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Linear stability analysis of an improved car-following model considering vehicle's inertia effect

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Abstract: *Car-following model and its variants are proposed to reproduce real traffic flow phenomena, but they ignore that different vehicles have different inertiasm, which is out of touch with reality. In this paper, a heterogeneous vehicular inertia (HVI) model is proposed by considering the vehicle's inertia effect based on the full velocity difference (FVD) model on a single-lane road. The linear stability theory is used to determine the stability condition for this model with time delay. The results demonstrate that the higher inertia effect and lower time delay effect would strengthen the stability of the proposed model.*

Keywords: Car-following modeling; Inertia effect; Stability condition ;Time delay effect; Traffic jam.

1. INTRODUCTION

Traffic jam has become a significant societal issue in contemporary life, affecting economic development and environmental pollution [1–5]. Many researchers have established various models, such as car-following, cellular automaton, gas kinetic, and hydrodynamic models, to reproduce the actual traffic flow better and explore the sociophysics mechanisms behind complex traffic phenomena [6–12]. Generally speaking, depending on the level of detail, these models can be classified as microscopic and macroscopic models. [13–16]. Microscopic models depict traffic flow dynamics at a high degree of detail, while macroscopic models aggregate traffic characteristics, including flow, mean speed, and density, to represent traffic flow at a low degree of detail. As a research trend, microscopic models are more concerned by researchers than macroscopic models, and the car-following model is the most concerned one [17][18].

The car-following model assumes that a focus vehicle follows the one in front of it in a single lane, keeping a minimum gap of time and space between them [19–23]. It and its variants focus more on the different ways vehicles interact with each other and ignore the diversities of vehicle properties, although these diversities can profoundly impact driving behaviors. The most typical example is that they assumed all vehicles have the same size, which obviously deviates from reality. Lighter vehicles, like compact cars, have less inertia and can be readily accelerated or decelerated. Their drivers tend to adjust their driving behaviors often to maximize traffic efficiency. Due to the high inertia, heavy vehicles like trucks and buses cannot accelerate or decelerate quickly, thus more likely to continue their driving behaviors out of safety concerns.

Another unrealistic aspect of the car-following model is that it ignores time delay caused by the driver's response to stimuli and the vehicle's mechanical response, which has been regarded as a crucial parameter to traffic flow stability since 1958 [24]. Since then, a growing number of scholars have started incorporating time delay into their models and found that taking it into account can better reproduce real traffic flow [25–30].

Based on the above, we proposed an improved car-following model considering the vehicle's inertia effect by introducing the inertia coefficient (I_c). For simplicity, vehicles are categorized into three types: high inertia vehicles (HV), medium inertia vehicles (MV), and low inertia vehicles (LV). We investigate the linear stability of the proposed model with and without time delay.

This paper's remaining sections are organized as follows. Section 2 introduces the conventional traffic models. The proposed model is presented in Section 3. Section 4 discussed its linear stability analysis. The main finding of this paper is concluded in Section 5.

2. BACKGROUND OF CAR-FOLLOWING MODELS

The Car-following model was first proposed by Pipes [31], and did not get much development in the following half century until Bando and her colleagues proposed a milestone model named the optimal velocity (O.V.) model in 1995 [32]. The governing equation for the O.V. model is as follows:

$$\frac{dv_n(t)}{dt} = a[V[\Delta x_n(t)] - v_n(t)], \qquad (1)$$

where a is the sensitivity coefficient of a driver, $v_n(t)$ and $x_n(t)$ are the velocity and position of vehicle n at time t, respectively. $\Delta x_n = x_{n+1} - x_n$ is the headway between two subsequent vehicles of the $(n+1)th$ and the nth vehicles at time t. The $V(\cdot)$ represents the optimal velocity function, which is upper-bounded monotonically decreasing, adopted as follows:

$$V(\Delta x_n(t)) = \frac{v_{max}}{2}[tanh(\Delta x_n(t) - h_c) + tanh(h_c)], \qquad (2)$$

where v_{max} and h_c are maximal velocity and safety distance, respectively.

The OV model can simulate various intricate traffic flow phenomena, including phase transitions and stop-and-go waves, but it will represent unrealistic acceleration and deceleration [33–35]. Helbing and Tilch [36] proposed a traffic flow model known as the G.F. model to get around this restriction. They introduced a negative speed difference into the OV model, and the dynamic equation is as follows:

$$\frac{dv_n(t)}{dt} = a[V[\Delta x_n(t)] - v_n(t)] + \lambda \Delta v_n(t) H(-v_n(t)), \quad (3)$$

where H denotes the Heaviside function, λ is the sensitivity coefficient (independent of a). $\Delta v_n(t) = v_{n+1} - v_n$ is the relative velocity of adjacent vehicles at time t. The results indicate that the G.F. model corresponds more closely to the field data than the OV model.

In 2001, Jiang et al. [37][38] believed that positive speed differences also impact traffic flow stability. By including both positive and negative velocity differences, they improved the GF model and proposed a full velocity difference model (for short, FVD model) as follows:

$$\frac{dv_n(t)}{dt} = a[V[\Delta x_n(t)] - v_n(t)] + \lambda \Delta v_n(t), \quad (4)$$

where each parameter is defined as previously. This FVD model is regarded as a typical model that can help with a traffic flow field investigation. Our main purpose is to incorporate the vehicle's inertia effect into FVD model and develop an extended car-following model. We derive the neutral stability condition from the linear stability theory and explain how the time delay and vehicle inertia effects affect the stability of the traffic flow system. The following sections will provide the specifics.

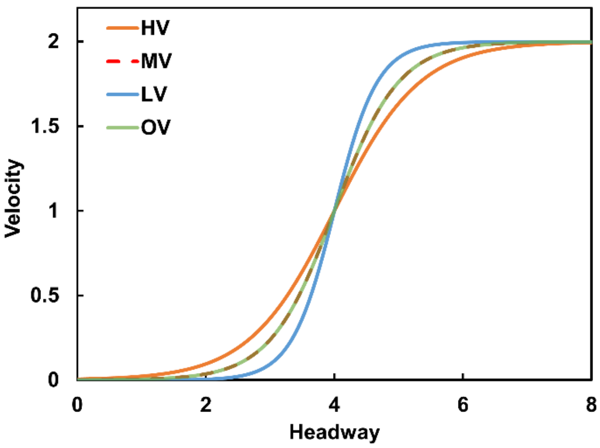

Fig. 1. Optimal velocity profiles of high-inertia vehicles (HV) with inertia coefficient, $I_c = 0.75$, medium inertia vehicles (MV) with $I_c = 1.00$, low inertia vehicles (LV) with $I_c = 1.50$, and the conventional OV model.

3. PROPOSED MODEL

By taking into account the vehicle's inertia and time delay effects, we proposed an improved car-following model based on FVD model, whose governing equation is

$$\frac{dv_n(t)}{dt} = a[V[\Delta x_n(t-\tau)] - v_n(t)] + \lambda \Delta v_n(t), \quad (5)$$

where τ is the time delay factor, while the vehicle's inertia effect is taken into account in the optimal velocity function adopted as follows:

$$V(\Delta x_n(t)) = \frac{v_{max}}{2}[tanh(I_c(\Delta x_n(t) - h_c)) + tanh(h_c)], \quad (6)$$

where I_c is the inertia coefficient, v_{max} and h_c are maximal velocity and safety distance, respectively. We classify vehicles into three types based on the value of I_c:

$$\begin{cases} 0 < I_c < 1 & \text{High inertia vehicles(HV)} \\ I_c = 1 & \text{Medium inertia vehicle(MV)} \\ I_c > 1 & \text{Low inertia vehicle(LV)} \end{cases} \quad (7)$$

For simplicity, the values of I_c for HV, MV, and LV are fixed at 0.75, 1.00, and 1.50, respectively. Their OV function diagrams are compared with the conventional OV model shown in Fig. 1.

4. LINEAR STABILITY ANALYSIS

All vehicles are supposed to run in line with constant headway b and the optimal velocity $V(b)$. Therefore, the steady-state solution of uniform flow is given by

$$x_n^0(t) = b \cdot n + V(b) \cdot t \quad \text{and} \quad b = L/N, \quad (8)$$

where L means the length of the domain (road) under the periodic boundary condition and N denotes the total number of vehicles.

The perturbed solution is as follows if $y_n(t)$ is assumed to represent a tiny deviation from the steady-state solution $x_n^0(t)$ [39–43]:

$$x_n(t) = x_n^0(t) + y_n(t), \quad (9)$$

By substituting Eq. (8) and (9) into Eq. (5), we get the model's linearized form. The remainder of it is as follows:

$$\frac{dv_n(t)}{dt} = a\left[V'(b)\Delta y_n(t-\tau) - \frac{dy_n(t)}{dt}\right] + \lambda \frac{d\Delta y_n(t)}{dt}, \quad (10)$$

where $\Delta y_n(t) = y_{n+1}(t) - y_n(t)$ and $V'(b) = \left.\frac{dV(\Delta x_n)}{d\Delta x_n}\right|_{\Delta x_n = b}$.

Expanding $y_n(t) = exp(ikn + zt)$, we obtain the following equation:

$$z^2 = a[V'(b)e^{-z\tau}(e^{ik} - 1) - z] + \lambda z(e^{ik} - 1), \quad (11)$$

Furthermore, letting $z = z_1(ik) + z_2(ik)^2 + \cdots$. The two roots of z that result from substituting this value into Eq. (11) and ignoring the terms with orders higher than 2 are:

$$z_1 = V'(b), \qquad z_2 = V'(b)\left[\frac{1}{2} + \frac{\lambda}{a} - V'(b)\left(\tau + \frac{1}{a}\right)\right], \quad (12)$$

The flow is unstable as $z_2 < 0$. On the contrary, the flow is stable as $z_2 > 0$. This leads to the following derivation of the neutral stability condition:

$$a = \frac{2[V'(b) - \lambda]}{1 - 2\tau V'(b)}, \tag{13}$$

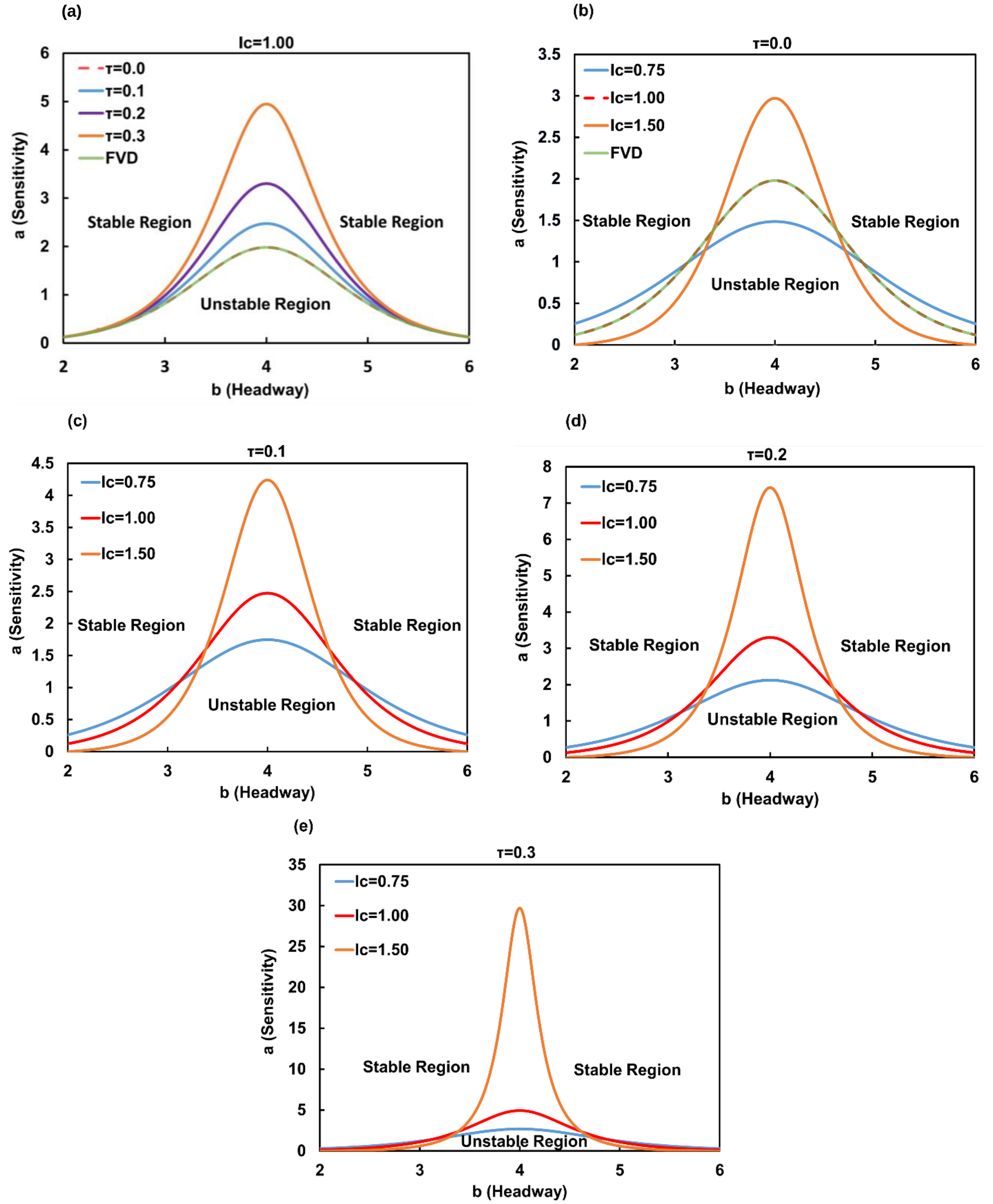

Fig. 2. Phase diagrams in headway-sensitivity (b, a) with $\lambda = 0.1$. Panel (a) compares the result from the FVD model with the HVI model, where the inertia coefficient I_c is fixed at 1.00, while the time delay effect τ varies in 0.0, 0.1, 0.2, 0.5. Panel (b) – (e) compare the result from the HVI model with varying I_c while the time delay effect τ is set at 0.0, 0.1, 0.2 and 0.3, respectively.

As a result, the condition meets the following formula, which ensures that the uniform traffic flow maintains stability under small perturbations:

$$a > \frac{2[V'(b) - \lambda]}{1 - 2\tau V'(b)}. \tag{14}$$

When I_c=1 and $\tau = 0$, neutral stability line remains the same as that of the FVD model.

Fig. 2 displays the flow stability states of the proposed model based on the neutral stability condition defined by Eq. (13). The proposed model was entirely consistent with the conventional FVD model while assuming $\tau = 0$, and Fig. 2. (a) demonstrates that stability would be lost as time delays increased. It is plausible that any time delay, whether brought on by the mechanical response time of a car or the physical reaction time of the driver, eventually introduces unwanted noise into the dynamic traffic flow system as a whole. This result can also be confirmed by comparing panels (b) to (e) in Fig. 2. The unstable regions increase significantly with more time delay. According to Fig. 2. (b), as I_c decreases, the stability area grows. This is because vehicles with high inertia cannot accelerate or decelerate frequently. Therefore, they prefer to maintain the status quo, thus making the dynamic traffic flow system more stable.

5. CONCLUSIONS

In this paper, we built an improved microscopic traffic flow model based on the FVD model by deliberately reviewing brilliant earlier studies on traffic flow modeling and taking into account the following two considerations: (1) the inertia coefficient is introduced in the OV function to take the inertia effect of vehicles into account, and (2) time delay effect. A linear stability condition is derived to show the model's capability of flow neutralization. The results confirm the stability of the proposed model will increase with the higher inertia effect but with the lower time delay effect. In the future, the Korteweg-de Vries-Burgers equation and its analytical solution will be derived using nonlinear analysis to observe the traffic flow behavior close to the neutral stability condition. Furthermore, we will conduct numerical simulations to reproduce the traffic phenomena presented by traffic flow consisting of three types of vehicles (HV, MV, and LV) in different proportions.

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