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Phenomenology in the Zee Model with the A_4 Symmetry

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Abstract

The Zee model generates neutrino masses at the one-loop level by adding charged $SU(2)_L$-singlet and extra $SU(2)_L$-doublet scalars to the standard model of particle physics. We introduce the softly broken A_4 symmetry to the Zee model as the origin of the nontrivial structure of the lepton flavor mixing. This model is compatible with the tribimaximal mixing which agrees well with neutrino oscillation measurements. Then, a sum rule $m_1 e^{i\alpha_{12}} + 2m_2 + 3m_3 e^{i\alpha_{32}} = 0$ is obtained and it results in $\Delta m^2_{31} < 0$ and $m_3 \geq 1.8 \times 10^{-2}\,$eV. The effective mass $|(M_\nu)_{ee}|$ for the neutrinoless double beta decay is predicted as $|(M_\nu)_{ee}| \geq 1.7 \times 10^{-2}\,$eV. The characteristic particles in this model are $SU(2)_L$-singlet charged Higgs bosons $s^+_\alpha (\alpha = \xi, \eta, \zeta)$ which are made from a **3** representation of A_4. Contributions of $s^\pm_\alpha$ to the lepton flavor violating decays of charged leptons are almost forbidden by an approximately remaining Z_3 symmetry; only $\mathrm{BR}(\tau \to \overline{e}\mu\mu)$ can be sizable by the flavor changing neutral current interaction with $SU(2)_L$-doublet scalars. Therefore, $s^\pm_\alpha$ can easily be light enough to be discovered at the LHC with satisfying current constraints. The flavor structures of $\mathrm{BR}(s^-_\alpha \to \ell\nu)$ are also discussed.

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I. INTRODUCTION

The standard model of particle physics (SM) can explain almost all of existing experimental results very well. However, the existence of masses of neutrinos, which are regarded as massless in the SM, was manifested in 1998 by the evidence of the atmospheric neutrino oscillation [1]. It is an important question how the SM should be extended to generate nonzero neutrino masses.

For massive neutrinos, the flavor eigenstates $\nu_{\ell L}$ ($\ell = e, \mu, \tau$), which are defined by the weak interaction, are given by superpositions of the mass eigenstates ν_{iL} as $\nu_{\ell L} = \sum_i (U_{\text{MNS}})_{\ell i}\, \nu_{iL}$. The mixing matrix U_{MNS} is referred to as the Maki-Nakagawa-Sakata (MNS) matrix [2]. In the standard parameterization for three neutrinos, U_{MNS} is expressed as

$$U_{\text{MNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta_D} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta_D} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where s_{ij} and c_{ij} denote $\sin\theta_{ij}$ and $\cos\theta_{ij}$, respectively. Brilliant successes of the neutrino oscillation measurements [1, 3–7] show

$$\Delta m^2_{21} \simeq 7.6 \times 10^{-5}\,\text{eV}^2, \quad |\Delta m^2_{31}| \simeq 2.4 \times 10^{-3}\,\text{eV}^2, \tag{2}$$

$$\sin^2 2\theta_{23} \simeq 1, \quad \sin^2 2\theta_{12} \simeq 0.87, \quad \sin^2 2\theta_{13} \lesssim 0.14, \tag{3}$$

where $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$, and m_i (i = 1-3) are the mass eigenvalues of ν_{iL}.

The simplest extension of the SM to accommodate neutrino masses would be to introduce right-handed neutrinos which were not necessary for massless neutrinos. If neutrino masses are given only by the SM Higgs field in the same way as the mass generations for charged fermions, their Dirac masses ($m_D \overline{\nu_L} \nu_R$ + h.c.) seem unnaturally smaller than other fermion masses. Then, we may rely on the seesaw mechanism where large Majorana mass terms for right-handed neutrinos ($M_R \overline{(\nu_R)^c} \nu_R/2$ + h.c.) are utilized to obtain very light neutrinos [8]. Such mass terms are allowed only for the Majorana particles which are identical to their antiparticles, and these terms break the lepton number conservation.

If neutrino masses are generated by a completely neutrino-specific mechanism, any values of neutrino masses seem acceptable even though they are very different from other fermion masses. The Zee model [9] shows an interesting possibility of such mechanisms. In the original Zee model, an extra $SU(2)_L$-doublet scalar field and an $SU(2)_L$-singlet charged scalar field are introduced to the SM. The mixing between these exotic charged scalars (from $SU(2)_L$-doublet and singlet fields) breaks the lepton number conservation. Then, Majorana mass terms of left-handed neutrinos ($m\,\overline{(\nu_L)^c}\nu_L/2$ + h.c.) are generated at the one-loop level without introducing the right-handed neutrino. Many works on the model have been done [10–15].

In the simplest version [10] of the Zee model, each of the fermions couples with only one of two $SU(2)_L$-doublet scalars in order to avoid simply the flavor-changing neutral current interaction (FCNC). The simplest Zee model was, however, ruled out at $3\,\sigma$ confidence level (CL) [15] by the accumulated knowledge from neutrino oscillation experiments. Therefore, the FCNC should exist in the Zee model as in the original Zee model with careful consideration about constraints from the lepton flavor violating (LFV) processes caused by the FCNC. It has been shown that the original Zee model can satisfy indeed constraints from neutrino oscillation measurements and LFV searches [13, 15].

On the other hand, the nontrivial structure of the lepton flavor mixing in Eq. (3) is mysterious because it is very different from the simple structure of the quark mixing. The lepton sector has two large mixings ($s_{23}^2 \simeq 0.5$ and $s_{12}^2 \simeq 0.3$) while the quark sector has small mixings only. It seems natural to expect that there is some underlying physics for the special feature of the lepton flavor. As a candidate for that, non-Abelian discrete symmetries have been studied (See, e.g., [16] and references therein). An interesting choice is the A_4 symmetry because the A_4 group is the minimal one which includes the three-dimensional irreducible representation; the representation seems suitable for three flavors of the lepton. Some simple models based on the A_4 symmetry can be found in, e.g., [17–22]. It is remarkable that so-called tribimaximal mixing [23], which agrees well with neutrino oscillation data, can be obtained in an excellent way [19]; the tribimaximal mixing is realized by left-handed lepton doublets in a three-dimensional representation if the mass eigenstates of charged leptons and neutrinos are eigenstates of Z_3 and Z_2 subgroups of A_4, respectively. It is very interesting that the nontrivial mixing structure is expressed in terms of the symmetry breaking pattern. The tribimaximal mixing in the standard parameterization of Eq. (1) is given by $s_{23} = 1/\sqrt{2}$

$(\sin^2 2\theta_{23} = 1)$, $s_{12} = 1/\sqrt{3}$ $(\sin^2 2\theta_{12} \simeq 0.89)$, and $s_{13} = 0$ as

$$U_{\text{TB}} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4}$$

Note that models with the A_4 symmetry include several $SU(2)_L$-doublet scalar fields and each of the leptons couples with more than one of them. Therefore, the lepton sector naturally has the FCNC which is required for the Zee model to agree with neutrino oscillation measurements.

In this article, we propose the simplest extension of the Zee model with softly broken A_4 symmetry (A4ZM). The soft breaking term of A_4 is required by the appropriate breaking pattern of A_4 to obtain the tribimaximal mixing. It is assumed that the soft breaking of A_4 is caused by the small breaking terms of the lepton number conservation in order to make the phenomenology simple and testable. We respect the renormalizability of interactions and do not introduce so-called flavons (singlet scalars of the SM gauge group); flavons are often introduced to Yukawa interactions in models with non-Abelian discrete symmetry as higher dimensional operators to produce the flavor structure of leptons (often of quarks also). The A4ZM is just for the lepton sector and the quark sector is almost identical to the SM one. Realizing the tribimaximal mixing, three neutrino masses satisfy a sum rule $m_1 e^{i\alpha_{12}} + 2m_2 + 3m_3 e^{i\alpha_{32}} = 0$. Then, masses and Majorana phases are governed by only one phase parameter in the A4ZM. We show $\Delta m_{31}^2 < 0$ and lower bounds on m_3 and $|(M_\nu)_{ee}|$. Although the FCNC is allowed, we see that most LFV decays of charged leptons are almost forbidden. This is because a Z_3 symmetry remains approximately and controls well the LFV processes. Thus, it would be expected that some of the exotic Higgs bosons are light enough to be discovered at the LHC. An $SU(2)_L$-singlet charged field s^+ is the characteristic particle in the original Zee model, and the A4ZM includes three fields s_α^+ $(\alpha = \xi, \eta, \zeta)$ which are made from a **3** representation of A_4. We discuss contributions of $s_\alpha^\pm$ to lepton flavor conserving processes; for example, $\mu \to e\overline{\nu_e}\nu_\mu$, $\nu_\ell e \to \nu_\ell e$. We present the characteristic flavor structures of the branching ratios (BRs) of $s_\alpha^- \to \ell\nu$ which will be useful for probing this model at the LHC.

This article is organized as follows. In Sec. II, we explain from A to Z of the A4ZM itself. It is demonstrated in Sec. III how the tribimaximal mixing can be realized in this model. Phenomenology is discussed in Sec. IV. Conclusions and discussions are given in Sec. V. Throughout this article, we use the words "singlet," etc. only for the representations of $SU(2)_L$ and "$\mathbf{1}$ representation," etc. for those of A_4 in order to avoid confusion.

II. THE MODEL

The A_4 symmetry is characterized by two elemental transformations S and T which satisfy

$$S^2 = T^3 = (ST)^3 = 1. \tag{5}$$

There are 3 one-dimensional and 1 three-dimensional irreducible representations. We use the following representations:

$$\mathbf{1} \; : \; S\,\mathbf{1} = \mathbf{1}, \quad T\,\mathbf{1} = \mathbf{1}, \tag{6}$$

$$\mathbf{1}' \; : \; S\,\mathbf{1}' = \mathbf{1}', \quad T\,\mathbf{1}' = \omega\mathbf{1}', \tag{7}$$

$$\mathbf{1}'' \; : \; S\,\mathbf{1}'' = \mathbf{1}'', \quad T\,\mathbf{1}'' = \omega^2\mathbf{1}'', \tag{8}$$

$$\mathbf{3} \; : \; S\,\mathbf{3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \mathbf{3}, \quad T\,\mathbf{3} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \mathbf{3}, \tag{9}$$

where $\omega \equiv \exp(2\pi i/3)$. Since all 12 elements of the A_4 group can be expressed as products of S and T, a model has the A_4 symmetry when the model is invariant under S and T. For $a = (a_x, a_y, a_z)^T$ and $b = (b_x, b_y, b_z)^T$ of $\mathbf{3}$, the following notations $(ab)_X$ for the decompositions of $\mathbf{3} \otimes \mathbf{3} \to X$ are used:

$$(ab)_{\mathbf{1}} \equiv a_x b_x + a_y b_y + a_z b_z, \tag{10}$$

$$(ab)_{\mathbf{1}'} \equiv a_x b_x + \omega^2 a_y b_y + \omega a_z b_z, \tag{11}$$

$$(ab)_{\mathbf{1}''} \equiv a_x b_x + \omega a_y b_y + \omega^2 a_z b_z, \tag{12}$$

$$(ab)_{\mathbf{3}_s} \equiv \Big(a_y b_z + a_z b_y, \; a_z b_x + a_x b_z, \; a_x b_y + a_y b_x\Big)^T, \tag{13}$$

$$(ab)_{\mathbf{3}_a} \equiv \Big(a_y b_z - a_z b_y, \; a_z b_x - a_x b_z, \; a_x b_y - a_y b_x\Big)^T. \tag{14}$$

The particle contents of the A4ZM are listed in Table I. The excellent realization of the tribimaximal mixing in models with A_4 is achieved by the breaking of A_4 into Z_3 for charged

	ψ^-_{1R}	ψ^-_{2R}	ψ^-_{3R}	$\Psi_{AL} = \begin{pmatrix} \psi^0_{AL} \\ \psi^-_{AL} \end{pmatrix}$	$\Phi_A = \begin{pmatrix} \phi^+_A \\ \phi^0_A \end{pmatrix}$	s^+_A	$\Phi_q = \begin{pmatrix} \phi^+_q \\ \phi^0_q \end{pmatrix}$
A_4	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
$SU(2)_L$	Singlet	Singlet	Singlet	Doublet	Doublet	Singlet	Doublet
$U(1)_Y$	-2	-2	-2	-1	1	2	1
$L\#$	1	1	1	1	0	-2	0

TABLE I: The leptons and the Higgs bosons in the A4ZM. The subscript $A = x, y, z$ denotes the index for $\mathbf{3}$ of A_4; for example, $(\Psi_{xL}, \Psi_{yL}, \Psi_{zL})$ belongs to $\mathbf{3}$ while each Ψ_{AL} are $SU(2)_L$-doublet fields. A doublet Higgs field Φ_q of $\mathbf{1}$ gives masses of quarks which are assigned to $\mathbf{1}$. The last row shows assignments of the lepton numbers.

leptons and into Z_2 for neutrinos [19]. For the appropriate A_4 breaking in the charged lepton sector, left-handed lepton doublets and scalar doublets should belong to $\mathbf{3}$ representations. The lepton doublets and scalar doublets are denoted as Ψ_{AL} and Φ_A, respectively. The subscript $A = x, y, z$ stands for the A_4 index of $\mathbf{3}$. Right-handed charged leptons ψ^-_{1R}, ψ^-_{2R}, and ψ^-_{3R} belong to $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{1}''$, respectively[1]. Charged singlet scalars s^+_A, which are the key particles in the Zee model, are components of a $\mathbf{3}$ representation. The quark sector is just like the SM with only particles of $\mathbf{1}$. The doublet scalar for quark masses is Φ_q of $\mathbf{1}$.

The Yukawa terms of leptons with scalar doublets Φ_A are given [17] by

$$\mathcal{L}_{\text{d-Yukawa}} \;=\; y_1\big(\,\overline{\Psi_L}\,\Phi\big)_{\mathbf{1}}\psi^-_{1R} + y_2\big(\,\overline{\Psi_L}\,\Phi\big)_{\mathbf{1}''}\psi^-_{2R} + y_3\big(\,\overline{\Psi_L}\,\Phi\big)_{\mathbf{1}'}\psi^-_{3R} + \text{h.c.} \tag{15}$$

where y_i are taken to be real by using redefinitions of phases of ψ^-_{iR}. The alignment of the vacuum expectation value (vev) is taken as

$$\langle\phi^0_x\rangle = \langle\phi^0_y\rangle = \langle\phi^0_z\rangle = \frac{v_3}{\sqrt{6}}, \quad \langle\phi^0_q\rangle = \frac{v_q}{\sqrt{2}}, \tag{16}$$

where $v_3^2 + v_q^2 = (246\text{GeV})^2$. Note that v_3 breaks A_4 into Z_3 subgroup while v_q preserves A_4.

[1] See Appendix A for another case where right-handed charged leptons are assigned to a $\mathbf{3}$ representation.

	e_R, e_L, ν_{eL} s^+_ξ, H^+_{D4}, H^+_{D3}	μ_R, μ_L, $\nu_{\mu L}$ s^+_η, H^+_{D2}	τ_R, τ_L, $\nu_{\tau L}$ s^+_ζ, H^+_{D1}
Z_3-charge	1	ω	ω^2

TABLE II: List of the Z_3-charge which is conserved approximately in the A4ZM.

The flavor eigenstates of leptons (the mass eigenstates of charged leptons) are given by

$$\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} = \begin{pmatrix} \psi^-_{1R} \\ \psi^-_{2R} \\ \psi^-_{3R} \end{pmatrix}, \tag{17}$$

$$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = U_L^\dagger \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix}, \quad L_\ell = \begin{pmatrix} \nu_{\ell L} \\ \ell_L \end{pmatrix}, \quad U_L^\dagger \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}. \tag{18}$$

The masses of charged leptons are

$$m_e = \frac{1}{\sqrt{2}} v_3\, y_1, \quad m_\mu = \frac{1}{\sqrt{2}} v_3\, y_2, \quad m_\tau = \frac{1}{\sqrt{2}} v_3\, y_3. \tag{19}$$

Since Z_3 symmetry remains for charged leptons, each flavor eigenstate has its Z_3-charge. Table II shows Z_3-charges of flavor eigenstates. The Z_3-charges for e, μ, and τ are 1, ω, and ω^2, respectively. In contrast with the usual flavor quantum number, $\overline{\mu}$ and τ have the same Z_3-charge ω^2.

The Yukawa terms with charged singlet scalars $s^\pm_A$ are expressed as

$$\mathcal{L}_{\text{s-Yukawa}} = f\Big(\Big(\overline{(\Psi_L)^c} i\sigma^2 \Psi_L\Big)_{\mathbf{3_a}} s^+\Big)_{\mathbf{1}} + \text{h.c.}, \tag{20}$$

where f is the Yukawa coupling constant, $\sigma^i (i = 1\text{-}3)$ are the Pauli matrices, and the superscript c means the charge conjugation. The antisymmetric nature of the coupling matrix in the original Zee model ($f_{e\mu} = -f_{\mu e}$, etc.) is replaced with the antisymmetric decomposition of two **3** representations. Note that we can not have such antisymmetric interactions if $s^\pm$ belongs to one-dimensional representations ($\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{1}''$).

In order to obtain the tribimaximal mixing, the mass eigenstates of neutrinos are required to be Z_2 eigenstates while flavor eigenstates are Z_3 eigenstates. Thus, soft breaking terms of A_4 are necessarily introduced to this model. It is assumed that the soft breaking of A_4

is connected to the breaking of the lepton number conservation[2]. Then, the soft breaking terms are

$$\tilde{V}_\mu = \sum_{A=x,y,z} \left(\Phi_x^T,\ \Phi_y^T,\ \Phi_z^T\right) \begin{pmatrix} 0 & (\mu_A)_{xy} & (\mu_A)_{xz} \\ -(\mu_A)_{xy} & 0 & (\mu_A)_{yz} \\ -(\mu_A)_{xz} & -(\mu_A)_{yz} & 0 \end{pmatrix} \begin{pmatrix} i\sigma^2\Phi_x \\ i\sigma^2\Phi_y \\ i\sigma^2\Phi_z \end{pmatrix} s_A^-$$
$$+ \left(\Phi_x^T,\ \Phi_y^T,\ \Phi_z^T\right) \begin{pmatrix} (\mu_q)_{xx} & (\mu_q)_{xy} & (\mu_q)_{xz} \\ (\mu_q)_{yx} & (\mu_q)_{yy} & (\mu_q)_{yz} \\ (\mu_q)_{zx} & (\mu_q)_{zy} & (\mu_q)_{zz} \end{pmatrix} \begin{pmatrix} s_x^- \\ s_y^- \\ s_z^- \end{pmatrix} i\sigma^2\Phi_q + \text{h.c.} \tag{21}$$

Note that $\tilde{V}_\mu$ does not destroy the vev alignment (16). Since the μ-parameters are the sources of the neutrino masses, it seems natural for them to be small[3]. The mixings between singlet and doublet scalars become small because they are controlled by the μ-parameter. With the vev's in Eq. (16), the soft breaking terms (21) give the small mixing term as

$$\left(s_x^-, s_y^-, s_z^-\right) M_{s\phi}^2 \left(\phi_x^+, \phi_y^+, \phi_z^+, \phi_q^+\right)^T, \tag{22}$$

$$M_{s\phi}^2 \equiv \frac{v_3}{\sqrt{6}} \begin{pmatrix} 2\left[(\mu_x)_{xy} + (\mu_x)_{xz}\right] & 2\left[(\mu_x)_{yz} - (\mu_x)_{xy}\right] & -2\left[(\mu_x)_{xz} + (\mu_x)_{yz}\right] & -\left[(\mu_q)_{xx} + (\mu_q)_{yx} + (\mu_q)_{zx}\right] \\ 2\left[(\mu_y)_{xy} + (\mu_y)_{xz}\right] & 2\left[(\mu_y)_{yz} - (\mu_y)_{xy}\right] & -2\left[(\mu_y)_{xz} + (\mu_y)_{yz}\right] & -\left[(\mu_q)_{xy} + (\mu_q)_{yy} + (\mu_q)_{zy}\right] \\ 2\left[(\mu_z)_{xy} + (\mu_z)_{xz}\right] & 2\left[(\mu_z)_{yz} - (\mu_z)_{xy}\right] & -2\left[(\mu_z)_{xz} + (\mu_z)_{yz}\right] & -\left[(\mu_q)_{xz} + (\mu_q)_{yz} + (\mu_q)_{zz}\right] \end{pmatrix}$$
$$+ \frac{v_q}{\sqrt{2}} \begin{pmatrix} (\mu_q)_{xx} & (\mu_q)_{yx} & (\mu_q)_{zx} & 0 \\ (\mu_q)_{xy} & (\mu_q)_{yy} & (\mu_q)_{zy} & 0 \\ (\mu_q)_{xz} & (\mu_q)_{yz} & (\mu_q)_{zz} & 0 \end{pmatrix}. \tag{23}$$

Since μ-parameters are assumed to be small, the Z_3 symmetry is preserved approximately also in the Higgs sector. Then, mass eigenstates of scalar fields are given approximately as

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[2] If the soft breaking of A_4 is caused by the quadratic terms of scalar doublets, the vacuum in Eq. (16) will not be natural. The quadratic terms of $s_A^\pm$ may be reliable. We do not take the option in this article in order to make phenomenology of $s_A^\pm$ simple.

[3] If μ-parameters are large, mass eigenstates of Higgs bosons are complicated, and phenomenology on them becomes less predictive.

eigenstates of Z_3. For singlet scalars, mass eigenstates are approximately given by

$$\begin{pmatrix} s_\xi^+ \\ s_\eta^+ \\ s_\zeta^+ \end{pmatrix} = U_s^\dagger \begin{pmatrix} s_x^+ \\ s_y^+ \\ s_z^+ \end{pmatrix}, \quad U_s^\dagger \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}. \tag{24}$$

Mass eigenstates of doubletlike charged Higgs bosons H_{Di}^+ are expressed with $\tan\beta \equiv v_q/v_3$ approximately as

$$\begin{pmatrix} H_{D1}^+ \\ H_{D2}^+ \\ H_{D3}^+ \\ G^+ \end{pmatrix} = U_{\phi^\pm}^\dagger \begin{pmatrix} \phi_x^+ \\ \phi_y^+ \\ \phi_z^+ \\ \phi_q^+ \end{pmatrix}, \quad U_{\phi^\pm}^\dagger \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -s_\beta & c_\beta \\ 0 & 0 & c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} 1 & \omega & \omega^2 & 0 \\ 1 & \omega^2 & \omega & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{3} \end{pmatrix}, \tag{25}$$

where c_β and s_β stand for $\cos\beta = v_3/\sqrt{v_3^2 + v_q^2}$ and $\sin\beta$, respectively. Table II shows Z_3-charges of these Higgs bosons also. Since a combination $(\Phi_x+\Phi_y+\Phi_z)/\sqrt{3}$ has a Z_3-charge 1, the combination can be mixed with Φ_q. Note that G^+ is identified to the Nambu-Goldstone (NG) boson because neutral partners of H_{Di}^+ have no vev. The Yukawa coupling constants $(Y_i)_{\ell\ell'}$ for $(Y_i)_{\ell\ell'}\, \overline{\nu_{\ell L}}\, \ell'_R\, H_{Di}^+$ are given by

$$Y_1 = \frac{\sqrt{2}}{v_3} \begin{pmatrix} 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \\ m_e & 0 & 0 \end{pmatrix}, \; Y_2 = \frac{\sqrt{2}}{v_3} \begin{pmatrix} 0 & 0 & m_\tau \\ m_e & 0 & 0 \\ 0 & m_\mu & 0 \end{pmatrix}, \; Y_3 = -\frac{\sqrt{2}\, s_\beta}{v_3} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}. \tag{26}$$

The matrix of Yukawa coupling constants for G^+ is

$$Y_G = \frac{\sqrt{2}\, c_\beta}{v_3} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}. \tag{27}$$

The Yukawa interactions $(F_\alpha)_{\ell\ell'}\, \overline{(\nu_{\ell L})^c}\, \ell'_L\, s_\alpha^+$ are governed by

$$F_\xi = 2if \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad F_\eta = 2if \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad F_\zeta = 2if \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{28}$$

The small mixing terms of singlet and doublet scalars are rewritten as $s_\alpha^-\, (U_s^\dagger M_{s\phi}^2 U_{\phi^\pm})_{\alpha i}\, H_{Di}^+$. Note that $s_\alpha^- (U_s^\dagger M_{s\phi}^2 U_{\phi^\pm})_{\alpha 4} G^+ = 0$.

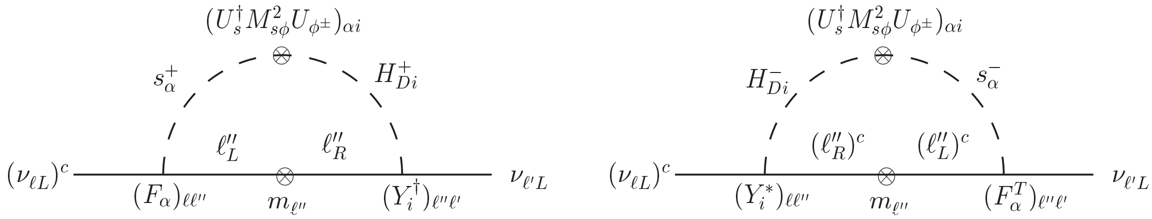

FIG. 1: One-loop diagrams which generate the mass matrix M_ν of neutrinos in the flavor basis in the A4ZM.

The neutrino masses are generated by one-loop diagrams in the Zee model and also in the A4ZM. Figure 1 shows the one-loop diagrams which generate M_ν in the flavor basis of neutrinos. The expression of M_ν is

$$(M_\nu)_{\ell\ell'} = \sum_{i=1}^{3}\sum_{\ell'',\alpha}(C_{\mathrm{loop}})_{\alpha i}\, m_{\ell''}\, (U_s^\dagger M_{s\phi}^2 U_{\phi^\pm})_{\alpha i}\Big\{(F_\alpha)_{\ell\ell''}(Y_i^\dagger)_{\ell''\ell'} + (Y_i^*)_{\ell\ell''}(F_\alpha^T)_{\ell''\ell'}\Big\}, \quad (29)$$

$$(C_{\mathrm{loop}})_{\alpha i} \equiv -\frac{1}{16\pi^2}\frac{1}{m_{s_\alpha^\pm}^2 - m_{H_{Di}^\pm}^2}\ln\frac{m_{s_\alpha^\pm}^2}{m_{H_{Di}^\pm}^2}, \quad (30)$$

where $m_{s_\alpha^\pm}$ and $m_{H_{Di}^\pm}$ are the masses of $s_\alpha^\pm$ and $H_{Di}^\pm$, respectively. There is no contribution from $G^\pm$ because of $(U_s^\dagger M_{s\phi}^2 U_{\phi^\pm})_{\alpha 4} = 0$.

III. TRIBIMAXIMAL MIXING IN THE A4ZM

In this section, we discuss how the tribimaximal mixing can be obtained in the A4ZM. Although other types of mixing can be obtained in this model[4], models with the A_4 symmetry will be motivated well only when the tribimaximal mixing is achieved in the leading order approximation. The A4ZM gives in general the following form of M_ν in the flavor basis:

$$M_\nu = \frac{m_\tau^2 f}{v_3}\begin{pmatrix} A_\tau & D_\tau & E_\tau \\ D_\tau & B_\tau & F_\tau \\ E_\tau & F_\tau & 0 \end{pmatrix} + \frac{m_\mu^2 f}{v_3}\begin{pmatrix} A_\mu & D_\mu & E_\mu \\ D_\mu & 0 & F_\mu \\ E_\mu & F_\mu & C_\mu \end{pmatrix} + \frac{m_e^2 f}{v_3}\begin{pmatrix} 0 & D_e & E_e \\ D_e & B_e & F_e \\ E_e & F_e & C_e \end{pmatrix}. \quad (31)$$

This form of M_ν is valid also for the original Zee model where the FCNC is allowed. Elements (A_τ, etc.) of the matrix are given by μ-parameters, vev's, and $(C_{\mathrm{loop}})_{\alpha i}$ which depends on

[4] If the mass matrix $M_{\nu 0}$ for ψ_{AL}^0 ("neutrinos" in our Lagrangian basis) is diagonalized by a real U_ν (orthogonal matrix), the form of U_L ensures $|(U_{\mathrm{MNS}})_{\mu 3}| = |(U_{\mathrm{MNS}})_{\tau 3}|$ which means $\theta_{23} = \pi/4$ in the standard form of U_{MNS}.

Higgs boson masses. Charged lepton masses appear as squared ones because of the chirality flip at internal lines and the forms of Y_i in Eq. (26). Parts of zeros are consequence of the antisymmetric nature of the singlet Yukawa coupling matrices F_α. The correlation between m_ℓ and vanishing elements is the characteristic feature. It is natural that contributions from m_e^2 and m_μ^2 are ignored. Then, we require that M_ν is diagonalized by $PU_{\rm TB}$ where $P \equiv {\rm diag}(e^{i\varphi_e}, e^{i\varphi_\mu}, e^{i\varphi_\tau})$ is just a redefinition of phases of flavor eigenstates to put the mixing matrix into the standard form. The conditions for the diagonalization are

$$B_\tau \;=\; 0, \tag{32}$$

$$E_\tau e^{i\varphi_\tau} \;=\; -D_\tau e^{i\varphi_\mu}, \tag{33}$$

$$F_\tau e^{i(\varphi_\mu+\varphi_\tau)} \;=\; -A_\tau e^{i\varphi_e} - D_\tau e^{i(\varphi_e+\varphi_\mu)}. \tag{34}$$

See [13, 14] for discussions on the original Zee model with a two-zeros texture ($(M_\nu)_{\mu\mu} = (M_\nu)_{\tau\tau} = 0$). See also e.g. [24] for model-independent discussions with two-zeros textures. With conditions (32)-(34), the mass eigenvalues can be expressed as

$$m_1 e^{i\alpha_{12}} \;=\; -|a| + 3|b|e^{i\varphi}, \tag{35}$$

$$m_2 \;=\; 2|a|, \tag{36}$$

$$m_3 e^{i\alpha_{32}} \;=\; -|a| - |b|e^{i\varphi}, \tag{37}$$

$$a \;\equiv\; \frac{m_\tau^2 f}{2v_3}\left(A_\tau e^{2i\varphi_e} + 2D_\tau e^{i(\varphi_e+\varphi_\mu)}\right), \tag{38}$$

$$b \;\equiv\; \frac{m_\tau^2 f}{2v_3} A_\tau e^{2i\varphi_e}, \quad \varphi \equiv \arg(b), \tag{39}$$

where m_i are real and positive. Two phases (α_{12} and α_{31}) are the Majorana phases which are physical parameters only for Majorana particles [25]. The predictions on neutrinos are discussed in Sect. IV A. Ignored masses, m_e^2 and m_μ^2 (or Yukawa coupling constants y_1^2 and y_2^2), may be regarded as breaking parameters of the Z_2 symmetry in the neutrino sector which give a deviation from the tribimaximal mixing. The deviation will provide a nonzero θ_{13}, and a naive expectation on the size of θ_{13} in this model will be $s_{13} \sim m_\mu^2/m_\tau^2 \simeq 3\times 10^{-3}$.

In the discussion above, it was implicitly assumed that there were sufficient number of parameters for the neutrino masses and the tribimaximal mixing. An example of undesired situations is the case where there is no soft breaking term of A_4. In this case, Z_3 symmetry remains in the neutrino sector also and results in $B_\tau = D_\tau = E_\tau = 0$ because $\overline{(\nu_{eL})^c}\,\nu_{\mu L}$, $\overline{(\nu_{eL})^c}\,\nu_{\tau L}$, and $\overline{(\nu_{\mu L})^c}\,\nu_{\tau L}$ are forbidden by Z_3. Then, M_ν is constrained too much to give the tribimaximal mixing although $\theta_{23} = \pi/4$ can be obtained. In addition, it is impossible to give a nonzero Δm^2_{32}.

Let us demonstrate the realization of the tribimaximal mixing in the A4ZM in a simple scenario where $\tan\beta$ is large. For example, the mass ratio of the top quark and the tau lepton, $m_t/m_\tau \simeq 100$, seems natural for $\tan\beta$. Then, the A_4 symmetry remains approximately in the Higgs sector. The mixing between Φ_q and $(\Phi_x + \Phi_y + \Phi_z)/\sqrt{3}$ becomes negligible in this case. The NG bosons are given dominantly by Φ_q. The remaining A_4 symmetry gives almost degenerate masses of exotic Higgs bosons as $m_{s^\pm} \simeq m_{s^\pm_\alpha}$ and $m_{\phi^\pm} \simeq m_{H^\pm_{Di}}$. The degenerate masses make the loop function as an overall factor $C_{\text{loop}} \simeq (C_{\text{loop}})_{\alpha i}$ of M_ν. As a result, a large $\tan\beta$ simplifies the conditions (32)-(34) as the ones just between μ_q. With $P = (1, 1, -1)$ for example[5], the conditions (32)-(34) result in

$$(\mu_q)_{zx} = \omega(\mu_q)_{xx} - \omega^2(\mu_q)_{yy} + (\mu_q)_{zy}, \tag{40}$$

$$(\mu_q)_{yx} = \omega^2(\mu_q)_{xx} + (\mu_q)_{yz} - \omega(\mu_q)_{zz}, \tag{41}$$

$$(\mu_q)_{xy} = \omega^2(\mu_q)_{zy} - \omega^2(\mu_q)_{xz} + (\mu_q)_{yz}. \tag{42}$$

Even in such a simplified case, a and b are expressed appropriately as two independent parameters:

$$a = \frac{2\omega f m_\tau^2}{\sqrt{3}} \frac{v_q}{v_3} C_{\text{loop}} \left\{-(\mu_q)_{yy} + \omega(\mu_q)_{zy} - (\mu_q)_{yz} + \omega(\mu_q)_{zz}\right\}, \tag{43}$$

$$b = \frac{2\omega f m_\tau^2}{\sqrt{3}} \frac{v_q}{v_3} C_{\text{loop}} \left\{-(\mu_q)_{xz} + \omega(\mu_q)_{yz}\right\}. \tag{44}$$

In the following discussions, we do not always assume a large $\tan\beta$. Strong degeneracy of Higgs boson masses with a large $\tan\beta$ will not be preferred for measuring characteristic flavor structures of their leptonic decays.

IV. PHENOMENOLOGY

Predictions in the A4ZM are discussed in this section. Results shown in Sec. IV A are valid not only in the A4ZM but also in the Zee model (not the simplest one) with the

[5] With $P = (1, 1, -1)$, the tribimaximal mixing requires the mass matrix $M_{\nu 0}$ of ψ^0_{AL} ("neutrinos" in our Lagrangian basis) to satisfy $(M_{\nu 0})_{xy} = (M_{\nu 0})_{xz} = 0$ and $(M_{\nu 0})_{yy} = (M_{\nu 0})_{zz}$. It is clear that A_4 is broken to Z_2 in the neutrino sector.

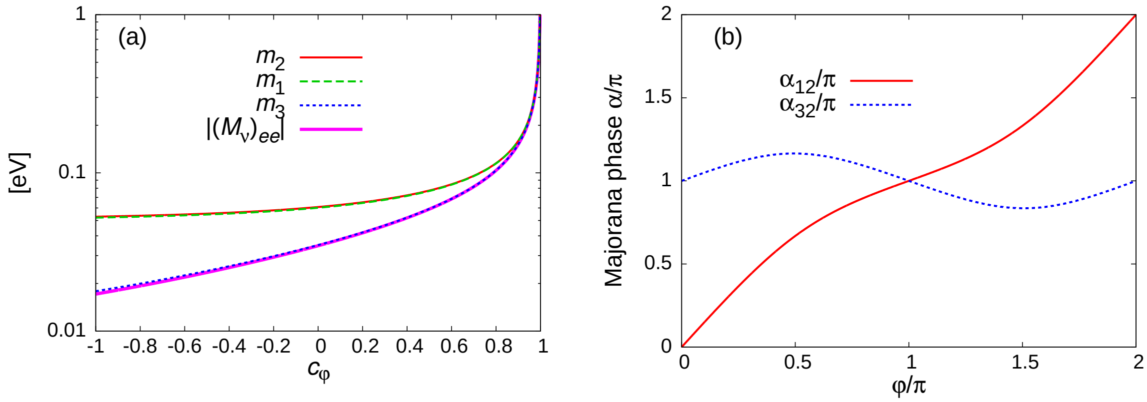

FIG. 2: (a) The c_φ dependences of m_1, m_2, m_3, and $|(M_\nu)_{ee}|$ are shown with red solid, green dashed, blue dotted, and bold solid magenta lines, respectively. Note that $m_2 \gtrsim m_1 > m_3 \gtrsim |(M_\nu)_{ee}|$. (b) The φ dependences of two Majorana phases α_{12} and α_{32} are shown with red solid and blue dotted lines, respectively.

tribimaximal ansatz. Therefore, in order to test the nature of the A_4 symmetry, decays of Higgs bosons into leptons and LFV decays of charged leptons are discussed in Secs. IV B and IV C, respectively. Constraints from lepton flavor-conserving decays of charged leptons are considered in Sec. IV D. Consequences for neutrino oscillation in matter are also shown in Sec. IV E.

A. Neutrino masses and Majorana phases

Neutrino masses in the A4ZM are expressed with two mass parameters $|a|$ and $|b|$ and a phase $\varphi \equiv \arg(b)$ as shown in Eqs. (35)-(37). It is clear that the A4ZM predicts a sum rule

$$m_1 e^{i\alpha_{12}} + 2m_2 + 3m_3 e^{i\alpha_{32}} = 0. \tag{45}$$

Note that the simplest Zee model predicts $m_1 + m_2 + m_3 = 0$ (no Majorana phases); for another example, the simplest version of the Higgs Triplet Model with softly broken A_4 symmetry (A4HTM) [22] shows $m_1 e^{i\alpha_{12}} - 2m_2 - m_3 e^{i\alpha_{32}} = 0$. Sum rules in other models are listed in [26]. Two mass parameters $|a|$ and $|b|$ are expressed as

$$\begin{aligned} |a|^2 &= \frac{-\Delta m_{31}^2}{24(1-c_\varphi^2)}\Big\{2(3+2r)+(1-c_\varphi^2)(3+4r) \\ &\qquad\qquad + c_\varphi\sqrt{4(3+2r)^2-(1-c_\varphi^2)(3+4r)^2}\Big\}, \end{aligned} \tag{46}$$

$$\begin{aligned} |b|^2 &= \frac{-\Delta m_{31}^2}{24(1-c_\varphi^2)}\Big\{2(3+2r)-(1-c_\varphi^2)(3+4r) \\ &\qquad\qquad + c_\varphi\sqrt{4(3+2r)^2-(1-c_\varphi^2)(3+4r)^2}\Big\}, \end{aligned} \tag{47}$$

where $c_\varphi \equiv \cos\varphi$ and $r \equiv \Delta m_{21}^2/(-\Delta m_{31}^2)$. We see that the A4ZM predicts $\Delta m_{31}^2 < 0$ which is so-called inverted mass ordering. Lower bounds on $|a|$ and $|b|$ are given by $c_\varphi = -1$. The lower bound on $|a|$ results in

$$m_3^2 = 4|a|^2 - \Delta m_{21}^2 + \Delta m_{31}^2 \geq (1.8\times 10^{-2}\,\mathrm{eV})^2. \tag{48}$$

The existence of the nontrivial lower bound on m_3 can be understood by the fact that $m_3 = 0$ in Eqs. (35)-(37) conflicts with $\Delta m_{21}^2 > 0$. Figure 2(a) shows behaviors of m_i with respect to c_φ. The red thin solid, green dashed, and blue dotted lines are for m_1, m_2, and m_3, respectively.

The neutrinoless double beta decay is the most promising phenomenon of the lepton number violation which is caused by Majorana neutrinos. The effective mass $|(M_\nu)_{ee}|$ which controls the decay is given by

$$|(M_\nu)_{ee}|^2 \;=\; 4|b|^2 = m_3^2 - \frac{\Delta m_{21}^2}{3} \geq (1.7\times 10^{-2}\,\mathrm{eV})^2. \tag{49}$$

The c_φ dependence of $|(M_\nu)_{ee}|$ is shown in Fig. 2(a) with the magenta bold solid line. Most of the region $|(M_\nu)_{ee}| \geq 1.7\times 10^{-2}\,\mathrm{eV}$ would be proved by the future experiments (See [27] for a review). Note that the simplest Zee model predicts $(M_\nu)_{ee} = 0$; for another example, the A4HTM gives a lower bound $|(M_\nu)_{ee}| \geq 0.0045\,\mathrm{eV}$ which allows rather smaller values than the expected sensitivities in the future experiments.

The φ dependences of two Majorana phases (α_{12} and α_{32}) in Eqs. (35) and (37) are shown in Fig. 2(b). Red solid and blue dotted lines are used for α_{12} and α_{32}, respectively. We see that α_{32} is restricted as $|\alpha_{32} - \pi| \leq 0.2\pi$ in this model.

	BR($s_\alpha^- \to \ell\nu$) $e\nu : \mu\nu : \tau\nu$	$\mu \to e\bar{\nu}_\ell \nu_{\ell'}$	$\tau \to \ell \bar{\nu}_{\ell'} \nu_{\ell''}$	Matter effect, $\nu e \to \nu e$
$s_\xi^\pm$	$0:1:1$	None	$\tau \to \mu\bar{\nu}_\mu \nu_\tau$	None
$s_\eta^\pm$	$1:0:1$	None	$\tau \to e\bar{\nu}_e \nu_\tau$	$\varepsilon_{\tau\tau}^{eP_L}$
$s_\zeta^\pm$	$1:1:0$	$\mu \to e\bar{\nu}_e \nu_\mu$	None	$\varepsilon_{\mu\mu}^{eP_L}$, ($\varepsilon_{\ell\ell}^{eP}$)

TABLE III: Phenomenological aspects of $s_\alpha^\pm$. The second column shows ratios of the leptonic decays of each $s_\alpha^\pm$, where the flavors of neutrinos are summed up. The third and fourth column present $\ell \to \ell' \bar{\nu}\nu$ which can be affected by $s_\alpha^\pm$ mediations. The last column shows contributions of $s_\alpha^\pm$ to effective four-Fermion couplings which relate to the nonstandard matter effect for the neutrino oscillation. The indirect contribution to the effect through the redefinition of G_F is indicated with parentheses. See the main text for the definition of $\varepsilon_{\ell\ell'}^{fP}$.

B. Higgs boson decays into leptons

The characteristic particles in the A4ZM are $s_\alpha^\pm$ of a **3** representation. The interactions of s_α^+ with leptons are given by singlet Yukawa coupling matrices F_α in Eq. (28). The second column of Table III shows the ratios of the branching ratios of leptonic decays of $s_\alpha^\pm$. The flavors of neutrinos are summed up because they will not be detected at collider experiments. Leptonic decays of $s_\alpha^\pm$ have characteristic flavor structures unless their masses degenerate (e.g., for a large $\tan\beta$). Each of $s_\alpha^\pm$ has only two modes as leptonic decays; for example, s_ξ^- decays into $\mu_L \nu_{\tau L}$ and $\tau_L \nu_{\mu L}$ with a common decay rate. Note that s_α^- can be easily distinguished from H_{Di}^- whose leptonic decays are dominated by the decay into τ. Therefore, if some of $s_\alpha^\pm$ are light enough to be produced at the LHC this model can be testable by measuring leptonic decays of $s_\alpha^\pm$.

The Yukawa couplings of H_{Di}^+ are shown in Eq. (26). Since Φ_q and a combination $\Phi_\xi \equiv (\Phi_x + \Phi_y + \Phi_z)/\sqrt{3}$ have the Z_3-charge 1, they behave as usual doublet scalar fields. The phenomenology of Φ_q and Φ_ξ (namely, $H_{D3}^\pm$, $G^\pm$, and neutral members) is almost identical to the one in a type of the two-Higgs-doublet models, which can be seen in [28–31]. Other two-linear combinations $\Phi_\eta \equiv (\Phi_x + \omega^2 \Phi_y + \omega\Phi_z)/\sqrt{3}$ and $\Phi_\zeta \equiv (\Phi_x + \omega\Phi_y + \omega^2\Phi_z)/\sqrt{3}$ have no vev and no contribution to the mass matrix of charged leptons. They can cause flavor violations in their Yukawa interactions. Phenomenology of Φ_η and Φ_ζ is the same as the one in a model discussed in [17] (See also [20]). Dominant leptonic decays of them are $\Phi_\eta \to \overline{\tau_R} L_e$ and $\Phi_\zeta \to \overline{\tau_R} L_\mu$.

C. Lepton flavor violating decays of charged leptons

The A4ZM does not give sizable BRs of $\mu \to \bar{e}ee$ and $\ell \to \ell'\gamma$ because they are forbidden by the remaining Z_3 symmetry; for example, $\mu \to e\gamma$ changes the Z_3-charge from ω (of μ) to 1 (of e and γ). The Z_3 symmetry allows only $\tau \to \overline{e}\mu\mu$ and $\tau \to \overline{\mu}ee$ among six $\tau \to \overline{\ell}\ell'\ell''$. Tree-level contributions to $\tau \to \overline{e}\mu\mu$ and $\tau \to \overline{\mu}ee$ are dominated by mediations of $(H^0_{D1})^\dagger \equiv (\phi^0_\zeta)^\dagger$ and $(H^0_{D2})^\dagger \equiv (\phi^0_\eta)^\dagger$, respectively[6]. The Yukawa couplings appear as $m_\mu^2 m_\tau^2/v_3^4$ for the decay rate of $\tau \to \overline{e_L}\mu_L\mu_R$ while the rate of $\tau \to \overline{\mu_R} e_L e_L$ is governed by $m_e^2 m_\tau^2/v_3^4$ [17]. Therefore, only $\tau \to \overline{e_L}\mu_L\mu_R$ can have a sizable decay rate in the A4ZM[7]. The signal of $\tau \to \overline{e}\mu\mu$ may exist just below the current experimental limit, $\mathrm{BR}(\tau \to \overline{e}\mu\mu) < 1.7 \times 10^{-8}$ at 90 % CL [32] (see also [33]), because constraints from other LFV processes are satisfied automatically. The lack of LFV in the A4ZM is a good feature of the model because the model can be excluded easily by the searches of the LFV processes. The branching ratio for $\tau \to \overline{e}\mu\mu$ is given by

$$\mathrm{BR}(\tau \to \overline{e}\mu\mu) = \mathrm{BR}(\tau \to \overline{e_L}\mu_L\mu_R) = \frac{m_\tau^2 m_\mu^2}{8 v_3^4 G_F^2 m^4_{H^0_{D1}}} \mathrm{BR}(\tau \to \mu\overline{\nu_\mu}\nu_\tau), \tag{50}$$

where $\mathrm{BR}(\tau \to \mu\overline{\nu_\mu}\nu_\tau) = 0.17$ and $\sqrt{2}G_F = 1/(v_3^2 + v_q^2)$. The bound $\mathrm{BR}(\tau \to \overline{e}\mu\mu) < 1.7 \times 10^{-8}$ results in

$$m_{H^0_{D1}} > 17\sqrt{1+\tan^2\beta}\,[\mathrm{GeV}]. \tag{51}$$

Note that $m_{H^0_{D1}}$ can not be $O(10)$ GeV because the LEP bound for $e^+e^- \to Z^* \to H_1 H_2$ [34] ($e^+e^- \to Z^* \to (H^0_{D1})^\dagger H^0_{D1}$ in our case) results in $m_{H^0_{D1}} \gtrsim 90\,\mathrm{GeV}$. If $\tan\beta$ is less than a several-times 10, H^0_{D1} can be light enough to be discovered at the LHC. Even for $\tan\beta \gtrsim 100$, H^0_{D2} can be light.

There is no remarkable constraint for $s^\pm_\alpha$ from the LFV decays of charged leptons because $\ell \to \ell'\gamma$ are forbidden as explained above. Thus, $s^\pm_\alpha$ can be light without caring about

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[6] Since H^0_{D1} and H^0_{D2} have Z_3-charges, they can be dealt with by keeping them as complex scalars. In other words, masses of $\mathrm{Re}(H^0_{D1})$ and $\mathrm{Im}(H^0_{D1})$ are the same.

[7] If both of left-handed lepton doublets and right-handed charged leptons are made from **3** representations, $\mathrm{BR}(\tau \to \overline{\mu}ee)$ can be also sizable as discussed in [21]. See also Appendix A.

constraints from $\ell \to \ell'\gamma$. We mention that the singlet scalar contribution to the anomalous magnetic dipole moment of muon has a minus sign[8] while a plus sign is favored to explain experimental results (See e.g., [35] and references therein). Other exotic phenomena (lepton flavor conserving) of $s_\alpha^\pm$ are discussed below.

D. Universality of $G_{\ell\ell'}$

In the A4ZM, $s_\alpha^\pm$ can contribute to $\ell \to \ell'\overline{\nu}\nu$ as shown in the third and fourth columns of Table III. See [12] for the case with the simplest Zee model. The effective coupling constants for $\ell \to \ell'\overline{\nu}\nu$ are denoted as $G_{\ell\ell'}$. Note that contributions of $s_\alpha^\pm$ are coherent with the exchange of W boson and can give large effects in principle. Such coherent effects of doublet scalars have the chirality suppression because they couple with ℓ_R. Contributions of $s_\xi^\pm$, $s_\eta^\pm$, and $s_\zeta^\pm$ to $G_{\ell\ell'}$ are denoted as $G_{\tau\mu}^{s_\xi^\pm}$, $G_{\tau e}^{s_\eta^\pm}$, and $G_{\mu e}^{s_\zeta^\pm}$, respectively. Explicit forms of them are

$$G_{\tau\mu}^{s_\xi^\pm} \equiv \frac{|(F_\xi)_{\tau\mu}|^2}{4\sqrt{2}\, m_{s_\xi^\pm}^2} = \frac{f^2}{\sqrt{2}\, m_{s_\xi^\pm}^2}, \quad G_{\tau e}^{s_\eta^\pm} \equiv \frac{f^2}{\sqrt{2}\, m_{s_\eta^\pm}^2}, \quad G_{\mu e}^{s_\zeta^\pm} \equiv \frac{f^2}{\sqrt{2}\, m_{s_\zeta^\pm}^2}. \tag{52}$$

Note that $2(\overline{\nu_\mu^c} P_L \tau)(\overline{\nu_\tau} P_R \mu^c) = (\overline{\nu_\tau}\gamma^\mu P_L \tau)(\overline{\mu}\gamma_\mu P_L \nu_\mu)$.

The Fermi coupling constant $G_F = 1.17 \times 10^{-5}\,\text{GeV}^{-2}$ is determined by $\mu \to e\overline{\nu}\nu$. Since $s_\zeta^\pm$ contributes to the decay, the value of G_F should be used for $G_{\mu e} = G^W + G_{\mu e}^{s_\zeta^\pm}$ where $G^W \equiv g^2/(4\sqrt{2}m_W^2)$ is the contribution from W boson. The extremely precise measurement of $\mu \to e\overline{\nu}\nu$ itself does not mean an extremely stringent constraint on $G_{\mu e}^{s_\zeta^\pm}$ although the interpretation of G_F changes. Following [12] where the exotic effect to the decay rate ($\propto G_{\mu e}^2$) was assumed to be smaller than 0.1% in order to avoid conflicting with the electroweak precision tests, we have

$$|f| < 2.7 \times 10^{-2} \left(\frac{m_{s_\zeta^\pm}}{300\,\text{GeV}} \right). \tag{53}$$

On the other hand, the contribution of W is universal for $\mu \to e\overline{\nu}\nu$, $\tau \to e\overline{\nu}\nu$, and $\tau \to \mu\overline{\nu}\nu$. Contributions of exotic particles may break the universality, $G_{\mu e} = G_{\tau e} = G_{\tau\mu}$.

[8] The sign of the contribution seems to be misunderstood as the plus sign sometimes.

Constraints from the test of the lepton universality of $G_{\ell\ell'}$ (p. 549 of [36]) can be written as

$$\frac{G_{\tau\mu}}{G_F} = 1 - \frac{G_{\mu e}^{s_\zeta^\pm}}{G_F} + \frac{G_{\tau\mu}^{s_\xi^\pm}}{G_F} = 0.981 \pm 0.018, \tag{54}$$

$$\frac{G_{\tau e}}{G_F} = 1 - \frac{G_{\mu e}^{s_\zeta^\pm}}{G_F} + \frac{G_{\tau e}^{s_\eta^\pm}}{G_F} = 1.0012 \pm 0.0053. \tag{55}$$

If we take a scenario of a large $\tan\beta$, masses of $s_\alpha^\pm$ are almost degenerate as $m_{s^\pm} \simeq m_{s_\alpha^\pm}$. These constraints are then satisfied automatically even for light $s_\alpha^\pm$ by virtue of the remaining A_4 symmetry in the Higgs sector. If $m_{s_\alpha^\pm}$ do not degenerate, $G_{\tau\mu}^{s_\xi^\pm}/G_F$ and $G_{\tau e}^{s_\eta^\pm}/G_F$ are constrained as $\lesssim O(0.01)$ by Eqs. (54) and (55). These constraints allow $f = O(0.01)$ for $m_{s_\alpha^\pm} = O(100)\,\text{GeV}$. Therefore, the A4ZM can be tested if $s_\alpha^- \to \ell\nu$ are measured precisely at the LHC.

E. Nonstandard interaction of neutrinos

During the propagation of neutrinos in the ordinary matter, the coherent forward scattering of them on the matter (e, u, and d) affects neutrino oscillations [37, 38]. The so-called nonstandard interaction (NSI) of neutrinos can give the nonstandard matter effect on the neutrino oscillation [37, 39]. The effective interaction for the exotic effect is expressed conventionally as

$$2\sqrt{2}\, G_F \varepsilon_{\ell\ell'}^{fP} \left(\overline{f}\gamma^\mu P f\right) \left(\overline{\nu_\ell}\gamma_\mu P_L \nu_{\ell'}\right), \tag{56}$$

where $f = e, u, d$ and $P = P_L, P_R$. Note that $s_\alpha^\pm$ can contribute to the interaction by using $2(\overline{\nu_{\ell'}^c} P_L f)(\overline{f} P_R \nu_\ell^c) = (\overline{f}\gamma^\mu P_L f)(\overline{\nu_\ell}\gamma_\mu P_L \nu_{\ell'})$. The last column of Table III shows possible $\varepsilon_{\ell\ell'}^{fP}$ in the A4ZM. There is no contribution of $s_\xi^\pm$ to $\varepsilon_{\ell\ell'}^{eP_L}$ because it does not couple with e. The contribution of $s_\eta^\pm$ to $\nu_\tau e \to \nu_\tau e$ is given by $G_{\tau\mu}^{s_\eta^\pm}/G_F$. On the other hand, $s_\zeta^\pm$ contributes directly to $\nu_\mu e \to \nu_\mu e$ with $G_{\mu e}^{s_\zeta^\pm}/G_F$. In addition, indirect contributions of $s_\zeta^\pm$ to $\varepsilon_{\ell\ell}^{eP}$ exist through G^W/G_F which is not the unity in this model but rather $1 - G_{\mu e}^{s_\zeta^\pm}/G_F$. We have

$$\varepsilon_{ee}^{eP_L} = 0 - (1 + g_L^{e(SM)})\frac{G_{\mu e}^{s_\zeta^\pm}}{G_F}, \tag{57}$$

$$\varepsilon_{\mu\mu}^{eP_L} = \frac{G_{\mu e}^{s_\zeta^\pm}}{G_F} - g_L^{e(SM)}\frac{G_{\mu e}^{s_\zeta^\pm}}{G_F}, \tag{58}$$

$$\varepsilon_{\tau\tau}^{eP_L} = \frac{G_{\tau\mu}^{s_\eta^\pm}}{G_F} - g_L^{e(SM)}\frac{G_{\mu e}^{s_\zeta^\pm}}{G_F}, \tag{59}$$

$$\varepsilon_{\ell\ell}^{eP_R} = 0 - g_R^{e(SM)}\frac{G_{\mu e}^{s_\zeta^\pm}}{G_F}, \tag{60}$$

where $g_L^{e(SM)} = -0.269$ and $g_R^{e(SM)} = 0.231$. The first and second terms in the right hand-side of Eqs. (57)-(60) correspond to the direct and indirect contributions of $s_\alpha^\pm$, respectively. See [40] for model-independent constraints on the NSI of neutrinos. In the A4ZM, $\varepsilon_{\ell\ell'}^{eP}$ are constrained by Eqs. (53)-(55) from $\ell \to \ell'\bar{\nu}\nu$. Values of $\varepsilon_{\ell\ell'}^{eP}$ turn out unfortunately to be $\lesssim O(0.01)$ and smaller than the expected sensitivity (~ 0.1) [41] at the neutrino factory in the future.

V. CONCLUSIONS AND DISCUSSIONS

In this article, we proposed the A4ZM in which the softly broken A_4 symmetry was introduced to the Zee model in the simplest way. The soft breaking term of A_4 is required by the appropriate breaking pattern of A_4 to obtain the tribimaximal mixing which agrees well with neutrino oscillation measurements. It was assumed that the soft breaking of A_4 came from the small breaking terms of the lepton number conservation. This assumption makes an approximate Z_3 symmetry remain in this model.

Realizing the tribimaximal mixing, the A4ZM gives a sum rule for mass eigenvalues of neutrinos, $m_1 e^{i\alpha_{12}} + 2m_2 + 3m_3 e^{i\alpha_{32}} = 0$. The sum rule results in $\Delta m_{31}^2 < 0$ (the inverted mass ordering) and gives the lower bound $m_3 \geq 1.8 \times 10^{-2}\,\text{eV}$. The effective mass for the neutrinoless double beta decay has a simple relation $|(M_\nu)_{ee}|^2 = m_3^2 - \Delta m_{21}^2/3$ and the lower bound $|(M_\nu)_{ee}| \geq 1.7 \times 10^{-2}\,\text{eV}$. Since most of the region of the $|(M_\nu)_{ee}|$ will be probed in the future experiments, this model presents a good prospect of affirmative results in the experiments.

The remaining Z_3 symmetry controls well the FCNC which is necessary for the Zee model to be consistent with neutrino oscillation data. Only $\text{BR}(\tau \to \overline{e}\mu\mu)$ which is caused by the FCNC can be sizable among LFV decays of charged leptons in this model. This model will be excluded easily if other LFV decays of charged leptons are discovered in the future.

The characteristic particles in the A4ZM are three $SU(2)_L$-singlet charged Higgs bosons s^+_α ($\alpha = \xi, \eta, \zeta$) which belong to a **3** representation of A_4. We showed predictions about the flavor structure of leptonic decays $s^-_\alpha \to \ell\nu$; for example, $\text{BR}(s^-_\xi \to \ell\nu)$ gives the ratios of the final states as $e\nu : \mu\nu : \tau\nu = 0 : 1 : 1$. Since $\ell \to \ell'\gamma$ are almost forbidden in the A4ZM by the remaining Z_3 symmetry, $s^\pm_\alpha$ are not constrained stringently. Therefore, it could be expected that some of $s^\pm_\alpha$ are light enough to be produced at the LHC. Then, the characteristic flavor structure of $\text{BR}(s^-_\alpha \to \ell\nu)$ will allow this model to be explored. There are mild constraints form $\ell \to \ell'\overline{\nu}\nu$. The constraints are, however, too strong to observe nonstandard effects for neutrino oscillations in matter with $\nu_\ell e \to \nu_\ell e$ in future experiments.

Finally, we mention some other models in which neutrino masses are generated by loop diagrams. Also in the Ma model [42], neutrino masses are given by one-loop diagrams which are different from the diagrams used in the Zee model. A version of the model with the softly broken A_4 symmetry was discussed in [20]. The two-loop and the three-loop diagrams are used for neutrino masses in the Zee-Babu model [43] and the Krauss-Nasri-Trodden model [44, 45], respectively. If we try to introduce the A_4 symmetry to these two models, there seems to be a difficulty; additional $SU(2)_L$-doublet scalar fields, which will be introduced always by the A_4 symmetry, allow that the neutrino masses in these models are generated at the one-loop level identically to the Zee model. Another three-loop diagram is used in the Aoki-Kanemura-Seto (AKS) model [46] which is compatible with multiple $SU(2)_L$-doublet scalars. The AKS model with the A_4 symmetry will be discussed elsewhere [47].

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Appendix A: right-handed charged leptons of a 3 representation

In the main part of this article, three right-handed charged leptons (ψ^-_{1R}, ψ^-_{2R}, and ψ^-_{3R}) belong to three one-dimensional representations of A_4. Here, we take another choice that right-handed charged leptons are in a $\mathbf{3}$ representation. They are expressed as $\psi^-_{AR}(A = x, y, z)$. See also [21] for a model with right-handed charged leptons of a $\mathbf{3}$ representation. The Yukawa terms of leptons with doublet scalar fields are modified as

$$\mathcal{L}_{\text{d-Yukawa}} = y_q \big(\overline{\Psi_L}\, \psi_R \big)_{\mathbf{1}} \Phi_q + y_s \big(\big(\overline{\Psi_L}\, \psi_R \big)_{\mathbf{3}_s} \Phi \big)_{\mathbf{1}} + y_a \big(\big(\overline{\Psi_L}\, \psi_R \big)_{\mathbf{3}_a} \Phi \big)_{\mathbf{1}} + \text{h.c.} \tag{A1}$$

With the vev alignment in Eq. (16), flavor eigenstates of right-handed charged leptons are given by

$$\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} = U_R^\dagger \begin{pmatrix} \psi^-_{xR} \\ \psi^-_{yR} \\ \psi^-_{zR} \end{pmatrix}, \quad U_R^\dagger \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \tag{A2}$$

while left-handed leptons are still obtained by Eq. (18). Their Z_3-charges are the same as before (Table II). The masses of charged leptons are

$$m_e = \frac{1}{\sqrt{2}} v_q\, y_q + \sqrt{\frac{2}{3}}\, v_3\, y_s, \tag{A3}$$

$$m_\mu = \frac{1}{\sqrt{2}} v_q\, y_q - \frac{1}{\sqrt{6}} v_3\, y_s - i \frac{1}{\sqrt{2}} v_3\, y_a, \tag{A4}$$

$$m_\tau = \frac{1}{\sqrt{2}} v_q\, y_q - \frac{1}{\sqrt{6}} v_3\, y_s + i \frac{1}{\sqrt{2}} v_3\, y_a. \tag{A5}$$

The Yukawa coupling constants can be expressed as

$$y_q = \frac{\sqrt{2}}{3 v_q} \left(m_e + m_\mu + m_\tau \right), \tag{A6}$$

$$y_s = \frac{1}{\sqrt{6}\, v_3} \left(2 m_e - m_\mu - m_\tau \right), \tag{A7}$$

$$y_a = \frac{i}{\sqrt{2}\, v_3} \left(m_\mu - m_\tau \right). \tag{A8}$$

The Yukawa matrices in Eq. (26) are replaced by

$$
\begin{aligned}
Y_1 &= \frac{\sqrt{2}}{3v_3}\begin{pmatrix} 0 & -m_e - m_\mu + 2m_\tau & 0 \\ 0 & 0 & 2m_e - m_\mu - m_\tau \\ -m_e + 2m_\mu - m_\tau & 0 & 0 \end{pmatrix}, \\
Y_2 &= \frac{\sqrt{2}}{3v_3}\begin{pmatrix} 0 & 0 & -m_e + 2m_\mu - m_\tau \\ -m_e - m_\mu + 2m_\tau & 0 & 0 \\ 0 & 2m_e - m_\mu - m_\tau & 0 \end{pmatrix}, \\
Y_3 &= -s_\beta Y_\xi + c_\beta Y_q,
\end{aligned}
\tag{A9}
$$

where

$$
Y_\xi \equiv \frac{\sqrt{2}}{3v_3}\begin{pmatrix} 2m_e - m_\mu - m_\tau & 0 & 0 \\ 0 & -m_e + 2m_\mu - m_\tau & 0 \\ 0 & 0 & -m_e - m_\mu + 2m_\tau \end{pmatrix}, \tag{A10}
$$

$$
Y_q \equiv \frac{\sqrt{2}}{3v_q}(m_e + m_\mu + m_\tau)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{A11}
$$

Note that $Y_G = c_\beta Y_\xi + s_\beta Y_q$ does not change from Eq. (27) as expected. Although M_ν has more complicated form than Eq. (31), it can be divided into six parts (matrices) which are proportional to $m_\ell m_{\ell'}$. Each of these parts has no $\ell\ell'$ element; for example, a part proportional to $m_\mu m_\tau$ has zeros at $\mu\tau$ and $\tau\mu$ elements. Therefore, $(M_\nu)_{\tau\tau}$ vanishes when m_e and m_μ are ignored. Then, we have again the sum rule in Eq. (45) for the tribimaximal mixing. Naive expectation on the size of θ_{13} in this case will be $s_{13} \sim m_\mu/m_\tau \simeq 6 \times 10^{-2}$ which is larger by 1 order of magnitude than the value for the case in the main text. Results in Sec. IV do not change except for the phenomenology of doubletlike Higgs bosons. Table IV shows ratios of $\mathrm{BR}(H_{Di}^- \to \ell\bar{\nu})$ where m_e and m_μ are neglected for simplicity. Neutrino flavors are summed up. The second column shows results for the case where right-handed charged leptons are of one-dimensional representations while the third column is for those of a **3** representation. Interactions of $H_{D1}^0 \equiv (\phi_x^0 + \omega\phi_y^0 + \omega^2\phi_z^0)/\sqrt{3}$ and $H_{D2}^0 \equiv (\phi_x^0 + \omega^2\phi_y^0 + \omega\phi_z^0)/\sqrt{3}$ with leptons are given also by Y_1 and Y_2 in Eq. (A9), respectively. These complex neutral scalars H_{D1}^0 and H_{D2}^0 are mass eigenstates with Z_3-charges of ω^2 and ω, respectively. Note that $\tau \to \overline{\mu} ee$ mediated by H_{D1}^0 and $(H_{D2}^0)^\dagger$ is not suppressed by

	$\mathrm{BR}(H^-_{Di} \to \ell\bar{\nu})$	
	$e\bar{\nu} : \mu\bar{\nu} : \tau\bar{\nu}$	
ℓ_R	$\mathbf{1}, \mathbf{1}', \mathbf{1}''$	$\mathbf{3}$
$H^\pm_{D1}$	$0:0:1$	$1:4:1$
$H^\pm_{D2}$	$0:0:1$	$4:1:1$
$H^\pm_{D3}$	$0:0:1$	$1:1:(1-3s_\beta^2)^2$

TABLE IV: Ratios of $\mathrm{BR}(H^-_{Di} \to \ell\bar{\nu})$ depending on representations of right-handed leptons. For simplicity $m_e = m_\mu = 0$ is used. Interactions of $G^\pm$ (NG boson) with charged leptons are dominated by the interaction with τ.

m_e/v_3 and m_μ/v_3 in this case. Thus not only $\mathrm{BR}(\tau \to \overline{e}\mu\mu)$, which is mediated by $(H^0_{D1})^\dagger$ and H^0_{D2}, but also $\mathrm{BR}(\tau \to \overline{\mu}ee)$ can be sizable as discussed in [21]. The current bound is $\mathrm{BR}(\tau \to \overline{\mu}ee) < 1.5 \times 10^{-8}$ at 90 % CL [32] (see also [33]). The branching ratios are

$$\mathrm{BR}(\tau \to \overline{\mu}ee) = \mathrm{BR}(\tau \to \overline{e}\mu\mu) \;=\; \frac{m_\tau^4}{162 v_3^4 G_F^2}\left(\frac{1}{m^4_{H^0_{D1}}} + \frac{1}{m^4_{H^0_{D2}}}\right)\mathrm{BR}(\tau \to \mu\overline{\nu_\mu}\nu_\tau), \tag{A12}$$

where $\mathrm{BR}(\tau \to \mu\overline{\nu_\mu}\nu_\tau) = 0.17$ and $\sqrt{2}G_F = 1/(v_3^2 + v_q^2)$. By using $\mathrm{BR}(\tau \to \overline{\mu}ee) < 1.5\times 10^{-8}$, we have

$$\left(\frac{1}{m^4_{H^0_{D1}}} + \frac{1}{m^4_{H^0_{D2}}}\right)^{-\frac{1}{4}} > 34\sqrt{1+\tan^2\beta}\,[\mathrm{GeV}]. \tag{A13}$$

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