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# Phenomenology in the Higgs Triplet Model with the $A_4$ Symmetry

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## Abstract

We discuss the phenomenology of doubly and singly charged Higgs bosons (of  $SU(2)_L$ -triplet fields) in the simplest  $A_4$ -symmetric version of the Higgs Triplet Model. Mass eigenstates of these Higgs bosons are obtained explicitly from the Higgs potential. It is shown that their decays into a pair of leptons have unique flavor structures which can be tested at the LHC if some of their masses are below the TeV scale. Sizable decay rates for  $\tau \rightarrow \bar{\mu}ee$  and  $\tau \rightarrow \bar{e}\mu\mu$  can be obtained naturally while other  $\tau \rightarrow \bar{\ell}\ell'\ell''$ ,  $\mu \rightarrow \bar{e}ee$ , and  $\ell \rightarrow \ell'\gamma$  are almost forbidden in this model. Contributions of these Higgs bosons to the non-standard interactions of neutrinos are also considered.

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## I. INTRODUCTION

Two curious features of the lepton sector have been clarified by neutrino oscillation measurements [1–5]. One feature is that neutrinos have nonzero masses which are extremely smaller than other fermion masses. This seems to indicate that neutrino masses are generated by a different mechanism from that for other fermions. In the Standard Model of particle physics (SM), fermion masses are obtained with the vacuum expectation value (vev) of an  $SU(2)_L$ -doublet scalar field while neutrinos are massless because of the absence of the right-handed neutrinos. The Higgs Triplet Model (HTM) [6, 7] is a simple extension of the SM with an  $SU(2)_L$ -triplet Higgs boson of hypercharge  $Y = 2$  whose vev provides Majorana neutrino masses without introducing right-handed neutrinos. The HTM has a predictive phenomenology because the matrix of triplet Yukawa couplings  $h_{\ell\ell'}$  is proportional to the neutrino mass matrix  $(M_\nu)_{\ell\ell'}$  in the flavor basis and  $M_\nu$  is very restricted now by neutrino oscillation data. The characteristic particle in the HTM is the doubly charged Higgs boson  $H^{\pm\pm}$  which will be discovered at hadron colliders (Tevatron and LHC) if it is light enough. Tevatron has been searching for  $H^{\pm\pm}$  and put lower bounds on its mass,  $m_{H^{\pm\pm}} > 112\text{--}150\text{ GeV}$  [8], where one of decay branching ratios (BRs) into same-signed charged leptons is assumed simply to be 100%. If  $\text{BR}(H^{--} \rightarrow \ell\ell')$  are measured, important information on the neutrino mass matrix will be obtained [9–12]. Even though  $H^{\pm\pm}$  is too heavy to be produced at collider experiments, lepton flavor violating processes ( $\mu \rightarrow \bar{e}ee$ ,  $\tau \rightarrow \bar{\ell}\ell'\ell''$ , etc.) are possible if  $h_{\ell\ell'}$  are sizable. Previous works for dependences of lepton flavor violating processes on the parameters in  $M_\nu$  can be found in [13, 14].

The other interesting feature of the lepton sector is the nontrivial structure of the lepton flavor mixing. The lepton flavors are mixed by two large mixing angles ( $\theta_{23} \simeq 45^\circ$  and  $\theta_{12} \simeq 34^\circ$ ) in contrast with the structure of the quark sector which has small mixings only. It seems natural to expect that there is some underlying physics for the special feature of the lepton flavor. As the candidate for that, non-Abelian discrete symmetries have been studied (See e. g., [15] and references therein). An interesting choice is the  $A_4$  symmetry because this is the minimal one including the 3-dimensional irreducible representation which seems suitable for three flavors of the lepton. Some simple models based on the  $A_4$  symmetry can be found in e. g., [16–23].

In this article, we deal with the simplest  $A_4$ -symmetric version of the Higgs Triplet Model

(A4HTM). The mass eigenstates of doubly charged Higgs bosons  $H_i^{\pm\pm}$  are obtained explicitly from the Higgs potential. We see the characteristic flavor structures of  $\text{BR}(H_i^{--} \rightarrow \ell\ell')$ . Other exotic processes like  $\tau \rightarrow \bar{\ell}\ell'\ell''$  are also considered. Similarly, we investigate also phenomenology of “triplet-like” singly charged Higgs bosons  $H_{Ti}^\pm$ ; we refer to the mass eigenstates which are made mainly from triplet scalar fields as the triple-like Higgs bosons.

This article is organized as follows. Section II is devoted to the explanation of the A4HTM. The Higgs sector is analyzed in Sec. III, and mass eigenstates of Higgs bosons are obtained there. Section IV shows phenomenology of the Higgs bosons: leptonic decays of the Higgs bosons, lepton flavor violating decays of charged leptons, non-standard interactions of neutrinos etc. We consider constraints on the model in Sec. V. Conclusions are given in Sec. VI. Throughout this article, we use the words “triplet” etc. only for the representations of  $SU(2)_L$  and “**3**-representation” etc. for the ones of  $A_4$  in order to avoid confusion.

## II. HIGGS TRIPLET MODEL WITH $A_4$ SYMMETRY

The  $A_4$  symmetry is characterized by two elemental transformations  $S$  and  $T$  which satisfy

$$S^2 = T^3 = (ST)^3 = 1. \quad (1)$$

There are three 1-dimensional and one 3-dimensional irreducible representations. We use the following representations:

$$\mathbf{1} : S \mathbf{1} = \mathbf{1}, \quad T \mathbf{1} = \mathbf{1}, \quad (2)$$

$$\mathbf{1}' : S \mathbf{1}' = \mathbf{1}', \quad T \mathbf{1}' = \omega \mathbf{1}', \quad (3)$$

$$\mathbf{1}'' : S \mathbf{1}'' = \mathbf{1}'', \quad T \mathbf{1}'' = \omega^2 \mathbf{1}'', \quad (4)$$

$$\mathbf{3} : S \mathbf{3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \mathbf{3}, \quad T \mathbf{3} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \mathbf{3}, \quad (5)$$

where  $\omega \equiv \exp(2\pi i/3)$ . We refer to the basis in eq. (5) as the  $S$ -diagonal basis. See appendix for another simple choice (the “ $T$ -diagonal basis”). Because of  $\mathbf{3}^* = \mathbf{3}$  in the  $S$ -diagonal basis, the basis seems better than the  $T$ -diagonal one for the construction of the  $A_4$ -symmetric Higgs potential.

	$\psi_{1R}^-$	$\psi_{2R}^-$	$\psi_{3R}^-$	$\Psi_{AL} = \begin{pmatrix} \psi_{AL}^0 \\ \psi_{AL}^- \end{pmatrix}$	$\Phi_A = \begin{pmatrix} \phi_A^+ \\ \phi_A^0 \end{pmatrix}$	$\delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}$	$\Delta_A = \begin{pmatrix} \frac{\Delta_A^+}{\sqrt{2}} & \Delta_A^{++} \\ \Delta_A^0 & -\frac{\Delta_A^+}{\sqrt{2}} \end{pmatrix}$
$A_4$	<b>1</b>	<b>1'</b>	<b>1''</b>	<b>3</b>	<b>3</b>	<b>1</b>	<b>3</b>
$SU(2)_L$	<b>1</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>3</b>	<b>3</b>
$U(1)_Y$	-2	-2	-2	-1	1	2	2

TABLE I: The leptons and the Higgs bosons in the A4HTM. The subscript  $A = x, y, z$  denotes the index for **3** of  $A_4$ ; for example,  $(\Psi_{xL}, \Psi_{yL}, \Psi_{zL})$  belongs to **3** while each  $\Psi_{AL}$  are  $SU(2)_L$ -doublet fields.

The particle contents in the A4HTM are listed in Table I. Singlet charged fermions  $\psi_{1R}^-$ ,  $\psi_{2R}^-$ , and  $\psi_{3R}^-$  belong to **1**, **1'**, and **1''**, respectively. Doublet fermions,  $\Psi_{xL}$ ,  $\Psi_{yL}$ , and  $\Psi_{zL}$  are members of **3**. A **3**-representation is composed of Higgs doublets,  $\Phi_x$ ,  $\Phi_y$ , and  $\Phi_z$ . A triplet field  $\delta$  of Higgs bosons is of **1**. Three Higgs triplets,  $\Delta_x$ ,  $\Delta_y$ , and  $\Delta_z$  construct a **3**-representation. Thus, the A4HTM is a four-Higgs-Triplet-Model and a three-Higgs-Doublet-Model (we may introduce an extra doublet boson for quarks). Other versions of  $A_4$ -symmetric HTM can be seen in [20, 21] which have six triplet fields of **1**, **1'**, **1''**, and **3**. The calculations in this section are almost identical to those for the model in [22] where  $A_4$  is broken by vev's of gauge singlet scalars (so-called flavon). The vev's of seven Higgs fields in the A4HTM are taken as follows:

$$\langle \phi_x^0 \rangle = \langle \phi_y^0 \rangle = \langle \phi_z^0 \rangle = \frac{v}{\sqrt{6}}, \quad (6)$$

$$\langle \delta^0 \rangle = \frac{v_\delta}{\sqrt{2}}, \quad \langle \Delta_x^0 \rangle = \frac{v_\Delta}{\sqrt{2}}, \quad \langle \Delta_y^0 \rangle = \langle \Delta_z^0 \rangle = 0, \quad (7)$$

where  $v = 246 \text{ GeV}$ . Similarly to the HTM, triplet vev's  $v_\delta$  and  $v_\Delta$  should be generated by explicit breaking terms of the lepton number conservation because spontaneous breaking of it [24] brings undesired Nambu-Goldston bosons (so-called Majoron). The triplet vev's (and explicit breaking parameters for them) are taken to be real positive by using two phase degrees of freedom of  $\delta$  and  $(\Delta_x, \Delta_y, \Delta_z)$ . Note that triplet vev's are constrained as  $v' \equiv \sqrt{v_\delta^2 + v_\Delta^2} < 3 \text{ GeV}$  by  $\rho_0 = 1.0004_{-0.0007}^{+0.0027}$  at  $2\sigma$  CL (page 137 of [25]). Since the alignment eq. (6) is invariant for acting  $T$  which satisfies  $T^3 = 1$ , the A4HTM has an approximate  $Z_3$  symmetry which is broken only by a small  $v_\Delta$ . The Yukawa terms for

doublet Higgs bosons are expressed as

$$\mathcal{L}_{\text{d-Yukawa}} = y_1(\overline{\Psi}_L \Phi)_{\mathbf{1}} \psi_{1R} + y_2(\overline{\Psi}_L \Phi)_{\mathbf{1}'} \psi_{2R} + y_3(\overline{\Psi}_L \Phi)_{\mathbf{1}''} \psi_{3R} + \text{h.c.} \quad (8)$$

The expressions  $(\mathbf{3} \mathbf{3})_{\mathbf{1}}$  etc. mean the decompositions of  $\mathbf{3} \otimes \mathbf{3} \rightarrow \mathbf{1}$  etc. among  $\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_s \oplus \mathbf{3}_a$  (See Appendix A). The flavor eigenstates of leptons<sup>1</sup> are given by

$$\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \equiv U_R^\dagger \begin{pmatrix} \psi_{1R}^- \\ \psi_{2R}^- \\ \psi_{3R}^- \end{pmatrix}, \quad \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \equiv U_L^\dagger \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix}, \quad L_\ell \equiv \begin{pmatrix} \nu_{\ell L} \\ \ell_L \end{pmatrix}, \quad (9)$$

$$U_R \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad U_L \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (10)$$

The masses of charged leptons are

$$m_e \equiv \frac{1}{\sqrt{2}} v y_1, \quad m_\mu \equiv \frac{1}{\sqrt{2}} v y_2, \quad m_\tau \equiv \frac{1}{\sqrt{2}} v y_3. \quad (11)$$

It is worth to note that  $L_e$ ,  $L_\mu$ , and  $L_\tau$  are eigenstates of  $T$  for eigenvalues 1,  $\omega$ , and  $\omega^2$ , respectively.

Neutrinos in the A4HTM are Majorana fermions. In general, the mass matrix  $M_\nu$  of Majorana neutrinos in the flavor basis can be expressed as

$$M_\nu = U_{\text{MNS}}^* \text{diag}(m_1 e^{i\alpha_{12}}, m_2, m_3 e^{i\alpha_{32}}) U_{\text{MNS}}^\dagger, \quad (12)$$

where  $m_i$  are real positive masses. The parameters  $\alpha_{12}$  and  $\alpha_{32}$  within  $[0, 2\pi)$  are Majorana phases [7, 26] which appear only for Majorana particles. The standard parametrization of the Maki-Nakagawa-Sakata (MNS) matrix [27],  $U_{\text{MNS}}$ , is

$$U_{\text{MNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_D} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_D} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (13)$$

where  $c_{ij}$  and  $s_{ij}$  stand for  $\cos \theta_{ij}$  and  $\sin \theta_{ij}$ , respectively. Neutrino oscillation measurements [1–5] show

$$\Delta m_{21}^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{31}^2| \simeq 2.4 \times 10^{-3} \text{ eV}^2, \quad (14)$$

$$\sin^2 2\theta_{23} \simeq 1, \quad \sin^2 2\theta_{12} \simeq 0.87, \quad \sin^2 2\theta_{13} \lesssim 0.14, \quad (15)$$

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<sup>1</sup> If  $\delta$  belongs to  $\mathbf{1}'$  instead of  $\mathbf{1}$ , the names of lepton flavors in eq. (9) are changed as  $(e, \mu, \tau) \rightarrow (\mu, \tau, e)$  in order to keep the structure of the neutrino mixing.

where  $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ .

In the A4HTM, neutrino masses are generated by the Yukawa terms of triplet Higgs bosons:

$$\mathcal{L}_{\text{t-Yukawa}} = h_\delta \left[ \overline{(\Psi_L)_\alpha^c} (\Psi_L)_\beta \right]_{\mathbf{1}} (i\sigma^2 \delta)_{\alpha\beta} + h_\Delta \left( \left( \overline{(\Psi_L)_\alpha^c} (\Psi_L)_\beta \right)_{\mathbf{3}_s} (i\sigma^2 \Delta)_{\alpha\beta} \right)_{\mathbf{1}} + \text{h.c.}, \quad (16)$$

where  $\alpha$  and  $\beta$  stand for the  $SU(2)_L$  index,  $\sigma^i$  are the Pauli matrices, and the superscript  $c$  means the charge conjugation. Without loss of generality,  $h_\delta$  can be taken as a real parameter by the redefinition of the phase of  $\Psi_L$ . The decomposition indicated by  $[\mathbf{3} \mathbf{3}]_{\mathbf{1}}$  is the one which depends on the representation ( $\mathbf{1}$  or  $\mathbf{1}'$  or  $\mathbf{1}''$ ) of  $\delta$ . By using  $U_L$  and triplet vev's for eq. (16), the mass matrix  $M_\nu$  of neutrinos is obtained. The mass matrix is expressed in the form of eq. (12) with

$$m_1 e^{i\alpha_{12}} = h_\delta v_\delta + h_\Delta v_\Delta, \quad m_2 = h_\delta v_\delta, \quad m_3 e^{i\alpha_{32}} = -h_\delta v_\delta + h_\Delta v_\Delta, \quad (17)$$

$$U_{\text{MNS}} = U_{\text{TB}} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (18)$$

$U_{\text{TB}}$  is the matrix of so-called tri-bimaximal mixing [28] which agrees with eq. (15). It is an attractive feature of the  $A_4$  symmetry that such a nontrivial mixing matrix can be given by a simple choice of the vev's in eq. (6) and (7). Two combinations of parameters are determined by eq. (14) as

$$|h_\Delta| v_\Delta = \frac{1}{\sqrt{2}} \sqrt{\Delta m_{31}^2 - 2\Delta m_{21}^2} \simeq 3.4 \times 10^{-2} \text{ eV}, \quad (19)$$

$$h_\delta v_\delta \cos \varphi_\Delta = -\frac{\Delta m_{31}^2}{2\sqrt{2} \sqrt{\Delta m_{31}^2 - 2\Delta m_{21}^2}} \simeq -1.8 \times 10^{-2} \text{ eV}, \quad (20)$$

where  $\varphi_\Delta \equiv \arg(h_\Delta)$ . It is apparent in eq. (19) that the A4HTM predicts  $\Delta m_{31}^2 > 0$ . Then,  $m_i$  are given<sup>2</sup> by

$$m_1^2 = \left\{ \frac{1}{8(1-2r) \cos^2 \varphi_\Delta} - r \right\} \Delta m_{31}^2 \geq (0.016 \text{ eV})^2, \quad (21)$$

$$m_2^2 = \frac{\Delta m_{31}^2}{8(1-2r) \cos^2 \varphi_\Delta} \geq (0.018 \text{ eV})^2, \quad (22)$$

$$m_3^2 = \left\{ \frac{1}{8(1-2r) \cos^2 \varphi_\Delta} + 1 - r \right\} \Delta m_{31}^2 \geq (0.051 \text{ eV})^2, \quad (23)$$

---

<sup>2</sup> Arbitrary  $m_i$  can be obtained if we introduce also  $\delta_2$  of  $\mathbf{1}'$  and  $\delta_3$  of  $\mathbf{1}''$  with a condition  $h_{\delta 2} v_{\delta 2} = h_{\delta 3} v_{\delta 3}$  for their Yukawa couplings and vev's [20].

where  $r \equiv \Delta m_{21}^2 / \Delta m_{31}^2$ . Majorana phases are

$$\tan \alpha_{12} = -\frac{(1-2r) \sin 2\varphi_\Delta}{1-2(1-2r) \cos^2 \varphi_\Delta}, \quad (24)$$

$$\tan \alpha_{32} = \frac{(1-2r) \sin 2\varphi_\Delta}{1+2(1-2r) \cos^2 \varphi_\Delta}, \quad \cos \alpha_{32} < 0. \quad (25)$$

Numerically,  $|\alpha_{32} - \pi| \lesssim 0.16\pi$ . The effective mass  $(M_\nu)_{ee}$  for the neutrinoless double beta decay (See [29] for a review) is expressed as

$$|(M_\nu)_{ee}|^2 = \left( \frac{1}{8(1-2r) \cos^2 \varphi_\Delta} - \frac{1+4r}{9} \right) \Delta m_{31}^2 \geq (0.0045 \text{ eV})^2. \quad (26)$$

### III. HIGGS SECTOR

It is necessary to take mass eigenstates of the Higgs bosons in order to consider their phenomenology which is our purpose in this article. The mass eigenstates can be obtained from the Higgs potential shown in the next subsection.

#### A. Higgs Potential

Let us first remind that an expression [13] of the Higgs potential in the HTM without the  $A_4$  symmetry is

$$\begin{aligned} V_{\text{HTM}} = & -m^2(\Phi^\dagger\Phi) + \lambda_1(\Phi^\dagger\Phi)^2 + M^2\text{Tr}(\Delta^\dagger\Delta) + \lambda_2[\text{Tr}(\Delta^\dagger\Delta)]^2 + \lambda_3\text{Det}(\Delta^\dagger\Delta) \\ & + \lambda_4(\Phi^\dagger\Phi)\text{Tr}(\Delta^\dagger\Delta) + \lambda_5(\Phi^\dagger\sigma^i\Phi)\text{Tr}(\Delta^\dagger\sigma^i\Delta) + \left( \frac{1}{\sqrt{2}}\mu(\Phi^T i\sigma^2\Delta^\dagger\Phi) + \text{h.c.} \right), \end{aligned} \quad (27)$$

where  $\Phi$  and  $\Delta$  are doublet and triplet Higgs bosons, respectively. Using these notations of coupling constants as the reference, we construct the  $A_4$ -symmetric potential for the A4HTM



as

$$V_{A4\text{HTM}} \equiv V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu, \quad (28)$$

$$V_m \equiv -m_\Phi^2 (\Phi^\dagger \Phi)_1 + M_\delta^2 \text{Tr}(\delta^\dagger \delta) + M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta)_1, \quad (29)$$

$$\begin{aligned} V_4 \equiv & \lambda_{4\delta} (\Phi^\dagger \Phi)_1 \text{Tr}(\delta^\dagger \delta) + \lambda_{4\Delta} (\Phi^\dagger \Phi)_1 \text{Tr}(\Delta^\dagger \Delta)_1 \\ & + \{ \lambda'_{4\Delta p} (\Phi^\dagger \Phi)_{1''} \text{Tr}(\Delta^\dagger \Delta)_{1'} + \text{h.c.} \} \\ & + \lambda_{4\Delta ss} (\Phi^\dagger \Phi)_{\mathbf{3}_s} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} + \lambda_{4\Delta aa} (\Phi^\dagger \Phi)_{\mathbf{3}_a} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a} \\ & + i\lambda_{4\Delta sa} (\Phi^\dagger \Phi)_{\mathbf{3}_s} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a} + i\lambda_{4\Delta as} (\Phi^\dagger \Phi)_{\mathbf{3}_a} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} \\ & + \{ \lambda'_{4s} \delta_{\beta\alpha}^* [\Delta_{\beta\alpha} (\Phi^\dagger \Phi)_{\mathbf{3}_s}]_1 + \lambda'_{4a} \delta_{\beta\alpha}^* [\Delta_{\beta\alpha} (\Phi^\dagger \Phi)_{\mathbf{3}_a}]_1 + \text{h.c.} \}, \end{aligned} \quad (30)$$

$$\begin{aligned} V_5 \equiv & \lambda_{5\delta} (\Phi^\dagger \sigma^i \Phi)_1 \text{Tr}(\delta^\dagger \sigma^i \delta) + \lambda_{5\Delta} (\Phi^\dagger \sigma^i \Phi)_1 \text{Tr}(\Delta^\dagger \sigma^i \Delta)_1 \\ & + \{ \lambda'_{5\Delta p} (\Phi^\dagger \sigma^i \Phi)_{1''} \text{Tr}(\Delta^\dagger \sigma^i \Delta)_{1'} + \text{h.c.} \} \\ & + \lambda_{5\Delta ss} (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_s} \text{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{3}_s} + \lambda_{5\Delta aa} (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_a} \text{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{3}_a} \\ & + i\lambda_{5\Delta sa} (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_s} \text{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{3}_a} + i\lambda_{5\Delta as} (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_a} \text{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{3}_s} \\ & + \{ \lambda'_{5s} (\delta^\dagger \sigma^i)_{\alpha\beta} [\Delta_{\beta\alpha} (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_s}]_1 \\ & \quad + \lambda'_{5a} (\delta^\dagger \sigma^i)_{\alpha\beta} [\Delta_{\beta\alpha} (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_a}]_1 + \text{h.c.} \}, \end{aligned} \quad (31)$$

where coupling constants  $\lambda'$  have complex values while  $\lambda$ 's are real<sup>3</sup>. The subscripts  $\alpha$  and  $\beta$  stand for the indices of  $SU(2)_L$ . Main parts of the squared mass matrices for triplet fields are induced by  $V_m$ ,  $V_4$ , and  $V_5$ , which give  $v^2 \Delta_x^{--} \Delta_x^{++}$  etc. Contributions from  $V_2$  and  $V_3$  can be ignored because they are suppressed by small triplet vev's. The expressions of  $V_1$ ,  $V_2$ ,  $V_3$ , and  $V_\mu$  are presented in Appendix C. Linear terms of triplet fields exist not only in  $V_\mu$  but also in  $V_4$  and  $V_5$ , which affect vacuum conditions for triplet vev's. Actually, the democratic alignment of doublet vev's in eq. (6) results in the democratic one for triplet  $\mathbf{3}$  also, which conflicts with eq. (7). Some solutions on the alignment problem were discussed in [22]. We may simply assume

$$v_\delta \text{Re}(\lambda'_{4s} + \lambda'_{5s}) + v_\Delta (\lambda_{4\Delta ss} + \lambda_{5\Delta ss}) = 0, \quad (32)$$

and use  $\tilde{V}_\mu$  with the soft breaking of  $A_4$  instead of the  $A_4$ -symmetric  $V_\mu$ ; for example,

$$\tilde{V}_\mu = \frac{1}{\sqrt{2}} \mu_\delta [\Phi_\alpha \Phi_\beta]_1 (i\sigma^2 \delta^\dagger)_{\alpha\beta} + \frac{1}{\sqrt{2}} \mu_\Delta (2\Phi_{y\alpha} \Phi_{z\beta}) (i\sigma^2 \Delta_x^\dagger)_{\alpha\beta} + \text{h.c.}, \quad (33)$$

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<sup>3</sup> One may rewrite  $V_5$  with  $(\Phi_A^\dagger \sigma^i \Phi_B) \text{Tr}(\Delta_C^\dagger \sigma^i \Delta_D) = 2\Phi_A^\dagger \Delta_D \Delta_C^\dagger \Phi_B - (\Phi_A^\dagger \Phi_B) \text{Tr}(\Delta_C^\dagger \Delta_D)$ .

where  $\mu_{\Delta_x}$  breaks softly the lepton number conservation and the  $A_4$  symmetry<sup>4</sup>. Redefinitions of phases of  $\delta$  and  $(\Delta_x, \Delta_y, \Delta_z)$  enable us to make  $\mu_\delta$  and  $\mu_{\Delta_x}$  real positive parameters. Ignoring corrections due to small triplet vev's, we can have

$$v = \sqrt{6}\langle\phi_x^0\rangle \simeq \sqrt{6}\langle\phi_y^0\rangle \simeq \sqrt{6}\langle\phi_z^0\rangle \simeq \frac{\sqrt{3}m_\Phi}{\sqrt{3\lambda_1 + 4\lambda_{1ss}}}, \quad (34)$$

$$\begin{pmatrix} v_\delta \\ v_\Delta \end{pmatrix} \simeq \begin{pmatrix} 6M_\delta^2 + 3\lambda_{\delta 45p}v^2 & 2\text{Re}(\lambda'_{s45p})v^2 \\ 2\text{Re}(\lambda'_{s45p})v^2 & 6M_\Delta^2 + 3\lambda_{\Delta 45p}v^2 \end{pmatrix}^{-1} \begin{pmatrix} 3v^2\mu_\delta \\ 2v^2\mu_{\Delta_x} \end{pmatrix}, \quad (35)$$

$$\langle\Delta_y^0\rangle = \langle\Delta_z^0\rangle = 0, \quad (36)$$

where  $\lambda_{45p}$ 's are defined by  $\lambda_4 + \lambda_5$  for each subscripts; for example,  $\lambda_{\delta 45p} \equiv \lambda_{4\delta} + \lambda_{5\delta}$ . Small triplet vev's may be also the origin of the small deviation from the tri-bimaximal mixing (small  $\theta_{13}$ ). In the following parts of this article, we just use the vacuum alignment in eq. (6) and (7) ignoring how to achieve them.

## B. Mass Eigenstates of Triplet Higgs Bosons

Ignoring small contributions from triplet vev's, the squared mass matrix of doubly charged Higgs bosons is obtained from  $V_m + V_4 + V_5$  as

$$\begin{pmatrix} \Delta_x^{--} & \Delta_y^{--} & \Delta_z^{--} & \delta^{--} \end{pmatrix} \times \begin{pmatrix} M_{\Delta 45m}^2 & [M_{\pm\pm}^2]_{21}^* & [M_{\pm\pm}^2]_{21} & \frac{1}{3}v^2(\lambda'_{s45m})^* \\ [M_{\pm\pm}^2]_{21} & M_{\Delta 45m}^2 & [M_{\pm\pm}^2]_{21}^* & \frac{1}{3}v^2(\lambda'_{s45m})^* \\ [M_{\pm\pm}^2]_{21}^* & [M_{\pm\pm}^2]_{21} & M_{\Delta 45m}^2 & \frac{1}{3}v^2(\lambda'_{s45m})^* \\ \frac{1}{3}v^2\lambda'_{s45m} & \frac{1}{3}v^2\lambda'_{s45m} & \frac{1}{3}v^2\lambda'_{s45m} & M_{\delta 45m}^2 \end{pmatrix} \begin{pmatrix} \Delta_x^{++} \\ \Delta_y^{++} \\ \Delta_z^{++} \\ \delta^{++} \end{pmatrix}, \quad (37)$$

$$[M_{\pm\pm}^2]_{21} \equiv \frac{1}{3}v^2(\lambda_{\Delta ss45m} + i\lambda_{\Delta sa45m}), \quad (38)$$

$$M_{\delta 45m}^2 \equiv M_\delta^2 + \frac{1}{2}v^2\lambda_{\delta 45m}, \quad M_{\Delta 45m}^2 \equiv M_\Delta^2 + \frac{1}{2}v^2\lambda_{\Delta 45m}, \quad (39)$$

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<sup>4</sup> If representation of  $\delta$  is  $\mathbf{1}'$ , also  $\mu_\delta$  must break  $A_4$  because  $(\Phi_\alpha\Phi_\beta)_{\mathbf{1}''}$  does not contain  $v^2$  term.

where  $\lambda_{45m}$ 's are defined by  $\lambda_4 - \lambda_5$  for each subscripts; for example,  $\lambda_{\Delta ss45m} \equiv \lambda_{4\Delta ss} - \lambda_{5\Delta ss}$ . Then, the mass eigenstates of doubly charged Higgs bosons are given by

$$\begin{pmatrix} H_1^{++} \\ H_2^{++} \\ H_3^{++} \\ H_4^{++} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \theta_{\pm\pm} & \sin \theta_{\pm\pm} \\ 0 & 0 & -\sin \theta_{\pm\pm} & \cos \theta_{\pm\pm} \end{pmatrix} \begin{pmatrix} 1 & \omega & \omega^2 & 0 \\ 1 & \omega^2 & \omega & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{3} e^{-i \arg(\lambda'_{s45m})} \end{pmatrix} \begin{pmatrix} \Delta_x^{++} \\ \Delta_y^{++} \\ \Delta_z^{++} \\ \delta^{++} \end{pmatrix}, \quad (40)$$

$$\tan 2\theta_{\pm\pm} \equiv \frac{2\sqrt{3} |\lambda'_{s45m}| v^2}{3M_{\Delta45m}^2 - 3M_{\delta45m}^2 + 2\lambda_{\Delta ss45m} v^2}, \quad (41)$$

where  $0 \leq \theta_{\pm\pm} \leq \pi/4$  for negative values ( $\leq 0$ ) of the denominator of eq. (41) and  $\pi/4 < \theta_{\pm\pm} \leq \pi/2$  for positive values ( $> 0$ ). It is understood by the approximate  $Z_3$  symmetry of the A4HTM that  $\delta$  is mixed with  $\Delta_\xi \equiv (\Delta_x + \Delta_y + \Delta_z)/\sqrt{3}$  for which acting  $T$  gives 1 as the eigenvalue<sup>5</sup>. The masses  $m_{H_i^{\pm\pm}}$  of  $H_i^{\pm\pm}$  are

$$m_{H_1^{\pm\pm}}^2 = M_{\Delta45m}^2 - \frac{1}{3} \lambda_{\Delta ss45m} v^2 + \frac{1}{\sqrt{3}} \lambda_{\Delta sa45m} v^2, \quad (42)$$

$$m_{H_2^{\pm\pm}}^2 = M_{\Delta45m}^2 - \frac{1}{3} \lambda_{\Delta ss45m} v^2 - \frac{1}{\sqrt{3}} \lambda_{\Delta sa45m} v^2, \quad (43)$$

$$m_{H_3^{\pm\pm}}^2 = \frac{1}{6} (3M_{\delta45m}^2 + 3M_{\Delta45m}^2 + 2\lambda_{\Delta ss45m} v^2 - 3\Delta m_{\pm\pm}^2), \quad (44)$$

$$m_{H_4^{\pm\pm}}^2 = \frac{1}{6} (3M_{\delta45m}^2 + 3M_{\Delta45m}^2 + 2\lambda_{\Delta ss45m} v^2 + 3\Delta m_{\pm\pm}^2), \quad (45)$$

$$3\Delta m_{\pm\pm}^2 \equiv \left\{ 12|\lambda'_{s45m}|^2 v^4 + (3M_{\Delta45m}^2 - 3M_{\delta45m}^2 + 2\lambda_{\Delta ss45m} v^2)^2 \right\}^{\frac{1}{2}}. \quad (46)$$

Note that  $m_{H_3^{\pm\pm}} \leq m_{H_4^{\pm\pm}}$  as the definition. These masses  $m_{H_i^{\pm\pm}}$  can be different enough from each other while the constraint from  $\rho$ -parameter does not prefer large mass differences between  $H_i^{\pm\pm}$  and their triplet-like partners ( $H_{Ti}^\pm$ ,  $H_{Ti}^0$ , and  $A_{Ti}^0$ ).

Decays of  $H_i^{\pm\pm}$  into same-signed charged leptons in the flavor basis are governed by the

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<sup>5</sup> If  $\delta$  belongs to  $\mathbf{1}'$ , the field is mixed with  $\Delta_\eta \equiv (\Delta_x + \omega^2 \Delta_y + \omega \Delta_z)/\sqrt{3}$  which is an eigenstate of  $T$  for an eigenvalue  $\omega$ . There will be no difficulty to obtain mass eigenstates of Higgs bosons even in the model of [20] where  $\delta_2$  of  $\mathbf{1}'$  and  $\delta_3$  of  $\mathbf{1}''$  are also introduced.

	$e_L$	$\mu_L$	$\tau_L$	$H_1^{++}$	$H_2^{++}$	$H_3^{++}, H_4^{++}$
$T$	1	$\omega$	$\omega^2$	$\omega^2$	$\omega$	1

TABLE II: Eigenstates and eigenvalues of  $T$ .

following couplings  $h_{i\pm\pm}$  for  $(h_{i\pm\pm})_{\ell\ell'} H^{++} \overline{(\ell_L)^c} \ell'_L$ :

$$h_{1\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad (47)$$

$$h_{2\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (48)$$

$$h_{3\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \cos \theta_{\pm\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \tilde{h}_\delta \sin \theta_{\pm\pm} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \quad (49)$$

$$h_{4\pm\pm} = -\frac{1}{\sqrt{3}} h_\Delta \sin \theta_{\pm\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \tilde{h}_\delta \cos \theta_{\pm\pm} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \quad (50)$$

$$\tilde{h}_\delta \equiv h_\delta e^{i \arg(\lambda'_{s45m})}. \quad (51)$$

The zeros in  $h_{i\pm\pm}$  can be understood easily by the eigenvalues of  $T$  for eigenstates  $H_i^{\pm\pm}$  and leptons (Table II); for example,  $(h_{1\pm\pm})_{ee}$  must vanish approximately because  $\overline{(e_L)^c} e_L H_1^{++}$  is not invariant for acting  $T$ . On the other hand, the values of nonzero elements of  $h_{i\pm\pm}$  are the consequence of the  $A_4$  symmetry.

Next, let us consider singly charged scalar fields also. We concentrate on the four triplet-like singly charged scalar,  $H_{Ti}^\pm$ . The mixing between doublet and triplet bosons is ignored because it is suppressed by small vev's of triplet fields<sup>6</sup>. Then, we can diagonalize the squared mass matrix for the singly charged ones of triplet fields similarly to the case for doubly charged ones. The mixing matrix with an angle  $\theta_\pm$ , masses  $m_{H_{Ti}^\pm}$ , and couplings  $h_{i\pm}$  for  $\sqrt{2} (h_{i\pm})_{\ell\ell'} H_{Ti}^+ \overline{(\nu_{\ell L})^c} \ell'_L$  are given simply by setting  $\lambda_5 = 0$  in eq. (40)-(51). Note that  $h_{3\pm}$

<sup>6</sup> The small vev can give a maximal mixing for neutral bosons in a special case but this does not happen for charged ones of the interest in this article. Phenomenology for the case in the HTM is shown in [30].

	BR( $H^{--} \rightarrow \ell\ell'$ ) $ee : \mu\mu : \tau\tau : e\mu : e\tau : \mu\tau$	$\tau \rightarrow \bar{\ell}\ell'\ell''$	others
$H_1^{\pm\pm}$	0 : 0 : 2 : 1 : 0 : 0	none	
$H_2^{\pm\pm}$	0 : 2 : 0 : 0 : 1 : 0	$\tau_L \rightarrow \bar{e}_L \mu_L \mu_L$	
$H_3^{\pm\pm}$	$R_3^{\pm\pm} : 0 : 0 : 0 : 0 : 1$	$\tau_L \rightarrow \bar{\mu}_L e_L e_L$	$\bar{e}_L e_L \rightarrow \bar{e}_L e_L$
$H_4^{\pm\pm}$	$R_4^{\pm\pm} : 0 : 0 : 0 : 0 : 1$	$\tau_L \rightarrow \bar{\mu}_L e_L e_L$	$\bar{e}_L e_L \rightarrow \bar{e}_L e_L$

TABLE III: Ratios of decays of  $H_i^{\pm\pm}$  into a pair of same-signed charged leptons in the A4HTM. Contributions of  $H_i^{\pm\pm}$  to  $\tau \rightarrow \bar{\ell}\ell'\ell''$  at the tree level are also shown. Note that all of  $H_i^{\pm\pm}$  does not contribute to  $\mu \rightarrow \bar{e}ee$  and  $\ell \rightarrow \ell'\gamma$  at the tree and one loop level, respectively. The Bhabha scattering can be affected by  $H_3^{\pm\pm}$  and  $H_4^{\pm\pm}$ .

and  $h_{4\pm}$  can be different from  $h_{3\pm\pm}$  and  $h_{4\pm\pm}$  in the A4HTM, respectively, while  $h_{\pm} = h_{\pm\pm}$  in the HTM. This is because  $\theta_{\pm}$  can be different from  $\theta_{\pm\pm}$  by the existence of  $\lambda_5$  in principle. However,  $\theta_{\pm} \simeq \theta_{\pm\pm}$  seems preferred because  $m_{H_{T_i}^{\pm}} \simeq m_{H_i^{\pm\pm}}$  (namely,  $|\lambda_5| \ll 1$ ) is favored by the  $\rho$  parameter.

The mass eigenstates of the triplet-like neutral Higgs bosons are shown in Appendix D for completeness.

#### IV. PHENOMENOLOGY OF HIGGS BOSONS

We assume that some of exotic Higgs bosons are light enough to be detected in collider experiments and to give sizable effects on some processes<sup>7</sup>. In this section, we first list up exotic processes which are possible with  $H_i^{\pm\pm}$  and the triplet-like  $H_{T_i}^{\pm}$ . Then, constraints from these processes are considered in the next section.

<sup>7</sup> Even if  $M_{\delta}$  and  $M_{\Delta}$  are very large, the leptogenesis with the decays of triplet bosons [31] does not happen in this model because their decays into  $\Psi_L$  have individual final states as we see in eq. (16).

### A. $H^{--} \rightarrow \ell\ell'$

The ratios of the branching ratios  $\text{BR}_{\ell\ell'} \equiv \text{BR}(H^{--} \rightarrow \ell\ell')$  are shown in Table III. We used

$$R_3^{\pm\pm} \equiv \frac{|2h_\Delta c_{\pm\pm} + \sqrt{3}\tilde{h}_\delta s_{\pm\pm}|^2}{2|h_\Delta c_{\pm\pm} - \sqrt{3}\tilde{h}_\delta s_{\pm\pm}|^2}, \quad R_4^{\pm\pm} \equiv \frac{|2h_\Delta s_{\pm\pm} - \sqrt{3}\tilde{h}_\delta c_{\pm\pm}|^2}{2|h_\Delta s_{\pm\pm} + \sqrt{3}\tilde{h}_\delta c_{\pm\pm}|^2}, \quad (52)$$

where  $c_{\pm\pm} \equiv \cos\theta_{\pm\pm}$  and  $s_{\pm\pm} \equiv \sin\theta_{\pm\pm}$ . It is clear that each of  $H_i^{\pm\pm}$  has only two decay modes into a pair of same-signed charged leptons. For a simple case with  $\tan 2\theta_{\pm\pm} = 0$ , one of  $R_3^{\pm\pm}$  and  $R_4^{\pm\pm}$  becomes 2 while the other is 1/2. Then, the  $H_i^{\pm\pm}$  which gives  $\text{BR}_{ee}/\text{BR}_{\mu\tau} = 1/2$  can be identified as the  $\delta^{\pm\pm}$  boson<sup>8</sup>. An interesting point is that decays of  $H_1^{\pm\pm}$  and  $H_2^{\pm\pm}$  give  $\text{BR}_{\mu\mu} \neq \text{BR}_{\tau\tau}$  and  $\text{BR}_{e\mu} \neq \text{BR}_{e\tau}$  in contrast with the case for the HTM in which  $\text{BR}_{\mu\mu} \simeq \text{BR}_{\tau\tau}$  and  $\text{BR}_{e\mu} \simeq \text{BR}_{e\tau}$ . If  $m_{H_1^{\pm\pm}} = m_{H_2^{\pm\pm}}$  which is realized at  $\lambda_{4\Delta sa} = \lambda_{5\Delta sa}$ , the sum of  $\text{BR}_{\ell\ell'}$  of  $H_1^{\pm\pm}$  and  $H_2^{\pm\pm}$  gives  $\text{BR}_{\mu\mu} = \text{BR}_{\tau\tau}$  and  $\text{BR}_{e\mu} = \text{BR}_{e\tau}$ .

If decays of  $H_i^{\pm\pm}$  are dominated by leptonic ones, the A4HTM gives sharp predictions for BR's themselves as

$$\text{BR}(H_1^{--} \rightarrow e\mu) = \frac{1}{3}, \quad (53)$$

$$\text{BR}(H_2^{--} \rightarrow \mu\mu) = \frac{2}{3}, \quad (54)$$

$$\text{BR}(H_3^{--} \rightarrow ee) = \frac{|2h_\Delta c_{\pm\pm} + \sqrt{3}\tilde{h}_\delta s_{\pm\pm}|^2}{6|h_\Delta|^2 c_{\pm\pm}^2 + 9|\tilde{h}_\delta|^2 s_{\pm\pm}^2}, \quad (55)$$

$$\text{BR}(H_4^{--} \rightarrow ee) = \frac{|2h_\Delta s_{\pm\pm} - \sqrt{3}\tilde{h}_\delta c_{\pm\pm}|^2}{6|h_\Delta|^2 s_{\pm\pm}^2 + 9|\tilde{h}_\delta|^2 c_{\pm\pm}^2}, \quad (56)$$

where modes involving  $\tau$  are omitted. Especially, 2/3 for  $\text{BR}_{\mu\mu}$  is too large to be reached in the HTM where  $\text{BR}_{\mu\mu} \lesssim 0.47$  [10]. It is possible to have a large  $\text{BR}_{ee}$  which can not be explained by the HTM where  $\text{BR}_{ee} \lesssim 0.49$ ; for example, the decay of  $H_3^{\pm\pm}$  gives  $\text{BR}_{ee} = 2/3$  for  $\theta_{\pm\pm} = 0$  and  $\text{BR}_{ee} = 1$  for  $h_\Delta c_{\pm\pm} = \sqrt{3}\tilde{h}_\delta s_{\pm\pm}$ . Even if  $\text{BR}_{ee}$  turns out to be very small, it does not result in a very small decay rate of the neutrinoless double beta decay (For the case in the HTM, see e. g., [32] and references therein). Unfortunately, it seems difficult to extract the information on  $\phi_\Delta \equiv \arg(h_\Delta)$  from  $\text{BR}_{\ell\ell'}$ .

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<sup>8</sup> Decays of  $\delta^{\pm\pm}$  of  $\mathbf{1}'$  or  $\mathbf{1}''$  also gives  $\text{BR}_{ee}/\text{BR}_{\mu\tau} = 1/2$ .

### B. $\tau \rightarrow \bar{\ell}\ell'\ell''$ and others

The third column of Table III shows possible  $\tau \rightarrow \bar{\ell}\ell'\ell''$  with  $H_i^{\pm\pm}$  mediation at the tree level. The most important point is that  $H_i^{\pm\pm}$  do not cause  $\mu \rightarrow \bar{e}ee$  at the tree level, for which the experimental constraint is very stringent as  $\text{BR}(\mu \rightarrow \bar{e}ee) < 1.0 \times 10^{-12}$  [33]. The radiative decays  $\ell \rightarrow \ell'\gamma$  at one loop level with  $H_i^{\pm\pm}$  are also forbidden. The eliminations of these lepton flavor violating decays can be understood as the consequence of the approximate  $Z_3$  symmetry of the A4HTM. Therefore, we can naturally expect signals of  $\tau \rightarrow \bar{\ell}\ell'\ell''$  in the future in collider experiments (Super-KEKB [34], super B factory [35], super flavor factory [36], and LHCb [37]) without caring about current constraints from  $\mu \rightarrow \bar{e}ee$  [33] and  $\ell \rightarrow \ell'\gamma$  [38, 39]. It is a good feature of the A4HTM that the model will be excluded if  $\mu \rightarrow e\gamma$  is observed in ongoing MEG experiment [40]. Only  $H_3^{\pm\pm}$  and  $H_4^{\pm\pm}$  can give a sizable  $\tau \rightarrow \bar{\mu}ee$  in this model while  $\tau \rightarrow \bar{e}\mu\mu$  (which is possible with  $H_2^{\pm\pm}$ ) can be affected also by neutral components of doublet fields [16]. Since  $H_1^{\pm\pm}$  does not contribute to  $\tau \rightarrow \bar{\ell}\ell'\ell''$  also, constraints on its coupling comes only from processes given by  $H_1^\pm$  if other  $H_i^{\pm\pm}$  are heavy enough. The Bhabha scattering ( $\bar{e}e \rightarrow \bar{e}e$ ) can be contributed by  $H_3^{\pm\pm}$  and  $H_4^{\pm\pm}$ .

### C. $H_T^- \rightarrow \ell\nu$

Table IV shows the processes to which the triplet-like  $H_{T_i}^\pm$  can contribute. The second column presents ratios of the branching ratios  $\text{BR}_{\ell\nu} \equiv \text{BR}(H_T^- \rightarrow \ell\nu)$  where the flavors of neutrinos in the final state are summed up. For decays of  $H_{T_3}^\pm$  and  $H_{T_4}^\pm$  we used

$$R_3^\pm \equiv \frac{|2h_\Delta c_\pm + \sqrt{3}\tilde{h}_\delta s_\pm|^2}{2|h_\Delta c_\pm - \sqrt{3}\tilde{h}_\delta s_\pm|^2}, \quad R_4^\pm \equiv \frac{|2h_\Delta s_\pm - \sqrt{3}\tilde{h}_\delta c_\pm|^2}{2|h_\Delta s_\pm + \sqrt{3}\tilde{h}_\delta c_\pm|^2}, \quad (57)$$

where  $c_\pm \equiv \cos\theta_\pm$  and  $s_\pm \equiv \sin\theta_\pm$ . Similarly to the case for  $H^{\pm\pm}$  decays,  $H_{T_1}^\pm$  and  $H_{T_2}^\pm$  give  $\text{BR}_{\mu\nu} \neq \text{BR}_{\tau\nu}$  while the HTM gives  $\text{BR}_{\mu\nu} \simeq \text{BR}_{\tau\nu}$ . Decays of degenerate  $H_{T_1}^\pm$  and  $H_{T_2}^\pm$  result in  $\text{BR}_{\mu\nu} = \text{BR}_{\tau\nu}$ . It is found that  $\text{BR}_{e\nu}$  can be larger than  $\text{BR}_{\mu\nu}$  ( $= \text{BR}_{\tau\nu}$ ) for  $H_{T_3}^\pm$  and  $H_{T_4}^\pm$  although the neutrino masses  $m_i$  in this model give  $\Delta m_{31}^2 \equiv m_3^2 - m_1^2 > 0$ . This is in contrast with  $\text{BR}_{e\nu} < \text{BR}_{\mu\nu}$  in the HTM for  $\Delta m_{31}^2 > 0$  [11].

	$\text{BR}(H_T^- \rightarrow \ell \nu)$ $e\nu : \mu\nu : \tau\nu$	$\mu \rightarrow e\bar{\nu}_\ell \nu_{\ell'}$	$\tau \rightarrow \ell\bar{\nu}_\ell \nu_\tau$ (coherent)	matter effect, $\nu e \rightarrow \nu e$
$H_{T1}^\pm$	1 : 1 : 4	$\mu \rightarrow e\bar{\nu}_e \nu_\mu$	none	$\epsilon_{\mu\mu}^e$
$H_{T2}^\pm$	1 : 4 : 1	$\mu \rightarrow e\bar{\nu}_\mu \nu_\tau$	$\tau \rightarrow e\bar{\nu}_e \nu_\tau$	$\epsilon_{\tau\tau}^e$
$H_{T3}^\pm$	$2R_3^\pm : 1 : 1$	$\mu \rightarrow e\bar{\nu}_\tau \nu_e$	$\tau \rightarrow \mu\bar{\nu}_\mu \nu_\tau$	$\epsilon_{ee}^e$
$H_{T4}^\pm$	$2R_4^\pm : 1 : 1$	$\mu \rightarrow e\bar{\nu}_\tau \nu_e$	$\tau \rightarrow \mu\bar{\nu}_\mu \nu_\tau$	$\epsilon_{ee}^e$

TABLE IV: Ratios of decays of the triplet-like  $H_{Ti}^\pm$  into a charged lepton and a neutrino are summarized, where the flavors of neutrinos are summed up. Possible decays of  $\mu$  with  $H_{Ti}^\pm$  mediation are also presented. The fourth column shows  $\tau$  decays which are coherent with the ones in the SM. The last column shows contributions of  $H_{Ti}^\pm$  to effective interactions which relate to the non-standard matter effect for the neutrino oscillation and the elastic scattering of  $\nu$  on the electron.

#### D. $\mu \rightarrow e\bar{\nu}\nu$ and $\tau \rightarrow \ell\bar{\nu}\nu$

The third column of Table IV shows  $\mu \rightarrow e\bar{\nu}_\ell \nu_{\ell'}$  which are possible with the  $H_{Ti}^\pm$  mediation. It is important to note that  $H_{Ti}^\pm$  can not contribute to  $\ell \rightarrow \ell' \gamma$  at one loop level. Only  $H_{T1}^\pm$  gives the coherent decay with the standard one of the  $W$  boson exchange, which can be larger effect than incoherent ones in principle. Of course, we can not find anything new in the standard  $\mu$  decay itself because new effects are absorbed by the experimental definition of the value of  $G_F$ . Incoherent ones given by other  $H_{Ti}^\pm$  affect measurements in the future neutrino factory where the neutrino beam is produced by the  $\mu$  decay. Neutrinos from the standard  $\mu^-$  decay give signals of  $\mu^-$  and  $e^+$  at the near detector. Non-standard effects on the neutrino production [41] will be observed at the near detector as the signals of the wrong-signed muon (for  $H_{T2}^\pm$ ) or the wrong-signed electron (for  $H_{T3}^\pm$  and  $H_{T4}^\pm$ ) if the detector can discriminate the charge and flavors.

The fourth column of Table IV is for  $\tau \rightarrow \ell\bar{\nu}_\ell \nu_\tau$  which are coherent with the decays via  $W^\pm$  mediation. Note that each  $H_{Ti}^\pm$  contribute to a decay of  $\mu$  or  $\tau$  coherently with the  $W^\pm$  contribution. Thus, there can be a sizable difference between effective couplings  $G_{\mu e}$  ( $\equiv G_F$ ) and  $G_{\tau\ell}$  which are determined by  $\mu \rightarrow e\bar{\nu}\nu$  and  $\tau \rightarrow \ell\bar{\nu}\nu$ , respectively. The effective



coupling  $G_{\mu e}^2 \equiv \sum G_{\mu \ell \ell'}^2$  is given by the effective interactions

$$2\sqrt{2} G_{\mu \ell \ell'} (\bar{\nu}_\ell \gamma_\mu P_L \mu) (\bar{e} \gamma^\mu P_L \nu_{\ell'}), \quad (58)$$

and  $G_{\tau \ell}$  are defined by the similar way. The contribution of  $W^\pm$  to  $G_{\mu \ell \ell'}$  is  $G_{\mu \ell \ell'}^W \equiv g^2/(4\sqrt{2} m_W^2)$ , where  $g$  denotes the gauge coupling constant of  $SU(2)_L$  and  $m_W$  is the mass of  $W^\pm$ . In the A4HTM, contributions of  $H_{T_i}^\pm$  to  $G_{\mu \ell \ell'}$  can be expressed<sup>9</sup> as

$$G_{\mu \ell \ell'}^{H_{T_i}^\pm} \equiv \sum_i \frac{(h_{i\pm})_{\ell' \mu} (h_{i\pm}^*)_{\ell e}}{2\sqrt{2} m_{H_{T_i}^\pm}^2}. \quad (59)$$

### E. Non-standard interactions of neutrinos

During the propagation of neutrinos in the ordinary matter, the coherent forward scattering of them on the matter ( $e$ ,  $u$ , and  $d$ ) affects neutrino oscillations [42, 43]. The so-called non-standard interaction (NSI) of neutrinos can give the non-standard matter effect on the neutrino oscillation [42, 44]. The relevant effective interaction for that is

$$2\sqrt{2} G_F \epsilon_{\ell \ell'}^{fP} (\bar{f} \gamma^\mu P f) (\bar{\nu}_\ell \gamma_\mu P_L \nu_{\ell'}), \quad (60)$$

where  $f = e, u, d$  and  $P = P_L, P_R$ . The interaction eq. (60) is defined just for the non-standard one, which should be added to the standard one of the weak interaction. Although eq. (60) is written in the form of the neutral current interaction, the effective interaction can be given by the charged scalar mediation also because of the Fierz transformation<sup>10</sup>. The triplet-like  $H_{T_i}^\pm$  in the A4HTM can generate  $\epsilon_{\ell \ell'}^{fP}$  with only the left-handed electron for only  $\ell = \ell'$  as

$$\epsilon_{\ell \ell}^{eP_L} = \sum_i \frac{|(h_{i\pm})_{e\ell}|^2}{2\sqrt{2} G_F m_{H_{T_i}^\pm}^2}. \quad (61)$$

The last column of Table IV shows  $\epsilon_{\ell \ell}^{eP_L}$  induced by each  $H_{T_i}^\pm$ . Possible sizes of  $\epsilon_{\ell \ell}^{eP_L}$  are shown in the next section by considering other constraints. Contributions of the doublet like charged Higgs fields to  $\epsilon_{\ell \ell}^{eP_R}$  are negligible because Yukawa couplings appear as  $m_e^2/v^2$ . The elastic scattering of neutrinos on the electron is affected also by  $\epsilon_{\ell \ell}^{eP_L}$ . A study on the NSI

<sup>9</sup> Note that  $2(\bar{\nu}_{\ell'}^c P_L \mu)(\bar{e} P_R \nu_\ell^c) = (\bar{\nu}_\ell \gamma^\mu P_L \mu)(\bar{e} \gamma_\mu P_L \nu_{\ell'})$ .

<sup>10</sup> Note that  $2(\bar{\nu}_{\ell'}^c P_L e)(\bar{e} P_R \nu_\ell^c) = (\bar{e} \gamma^\mu P_L e)(\bar{\nu}_\ell \gamma_\mu P_L \nu_{\ell'})$ .

in the HTM for the matter effect and the neutrino production (See the previous subsection also) can be seen in [45]. Model-independent constraints on the NSI for the matter effect can be found in [46].

## F. Doublet Higgs sector

Contributions of doublet-like Higgs bosons to the flavor violating decays of charged leptons are the same as the ones in a model discussed in [16] (See also [23]). Two combinations  $\Phi_\eta \equiv (\Phi_x + \omega^2\Phi_y + \omega\Phi_z)/\sqrt{3}$  and  $\Phi_\zeta \equiv (\Phi_x + \omega\Phi_y + \omega^2\Phi_z)/\sqrt{3}$ , which have no vev and no contribution to the mass matrix of charged leptons, can cause flavor changing neutral currents. The largest contribution of doublet-like neutral Higgs bosons (real and imaginary parts of  $(\phi_\eta^0 + \phi_\zeta^0)/\sqrt{2}$  and  $(-i\phi_\eta^0 + i\phi_\zeta^0)/\sqrt{2}$ ) is to  $\tau_R \rightarrow \bar{e}_L\mu_L\mu_R$  for which the Yukawa coupling appears as  $m_\mu m_\tau/v^2$ . There is no contribution to  $\mu \rightarrow \bar{e}ee$  and  $\ell \rightarrow \ell'\gamma$  because of an approximate  $Z_3$  symmetry. The quark sector can be just like the SM one, which is described by only **1**-representations with an additional Higgs doublet field  $\Phi_q$  as mentioned in [16]. The phenomenology of  $\Phi_q$  and  $\Phi_\xi \equiv (\Phi_x + \Phi_y + \Phi_z)/\sqrt{3}$  is almost identical to a type of the two-Higgs-doublet-models, which can be seen in [47–50].

## V. CONSTRAINTS

In this section, constraints on the model and future prospects are considered. We assume that one of  $H_i^{\pm\pm}$  is much lighter than the others for simplicity. Then, one of  $H_{T1}^\pm$  should be light also because large mass splittings are disfavored by the  $\rho$  parameter.

### A. Case of light $H_1^{\pm\pm}$ and $H_{T1}^\pm$

If only  $H_1^{\pm\pm}$  is light enough among  $H_i^{\pm\pm}$ , there is no constraint on the model from  $\tau \rightarrow \bar{\ell}\ell'\ell''$ . Since  $H_{T1}^\pm$  also must be light enough in this case, a constraint comes from

$$\frac{G_{\tau e}^2}{G_F^2} = \frac{(G^W)^2}{(G^W + G_{\mu e}^{H_T^\pm})^2} = \frac{(G_F - G_{\mu e}^{H_T^\pm})^2}{G_F^2} = 1.0012 \pm 0.0053 \quad (\text{p. 512 of [25]}), \quad (62)$$

where  $G^W$  and  $G_{\ell\ell'}^{H_T^\pm}$  indicate contributions of  $W$  and  $H_T^\pm$  to  $G_{\ell\ell'}$ , respectively. We obtain

$$|h_\Delta|^2 < 3.4 \times 10^{-2} \left( \frac{m_{H_{T1}^\pm}}{300 \text{ GeV}} \right)^2 \quad (90\% \text{CL}) \quad (63)$$

The coefficient of NSI relevant to the matter effect for the neutrino oscillation is constrained by eq. (63) as  $|\epsilon_{\mu\mu}^e| = |G_{\mu e}^{H_T^\pm}/G_F| < 3.8 \times 10^{-3}$  which is smaller than the expected sensitivity ( $\sim 0.1$ ) [51] in the neutrino factory. There is no effect on the production of the neutrino beam.

### B. Case of light $H_2^{\pm\pm}$ and $H_{T2}^\pm$

If  $H_2^{\pm\pm}$  is lighter enough than other  $H_i^{\pm\pm}$ , a constraint on the model is given by

$$\text{BR}(\tau \rightarrow \bar{e}\mu\mu) = \frac{|h_\Delta|^4}{36G_F^2 m_{H_{T2}^\pm}^4} \text{BR}(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau) < 1.7 \times 10^{-8} \quad (90\% \text{CL}) \quad [52], \quad (64)$$

where  $\text{BR}(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau) \simeq 0.17$ . We have

$$|h_\Delta|^2 < 2.0 \times 10^{-3} \left( \frac{m_{H_2^{\pm\pm}}}{300 \text{ GeV}} \right)^2 \quad (90\% \text{CL}). \quad (65)$$

Another constraint on  $|h_\Delta|$  can be obtained by  $G_{\tau e}^2/G_F^2 = 1.0012 \pm 0.0053$  as

$$|h_\Delta|^2 < 4.4 \times 10^{-2} \left( \frac{m_{H_{T2}^\pm}}{300 \text{ GeV}} \right)^2 \quad (90\% \text{CL}), \quad (66)$$

although this is weaker than eq. (65) because  $m_{H_{T2}^\pm}$  should not be very different from  $m_{H_2^{\pm\pm}}$ .

The effective coupling  $G_{\mu ee\tau}$  for  $\mu \rightarrow e\bar{\nu}_e\nu_\tau$  is constrained by eq. (65) with  $m_{H_{T2}^\pm} \simeq m_{H_2^{\pm\pm}}$  as  $|G_{\mu ee\tau}/G_F| \lesssim 2 \times 10^{-4}$  which can be around the expected sensitivity at a near detector of the neutrino factory [45]. The non-standard matter effect with  $\epsilon_{\tau\tau}^e$  is too small to be observed in the neutrino factory because eq. (65) results in  $\epsilon_{\tau\tau}^e \lesssim 10^{-3}$ .

### C. Case of light $H_3^{\pm\pm}$ and $H_{T3}^\pm$

Let us remind that we have defined as  $m_{H_3^{\pm\pm}} \leq m_{H_4^{\pm\pm}}$ . If  $H_4^{\pm\pm}$  is very heavy, a relevant constraint is

$$\text{BR}(\tau \rightarrow \bar{\mu}ee) = \frac{|(h_{3\pm\pm})_{\tau\mu}(h_{3\pm\pm})_{ee}|^2}{4G_F^2 m_{H_3^{\pm\pm}}^4} \text{BR}(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau) < 1.5 \times 10^{-8} \quad (90\% \text{CL}) \quad [52], \quad (67)$$

which results in

$$|(h_{3\pm\pm})_{\tau\mu}(h_{3\pm\pm})_{ee}| < 6.3 \times 10^{-4} \left( \frac{m_{H_3^{\pm\pm}}}{300 \text{ GeV}} \right)^2 \quad (90\% \text{CL}). \quad (68)$$

The constraint on  $|(h_{3\pm\pm})_{ee}|$  itself is given by the Bhabha scattering [53]. For example, we have<sup>11</sup>

$$|(h_{3\pm\pm})_{ee}| \lesssim 0.3 \quad (90\% \text{CL}, m_{H_3^{\pm\pm}} = 300 \text{ GeV}). \quad (69)$$

For  $h_{3\pm}$ , a constraint comes from  $G_{\tau\mu}^2/G_F^2 = 0.981 \pm 0.018$  (p. 512 of [25]), and we have

$$|(h_{3\pm})_{\tau\mu}|^2 < 1.6 \times 10^{-3} \left( \frac{m_{H_{T3}^{\pm}}}{300 \text{ GeV}} \right)^2 \quad (90\% \text{CL}). \quad (70)$$

The LSND result [54] on  $\nu_e e$  elastic scattering,  $\sigma_{\nu_e e}^{\text{LSND}} = (10.1 \pm 1.5) E_{\nu_e} (\text{MeV}) \times 10^{-45} \text{ cm}^2$ , can be translated into a constraint  $\epsilon_{ee}^{ePL} < 0.11$  at 90%CL [46]. A comparable constraint on  $\epsilon_{ee}^{ePL}$  was obtained with solar and reactor neutrinos [55]. The constraint  $\epsilon_{ee}^{ePL} < 0.11$  can be written as

$$|(h_{3\pm})_{ee}|^2 < 0.33 \left( \frac{m_{H_{T3}^{\pm}}}{300 \text{ GeV}} \right)^2 \quad (90\% \text{CL}). \quad (71)$$

For  $m_{H_{T3}^{\pm}} \simeq m_{H_3^{\pm\pm}}$  (namely,  $|\lambda_5| \ll 1$  and then  $h_{i\pm} \simeq h_{i\pm\pm}$ ), the effective coupling  $G_{\mu\tau e}$  for  $\mu \rightarrow e \bar{\nu}_\tau \nu_e$  is constrained by eq. (68) as  $G_{\mu\tau e}/G_F \lesssim 2 \times 10^{-4}$ . Constraints of eq. (69) and (71) on the non-standard matter effect are comparable ( $\epsilon_{ee}^{ePL} \lesssim 0.1$ ). These non-standard effects can be close to the expected sensitivity in the neutrino factory.

## VI. CONCLUSIONS

In this article, we investigated the phenomenology of triplet Higgs bosons in the simplest  $A_4$ -symmetric version of the Higgs Triplet Model (A4HTM). The A4HTM is a four-Higgs-Triplet-Model ( $\delta$  of **1** and  $(\Delta_x, \Delta_y, \Delta_z)$  of **3**). Four mass eigenstates of doubly charged Higgs bosons,  $H_i^{\pm\pm}$ , are obtained explicitly from the Higgs potential. We also obtained four mass eigenstates of the triplet-like singly charged Higgs bosons,  $H_{Ti}^{\pm}$ , for which doublet components can be ignored because of small triplet vev's.

It was shown that the A4HTM gives unique predictions about their decay branching ratios into two leptons ( $H_i^{--} \rightarrow \ell\ell'$  and  $H_{iT}^- \rightarrow \ell\nu$ ); for example, the leptonic decays of  $H_2^{--}$  are only into  $\mu\mu$  and  $e\tau$  because an approximate  $Z_3$  symmetry remains, and the ratio of the

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<sup>11</sup> The bound at 95%CL in [53] is translated naively to the bound at 90%CL by a factor of 1.9/1.6, where 95%CL and 90%CL correspond to  $1.9\sigma$  and  $1.6\sigma$ , respectively.

branching ratios is  $2 : 1$  as a consequence of the  $A_4$  symmetry in the original Lagrangian. Therefore, it will be possible to test the model at hadron colliders (Tevatron and LHC) if some of these Higgs bosons are light enough to be produced.

Even if these Higgs bosons are too heavy to be produced at hadron colliders, they can affect the lepton flavor violating decays of charged leptons if the triplet Yukawa coupling constants are large enough. It was shown that there is no contribution of these Higgs bosons to  $\mu \rightarrow \bar{e}ee$  and  $\ell \rightarrow \ell'\gamma$ . Thus, we can naturally expect signals of  $\tau \rightarrow \bar{\mu}ee$  and  $\tau \rightarrow \bar{e}\mu\mu$  (which are possible in this model among six  $\tau \rightarrow \bar{\ell}\ell'\ell''$ ) in the future in collider experiments (Super-KEKB, super B factory, super flavor factory, and LHCb) without interfering with a stringent experimental bound on  $\mu \rightarrow \bar{e}ee$ . This model will be excluded if  $\ell \rightarrow \ell'\gamma$  is observed.

We considered current experimental constraints on the model and prospects of the measurement of the non-standard neutrino interactions (NSI) in the neutrino factory. If  $H_2^{\pm\pm}$  or  $H_3^{\pm\pm}$  is lighter enough than other  $H_i^{\pm\pm}$ , effects of the NSI can be around the expected sensitivity in the neutrino factory.

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## Appendix A: Decompositions

For  $a = (a_x, a_y, a_z)^T$  and  $b = (b_x, b_y, b_z)^T$  of  $\mathbf{3}$  in the  $S$ -diagonal basis of eq. (5), we used

$$(ab)_{\mathbf{1}} \equiv a_x b_x + a_y b_y + a_z b_z, \quad (\text{A1})$$

$$(ab)_{\mathbf{1}'} \equiv a^T X' b = a_x b_x + \omega^2 a_y b_y + \omega a_z b_z, \quad X' \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad (\text{A2})$$

$$(ab)_{\mathbf{1}''} \equiv a^T X'' b = a_x b_x + \omega a_y b_y + \omega^2 a_z b_z, \quad X'' \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad (\text{A3})$$

$$(ab)_{\mathbf{3}_s} \equiv \left( a^T V_{sx} b, a^T V_{sy} b, a^T V_{sz} b \right)^T \\ = \left( a_y b_z + a_z b_y, a_z b_x + a_x b_z, a_x b_y + a_y b_x \right)^T, \quad (\text{A4})$$

$$V_{sx} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad V_{sy} \equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad V_{sz} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\text{A5})$$

$$(ab)_{\mathbf{3}_a} \equiv \left( a^T V_{ax} b, a^T V_{ay} b, a^T V_{az} b \right)^T \\ = \left( a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x \right)^T, \quad (\text{A6})$$

$$V_{ax} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad V_{ay} \equiv \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad V_{az} \equiv \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (\text{A7})$$

If we use a  $T$ -diagonal basis defined as

$$\mathbf{3}_T \equiv U_T \mathbf{3}, \quad U_T \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad (\text{A8})$$

$$\tilde{S} \mathbf{3}_T = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \mathbf{3}_T, \quad \tilde{T} \mathbf{3}_T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \mathbf{3}_T, \quad (\text{A9})$$

there are two kinds of decompositions:  $\mathbf{3}_T \otimes \mathbf{3}_T = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_{Ts} \oplus \mathbf{3}_{Ta}$  and  $\mathbf{3}_T^* \otimes \mathbf{3}_T = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_T \oplus \mathbf{3}_T^*$ . Note that  $\mathbf{3}_T^* \otimes \mathbf{3}_T^* = (\mathbf{3}_T \otimes \mathbf{3}_T)^*$ . For  $a_T \equiv (a_\xi, a_\eta, a_\zeta)^T$  and

$b_T \equiv (b_\xi, b_\eta, b_\zeta)^T$  in the  $T$ -diagonal basis, decompositions for  $\mathbf{3}_T \otimes \mathbf{3}_T$  are given by

$$\mathbf{3}_T \otimes \mathbf{3}_T \rightarrow \mathbf{1} : a_T^T \Xi_s b_T = a_\xi b_\xi + a_\eta b_\zeta + a_\zeta b_\eta, \quad \Xi_s \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (\text{A10})$$

$$\mathbf{3}_T \otimes \mathbf{3}_T \rightarrow \mathbf{1}' : a_T^T \Xi'_s b_T = a_\xi b_\eta + a_\eta b_\xi + a_\zeta b_\zeta, \quad \Xi'_s \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (\text{A11})$$

$$\mathbf{3}_T \otimes \mathbf{3}_T \rightarrow \mathbf{1}'' : a_T^T \Xi''_s b_T = a_\xi b_\zeta + a_\eta b_\eta + a_\zeta b_\xi, \quad \Xi''_s \equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (\text{A12})$$

$$\mathbf{3}_T \otimes \mathbf{3}_T \rightarrow \mathbf{3}_{Ts} : \left( a_T^T V_{s\xi} b_T, a_T^T V_{s\eta} b_T, a_T^T V_{s\zeta} b_T \right)^T, \quad (\text{A13})$$

$$V_{s\xi} \equiv \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \quad V_{s\eta} \equiv \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad V_{s\zeta} \equiv \begin{pmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad (\text{A14})$$

$$\mathbf{3}_T \otimes \mathbf{3}_T \rightarrow \mathbf{3}_{Ta} : \left( a_T^T V_{a\xi} b_T, a_T^T V_{a\eta} b_T, a_T^T V_{a\zeta} b_T \right)^T, \quad (\text{A15})$$

$$V_{a\xi} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad V_{a\eta} \equiv \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad V_{a\zeta} \equiv \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}. \quad (\text{A16})$$

On the other hand, decompositions for  $\mathbf{3}_T^* \otimes \mathbf{3}_T$  are given by

$$\mathbf{3}_T^* \otimes \mathbf{3}_T \rightarrow \mathbf{1} : a_T^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} b_T = a_\xi^* b_\xi + a_\eta^* b_\eta + a_\zeta^* b_\zeta, \quad (\text{A17})$$

$$\mathbf{3}_T^* \otimes \mathbf{3}_T \rightarrow \mathbf{1}' : a_T^\dagger \Xi' b_T = a_\xi^* b_\eta + a_\eta^* b_\zeta + a_\zeta^* b_\xi, \quad \Xi' \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (\text{A18})$$

$$\mathbf{3}_T^* \otimes \mathbf{3}_T \rightarrow \mathbf{1}'' : a_T^\dagger \Xi'' b_T = a_\xi^* b_\zeta + a_\eta^* b_\xi + a_\zeta^* b_\eta, \quad \Xi'' \equiv \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad (\text{A19})$$

$$\mathbf{3}_T^* \otimes \mathbf{3}_T \rightarrow \mathbf{3}_T : \left( a_T^\dagger V_\xi b_T, a_T^\dagger V_\eta b_T, a_T^\dagger V_\zeta b_T \right)^T, \quad (\text{A20})$$

$$\mathbf{3}_T^* \otimes \mathbf{3}_T \rightarrow \mathbf{3}_T^* : \left( a_T^\dagger V_\xi^* b_T, a_T^\dagger V_\eta^* b_T, a_T^\dagger V_\zeta^* b_T \right)^T, \quad (\text{A21})$$

$$V_\xi \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad V_\eta \equiv \begin{pmatrix} 0 & \omega^2 & 0 \\ 0 & 0 & 1 \\ \omega & 0 & 0 \end{pmatrix}, \quad V_\zeta \equiv \begin{pmatrix} 0 & 0 & \omega \\ \omega^2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \quad (\text{A22})$$

## Appendix B: "Fierz transformation"

We show useful relations to construct the  $A_4$ -symmetric Higgs potential, which are similar to the famous Fierz transformation for the four-fermion interactions. We need not to use the relations explicitly but we should keep the existence in our mind in order to reduce the number of terms in the Higgs potential. Let  $\phi_i$  ( $i = 1-4$ ) wave functions of  $\mathbf{3}$  in the basis of eq. (5). For terms involving three  $\phi_i$ , we have

$$\begin{pmatrix} [\phi_1(\phi_2\phi_3)\mathbf{3}_s]_{\mathbf{1}} \\ [\phi_1(\phi_2\phi_3)\mathbf{3}_a]_{\mathbf{1}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} [(\phi_1\phi_2)\mathbf{3}_s\phi_3]_{\mathbf{1}} \\ [(\phi_1\phi_2)\mathbf{3}_a\phi_3]_{\mathbf{1}} \end{pmatrix}. \quad (\text{B1})$$



Thus, we can concentrate ourselves to one of the sets of the decompositions,  $\phi_1(\phi_2\phi_3)$  or  $(\phi_1\phi_2)\phi_3$ . Similar relations for the term involving four  $\phi_i$  are obtained as

$$\begin{aligned}
& \left( (\phi_1\phi_2)_1(\phi_3\phi_4)_1, (\phi_1\phi_2)_{1'}(\phi_3\phi_4)_{1''}, (\phi_1\phi_2)_{1''}(\phi_3\phi_4)_{1'}, \left( (\phi_1\phi_2)_{\mathbf{3}_s}(\phi_3\phi_4)_{\mathbf{3}_s} \right)_1, \right. \\
& \quad \left. \left( (\phi_1\phi_2)_{\mathbf{3}_a}(\phi_3\phi_4)_{\mathbf{3}_a} \right)_1, \left( (\phi_1\phi_2)_{\mathbf{3}_s}(\phi_3\phi_4)_{\mathbf{3}_a} \right)_1, \left( (\phi_1\phi_2)_{\mathbf{3}_a}(\phi_3\phi_4)_{\mathbf{3}_s} \right)_1 \right)^T \\
&= \frac{1}{12} \begin{pmatrix} 4 & 4 & 4 & 6 & -6 & 0 & 0 \\ 4 & 4 & 4 & -3 & 3 & -3i\sqrt{3} & 3i\sqrt{3} \\ 4 & 4 & 4 & -3 & 3 & 3i\sqrt{3} & -3i\sqrt{3} \\ 8 & -4 & -4 & 6 & 6 & 0 & 0 \\ -8 & 4 & 4 & 6 & 6 & 0 & 0 \\ 0 & 4i\sqrt{3} & -4i\sqrt{3} & 0 & 0 & 6 & 6 \\ 0 & -4i\sqrt{3} & 4i\sqrt{3} & 0 & 0 & 6 & 6 \end{pmatrix} \begin{pmatrix} (\phi_1\phi_4)_1(\phi_3\phi_2)_1 \\ (\phi_1\phi_4)_{1'}(\phi_3\phi_2)_{1''} \\ (\phi_1\phi_4)_{1''}(\phi_3\phi_2)_{1'} \\ \left( (\phi_1\phi_4)_{\mathbf{3}_s}(\phi_3\phi_2)_{\mathbf{3}_s} \right)_1 \\ \left( (\phi_1\phi_4)_{\mathbf{3}_a}(\phi_3\phi_2)_{\mathbf{3}_a} \right)_1 \\ \left( (\phi_1\phi_4)_{\mathbf{3}_s}(\phi_3\phi_2)_{\mathbf{3}_a} \right)_1 \\ \left( (\phi_1\phi_4)_{\mathbf{3}_a}(\phi_3\phi_2)_{\mathbf{3}_s} \right)_1 \end{pmatrix}. \quad (\text{B2})
\end{aligned}$$

These relations are obtained by the "Fierz transformation" for  $3 \times 3$  matrices:

$$(\phi_1\Gamma^i\phi_2)(\phi_3(\Gamma^j)^\dagger\phi_4) = \sum_k (\phi_1 M_{ij}^k \phi_4)(\phi_3(\Gamma^k)^\dagger\phi_2), \quad M_{ij}^k \equiv \Gamma^i\Gamma^k(\Gamma^j)^\dagger, \quad (\text{B3})$$

$$\begin{aligned}
\Gamma^i \equiv \left\{ \frac{1}{\sqrt{3}}I, \frac{1}{\sqrt{3}}X', \frac{1}{\sqrt{3}}X'', \frac{1}{\sqrt{2}}V_{sx}, \frac{1}{\sqrt{2}}V_{sy}, \frac{1}{\sqrt{2}}V_{sz}, \right. \\
\left. \frac{1}{\sqrt{2}}V_{ax}, \frac{1}{\sqrt{2}}V_{ay}, \frac{1}{\sqrt{2}}V_{az} \right\}, \quad (\text{B4})
\end{aligned}$$

where  $I$  is the identity matrix and  $\Gamma^i$  give the complete set of  $3 \times 3$  matrices which satisfy  $\text{Tr}(\Gamma^i(\Gamma^j)^\dagger) = \delta^{ij}$ . Definitions of the matrices of  $\Gamma^i$  are shown in Appendix A.

### Appendix C: Higgs Potential

We show for completeness the parts of the  $A_4$ -symmetric Higgs potential, which are not used in the main part of this article:

$$\begin{aligned}
V_1 = & \lambda_1 [(\Phi^\dagger\Phi)_1]^2 + \lambda_{1p}(\Phi^\dagger\Phi)_{1'}(\Phi^\dagger\Phi)_{1''} \\
& + \lambda_{1ss}((\Phi^\dagger\Phi)_{\mathbf{3}_s}(\Phi^\dagger\Phi)_{\mathbf{3}_s})_1 + \lambda_{1aa}((\Phi^\dagger\Phi)_{\mathbf{3}_a}(\Phi^\dagger\Phi)_{\mathbf{3}_a})_1 \\
& + i\lambda_{1sa}(\Phi^\dagger\Phi)_{\mathbf{3}_s}(\Phi^\dagger\Phi)_{\mathbf{3}_a}, \quad (\text{C1})
\end{aligned}$$

$$\begin{aligned}
V_2 = & \lambda_{2\delta} [\text{Tr}(\delta^\dagger \delta)]^2 \\
& + \lambda_{2\Delta} [\text{Tr}(\Delta^\dagger \Delta)_1]^2 + \lambda_{2\Delta p} \text{Tr}(\Delta^\dagger \Delta)_{1'} \text{Tr}(\Delta^\dagger \Delta)_{1''} \\
& + \lambda_{2\Delta ss} \left( \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} \right)_1 + \lambda_{2\Delta aa} \left( \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a} \right)_1 \\
& + i\lambda_{2\Delta sa} \left( \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a} \right)_1 \\
& + \lambda_{2\delta\Delta 1} \text{Tr}(\delta^\dagger \delta) \text{Tr}(\Delta^\dagger \Delta)_1 + \lambda_{2\delta\Delta 2} (\delta_{\beta\alpha}^* \delta_{\omega\gamma}) (\Delta_{\beta\alpha} \Delta_{\omega\gamma}^*)_1 \\
& + \left\{ \lambda'_{2\delta\Delta 3} (\delta_{\beta\alpha}^* \delta_{\omega\gamma}^*) [\Delta_{\beta\alpha} \Delta_{\omega\gamma}]_1 + \text{h.c.} \right\} \\
& + \left\{ \lambda'_{2\delta\Delta s} \delta_{\beta\alpha}^* [\Delta_{\beta\alpha} (\Delta_{\omega\gamma}^* \Delta_{\omega\gamma})_{\mathbf{3}_s}]_1 + \text{h.c.} \right\} \\
& + \left\{ \lambda'_{2\delta\Delta a} \delta_{\beta\alpha}^* [\Delta_{\beta\alpha} (\Delta_{\omega\gamma}^* \Delta_{\omega\gamma})_{\mathbf{3}_a}]_1 + \text{h.c.} \right\}, \tag{C2}
\end{aligned}$$

$$\begin{aligned}
V_3 = & \frac{1}{2} \lambda_{3\delta} \left\{ [\text{Tr}(\delta^\dagger \delta)]^2 - \text{Tr}([\delta^\dagger \delta]^2) \right\} \\
& + \frac{1}{2} \lambda_{3\Delta} \left\{ [\text{Tr}(\Delta^\dagger \Delta)_1]^2 - \text{Tr}([\Delta^\dagger \Delta]_1^2) \right\} \\
& + \frac{1}{2} \lambda_{3\Delta p} \left\{ \text{Tr}(\Delta^\dagger \Delta)_{1'} \text{Tr}(\Delta^\dagger \Delta)_{1''} - \text{Tr}((\Delta^\dagger \Delta)_{1'} (\Delta^\dagger \Delta)_{1''}) \right\} \\
& + \frac{1}{2} \lambda_{3\Delta ss} \left\{ (\text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s})_1 - \text{Tr}((\Delta^\dagger \Delta)_{\mathbf{3}_s} (\Delta^\dagger \Delta)_{\mathbf{3}_s})_1 \right\} \\
& + \frac{1}{2} \lambda_{3\Delta aa} \left\{ (\text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a})_1 - \text{Tr}((\Delta^\dagger \Delta)_{\mathbf{3}_a} (\Delta^\dagger \Delta)_{\mathbf{3}_a})_1 \right\} \\
& + \frac{1}{2} i\lambda_{3\Delta sa} \left\{ (\text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} \text{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a})_1 - \text{Tr}((\Delta^\dagger \Delta)_{\mathbf{3}_s} (\Delta^\dagger \Delta)_{\mathbf{3}_a})_1 \right\} \\
& + \frac{1}{2} \lambda_{3\delta\Delta 1} \left\{ \text{Tr}(\delta^\dagger \delta) \text{Tr}(\Delta^\dagger \Delta)_1 - \text{Tr}((\delta^\dagger \delta) (\Delta^\dagger \Delta)_1) \right\} \\
& + \frac{1}{2} \lambda_{3\delta\Delta 2} \left\{ \delta_{\beta\alpha}^* (\Delta_{\beta\alpha} \Delta_{\omega\gamma}^*)_1 \delta_{\omega\gamma} - \text{Tr}(\delta^\dagger (\Delta \Delta^\dagger)_1 \delta) \right\} \\
& + \left\{ \frac{1}{2} \lambda'_{3\delta\Delta 3} \left( (\delta_{\beta\alpha}^* \delta_{\omega\gamma}^*) [\Delta_{\beta\alpha} \Delta_{\omega\gamma}]_1 - \delta_{\beta\alpha}^* \delta_{\omega\gamma}^* [\Delta_{\beta\gamma} \Delta_{\omega\alpha}]_1 \right) + \text{h.c.} \right\} \\
& + \left\{ \frac{1}{2} \lambda'_{3\delta\Delta s} \left( \delta_{\beta\alpha}^* [\Delta_{\beta\alpha} (\Delta_{\omega\gamma}^* \Delta_{\omega\gamma})_{\mathbf{3}_s}]_1 - \delta_{\beta\alpha}^* [\Delta_{\beta\gamma} (\Delta_{\omega\gamma}^* \Delta_{\omega\alpha})_{\mathbf{3}_s}]_1 \right) + \text{h.c.} \right\} \\
& + \left\{ \frac{1}{2} \lambda'_{3\delta\Delta a} \delta_{\beta\alpha}^* [\Delta_{\beta\alpha} (\Delta_{\omega\gamma}^* \Delta_{\omega\gamma})_{\mathbf{3}_a}]_1 + \text{h.c.} \right\}, \tag{C3}
\end{aligned}$$

$$V_\mu = \frac{1}{\sqrt{2}} \mu_\delta [\Phi_\alpha \Phi_\beta]_1 (i\sigma^2 \delta^\dagger)_{\alpha\beta} + \frac{1}{\sqrt{2}} \mu_\Delta \left( (\Phi_\alpha \Phi_\beta)_{\mathbf{3}_s} (i\sigma^2 \Delta^\dagger)_{\alpha\beta} \right)_1 + \text{h.c.} \tag{C4}$$

Note that  $V_3$  can be rewritten in term of the determinant by using

$$\begin{aligned} & \text{Tr}(\Delta_A^\dagger \Delta_B) \text{Tr}(\Delta_C^\dagger \Delta_D) - \text{Tr}\left((\Delta_A^\dagger \Delta_B)(\Delta_C^\dagger \Delta_D)\right) \\ &= \begin{vmatrix} (\Delta_A^\dagger \Delta_B)_{11} & (\Delta_A^\dagger \Delta_B)_{12} \\ (\Delta_C^\dagger \Delta_D)_{21} & (\Delta_C^\dagger \Delta_D)_{22} \end{vmatrix} + \begin{vmatrix} (\Delta_C^\dagger \Delta_D)_{11} & (\Delta_C^\dagger \Delta_D)_{12} \\ (\Delta_A^\dagger \Delta_B)_{21} & (\Delta_A^\dagger \Delta_B)_{22} \end{vmatrix}. \end{aligned} \quad (\text{C5})$$

## Appendix D: Masses of triplet-like neutral Higgs bosons

Since fields in the  $T$ -diagonal basis have  $Z_3$ -charges, they can not be the mass eigenstates for neutral particles while they turn out to be the ones for charged particles. We show here that the mass eigenstates of the triplet-like neutral Higgs bosons just for the completeness, which seem the most complicated ones in the A4HTM. We assume that there is no large mixing between triplet and doublet fields, which is possible with small triplet vev's in principle (See [30] for the case in the HTM). The squared mass matrix for  $(\text{Re}(\Delta_x), \dots, \text{Re}(\delta), \text{Im}(\Delta_x), \dots, \text{Im}(\delta))$  is given by

$$M_{T0}^2 \equiv \begin{pmatrix} M_{\text{TCPC}}^2 & M_{\text{TCPV}}^2 \\ (M_{\text{TCPV}}^2)^T & M_{\text{TCPC}}^2 \end{pmatrix}, \quad (\text{D1})$$

$$M_{\text{TCPC}}^2 \equiv \begin{pmatrix} M_{\Delta 45p}^2 & \frac{1}{3}v^2\lambda_{\Delta ss45p} & \frac{1}{3}v^2\lambda_{\Delta ss45p} & \frac{1}{3}v^2\text{Re}(\lambda'_{s45p}) \\ \frac{1}{3}v^2\lambda_{\Delta ss45p} & M_{\Delta 45p}^2 & \frac{1}{3}v^2\lambda_{\Delta ss45p} & \frac{1}{3}v^2\text{Re}(\lambda'_{s45p}) \\ \frac{1}{3}v^2\lambda_{\Delta ss45p} & \frac{1}{3}v^2\lambda_{\Delta ss45p} & M_{\Delta 45p}^2 & \frac{1}{3}v^2\text{Re}(\lambda'_{s45p}) \\ \frac{1}{3}v^2\text{Re}(\lambda'_{s45p}) & \frac{1}{3}v^2\text{Re}(\lambda'_{s45p}) & \frac{1}{3}v^2\text{Re}(\lambda'_{s45p}) & M_{\delta 45p}^2 \end{pmatrix}, \quad (\text{D2})$$

$$M_{\text{TCPV}}^2 \equiv \begin{pmatrix} 0 & -\frac{1}{3}v^2\lambda_{\Delta sa45p} & -\frac{1}{3}v^2\lambda_{\Delta sa45p} & \frac{1}{3}v^2\text{Im}(\lambda'_{s45p}) \\ -\frac{1}{3}v^2\lambda_{\Delta sa45p} & 0 & -\frac{1}{3}v^2\lambda_{\Delta sa45p} & \frac{1}{3}v^2\text{Im}(\lambda'_{s45p}) \\ -\frac{1}{3}v^2\lambda_{\Delta sa45p} & -\frac{1}{3}v^2\lambda_{\Delta sa45p} & 0 & \frac{1}{3}v^2\text{Im}(\lambda'_{s45p}) \\ -\frac{1}{3}v^2\text{Im}(\lambda'_{s45p}) & -\frac{1}{3}v^2\text{Im}(\lambda'_{s45p}) & -\frac{1}{3}v^2\text{Im}(\lambda'_{s45p}) & 0 \end{pmatrix}, \quad (\text{D3})$$

$$M_{\delta 45p}^2 \equiv M_\delta^2 + \frac{1}{2}v^2\lambda_{\delta 45p}, \quad M_{\Delta 45p}^2 \equiv M_\Delta^2 + \frac{1}{2}v^2\lambda_{\Delta 45p}, \quad (\text{D4})$$

where  $\lambda_{45p}$  are defined as  $\lambda_4 + \lambda_5$  for each subscripts. The squared mass matrix  $M_{T0}^2$  can be diagonalized as  $O_{T0}M_{T0}^2O_{T0}^T$  by the orthogonal matrix  $O_{T0}$ :

$$O_{T0} \equiv O_{Ts}O_{\Delta sa} \begin{pmatrix} O_{\text{TCPC}} & 0_{4 \times 4} \\ 0_{4 \times 4} & O_{\text{TCPC}} \end{pmatrix}, \quad (\text{D5})$$

$$O_{\text{TCPC}} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \cos \theta_{T0} & \sin \theta_{T0} \\ 0 & 0 & -\sin \theta_{T0} & \cos \theta_{T0} \end{pmatrix} \begin{pmatrix} 1 & \omega & \omega^2 & 0 \\ 1 & \omega^2 & \omega & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{3} \end{pmatrix}, \quad (\text{D6})$$

$$O_{T_s} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos \theta_{T_s} & 0 & 0 & 0 & \sin \theta_{T_s} \\ 0 & 0 & 0 & \cos \theta_{T_s} & 0 & 0 & \sin \theta_{T_s} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin \theta_{T_s} & 0 & 0 & \cos \theta_{T_s} \\ 0 & 0 & -\sin \theta_{T_s} & 0 & 0 & 0 & \cos \theta_{T_s} \end{pmatrix}, \quad (\text{D7})$$

$$O_{\Delta sa} \equiv \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (\text{D8})$$

The mixing angles are defined as

$$\tan 2\theta_{T0} \equiv \frac{2\sqrt{3} v^2 \text{Re}(\lambda'_{s45p})}{3M_{\Delta 45p}^2 - 3M_{\delta 45p}^2 + 2\lambda_{\Delta ss45p} v^2}, \quad (\text{D9})$$

$$\tan 2\theta_{T_s} \equiv \frac{2\sqrt{3} v^2 \text{Im}(\lambda'_{s45p})}{(3M_{\Delta 45p}^2 - 3M_{\delta 45p}^2 + 2v^2 \lambda_{\Delta ss45p}) \cos 2\theta_{T0} + 2\sqrt{3} v^2 \text{Re}(\lambda'_{s45p}) \sin 2\theta_{T0}}. \quad (\text{D10})$$

Note that maximal mixings in  $O_{\Delta sa}$  appear only for the case that the squared triplet vev's (which we ignored here) are much smaller than  $v^2 \lambda_{\Delta sa45p}$ ; if not,  $O_{\Delta sa}$  is almost the unit matrix.

The mass eigenstates and their masses are obtained as

$$\begin{aligned} & \left( H_{T1}^0, \dots, H_{T4}^0, A_{T1}^0, \dots, A_{T4}^0 \right)^T \\ &= O_{T0} \left( \text{Re}(\Delta_x^0), \dots, \text{Re}(\delta^0), \text{Im}(\Delta_x^0), \dots, \text{Im}(\delta^0) \right)^T, \end{aligned} \quad (\text{D11})$$

$$m_{H_{T1}^0}^2 = m_{A_{T1}^0}^2 = M_{\Delta_{45p}}^2 - \frac{1}{3}v^2 \left( \lambda_{\Delta_{ss45p}} + \sqrt{3}\lambda_{\Delta_{sa45p}} \right), \quad (\text{D12})$$

$$m_{H_{T2}^0}^2 = m_{A_{T2}^0}^2 = M_{\Delta_{45p}}^2 - \frac{1}{3}v^2 \left( \lambda_{\Delta_{ss45p}} - \sqrt{3}\lambda_{\Delta_{sa45p}} \right), \quad (\text{D13})$$

$$m_{H_{T3}^0}^2 = m_{A_{T3}^0}^2 = \frac{1}{6} \left( M_{\delta_{45p}}^2 + M_{\Delta_{45p}}^2 + 2\lambda_{\Delta_{ss45p}}v^2 - 3\Delta m_0^2 \right), \quad (\text{D14})$$

$$m_{H_{T3}^0}^2 = m_{A_{T3}^0}^2 = \frac{1}{6} \left( M_{\delta_{45p}}^2 + M_{\Delta_{45p}}^2 + 2\lambda_{\Delta_{ss45p}}v^2 + 3\Delta m_0^2 \right), \quad (\text{D15})$$

$$3\Delta m_0^2 \equiv \left\{ \left( 3M_{\Delta_{45p}} - 3M_{\delta_{45p}} + 2\lambda_{\Delta_{ss45p}}v^2 \right)^2 + 12|\lambda'_{s45p}|^2v^4 \right\}^{\frac{1}{2}}. \quad (\text{D16})$$

Of course,  $H_{Ti}^0$  and  $A_{Ti}^0$  become the CP-even and odd neutral Higgs bosons, respectively, if  $M_{\text{TCPV}}^2$  vanishes. It is clear that eq. (D12)-(D16) can be given by replacing  $\lambda_5$  with  $-\lambda_5$  in eq. (42)-(51).

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- [1] B. T. Cleveland *et al.*, *Astrophys. J.* **496**, 505 (1998); W. Hampel *et al.* [GALLEX Collaboration], *Phys. Lett. B* **447**, 127 (1999); J. N. Abdurashitov *et al.* [SAGE Collaboration], *J. Exp. Theor. Phys.* **95**, 181 (2002) [*Zh. Eksp. Teor. Fiz.* **122**, 211 (2002)] [arXiv:astro-ph/0204245]; J. Hosaka *et al.* [Super-Kamkiokande Collaboration], *Phys. Rev. D* **73**, 112001 (2006); B. Aharmim *et al.* [SNO Collaboration], *Phys. Rev. Lett.* **101**, 111301 (2008); C. Arpesella *et al.* [The Borexino Collaboration], *Phys. Rev. Lett.* **101**, 091302 (2008)
  - [2] Y. Ashie *et al.* [Super-Kamiokande Collaboration], *Phys. Rev. D* **71**, 112005 (2005); J. L. Raaf [Super-Kamiokande Collaboration], *J. Phys. Conf. Ser.* **136**, 022013 (2008).
  - [3] M. H. Ahn *et al.* [K2K Collaboration], *Phys. Rev. D* **74**, 072003 (2006); P. Adamson *et al.* [MINOS Collaboration], *Phys. Rev. Lett.* **101**, 131802 (2008)
  - [4] M. Apollonio *et al.* [CHOOZ Collaboration], *Eur. Phys. J. C* **27**, 331 (2003).
  - [5] S. Abe *et al.* [KamLAND Collaboration], *Phys. Rev. Lett.* **100**, 221803 (2008).
  - [6] W. Konetschny and W. Kummer, *Phys. Lett. B* **70**, 433 (1977); T. P. Cheng and L. F. Li, *Phys. Rev. D* **22**, 2860 (1980); M. Magg and C. Wetterich, *Phys. Lett. B* **94**, 61 (1980).
  - [7] J. Schechter and J. W. F. Valle, *Phys. Rev. D* **22**, 2227 (1980).

- [8] D. E. Acosta *et al.* [CDF Collaboration], Phys. Rev. Lett. **93**, 221802 (2004); T. Aaltonen *et al.* [The CDF Collaboration], Phys. Rev. Lett. **101**, 121801 (2008); V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **93**, 141801 (2004); Phys. Rev. Lett. **101**, 071803 (2008).
- [9] J. Garayoa and T. Schwetz, JHEP **0803**, 009 (2008) [arXiv:0712.1453 [hep-ph]]; M. Kadastik, M. Raidal and L. Rebane, Phys. Rev. D **77**, 115023 (2008) [arXiv:0712.3912 [hep-ph]];
- [10] A. G. Akeroyd, M. Aoki and H. Sugiyama, Phys. Rev. D **77**, 075010 (2008) [arXiv:0712.4019 [hep-ph]];
- [11] P. Fileviez Perez, T. Han, G. y. Huang, T. Li and K. Wang, Phys. Rev. D **78**, 015018 (2008) [arXiv:0805.3536 [hep-ph]].
- [12] H. Nishiura and T. Fukuyama, Phys. Rev. D **80**, 017302 (2009) [arXiv:0905.3963 [hep-ph]]; arXiv:0909.0595 [hep-ph].
- [13] E. J. Chun, K. Y. Lee and S. C. Park, Phys. Lett. B **566**, 142 (2003);
- [14] M. Kakizaki, Y. Ogura and F. Shima, Phys. Lett. B **566**, 210 (2003); A. G. Akeroyd, M. Aoki and H. Sugiyama, Phys. Rev. D **79**, 113010 (2009); T. Fukuyama, H. Sugiyama and K. Tsumura, JHEP **1003**, 044 (2010).
- [15] E. Ma, arXiv:0705.0327 [hep-ph]; H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. Shimizu and M. Tanimoto, arXiv:1003.3552 [hep-th].
- [16] E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001) [arXiv:hep-ph/0106291]; E. Ma, Mod. Phys. Lett. A **17**, 289 (2002) [arXiv:hep-ph/0201225].
- [17] S. L. Chen, M. Frigerio and E. Ma, Nucl. Phys. B **724**, 423 (2005) [arXiv:hep-ph/0504181].
- [18] E. Ma, Mod. Phys. Lett. A **20**, 2601 (2005) [arXiv:hep-ph/0508099].
- [19] M. Hirsch, A. S. Joshipura, S. Kaneko and J. W. F. Valle, Phys. Rev. Lett. **99**, 151802 (2007) [arXiv:hep-ph/0703046].
- [20] E. Ma, Phys. Rev. D **70**, 031901 (2004) [arXiv:hep-ph/0404199].
- [21] M. Hirsch, A. Villanova del Moral, J. W. F. Valle and E. Ma, Phys. Rev. D **72**, 091301 (2005) [Erratum-ibid. D **72**, 119904 (2005)] [arXiv:hep-ph/0507148].
- [22] G. Altarelli and F. Feruglio, Nucl. Phys. B **720**, 64 (2005) [arXiv:hep-ph/0504165]; Nucl. Phys. B **741**, 215 (2006) [arXiv:hep-ph/0512103].
- [23] E. Ma, Phys. Lett. B **671**, 366 (2009) [arXiv:0808.1729 [hep-ph]].
- [24] G. B. Gelmini and M. Roncadelli, Phys. Lett. B **99**, 411 (1981).
- [25] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).

- [26] S. M. Bilenky, J. Hosek and S. T. Petcov, Phys. Lett. B **94**, 495 (1980); M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. B **102**, 323 (1981).
- [27] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
- [28] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002) [arXiv:hep-ph/0202074].
- [29] F. T. Avignone, S. R. Elliott and J. Engel, Rev. Mod. Phys. **80**, 481 (2008) [arXiv:0708.1033 [nucl-ex]].
- [30] A. G. Akeroyd and C. W. Chiang, arXiv:1003.3724 [hep-ph].
- [31] E. Ma and U. Sarkar, Phys. Rev. Lett. **80**, 5716 (1998) [arXiv:hep-ph/9802445].
- [32] S. T. Petcov, H. Sugiyama and Y. Takanishi, Phys. Rev. D **80**, 015005 (2009)
- [33] U. Bellgardt *et al.* [SINDRUM Collaboration], Nucl. Phys. B **299**, 1 (1988).
- [34] A. G. Akeroyd *et al.* [SuperKEKB Physics Working Group], arXiv:hep-ex/0406071.
- [35] J. L. Hewett *et al.*, arXiv:hep-ph/0503261.
- [36] T. Browder, M. Ciuchini, T. Gershon, M. Hazumi, T. Hurth, Y. Okada and A. Stocchi, JHEP **0802**, 110 (2008); T. E. Browder, T. Gershon, D. Pirjol, A. Soni and J. Zupan, arXiv:0802.3201 [hep-ph].
- [37] R. Santinelli, eConf **C0209101**, WE14 (2002) [Nucl. Phys. Proc. Suppl. **123**, 234 (2003)]  
M. Giffels, J. Kallarackal, M. Kramer, B. O’Leary and A. Stahl, Phys. Rev. D **77**, 073010 (2008)
- [38] M. L. Brooks *et al.* [MEGA Collaboration], Phys. Rev. Lett. **83**, 1521 (1999).
- [39] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **95**, 041802 (2005); Phys. Rev. Lett. **96**, 041801 (2006); K. Hayasaka *et al.* [Belle Collaboration], Phys. Lett. B **666**, 16 (2008).
- [40] P. W. Cattaneo [MEG Collaboration], arXiv:0909.0199 [physics.ins-det].
- [41] Y. Grossman, Phys. Lett. B **359**, 141 (1995) [arXiv:hep-ph/9507344].
- [42] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978);
- [43] S. P. Mikheev and A. Y. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985) [Yad. Fiz. **42**, 1441 (1985)].
- [44] E. Roulet, Phys. Rev. D **44**, 935 (1991); M. M. Guzzo, A. Masiero and S. T. Petcov, Phys. Lett. B **260**, 154 (1991).
- [45] M. Malinsky, T. Ohlsson and H. Zhang, Phys. Rev. D **79**, 011301 (2009)
- [46] S. Davidson, C. Pena-Garay, N. Rius and A. Santamaria, JHEP **0303**, 011 (2003)

- [47] V. D. Barger, J. L. Hewett and R. J. N. Phillips, Phys. Rev. D **41**, 3421 (1990).
- [48] Y. Grossman, Nucl. Phys. B **426**, 355 (1994) [arXiv:hep-ph/9401311].
- [49] A. G. Akeroyd and W. J. Stirling, Nucl. Phys. B **447**, 3 (1995); A. G. Akeroyd, Phys. Lett. B **377**, 95 (1996) [arXiv:hep-ph/9603445]; J. Phys. G **24**, 1983 (1998) [arXiv:hep-ph/9803324]; Nucl. Phys. B **544**, 557 (1999) [arXiv:hep-ph/9806337].
- [50] M. Aoki, S. Kanemura, K. Tsumura and K. Yagyu, Phys. Rev. D **80**, 015017 (2009) [arXiv:0902.4665 [hep-ph]].
- [51] N. C. Ribeiro, H. Minakata, H. Nunokawa, S. Uchinami and R. Zukanovich-Funchal, JHEP **0712**, 002 (2007); J. Kopp, T. Ota and W. Winter, Phys. Rev. D **78**, 053007 (2008).
- [52] K. Hayasaka, K. Inami and Y. Miyazaki, Phys. Lett. B **687**, 139 (2010).
- [53] P. Achard *et al.* [L3 Collaboration], Phys. Lett. B **576**, 18 (2003) G. Abbiendi *et al.* [OPAL Collaboration], Phys. Lett. B **577**, 93 (2003)
- [54] L. B. Auerbach *et al.* [LSND Collaboration], Phys. Rev. D **63**, 112001 (2001)
- [55] A. Bolanos, O. G. Miranda, A. Palazzo, M. A. Tortola and J. W. F. Valle, Phys. Rev. D **79**, 113012 (2009)