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Kanemura, Shinya
Department of Physics, Osaka University

Matsuda, Koichi
Department of Physics, Osaka University

Ota, Toshihiko
Department of Physics, Osaka University

Shindou, Tetsuo
Scuola Internazionale Superiore di Studi Avanzati

他

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Phase effects from the general neutrino Yukawa matrix on lepton flavor violation

Shinya Kanemura,[1,*] Koichi Matsuda,[1,†] Toshihiko Ota,[1,‡]
Tetsuo Shindou,[2,§] Eiichi Takasugi,[1,¶] and Koji Tsumura[1,**]

[1]*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*

[2]*Scuola Internazionale Superiore di Studi Avanzati Via Beirut 2–4, I-34014, Trieste, Italy*

Abstract

We examine contributions from Majorana phases to lepton flavor violating processes in the framework of the minimal supersymmetric standard model with heavy right-handed neutrinos. All phases in the complex neutrino Yukawa matrix are taken into account in our study. We find that in the scenario with universal soft-breaking terms sizable phase effects can appear on the lepton flavor violating processes such as $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$. In particular, the branching ratio of $\mu \to e\gamma$ can be considerably enhanced due to the Majorana phases, so that it can be much greater than that of $\tau \to \mu\gamma$.

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*Electronic address: kanemu@het.phys.sci.osaka-u.ac.jp
†Electronic address: matsuda@het.phys.sci.osaka-u.ac.jp
‡Electronic address: toshi@het.phys.sci.osaka-u.ac.jp
§Electronic address: shindou@sissa.it
¶Electronic address: takasugi@het.phys.sci.osaka-u.ac.jp
**Electronic address: ko2@het.phys.sci.osaka-u.ac.jp

1. INTRODUCTION

In the standard model lepton flavor violation (LFV) is negligible, while it can be sizable in new physics models such as those based on supersymmetry (SUSY). Therefore search for LFV can be a good probe of new physics. Observed tiny neutrino masses may be explained by the seesaw mechanism[1], assuming heavy right-handed Majorana neutrinos, which are compatible with the scenario of grand unified theories (GUTs). In the framework of SUSY models, LFV is induced through one-loop diagrams with slepton mixing[2]. In the SUSY model with right-handed neutrinos, the slepton mixing can be induced from the renormalization group effect of the neutrino Yukawa interaction between the scale of right-handed neutrino masses and the GUT scale, even when soft-SUSY-breaking terms are universal at the GUT scale.

The neutrino mass matrix obtained via the seesaw mechanism generally includes two Majorana phases[3]. They can be directly searched through neutrinoless double beta decays[4]. The existence of these Majorana phases can play an important role in various phenomena such as leptogenesis[5], lepton number violating processes and so on. Searches for these phenomena could provide a hint for the neutrino Majorana mass matrix. Furthermore, as we shall show below, the prediction on LFV can be drastically changed by the Majorana phases.

In the present paper, we explore LFV processes such as $\mu \to e\gamma$ in the framework of the minimal supersymmetric standard model with right-handed Majorana neutrinos (MSSMRN) under the assumption of universal soft-SUSY-breaking terms at the GUT scale M_{GUT}. Neutrino mass matrix m_ν is given by $m_\nu = Y_\nu^T D_R^{-1} Y_\nu \langle \phi_u^0 \rangle^2$, where Y_ν is the neutrino Yukawa matrix, D_R is the right-handed neutrino mass matrix which is diagonal, and ϕ_u^0 is the neutral component of the Higgs doublet with hypercharge $-1/2$. In the basis where the charged lepton mass matrix is diagonal, the neutrino Dirac mass matrix $m_D \equiv Y_\nu \langle \phi_u^0 \rangle$ can be parameterized by[6, 7]

$$m_D = \sqrt{D_R} R \sqrt{D_\nu} U^\dagger \ , \tag{1}$$

where D_ν is the eigenmatrix of neutrino masses, R is a complex orthogonal matrix ($R^T R = R R^T = \mathbf{1}$), and U is the neutrino mixing matrix. In Refs. [7, 8], the decay rates of $\ell_i \to \ell_j\gamma$ $(i \neq j)$ are evaluated by assuming that R is a real orthogonal matrix and that the right-handed neutrino masses are degenerate; i.e., $D_R = M \times \mathbf{1}$ where M is the heavy Majorana mass scale. Under this assumption, the effect of Majorana phases on the low energy phenomena is screened. The relation among the branching ratios is given by

$$\mathrm{Br}(\mu \to e\gamma) \simeq \frac{\mathrm{Br}(\tau \to e\gamma)}{\mathrm{Br}(\tau \to \bar{\nu}_e \nu_\tau e)} \ll \mathrm{Br}(\tau \to \mu\gamma) \ , \tag{2}$$

where current neutrino data have been used. The hierarchical D_R case with a real R has been analyzed in Ref. [6]. On the other hand, the importance of the treatment of R as a complex matrix has been pointed out in Ref. [7], by showing that phases in R can give a substantial effect on low energy phenomena.

In this paper, we discuss the role of the imaginary part of R, and study the combined effect with Majorana phases in neutrino mixing matrix on the branching ratios of the LFV processes. We assume that $D_R = M \times \mathbf{1}$. We obtain analytic expressions of the branching ratios in two limiting cases: i.e., one is the case with R being approximately a real orthogonal matrix, and the other is with R being a typical complex matrix. We find that

$$\mathrm{Br}(\mu \to e\gamma) \simeq \frac{\mathrm{Br}(\tau \to e\gamma)}{\mathrm{Br}(\tau \to \bar{\nu}_e \nu_\tau e)} \gg \mathrm{Br}(\tau \to \mu\gamma) \ , \tag{3}$$

in the wide range of the parameter space for a typical complex matrix R. The branching ratio of $\mu \to e\gamma$ can be enhanced in comparison with that of $\tau \to \mu\gamma$. This is a novel feature with a complex R. We also give numerical calculations in order to see how these two limiting cases are extrapolated.

2. EVALUATION OF LFV BRANCHING RATIOS

In this section, we briefly review LFV in the MSSMRN, and discuss the Majorana phase effects on LFV processes.

In the model based on SUSY, LFV processes can occur at the low energy scale through the slepton mixing. In the MSSMRN, sizable off-diagonal elements of the slepton mass matrix can be induced by renormalization group effects due to the neutrino Yukawa interaction between M_{GUT} and M, even when universal soft-breaking masses are assumed at M_{GUT}. The induced off-diagonal elements are approximately expressed as[2]

$$(m_{\tilde{L}}^2)_{ij} \simeq \frac{6m_0^2 + |A_0|^2}{16\pi^2} \ln \frac{M_{GUT}}{M} (Y_\nu^\dagger Y_\nu)_{ij} \quad (i \neq j) \ , \tag{4}$$

where m_0 and A_0 are universal soft-SUSY-breaking parameters. The decay rates for LFV processes $\ell_i \to \ell_j \gamma$ $(i \neq j)$ are given by

$$\Gamma(\ell_i \to \ell_j \gamma) \simeq \frac{\alpha_{EM}^3 m_{\ell_i}^5}{192\pi^3} \frac{|(m_{\tilde{L}}^2)_{ij}|^2}{m_{SUSY}^8} \tan^2 \beta \ , \tag{5}$$

where α_{EM} is the fine structure constant, m_{SUSY} represents the typical mass scale of SUSY particles, and $\tan\beta$ is the ratio of vacuum expectation values of the two Higgs doublets. The branching ratios are related to each other as

$$\begin{aligned} \frac{\mathrm{Br}(\mu \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} &\simeq \frac{1}{\mathrm{Br}(\tau \to \bar{\nu}_e \nu_\tau e)} \frac{|(m_D^\dagger m_D)_{12}|^2}{|(m_D^\dagger m_D)_{23}|^2} \sim 5.6 \times \frac{|(m_D^\dagger m_D)_{12}|^2}{|(m_D^\dagger m_D)_{23}|^2} \ , \\ \frac{\mathrm{Br}(\tau \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} &\simeq \frac{|(m_D^\dagger m_D)_{13}|^2}{|(m_D^\dagger m_D)_{23}|^2} \ , \end{aligned} \tag{6}$$

where experimental result $\mathrm{Br}(\tau \to \bar{\nu}_e \nu_\tau e) = 0.1784$ is used. These ratios are determined only by the neutrino Yukawa matrix.

We work on the basis that the right-handed neutrino mass matrix is diagonal, and assume that the matrix is approximately proportional to the identity matrix; i.e. $D_R \simeq M \times \mathbf{1}$. By using Eq. (1) we obtain

$$m_D^\dagger m_D \simeq MU\sqrt{D_\nu} R^\dagger R \sqrt{D_\nu} U^\dagger = MU\sqrt{D_\nu} Q^\dagger Q \sqrt{D_\nu} U^\dagger \ . \tag{7}$$

Here we have introduced a real orthogonal matrix O by $R = OQ$, where Q is a product of $Q_a (a = 1\text{-}3)$ with

$$Q_1 = \begin{pmatrix} \cosh y_1 & i\sinh y_1 & 0 \\ -i\sinh y_1 & \cosh y_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, Q_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cosh y_2 & i\sinh y_2 \\ 0 & -i\sinh y_2 & \cosh y_2 \end{pmatrix}, Q_3 = \begin{pmatrix} \cosh y_3 & 0 & i\sinh y_3 \\ 0 & 1 & 0 \\ -i\sinh y_3 & 0 & \cosh y_3 \end{pmatrix} . \tag{8}$$

The matrices Q_a satisfy that $Q_a^\dagger = Q_a$ and $Q_a^2(y_a) = Q_a(2y_a)$. The matrix Q plays a role not only to introduce the complex phases but also to change the size of Yukawa couplings.*

The neutrino mixing matrix U is separated into two parts, $U = U_{MNS}P$, where U_{MNS} is the Maki-Nakagawa-Sakata matrix[9] in the phase convention of Ref. [10] and P is the Majorana phase matrix given by $P = \mathrm{diag}(1, e^{i\alpha_0}, e^{i\beta_0})$ with α_0 and β_0 being Majorana CP violation phases [3]. In order to see qualitative features, we here take the Bi-maximal mixing solution[11]

$$U_{MNS}^{Bi-max} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} , \tag{9}$$

for analytic calculations. In particular, we consider the following three cases for D_ν; the normal hierarchical (NH) case ($m_1 \ll m_2 \ll m_3$), the inverse hierarchical (IH) case ($m_3 \ll m_1 \sim m_2$), and the quasi-degenerate (QD) case ($m_1 \simeq m_2 \sim m_3$);

$$\text{NH}: \quad m_1 \simeq 0\ ,\ m_2 \simeq \sqrt{\Delta m_\odot^2}\ ,\ m_3 \simeq \sqrt{\Delta m_{\rm atm}^2}\ , \tag{10}$$

$$\text{IH}: \quad m_1 \simeq \sqrt{\Delta m_{\rm atm}^2}\left(1 - \frac{1}{2}\frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2}\right)\ ,\ m_2 \simeq \sqrt{\Delta m_{\rm atm}^2}\ ,\ m_3 \simeq 0\ , \tag{11}$$

$$\text{QD}: \quad m_1 \equiv m\ ,\ m_2 \simeq m + \frac{\Delta m_\odot^2}{2m}\ ,\ m_3 \simeq m + \frac{\Delta m_{\rm atm}^2}{2m}\ . \tag{12}$$

Here, $\Delta m_\odot^2 \equiv m_2^2 - m_1^2 (= 8.0 \times 10^{-5}\mathrm{eV}^2)$[12] is the squared mass difference for the solar neutrino mixing, and $\Delta m_{\rm atm}^2 \equiv |m_3^2 - m_2^2| (= 2.5 \times 10^{-3}\mathrm{eV}^2)$[13] is that for the atmospheric neutrino mixing.

To evaluate $m_D^\dagger m_D$, we consider the following two limiting cases.

(a) The small y_a limit (R is real.) :

We have $Q = \mathbf{1}$, and thus $m_D^\dagger m_D = MU_{MNS}^{Bi-max} D_\nu U_{MNS}^{Bi-max\,\dagger}$, where the elements of

* We note that fine tuning of order $\mathcal{O}(e^{y_a})$ is necessary to obtain the light neutrino mass scale in the case of $y_a > 1$.

$m_D^\dagger m_D$ are determined by neutrino masses and the mixing matrix as

$$(m_D^\dagger m_D)_{12} = (m_D^\dagger m_D)_{13} = -\frac{M}{2\sqrt{2}}(m_2 - m_1) \ ,$$
$$(m_D^\dagger m_D)_{23} = \frac{M}{4}(m_1 + m_2 - 2m_3) \ . \tag{13}$$

We then obtain from Eq. (6) that

$$\frac{\mathrm{Br}(\mu \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} \simeq \begin{cases} 5.6 \times \frac{1}{2}\left(\frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2}\right) & \simeq 0.23 \quad \text{for NH} \\ 5.6 \times \frac{1}{8}\left(\frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2}\right)^2 & \simeq 7.7 \times 10^{-4} \quad \text{for IH} \\ 5.6 \times \frac{1}{2}\left(\frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2}\right)^2 & \simeq 3.1 \times 10^{-3} \quad \text{for QD} \end{cases} . \tag{14}$$

For all the cases, it turns out that $\mathrm{Br}(\mu \to e\gamma) \simeq 5.6 \times \mathrm{Br}(\tau \to e\gamma) \ll \mathrm{Br}(\tau \to \mu\gamma)$, as pointed out in Refs. [7, 8]. In this limit, the Majorana phases do not affect the LFV processes.

(b) The large y_a case :

The matrix Q has a simple form. First, the matrices Q_a behave as

$$Q_a \simeq \frac{e^{y_a}}{\sqrt{2}}\mathcal{Q}_a \ , \tag{15}$$

where

$$\mathcal{Q}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i & 0 \\ -i & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \mathcal{Q}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & i \\ 0 & -i & 1 \end{pmatrix} , \mathcal{Q}_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 1 \end{pmatrix} . \tag{16}$$

They satisfy $\mathcal{Q}_a^\dagger = \mathcal{Q}_a$ and $\mathcal{Q}_a^2 = \sqrt{2}\mathcal{Q}_a$. As for the product of $\mathcal{Q}_a$ such as $\mathcal{Q} \in \{\mathcal{Q}_a, \mathcal{Q}_b\mathcal{Q}_a, \mathcal{Q}_c\mathcal{Q}_b\mathcal{Q}_a\}$, we find an interesting relation as

$$\mathcal{Q}^\dagger\mathcal{Q} = \sqrt{2}\mathcal{Q}_a \ . \tag{17}$$

By using Eq. (17) $Q^\dagger Q$ is expressed by

$$Q^\dagger Q \simeq \frac{e^{2(y_1+y_2+y_3)}}{4\sqrt{2}}\mathcal{Q}_a \ . \tag{18}$$

This means that $Q^\dagger Q$ is characterized by three independent matrices $\mathcal{Q}_a (a = 1$-$3)$ for large y_a. Thus, we examine the following three cases, taking $R = OQ_a \simeq e^{y_a} O\mathcal{Q}_a/\sqrt{2}$.

(b-1) $R = OQ_1$

We have

$$(m_D^\dagger m_D)_{12} = -(m_D^\dagger m_D)_{13} = \left(\frac{Me^{2y_1}}{2}\right) \frac{m_2 - m_1 + i2\sqrt{m_1 m_2}\cos\alpha_0}{2\sqrt{2}} ,$$
$$(m_D^\dagger m_D)_{23} = -\left(\frac{Me^{2y_1}}{2}\right) \frac{m_1 + m_2 - 2\sqrt{m_1 m_2}\sin\alpha_0}{4} . \tag{19}$$

Thus the LFV branching ratios are related as $\mathrm{Br}(\mu \to e\gamma) \simeq 5.6 \times \mathrm{Br}(\tau \to e\gamma)$ for all cases. For the NH case, we obtain $\mathrm{Br}(\mu \to e\gamma) \simeq 11.2 \times \mathrm{Br}(\tau \to \mu\gamma)$. For the IH and the QD cases, one finds

$$\frac{\mathrm{Br}(\mu \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} \simeq 11.2 \times \frac{\cos^2\alpha_0}{(1+\sin\alpha_0)^2} . \tag{20}$$

This ratio is a function of α_0. It is 11.2 for $\alpha_0 = 0$ or π, and 0 for $\alpha_0 = \pi/2$.

(b-2) $R = OQ_2$

The difference of the Majorana phases $\alpha_0 - \beta_0$ enters into $m_D^\dagger m_D$,

$$(m_D^\dagger m_D)_{12} = \left(\frac{Me^{2y_2}}{2}\right) \frac{m_2 + i\sqrt{2}\sqrt{m_2 m_3}e^{i(\alpha_0-\beta_0)}}{2\sqrt{2}} ,$$
$$(m_D^\dagger m_D)_{13} = \left(\frac{Me^{2y_2}}{2}\right) \frac{-m_2 + i\sqrt{2}\sqrt{m_2 m_3}e^{i(\alpha_0-\beta_0)}}{2\sqrt{2}} ,$$
$$(m_D^\dagger m_D)_{23} = \left(\frac{Me^{2y_2}}{2}\right) \frac{-m_2 + 2\,m_3 + 2\sqrt{2}i\sqrt{m_2 m_3}\cos(\alpha_0-\beta_0)}{4} . \tag{21}$$

For the NH case and the IH case, the branching ratios of $\ell_i \to \ell_j\gamma$ are related to each other as $\mathrm{Br}(\mu \to e\gamma) \simeq 5.6 \times \mathrm{Br}(\tau \to e\gamma)$, and

$$\frac{\mathrm{Br}(\mu \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} \simeq \begin{cases} 5.6 \times \sqrt{\frac{\Delta m_\odot^2}{\Delta m_{\mathrm{atm}}^2}} \simeq 1.0 & \text{for NH} \\ 11.2 & \text{for IH} \end{cases} . \tag{22}$$

For the QD case, we obtain

$$\frac{\mathrm{Br}(\mu \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} \simeq 11.2 \times \frac{3 + 2\sqrt{2}\sin(\alpha_0-\beta_0)}{1 + 8\cos^2(\alpha_0-\beta_0)} ,$$
$$\frac{\mathrm{Br}(\tau \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} \simeq \frac{2(3 - 2\sqrt{2}\sin(\alpha_0-\beta_0))}{1 + 8\cos^2(\alpha_0-\beta_0)} . \tag{23}$$

We have $\mathrm{Br}(\mu \to e\gamma) \simeq 5.6 \times \mathrm{Br}(\tau \to e\gamma) \simeq 3.7 \times \mathrm{Br}(\tau \to \mu\gamma)$ for $\alpha_0 - \beta_0 = 0$ or π. The ratio $\mathrm{Br}(\mu \to e\gamma)/\mathrm{Br}(\tau \to \mu\gamma)$ takes its minimum value 1.9 at $\alpha_0 - \beta_0 \simeq -\pi/2$.

<table>
<tr><td colspan="2" rowspan="2"></td><td>small y_a</td><td colspan="3">large y_a</td></tr>
<tr><td>$Q=\mathbf{1}$</td><td>Q_1</td><td>Q_2</td><td>Q_3</td></tr>
<tr><td rowspan="3">$\frac{\mathrm{Br}(\mu\to e\gamma)}{\mathrm{Br}(\tau\to\mu\gamma)}$</td><td>NH</td><td>0.23</td><td>11.2</td><td>1.0</td><td>$\ll 1$</td></tr>
<tr><td>IH</td><td>7.7×10^{-4}</td><td rowspan="2">$11.2\times\frac{\cos^2\alpha_0}{(1+\sin\alpha_0)^2}$</td><td>11.2</td><td>11.2</td></tr>
<tr><td>QD</td><td>3.1×10^{-3}</td><td>$11.2\times\frac{3-2\sqrt{2}\sin(\alpha_0-\beta_0)}{1+8\cos^2(\alpha_0-\beta_0)}$</td><td>$11.2\times\frac{3+2\sqrt{2}\sin\beta_0}{1+8\cos^2\beta_0}$</td></tr>
<tr><td rowspan="3">$\frac{\mathrm{Br}(\mu\to e\gamma)}{\mathrm{Br}(\tau\to e\gamma)}$</td><td>NH</td><td rowspan="3">5.6</td><td rowspan="3">5.6</td><td rowspan="2">5.6</td><td rowspan="2">5.6</td></tr>
<tr><td>IH</td></tr>
<tr><td>QD</td><td>$5.6\times\frac{3-2\sqrt{2}\sin(\alpha_0-\beta_0)}{3+2\sqrt{2}\sin(\alpha_0-\beta_0)}$</td><td>$5.6\times\frac{3+2\sqrt{2}\sin\beta_0}{3-2\sqrt{2}\sin\beta_0}$</td></tr>
</table>

TABLE I: Summary of the ratios of the LFV processes.

(b-3) $R = OQ_3$

The Majorana phase β_0 enters into $m_D^\dagger m_D$. We obtain

$$\begin{aligned}
(m_D^\dagger m_D)_{12} &= \left(\frac{Me^{2y_3}}{2}\right)\frac{-m_1+i\sqrt{2}\sqrt{m_1m_3}e^{-i\beta_0}}{2\sqrt{2}} \ , \\
(m_D^\dagger m_D)_{13} &= \left(\frac{Me^{2y_3}}{2}\right)\frac{m_1+i\sqrt{2}\sqrt{m_1m_3}e^{-i\beta_0}}{2\sqrt{2}} \ , \\
(m_D^\dagger m_D)_{23} &= \left(\frac{Me^{2y_3}}{2}\right)\frac{-m_1+2m_3-i2\sqrt{2}\sqrt{m_1m_3}\cos\beta_0}{4} \ .
\end{aligned} \tag{24}$$

In this case $\left|(m_D^\dagger m_D)_{ij}\right|^2$ can be obtained from case (b-2) by replacing m_2 with m_1 and $\alpha_0-\beta_0$ with $\pi-\beta_0$. The branching ratio $\mathrm{Br}(\mu\to e\gamma)$ is suppressed in the NH case because of $\mathrm{Br}(\mu\to e\gamma)\simeq 5.6\times\mathrm{Br}(\tau\to e\gamma)\simeq 5.6\times(m_1^2/\Delta m_{\mathrm{atm}}^2)\mathrm{Br}(\tau\to\mu\gamma)\ll\mathrm{Br}(\tau\to\mu\gamma)$. For the IH case, the branching ratios are related to each other as $\mathrm{Br}(\mu\to e\gamma)\simeq 5.6\times\mathrm{Br}(\tau\to e\gamma)\simeq 11.2\times\mathrm{Br}(\tau\to\mu\gamma)$. For the QD case, relation among the ratios of branching ratios is obtained from Eq.(23) by changing $\alpha_0-\beta_0$ to $\pi-\beta_0$.

The results are summarized in Table 1.

For the ratio $\mathrm{Br}(\mu\to e\gamma)/\mathrm{Br}(\tau\to e\gamma)$, Q does not contribute except for the QD case with $Q=Q_2$ or Q_3 where the Majorana phases give a significant effect. The drastic effect occurs for $\mathrm{Br}(\mu\to e\gamma)/\mathrm{Br}(\tau\to\mu\gamma)$ by Q or by the interplay between Q and the Majorana phases. The substantial enhancement arises in $\mathrm{Br}(\mu \to e\gamma)$, which is a quite different feature from the case with $Q = \mathbf{1}$. By the introduction of $Q(\neq \mathbf{1})$, the Majorana phases can affect the LFV processes. This fact is a quite interesting because the observation of the LFV processes would give useful information of Majorana phases.

3. NUMERICAL RESULTS

In the previous section, we consider the two limiting cases for the parameter y_a. When y_a take the intermediate values, we may guess the result by extrapolating from the two limits, but some non-trivial structure might appear. Therefore, we perform the numerical evaluation of the LFV branching ratios for three typical cases, $R = OQ_a (a = 1\text{-}3)$. Neutrino mixing parameters are taken to be $\tan^2\theta_\odot = 0.45$[12], $\sin 2\theta_{atm} = 1$[13], and $\sin\theta_{13} = 0$. The values for M and M_{GUT} are taken as $M = 10^{10}$GeV and $M_{GUT} = 2 \times 10^{16}$GeV. The SUSY parameters are taken to be $m_0 = A_0 = m_{SUSY} = 100$GeV and $\tan\beta = 10$. For standard model parameters $\alpha_{EM} = 1/137$ and $v = 246$GeV are used. It will be shown that the ratios of the branching ratios are not sensitive to SUSY parameters, right-handed neutrino mass scale, and the GUT scale.

We analyze the y_a dependences of $\mathrm{Br}(\mu \to e\gamma)/\mathrm{Br}(\tau \to \mu\gamma)$. The result for the NH case is shown in Fig. 1. We find the smooth extrapolation in $\mathrm{Br}(\mu \to e\gamma)/\mathrm{Br}(\tau \to \mu\gamma)$ between $\mathcal{O}(0.1)$ and $\mathcal{O}(1)$ for $R = OQ_2$ with $\alpha_0 - \beta_0 = 0$ and between $\mathcal{O}(0.1)$ to $\mathcal{O}(10^{-6}) \ll 1$ for $R = OQ_3$. For $R = OQ_1$, some structure is observed between $\mathcal{O}(0.1)$ and $\mathcal{O}(10)$. The ratio blows up around $y \sim 1.3$ due to the vanishing $\mathrm{Br}(\tau \to \mu\gamma)$. There is no α_0 dependence.

The IH case is shown in Fig. 2. The dotted (dashed) curve represents $\mathrm{Br}(\mu \to e\gamma)/\mathrm{Br}(\tau \to \mu\gamma)$ for $R = OQ_2$ (OQ_3) where the smooth extrapolation is found between very small value to about 50 (2), where there is no α_0 dependence. The case $R = OQ_1$ is shown for solid curves, which has the Majorana phase α_0 dependence. For all cases, we find the smooth extrapolations between two limiting values, the small y_a and the large y_a.

For the QD case with $R = OQ_2$, the ratio of the branching ratios depends on $\alpha_0 - \beta_0$,

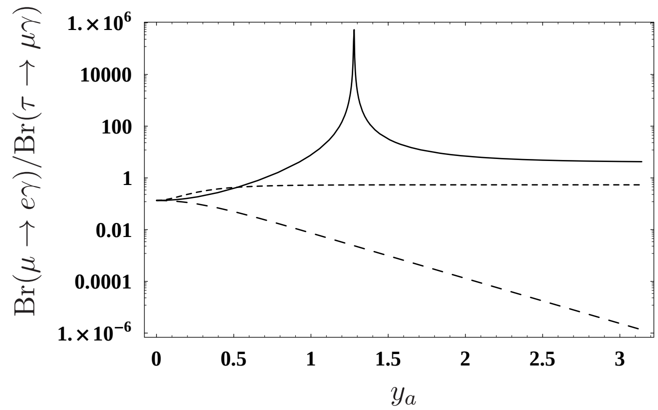

FIG. 1: The ratio of the branching ratios is shown in the NH case for Q_1 (solid curve), for Q_2 with $\alpha_0 - \beta_0 = 0$ (dotted curve), and for Q_3 (dashed curve).

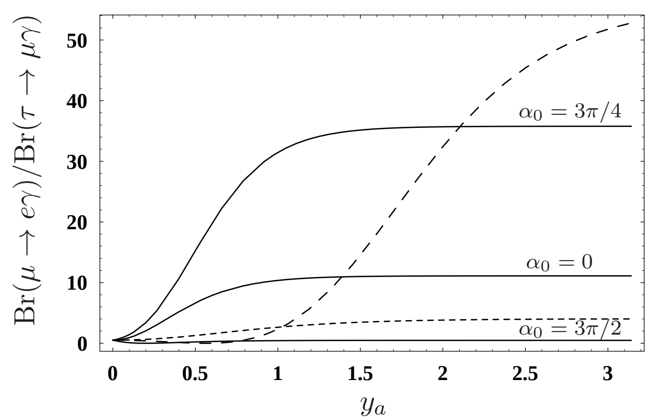

FIG. 2: The ratio of the branching ratios is shown in the IH case for Q_1 with $\alpha_0 = 0, 3\pi/4, 3\pi/2$ (solid curve), for Q_2 (dotted curve), and for Q_3 (dashed curve).

and is roughly obtained by replacing β_0 to $\pi - (\alpha_0 - \beta_0)$ in the formula for $R = OQ_3$. The results for $R = OQ_1$ are similar to those for the IH case with $R = OQ_1$. In Fig. 3, we show the y_3 dependence for the case with $R = OQ_3$ for $\alpha_0 - \beta_0 = 0, 3\pi/4, 3\pi/2$. The enhancement occurs for $\alpha_0 - \beta_0 = 3\pi/4$ because $\mathrm{Br}(\tau \to \mu\gamma)$ is suppressed.

In Fig. 4, $\mathrm{Br}(\mu \to e\gamma)$ with $R = OQ_1$ is shown as a function of y_1. As y_1 grows, the neutrino Yukawa couplings become large for all the neutrino mass spectrum. Thus, the smooth extrapolation is obtained, so that the two limiting cases give the general trend

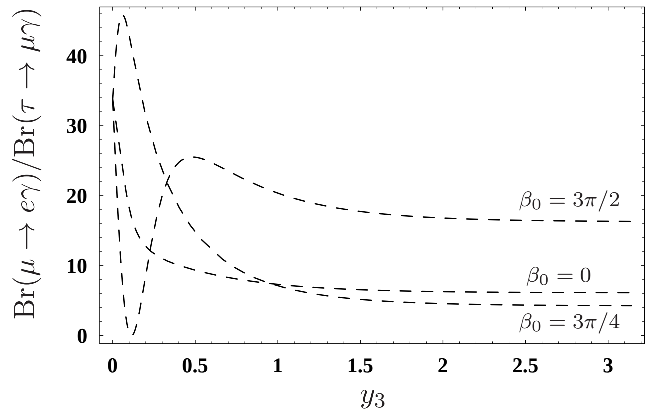

FIG. 3: The ratio of the branching ratios is shown in the QD case for Q_3 with $\beta_0 = 0, 3\pi/4, 3\pi/2$.

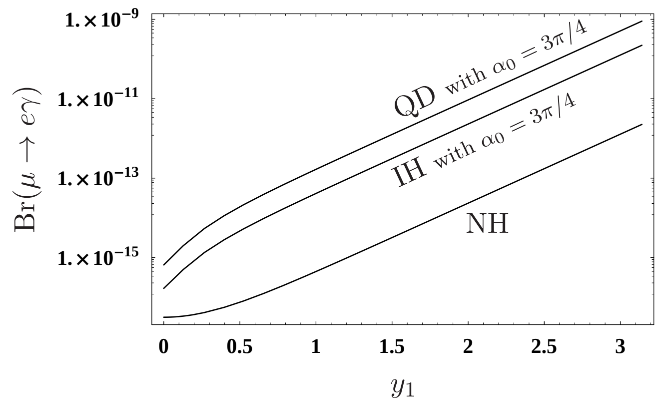

FIG. 4: The LFV branching ratios $\mathrm{Br}(\mu \to e\gamma)$ are shown in the NH case, the IH case with $\alpha_0 = 3\pi/4$ and the QD case with $\alpha_0 = 3\pi/4$ for $R = OQ_1$.

of the y_a dependence. In many cases, the Majorana phases play an important role on the prediction of the LFV processes. Therefore, we can obtain useful information of the Majorana phases from the experimental data of the LFV processes.

4. CONCLUSION

We have shown the importance of the complex nature of the neutrino Yukawa matrix for the case of the degenerate right-handed neutrino masses. With the complex R, the

Majorana phases play an important role for the prediction of the LFV processes. In order to see the effect analytically, we have taken the parameterization, $R = OQ$. We have considered the two limiting cases; the small y_a case with $Q = \mathbf{1}$ and the large y_a case with complex matrix Q. We have obtained the analytic expressions for ratios of the branching ratios of $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$, which are shown in Table 1. The effect of Q is sizable and gives enhancement of $\mathrm{Br}(\mu \to e\gamma)/\mathrm{Br}(\tau \to \mu\gamma)$ in many cases. In particular, the Majorana phases contribute to some cases. This would give a possibility to obtain useful information of Majorana phases by observing the LFV processes. This is quite interesting and important because extracting the information for the Majorana phases can be used to examine the nature of neutrinos.

It may also be interesting to discuss the possibility to determine the neutrino Yukawa matrix by analysing the double beta decay, the $\mu^- \to e^+$ [4, 14] and $\mu^- \to \mu^+$ conversion [15], the LFV processes which occur through SUSY contributions, and the leptogenesis.

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