d\vec{X}}{dt} = \frac{\vec{B}^*}{B_{\parallel}^*}\frac{p_{\parallel}}{m\gamma} + \frac{\mu}{m\Omega B_{\parallel}^*}\vec{B}\times\nabla B \qquad (9)$$

$$\frac{dp_{\parallel}}{dt} = -\mu\frac{\vec{B}^*}{B_{\parallel}^*}\cdot\nabla B \qquad (10)$$

where $p_{\parallel} = \gamma m v_{\parallel}$ is the relativistic parallel momentum, $\vec{B}^*$, $B_{\parallel}^*$ and relativistic factor γ are defined by

$$\vec{B}^* = \vec{B} + B\frac{p_{\parallel}}{m\gamma\Omega}\nabla\times\vec{b} + \frac{1}{BB_{\parallel}^*}\vec{E}\times\vec{B} \qquad (11)$$

$$B_{\parallel}^* = \vec{b}\cdot\vec{B}^* = B\left(1+\frac{p_{\parallel}}{m\gamma\Omega}\vec{b}\cdot\nabla\times\vec{b}\right) \qquad (12)$$

$$\gamma = \sqrt{1+\frac{2\mu B}{mc^2}+\frac{p_{\parallel}^2}{m^2c^2}} \qquad (13)$$

These above equations are used in the code to track the particle orbits, where equation 9 and 10 are typical ordinary differential equation (ODE) and the fourth-order Runge–Kutta method (RK4) is used to solve these ODEs to ensure a small numerical error at each time step. The basic formula of RK4 is shown as below

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n+\frac{h}{2}, y_n+\frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n+\frac{h}{2}, y_n+\frac{k_2}{2}\right)$$

$$k_4 = hf\left(x_n+\frac{h}{2}, y_n+\frac{k_3}{2}\right)$$

$$y_{n+1} = y_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_1}{6} + O(h^5) \qquad (14)$$

where h is the step length. In order to ensure the accuracy of the orbit calculation, the rectangular mesh grid of 200×200 is created in the R-Z plane, and the magnetic field information of the particle location is obtained by the cubic spline interpolation method. By now, the particle orbit can be calculated step by step.

3.3 Benchmark of the code

The first step is to use a guiding center orbit simulation code (without relativistic effect) [10] to check the correctness of RGCO. Take deuterium with energy of 10 keV as a test particle (the relativistic effect can be neglected), calculate the motion orbit of the test particle in the EAST tokamak configuration with RGCO and the test code, respectively. Decompose the orbits into three orthogonal components of cylindrical coordinates, $R-t$, $\phi-t$ and $Z-t$, as shown in Fig. 3. It can be seen that the orbits calculated by RGCO and the test code in three directions are almost the same, indicating that the RGCO can be used to calculate particle orbits accurately.

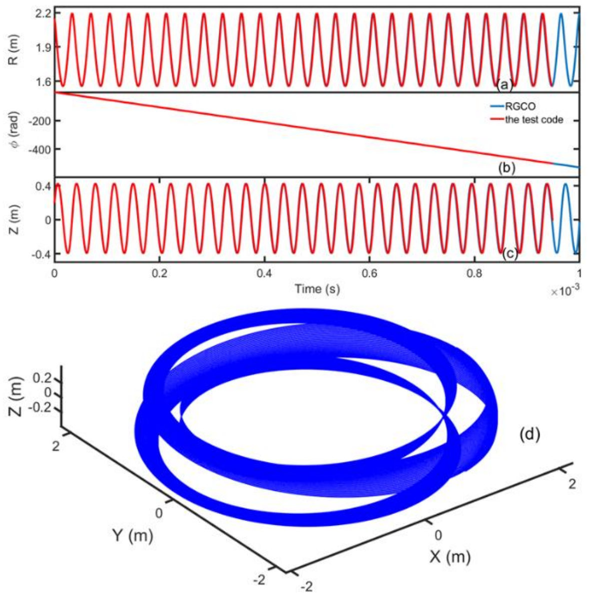

Fig. 3. The motion orbit of deuterium in EAST, calculated by RGCO (blue) and the test code (red). Initial conditions of the test particle: position $(R,\phi,Z)=(2.15,0,0.2)$, pitch angle 10°, particle energy 10 keV. The orbits are decomposed into (a): $R-t$, (b): $\phi-t$, and (c): $Z-t$. (d): The orbit calculated by RGCO in 3D view.

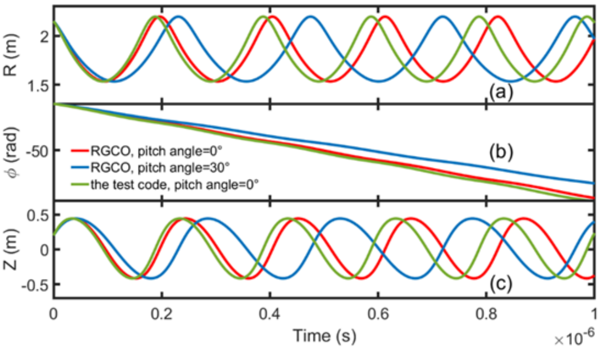

Fig. 4. The motion orbit of fast electron in EAST. Initial conditions of the test particle: position $(R,\phi,Z)=(2.15,0,0.2)$, particle energy 100 keV. The orbit calculated by RGCO with initial pitch angle 0° (red), calculated by RGCO with initial pitch angle 30° (blue), calculated by the test code with initial pitch angle 0° (green). The orbits are decomposed into (a): $R-t$, (b): $\phi-t$, and (c): $Z-t$.

The second step is to check the correctness of relativistic effect in RGCO. Take electron as a test particle with initial pitch angle 0° and 30°, with high energy 100 keV (typical energy of fast electron in EAST), the relativistic effect of which cannot be neglected. Then use these two codes to calculate the motion orbit of the test particle, the results are shown in Fig. 4. It can be clearly seen that if the relativistic effect is not taken into account, the electron velocity will become larger, and the error will accumulate with the time step of the simulation. And the pitch angle plays an important role in the particle motion orbit. The above two checks proved that RGCO can be used for the particle orbit simulation.

3.4 Geometric boundary and orbit simulation

Although the EFIT equilibrium is symmetric in toroidal direction, the electron motion in a real tokamak is 3-dimensional. The different kinds of auxiliary systems for plasma operation (such as auxiliary heating system, RMP coils) are arranged in different toroidal positions of tokamak, which can influence the electron motion in local position. Therefore, a 3-dimensional flexible geometric model is constructed in the code. For example, Fig. 5 shows the backward-tracking of electron orbit from the hot spot location in EAST, with the aim of finding the starting position of the electrons and thus exploring the cause of the hot spot. The geometric model of two LHW antennas on EAST have been constructed, when the particle moves to the SOL in front of the LHW antennas, the orbit tracking is automatically stopped and the termination position is records, because the electron in front of the LHW antenna have the opportunity to be accelerated by the LHW power. By setting different boundary conditions in different simulation cases, different physics topics can be investigated, such as the generation of hot spots and the confinement of energetic particles.

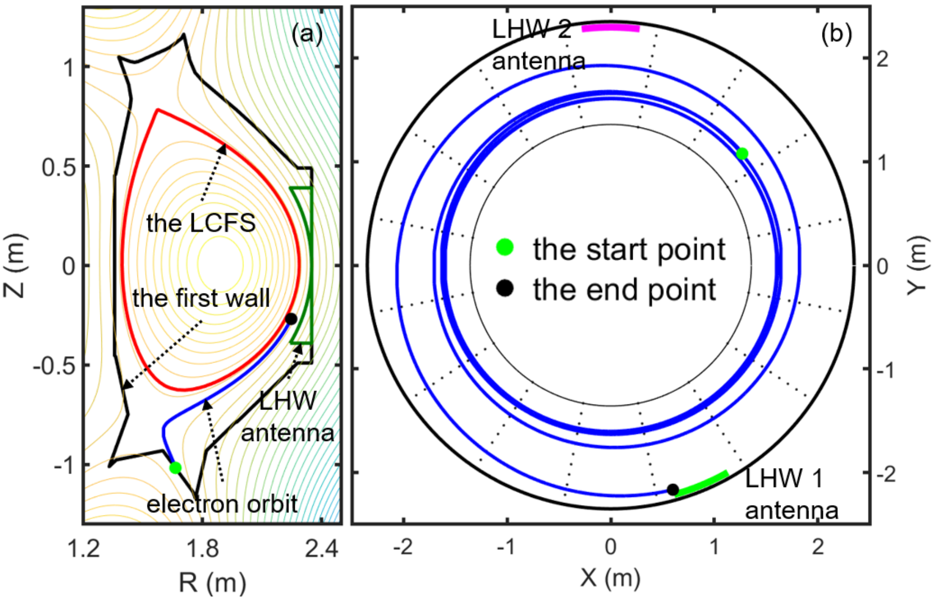

Fig. 5. The geometric model of two LHW antennas and backward-tracking of electron orbit in (a): R-Z view and (b): top view.

4. EXPERIMENT RESULTS

Using the new guiding center orbit code, a series of simulation can be performed. It has been proved that energetic electrons are excited and sustained in QUEST plasma under the 2nd harmonic electron cyclotron resonance heating (ECRH) [11]. Then the electron motion orbit simulation based on the RGCO code is performed to explore the confinement of energetic electrons in above experiments, and the equilibrium magnetic reconstruction required by the RGCO code are provided by the EFIT code. Based on the magnetic equilibrium of shot 35442 at 2.5 s, the forward-tracking of electron orbit from inside and outside of the LCFS were performed. The pitch angle is scanned from 0° to 180° at every 10°. And the initial energy of electron is set to 20 keV, which is the average energy of the tail electron in this shot [11].

The initial position of the test electron inside the LCFS is $(R,\phi,Z)=(0.55,0,0)$. And the electron orbit simulation by the RGCO with each pitch angle can be seen in Fig. 6. It can be seen that when the pitch angle between 0° and 40° and between 140° and 180°, the electron will move along the passing orbit, when the pitch angle between 50° and 130°, the electron will move along the banana orbit. The shape of the banana orbit shows regular change with the alteration of the pitch angle. As the pitch angle changes from 50° to 90°, the banana width of the orbit become gradually smaller from ~1 cm to ~0.02 cm, the poloidal range occupied by the banana orbit become gradually smaller, and all the orbits are inward shifted. Correspondingly, as the pitch angle changes from 90° to 130°, the banana width of the orbit become gradually larger from ~0.02 cm to ~1 cm, the poloidal range occupied by the banana orbit become gradually larger, and all the orbits are outward shifted. Electrons with all pitch angles can be well confined in the magnetic field.

Then perform the electron orbit simulation outside the LCFS. The initial position of the test electron is $(R,\phi,Z)=(0.8,0,0)$. And the electron orbit simulation by the RGCO with each pitch angle can be seen in Fig. 7. It can be seen that when the pitch angle between 0° and 50°, the electrons will move upward and eventually attack the PFCs, when the pitch angle between 130° and 180°, the electrons will move downward and eventually attack the PFCs, when the pitch angle between 60° and 120°, the electron will move along the banana orbit, being well confined by the magnetic field. With the change of the pitch angle, the shape of the banana orbit has the same pattern as the banana orbit inside the LCFS. These simulation results confirmed that energetic electrons can be well confined in the plasma, which contribute to the non-inductive current driving of the QUEST tokamak.

5. SUMMARY AND DISCUSSION

The EP physics is one of the most important key issues in magnetic confinement fusion. To better understand the confinement mechanism of energetic electrons in sphere tokamak, a new numerical code (RGCO) with relativistic effect has been developed based on the test particle method. The RGCO code simulates the guiding center orbit of particle motion in cylindrical coordinates, which taking into account the realistic geometry of the tokamak, such as the first wall as the motion boundary and the 3-dimensional magnetic field that includes the ripple field. The simulation results presented in this study proved that energetic electrons can be well confined in the tokamak magnetic configuration both inside and outside the LCFS with different pitch angle.

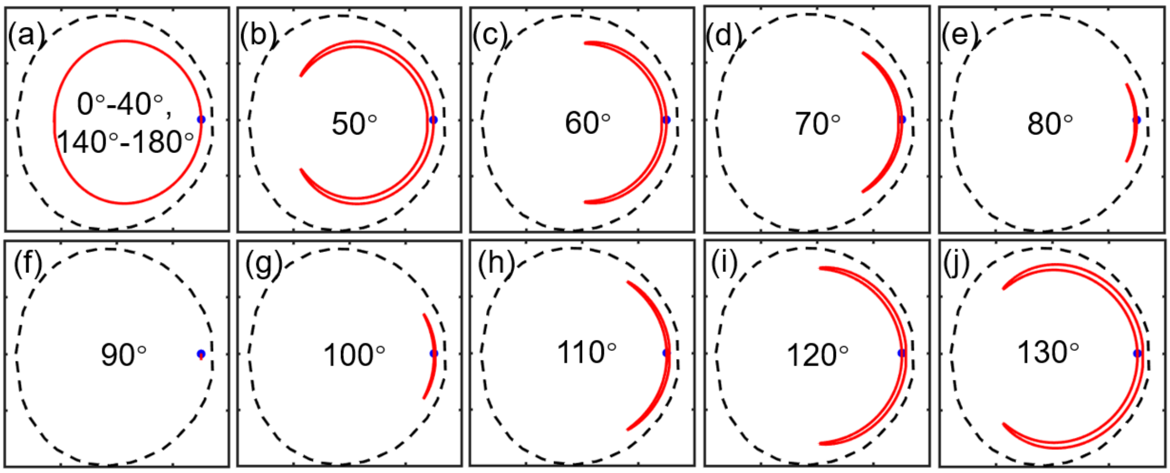

Fig. 6. The motion orbits of energetic electron inside the LCFS. The red line in each subplot represents the electron orbit with different pitch angle. The black dotted line represents the LCFS. The blue dot represents the start point of the orbit.

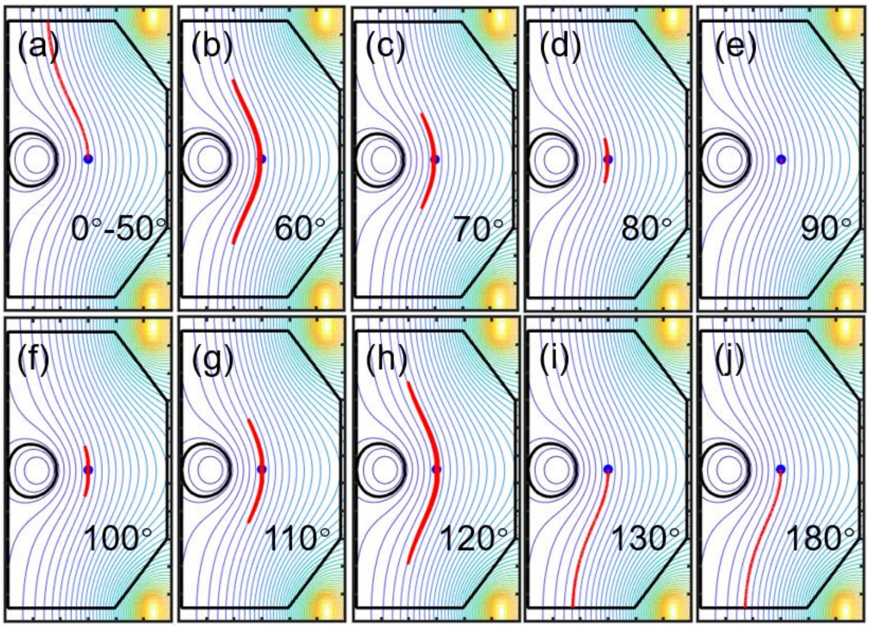

Fig. 7. The motion orbits of energetic electron outside the LCFS. The red line in each subplot represents the electron orbit with different pitch angle. The black line represents the LCFS. The blue dot represents the start point of the orbit.

As one of the most important products in deuterium (D) - tritium (T) nuclear fusion reaction, alpha particles with 3.5 MeV play an important role in self-sustained burning of future fusion power plant. Therefore, the confinement and loss mechanism of the alpha particle is a key issue and will be researched in QUEST by physics experiment combined with the RGCO code simulation in the near future.

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