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バージョン：

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Abstract: A two level storage inventory model is constructed in this article. It is well known that the demand for seasonal products (such as fur coats) increases at the beginning of the season until a certain period of time and stabilizes into a fixed amount of time for the rest of the season. To store these extra parts for the buyer arrange additional storage space. This model uses a ramp type demand rate, variable deterioration and shortages are partially backlogged using a variable backordering rate. The entire research is conducted in an inflationary environment. The goal of this model is to reduce the system's total average cost. A numerical assessment and sensitivity analysis are used to verify the suggested model's optimal solution.

Keywords: Two-warehouse, ramp type demand rate, partial backlogging, inflation, time varying deterioration.

1. Introduction

The majority of inventory issues are founded on the assumption that you have access to an owned warehouse with infinite storage. In practice, however, because warehouses often have limited storage space, this assumption is untrue. Inventory management purchases a large number of goods all at once when the cost of sourcing products is higher than the cost of inventory, or when a favorable price discount for bulk purchases is available, or when the demand for the product is high. The present owned warehouse (OW), which has limited storage capacity, will be unable to accommodate such a large amount of items. After that, the excess items are stored in a rented warehouse (RW), which is located either far away or close to OW and these items are only sold to clients at OW. The cost of inventories at RW is often higher than at OW. As a result, goods are placed first in OW, followed by excess stock in RW, in order to reduce inventory carrying costs. RW stocks are also cleared first, with stock shifted from RW to OW in a continuous or bulk release pattern. This inventory system is known as a two-storage inventory system.

In supermarkets, when attractive discounts are available for bulk purchases or when the purchase price of goods exceeds other inventory-related costs. The company's management chooses to buy a significant quantity of goods all at once. These goods cannot be housed in a crowded market area's existing storehouse (i.e. OW). In this circumstance, one (often more than one) additional godown (i.e., RW) is hired on a rental basis for the storage of more goods. Hartley[4)] invented the two-warehouse inventory system first. Hartley[4)] provided a basic two-story model, in which the cost of transporting a unit from a rented area (RW) to a warehouse (OW) was not considered. Sarma[8)] gives a two-level storage deterministic inventory model with infinite refilling rate. In this model, he extends the Hartley[4)] model by introducing a transport value. For a linear trend in demand with two levels of storage, Goswami and Chaudhuri[9)] provide an EOQ model. For degrading goods, Bhunia and Maiti[13)] formulated a two-warehouse inventory model with linearly increasing demand over time, shortages were permitted and excess demand was backlogged. Due to the restricted capacity of existing storage Kar et al.[14)] suggested a deterministic inventory model with two level storage facilities over a finite time horizon. Zhou and Yang[16)] gives a two-level storage inventory model with a stock-level dependent demand rate. Hsieh et al.[17)] proposed a two-warehouse deterministic inventory model for degrading goods with shortages by reducing the net present value of the entire cost. Recently, Skouri and Konstantaras[24)] created two warehouse inventory models for decaying commodities with ramp demand rates.

As the list deteriorates in nature, the problem becomes more complicated. In the inventory system, list depletion is a critical factor. In recent years, several researchers have worked to compile a list of degrading goods, as most physical objects degrade over time. Ghare and Schrader[1] described an inventory model with exponentially deteriorating goods. Covert and Philip[2] proposed EOQ model Weibull distribution deterioration. Nahmias[7], Wee[10], Sarker et al.[12], have done some important work with the fall and trend demand in this area of architectural features of the inventory system. Singh et al.[18] gives a method for ordering goods with stock-dependent demand, partial backlog and inflation. Kumar and Singh[21] present a perishable inventory model with time-dependent demand and lost sales. Chaudhary and Sharma[20] gives an inventory model with Weibull distribution deterioration, time varying demand with shortages. Singh and Sharma[25] again introduced a model with the facility of allowable delay in payment with variable deterioration and shortages. Jaggi et al.[26] gives the effect of deterioration on two-warehouse inventory model with imperfect quality. Jing and Chao[28] establish a dynamic lot size model with perishable inventory and stockout.

In today's climate the time value of money cannot be further ignored due to the impact of inflation and high inflation and as a result the purchasing power of money is greatly reduced. Initially, To alleviate the belief in the absence of inflation outcomes, Buzacott[3] discussed EOQ models with continuous demand and a single inflation rate across all related. Bierman and Thomas[5] developed an inventory model under the inflationary conditions. Misra[6] introduced an inventory model for different inflation rates with various costs. Bose et al.[11] formulated an EOQ inventory model with inflation and time discounting. Later on, Yang et al.[15] gives a inventory models under inflation with fluctuating demand. Singh et al.[19] developed a model for depreciation of deficits and demand based on stocks under inflation in two stores under one management. Singh et al.[22], provided a modified model of two deteriorating warehouses with need based on socks and shortages. Singh and Sharma[23] have explored the effects of inflation in inventory models. Chakraborty et al.[27] developed two-warehouse partial backlogging inventory model with ramp type demand rate, three-parameter Weibull distribution deterioration under inflation and permissible delay in payments.

We have constructed a listing model in which the depreciation rate is time dependant and the shortage is partially accounted for as a backlog in the current paper. A ramp-type demand function of time is assumed. Inflationary effects are also considered. The suggested model is subjected to a thorough sensitivity analysis, which is demonstrated using numerical examples. The total cost function's convexity per unit time is depicted graphically.

2. Notations and Assumptions

The notations in the model are used as:

1. $I_O(t)$ and $I_R(t)$ is having the OW and RW inventory over time t $(0 \le t \le T)$ and $(0 \le t \le x_1)$ respectively.
2. In OW and RW, t_1 and x_1 are the times when the inventory level becomes zero.
3. The inflation rate is r and capacity of OW is W.
4. In RW and OW, holding cost of inventory per unit item per unit time are c_1 and c_2 respectively.
5. The deterioration and shortage costs per unit item per unit time are represented by c_3 and c_4, respectively.
6. Due to reduced sales, the opportunity cost per unit item is c_5.
7. Per replenishment cycle, the ordering cost is A_0 and μ represents the point at which rising demand stabilizes.

The assumptions of the developed model are as follow.

1. The cycle duration is T. There is no lead time and the replenishment rate is limitless.
2. Shortages are partially backlogged. The backlogging rate is $e^{-\delta t}$, where t is the constant waiting time until the next replenishment and δ is a positive constant backlogging parameter.
3. The rate of degradation is time-dependent. In RW, the degradation rate is $\theta_1(t) = \alpha t$ with $\alpha > 0$ as the deterioration rate parameter. The degradation rate in OW is $\theta_2(t) = \beta t$ with $\beta > 0$ as deterioration rate parameter.
4. $D(t)$ is a function of Ramp type demand and defined by $D(t) = \begin{cases} f(t) = a + bt, & t < \mu \\ f(\mu) = a + b\mu, & t \ge \mu \end{cases}$

where $f(t)$ is linear equation of time and $f(\mu)$ is the linear equation of μ, which is constant.

3. Formulation and Solution

The inventory level in RW depletes owing to demand and deterioration in the period $(0, x_1)$ and vanishes at $t = x_1$. In OW, the inventory level W drops owing to degradation solely in the period $(0, x_1)$ and the inventory level reduces owing to demand and deterioration throughout the interval (x_1, t_1) and vanishes at $t = t_1$. Following that, in the period (t_1, T) shortages start occurring and are partially backlogged with time varying backlogging rate. Therefore, the inventory levels in the RW and OW are determined by the differential equations given below at any time t in the period $(0, x_1)$

$$\frac{dI_R(t)}{dt} + \alpha t . I_R(t) = -D(t), \ 0 \le t \le x_1 \quad \textbf{……(1)}$$

By using B. C. (boundary condition) $I_R(x_1) = 0$

$$\frac{dI_O(t)}{dt} + \beta t . I_O(t) = 0, \quad 0 \le t \le x_1 \quad \textbf{……(2)}$$

By using I. C. (initial condition) $I_O(0) = W$

While, the inventory level in the OW is determined by the differential equation given below at any time t in the period (x_1, t_1):

$$\frac{dI_O(t)}{dt} + \beta t . I_O(t) = -D(t) \quad x_1 \le t \le t_1 \quad \textbf{……(3)}$$

By using B. C. (boundary condition) $I_O(t_1) = 0$

Similarly, the inventory level is determined by the differential equation given below at any time t in the period (t_1, T):

$$\frac{dI_O(t)}{dt} = -e^{-\delta(T-t)}D(t) \quad t_1 \le t \le T \qquad \text{......(4)}$$

By using B. C. (boundary condition) $I_O(t_1) = 0$

The following three cases may arise according to the position of μ:

Case I: $x_1 \le t_1 \le \mu$

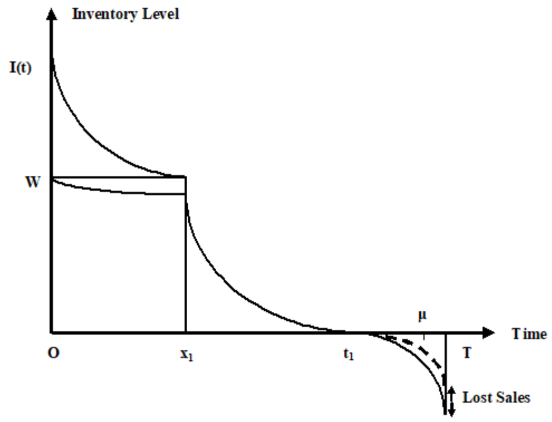


Fig. 1

The following equations (1) to (4) are defined as follows in this case:

$$\frac{dI_R(t)}{dt} + \alpha t . I_R(t) = -(a+bt), \ 0 \le t \le x_1 \qquad \text{......(5)}$$

By using B. C. (boundary condition) $I_R(x_1) = 0$

$$\frac{dI_O(t)}{dt} + \beta t . I_R(t) = 0, \qquad 0 \le t \le x_1 \qquad \text{......(6)}$$

By using I. C. (initial condition) $I_O(0) = W$

$$\frac{dI_O(t)}{dt} + \beta t . I_O(t) = -(a+bt), \ x_1 \le t \le t_1 \qquad \text{......(7)}$$

By using B. C. (boundary condition) $I_O(t_1) = 0$

$$\frac{dI_O(t)}{dt} = -e^{-\delta(T-t)}(a+bt), \quad t_1 \le t \le \mu \qquad \text{......(8)}$$

By using B. C. (boundary condition) $I_O(t_1) = 0$

$$\frac{dI_O(t)}{dt} = -e^{-\delta(T-t)}(a+b\mu), \ \mu \le t \le T \qquad \text{......(9)}$$

The solution to the above mentioned equations are as follows

$$I_R(t) = \Big[a(x_1 - t) + \frac{b}{2}(x_1^2 - t^2) + \frac{\alpha a}{6}(x_1^3 - t^3) + \frac{\alpha b}{8}(x_1^4 - t^4)\Big] e^{-\alpha t^2/2}, 0 \le t \le x_1 \qquad \text{......(10)}$$

$$I_O(t) = We^{-\beta t^2/2}, \quad 0 \le t \le x_1 \qquad \text{......(11)}$$

$$I_O(t) = \Big[a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{\beta a}{6}(t_1^3 - t^3) + \frac{\beta b}{8}({t_1}^4 - t^4)\Big] e^{-\beta t^2/2}, \ x_1 \le t \le t_1 \qquad \text{......(12)}$$

$$I_O(t) = -\Big[(1-\delta T)\Big\{a(t - t_1) + \frac{b}{2}(t^2 - t_1^2)\Big\} + \delta \Big\{\frac{a}{2}(t^2 - t_1^2) + \frac{b}{3}(t^3 - t_1^3)\Big\}\Big], t_1 \le t \le \mu \qquad \text{......(13)}$$

$$I_O(t) = -\Big[(a + b\mu)\Big\{(1-\delta T)(t-\mu) + \frac{\delta}{2}(t^2 - \mu^2)\Big\} + M\Big], \ \mu \le t \le T \qquad \text{......(14)}$$

where $M = (1-\delta T)\Big\{a(\mu - t_1) + \frac{b}{2}(\mu^2 - t_1^2)\Big\} + \delta\Big\{\frac{a}{2}(\mu^2 - t_1^2) + \frac{b}{3}(\mu^3 - t_1^3)\Big\}$

For the time period 0 to x_1, the present cost H_R (Holding cost for RW) is

$$H_R = c_1 \int_0^{x_1} I_R(t) e^{-rt} dt$$

$$H_R = c_1 \Big[\frac{a x_1^2}{2} + \frac{b x_1^3}{3} - r\Big(\frac{a x_1^3}{6} + \frac{b x_1^4}{8} + \frac{a\alpha x_1^5}{40} + \frac{b\alpha x_1^6}{48} - \frac{a\alpha^2 x_1^7}{112} - \frac{b\alpha^2 x_1^8}{128}\Big) + \alpha\Big(\frac{a x_1^4}{12} + \frac{b x_1^5}{15}\Big) - \frac{a\alpha^2 x_1^6}{72} - \frac{b\alpha^2 x_1^7}{84}\Big] \qquad \text{......(15)}$$

For the time period 0 to x_1, the present cost H_{O1} (Holding cost for OW) is

$$H_{O1} = c_2 \int_0^{x_1} I_O(t) e^{-rt} dt$$

$$H_{O1} = c_2 W \Big[x_1 - \frac{r x_1^2}{2} - \frac{\beta x_1^3}{6} + \frac{\beta r x_1^4}{8}\Big] \qquad \text{......(16)}$$

For the time period x_1 to t_1, the present cost H_{O2} (Holding cost for OW) is

$$H_{O2} = c_2 \int_{x1}^{t_1} I_O(t) e^{-rt} dt$$

$$H_{O2} = c_2 \Big[\Big(a t_1 + \frac{b t_1^2}{2} + \frac{a\beta t_1^3}{6} + \frac{b\beta t_1^4}{8}\Big)\Big((t_1 - x_1) - r\Big(\frac{(t_1^2 - x_1^2)}{2} - \frac{\beta(t_1^4 - x_1^4)}{8}\Big) - \frac{\beta}{2}\frac{(t_1^3 - x_1^3)}{3}\Big) + r\Big(\frac{a(t_1^3 - x_1^3)}{3} + \frac{b(t_1^4 - x_1^4)}{8} - \frac{a\beta(t_1^5 - x_1^5)}{15} - \frac{\beta(t_1^6 - x_1^6)}{48} - \frac{a\beta^2(t_1^7 - x_1^7)}{84} - \frac{b\beta^2(t_1^8 - x_1^8)}{128}\Big) + \frac{\beta}{2}\Big(\frac{a(t_1^4 - x_1^4)}{4} + \frac{b(t_1^5 - x_1^5)}{10} + \frac{a\beta(t_1^6 - x_1^6)}{36} + \frac{b\beta(t_1^7 - x_1^7)}{56}\Big) - \frac{a(t_1^2 - x_1^2)}{2} - \frac{b(t_1^3 - x_1^3)}{6} - \frac{a\beta(t_1^4 - x_1^4)}{24} - \frac{b\beta(t_1^5 - x_1^5)}{40}\Big] \qquad \text{......(17)}$$

Now, for the time period 0 to t_1, current cost H_O (Total holding cost for OW) is

$$H_O = H_{O1} + H_{O2} \qquad \text{......(18)}$$

For the time period 0 to x_1, the present value D_R (deterioration cost for RW) is

$$D_R = c_3 \int_0^{x_1} \alpha t I_R(t) e^{-rt} dt$$

$$D_R = c_3 \alpha \Big[\frac{a}{6} x_1^3 + (3b - 2ar)\frac{x_1^4}{24} + \Big(\frac{a\alpha}{40} - \frac{br}{15}\Big) x_1^5 + \Big(\frac{b}{48} - \frac{ar}{90}\Big)\alpha x_1^6 - \alpha\Big(\frac{1}{105} br + \frac{1}{112}\alpha a\Big) x_1^7 + \alpha^2\Big(\frac{1}{160} ar - \frac{1}{128} b\Big) x_1^8 + \Big(\frac{1}{180}\alpha^2 rb\Big) x_1^9\Big] \qquad \text{......(19)}$$

For the time period 0 to x_1, the present value D_{O1} (Deterioration cost for OW) is

$$D_{O1} = c_3 \int_0^{x_1} \beta t I_O(t) e^{-rt} dt$$

$$D_{O1} = c_3 W\beta\left[\frac{x_1^2}{2} - \frac{rx_1^3}{3} - \frac{\beta x_1^4}{8} + \frac{\beta r x_1^5}{10}\right] \qquad \text{......(20)}$$

For the time period x_1 to t_1, the present value D_{O2} (Deterioration cost for OW) is

$$D_{O2} = c_3 \int_{x1}^{t_1} \beta t I_O(t) e^{-rt} dt$$

$$D_{O2} = c_3\beta\left[\left(at_1 + \frac{bt_1^2}{2} + \frac{a\beta t_1^3}{6} + \frac{b\beta t_1^4}{8}\right)\left(\frac{(t_1^2 - x_1^2)}{2} - \frac{r(t_1^3 - x_1^3)}{3} - \frac{\beta(t_1^4 - x_1^4)}{8} + \frac{\beta r(t_1^5 - x_1^5)}{10}\right) - \frac{a}{3}(t_1^3 - x_1^3) + (2ar - b)\frac{(t_1^4 - x_1^4)}{8} + (2a\beta - 3br)\frac{(t_1^5 - x_1^5)}{30} - \beta\left(\frac{b}{48} - \frac{ar}{18}\right)(t_1^6 - x_1^6) - \beta\left(\frac{br}{56} - \frac{a\beta}{84}\right)(t_1^7 - x_1^7) - \beta^2\left(\frac{ar}{96} - \frac{b}{128}\right)(t_1^8 - x_1^8) - \left(\frac{\beta^2 rb}{144}\right)(t_1^9 - x_1^9)\right] \qquad \text{......(21)}$$

Now, for the time period 0 to t_1, the present cost D_O (Total deterioration cost for OW) is

$$D_O = D_{O1} + D_{O2} \qquad \text{.......(22)}$$

For the time period t_1 to μ, the present value Sh_{O1} (Shortage cost for OW) is

$$Sh_{O1} = -c_4 \int_{t_1}^{\mu} I_O(t) e^{-rt} dt$$

$$Sh_{O1} = c_4\left[(1 - \delta T)\left\{a\left(\frac{\mu^2}{2} - t_1\mu\right) + \frac{b}{2}\left(\frac{\mu^3}{3} - t_1^2\mu\right)\right\} + \delta\left\{\frac{a}{2}\left(\frac{\mu^3}{3} - t_1^2\mu\right) + \frac{b}{3}\left(\frac{\mu^4}{4} - t_1^3\mu\right)\right\} - r(1 - \delta T)\left\{a\left(\frac{\mu^3}{3} - \frac{t_1\mu^2}{2}\right) + \frac{b}{2}\left(\frac{\mu^4}{4} - \frac{t_1^2\mu^2}{2}\right)\right\} - r\delta\left\{\frac{a}{2}\left(\frac{\mu^4}{4} - \frac{t_1^2\mu^2}{2}\right) + \frac{b}{3}\left(\frac{\mu^5}{5} - \frac{t_1^3\mu^2}{2}\right)\right\} + (1 - \delta T)\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3}\right) + \delta\left(\frac{at_1^3}{3} + \frac{bt_1^4}{4}\right) - r(1 - \delta T)\left(\frac{at_1^3}{6} + \frac{bt_1^4}{4}\right) - r\delta\left(\frac{at_1^4}{4} + \frac{bt_1^5}{10}\right)\right] \qquad \text{......(23)}$$

For the time period μ to T, the present cost Sh_{O2} (Shortage cost for OW) is

$$Sh_{O2} = -c_4 \int_{\mu}^{T} I_O(t) e^{-rt} dt$$

$$Sh_{O2} = c_4\left[(a + b\mu)\left\{(1 - \delta T)\left(\frac{T^2}{2} - T\mu\right) + \frac{\delta}{2}\left(\frac{T^3}{3} - \mu^2 T\right)\right\} + M(T - \mu) - \frac{rM(T^2 - \mu^2)}{2} - r(a + b\mu)\left\{(1 - \delta T)\left(\frac{T^3}{3} - \frac{\mu T^2}{2}\right) + \frac{\delta}{2}\left(\frac{T^4}{4} - \frac{\mu^2 T^2}{2}\right)\right\} + (a + b\mu)\left\{(1 - \delta T)\frac{\mu^2}{2} + \frac{\delta\mu^3}{3}\right\} - r(a + b\mu)\left\{(1 - \delta T)\frac{\mu^3}{6} + \frac{\delta\mu^4}{4}\right\}\right] \qquad \text{......(24)}$$

Now, for the time period t_1 to T, the present value Sh_O (Total shortage cost for OW) is

$$Sh_O = Sh_{O1} + Sh_{O2} \qquad \text{......(25)}$$

Due to partial backlogging, the present value L_{S1} (Lost sales cost) for the time interval t_1 to μ is

$$L_{S1} = c_5\left[\int_{t_1}^{\mu}\left(1 - e^{-\delta(T-t)}\right)(a + bt)e^{-rt} dt\right]$$

$$L_{S1} = c_5\left[\delta a\left(T(\mu - t_1) - \frac{(1+rT)(\mu^2 - t_1^2)}{2} + \frac{r(\mu^3 - t_1^3)}{3}\right) + \delta b\left(\frac{T(\mu^2 - t_1^2)}{2} - \frac{(1+rT)(\mu^3 - t_1^3)}{3} + \frac{r(\mu^4 - t_1^4)}{4}\right)\right] \qquad \text{......(26)}$$

The present value L_{S2} (Lost sales cost) for the time interval μ to T is

$$L_{S2} = c_5\left[\int_{\mu}^{T}\left(1 - e^{-\delta(T-t)}\right)(a + b\mu)e^{-rt} dt\right]$$

$$L_{S2} = c_5\delta(a + b\mu)\left[\left(\frac{T^2}{2} - \frac{rT^3}{6}\right) - \left(T\mu - \frac{\mu^2}{2} - \frac{rT\mu^2}{2} + \frac{r\mu^3}{3}\right)\right] \qquad \text{......(27)}$$

The present value L_S (Total lost sales cost) for the time interval t_1 to T is

$$L_S = L_{S1} + L_{S2} \qquad \text{......(28)}$$

The current cost of total inventory per unit of time can be calculated as follows:

$$T_{C1}(t_1, T) = \frac{1}{T}\left[A_0 + H_R + H_O + D_R + D_O + Sh_O + L_S\right] \qquad \text{......(29)}$$

Case II: $x_1 \le \mu \le t_1$

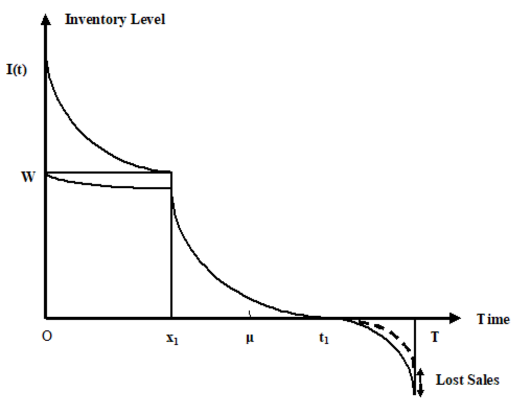


Fig. 2

The following equations (1) to (4) are defined as follows in this case:

$$\frac{dI_R(t)}{dt} + \alpha t . I_R(t) = -(a + bt), 0 \le t \le x_1 \qquad \text{......(30)}$$

By using B. C. (boundary condition) $I_R(x_1) = 0$

$$\frac{dI_O(t)}{dt} + \beta t . I_O(t) = 0, 0 \le t \le x_1 \qquad \text{......(31)}$$

By using I. C. (initial condition) $I_O(0) = W$

$$\frac{dI_O(t)}{dt} + \beta t . I_O(t) = -(a + bt), \ x_1 \le t \le \mu \qquad \text{......(32)}$$

$\frac{dI_O(t)}{dt} + \beta t.I_O(t) = -(a+b\mu),\ \mu \le t \le t_1$ **……(33)**

By using B. C. (boundary condition) $I_O(t_1) = 0$

$\frac{dI_O(t)}{dt} = e^{-\delta(T-t)}(a+b\mu),\ t_1 \le t \le T$ **……(34)**

By using B. C. (boundary condition) $I_O(t_1) = 0$

The solution to the above mentioned equations are as follows

$I_R(t) = \left[a(x_1 - t) + \frac{b}{2}(x_1^2 - t^2) + \frac{\alpha a}{6}(x_1^3 - t^3) + \frac{\alpha b}{8}({x_1}^4 - t^4)\right] e^{-\alpha t^2/2}, 0 \le t \le x_1$ **……(35)**

$I_O(t) = We^{-\beta t^2/2}, 0 \le t \le x_1$ **……(36)**

$I_O(t) = \left[W + a(x_1 - t) + \frac{b}{2}({x_1}^2 - t^2) + \frac{\beta a}{6}({x_1}^3 - t^3) + \frac{\beta b}{8}({x_1}^4 - t^4)\right] e^{-\beta t^2/2}, x_1 \le t \le \mu$ **……(37)**

$I_O(t) = \left[(a+b\mu)\left((t_1 - t) + \frac{\beta}{6}(t_1^3 - t^3)\right)\right] e^{-\beta t^2/2},$

$\mu \le t \le t_1$ **……(38)**

$I_O(t) = -(a+b\mu)\left[(1-\delta T)(t - t_1) + \frac{\delta}{2}(t^2 - {t_1}^2)\right], t_1 \le t \le T$ **……(39)**

For the time period 0 to x_1, the present cost H_R (Holding cost for RW) is

$H_R = c_1 \int_0^{x_1} I_R(t) e^{-rt} dt$

$$H_R = \left[\frac{ac_1x_1^2}{2} + \left(\frac{bc_1x_1^3}{3}\right) + (\eta_1 - rc_1)\left(\frac{ax_1^3}{6} + \frac{bx_1^4}{8} + \frac{a\alpha x_1^5}{40} + \frac{b\alpha x_1^6}{48} - \frac{a\alpha^2x_1^7}{112} - \frac{b\alpha^2x_1^8}{128}\right) + (\alpha c_1 - r\eta_1)\left(\frac{ax_1^4}{12} + \frac{bx_1^5}{15}\right) - \frac{a\alpha^2c_1x_1^6}{72} - \frac{b\alpha^2c_1x_1^7}{84} - \frac{a\alpha r\eta_1x_1^6}{90} - \frac{b\alpha r\eta_1x_1^7}{105} + \frac{a\alpha^2r\eta_1x_1^8}{160} + \frac{b\alpha^2r\eta_1x_1^9}{180}\right] \quad \textbf{……(40)}$$

For the time period 0 to x_1, the present cost H_{O1} (Holding cost for OW) is

$H_{O1} = c_2 \int_0^{x_1} I_O(t) e^{-rt} dt$

$$H_{O1} = c_2W\left[x_1 - \frac{rx_1^2}{2} - \frac{\beta x_1^3}{6} + \frac{\beta rx_1^4}{8}\right] \quad \textbf{……(41)}$$

For the time period x_1 to μ, the present cost H_{O2} (Holding cost for OW) is

$H_{O2} = c_2 \int_{x1}^{\mu} I_O(t) e^{-rt} dt$

$$H_{O2} = c_2\left[\left(W + ax_1 + \frac{bx_1^2}{2} + \frac{a\beta x_1^3}{6} + \frac{b\beta x_1^4}{8}\right)\left((\mu - x_1) - r\left(\frac{(\mu^2 - x_1^2)}{2} - \frac{\beta(\mu^4 - x_1^4)}{8}\right) - \frac{\beta(\mu^3 - x_1^3)}{6}\right) + r\left(\frac{a(\mu^3 - x_1^3)}{3} + \frac{b(\mu^4 - x_1^4)}{8} - \frac{a\beta(\mu^5 - x_1^5)}{15} - \frac{\beta(\mu^6 - x_1^6)}{48} - \frac{a\beta^2(\mu^7 - x_1^7)}{84} - \frac{b\beta^2(\mu^8 - x_1^8)}{128}\right) + \frac{\beta}{2}\left(\frac{a(\mu^4 - x_1^4)}{4} + \frac{b(\mu^5 - x_1^5)}{10} + \frac{a\beta(\mu^6 - x_1^6)}{36} + \frac{b\beta(\mu^7 - x_1^7)}{56}\right) - \frac{a(\mu^2 - x_1^2)}{2} - \frac{b(\mu^3 - x_1^3)}{6} - \frac{a\beta(\mu^4 - x_1^4)}{24} - \frac{b\beta(\mu^5 - x_1^5)}{40}\right]$$

……(42)

For the time period μ to t_1, the present cost H_{O3} (Holding cost for OW) is

$$H_{O3} = c_2 \int_{\mu}^{t_1} I_O(t) e^{-rt} dt$$

$$H_{O3} = c_2(a+b\mu)\left[\left(t_1 + \frac{\beta t_1^3}{6}\right)\left((t_1 - \mu) - c_2r\left(\frac{(t_1^2 - \mu^2)}{2} - \frac{\beta(t_1^4 - \mu^4)}{8}\right) - \frac{\beta(t_1^3 - \mu^3)}{6}\right) + r\left(\frac{(t_1^3 - \mu^3)}{3} - \frac{\beta(t_1^5 - \mu^5)}{15} - \frac{\beta^2(t_1^7 - \mu^7)}{84}\right) + \frac{\beta}{2}\left(\frac{(t_1^4 - \mu^4)}{4} + \frac{\beta(t_1^6 - \mu^6)}{36}\right) - \frac{a(t_1^2 - \mu^2)}{2} - \frac{\beta(t_1^4 - \mu^4)}{24}\right] \quad \textbf{……(43)}$$

Now, for the time period 0 to t_1, current cost H_O (Total holding cost for OW) is

$H_O = H_{O1} + H_{O2} + H_{O3}$ **……(44)**

For the time period 0 to x_1, the present value D_R (Deterioration cost for RW) is

$$D_R = c_3 \int_0^{x_1} \alpha t I_R(t) e^{-rt} dt$$

$$D_R = c_3\alpha\left[\frac{a}{6}x_1^3 + (3b - 2ar)\frac{x_1^4}{24} + \left(\frac{a\alpha}{40} - \frac{br}{15}\right)x_1^5 + \left(\frac{b}{48} - \frac{ar}{90}\right)\alpha x_1^6 - \alpha\left(\frac{1}{105}br + \frac{1}{112}\alpha a\right)x_1^7 + \alpha^2\left(\frac{1}{160}ar - \frac{1}{128}b\right)x_1^8 + \left(\frac{1}{180}\alpha^2 rb\right)x_1^9\right] \quad \textbf{……(45)}$$

For the time period 0 to x_1, the present value D_{O1} (Deterioration cost for OW) is

$D_{O1} = c_3 \int_0^{x_1} \beta t I_O(t) e^{-rt} dt$

$$D_{O1} = c_3W\beta\left[\frac{x_1^2}{2} - \frac{rx_1^3}{3} - \frac{\beta x_1^4}{8} + \frac{\beta rx_1^5}{10}\right] \quad \textbf{……(46)}$$

For the time period x_1 to μ, the present value D_{O2} (Deterioration cost for OW) is

$D_{O2} = c_3 \int_{x1}^{\mu} \beta t I_O(t) e^{-rt} dt$

$$D_{O2} = c_3\beta\left[\left(W + ax_1 + \frac{bx_1^2}{2} + \frac{a\beta x_1^3}{6} + \frac{b\beta x_1^4}{8}\right)\left(\frac{(\mu^2 - x_1^2)}{2} - \frac{r(\mu^3 - x_1^3)}{3} - \frac{\beta(\mu^4 - x_1^4)}{8} + \frac{\beta r(\mu^5 - x_1^5)}{10}\right) - \frac{a}{3}(\mu^3 - x_1^3) + (2ar - b)\frac{(\mu^4 - x_1^4)}{8} + (2a\beta - 3br)\frac{(\mu^5 - x_1^5)}{30} - \beta\left(\frac{b}{48} - \right.\right.$$

$$\left.\frac{ar}{18}\right)(\mu^6 - x_1^6) - \beta\left(\frac{br}{56} - \frac{a\beta}{84}\right)(\mu^7 - x_1^7) - \beta^2\left(\frac{ar}{96} - \frac{b}{128}\right)(\mu^8 - x_1^8) - \left(\frac{\beta^2 rb}{144}\right)(\mu^9 - x_1^9)\Bigg] \qquad \textbf{......(47)}$$

For the time period μ to t_1, the present value D_{O3} (Deterioration cost for OW) is

$$D_{O3} = c_3\int_{\mu}^{t_1} \beta t I_O(t)e^{-rt}dt$$

$$D_{O3} = c_3\beta(a + b\mu)\left[\left(t_1 + \frac{\beta t_1^3}{6}\right)\left(\frac{(t_1^2 - \mu^2)}{2} - \frac{r(t_1^3 - \mu^3)}{3} - \frac{\beta(t_1^4 - \mu^4)}{8} + \frac{\beta r(t_1^5 - \mu^5)}{10}\right) - \frac{(t_1^3 - \mu^3)}{3} + \frac{r(t_1^4 - \mu^4)}{4} + \frac{\beta(t_1^5 - \mu^5)}{15} - \frac{\beta r(t_1^6 - \mu^6)}{18} + \frac{\beta^2(t_1^7 - \mu^7)}{84} - \frac{\beta^2 r(t_1^8 - \mu^8)}{96}\right] \qquad \textbf{......(48)}$$

Now, for the time period 0 to t_1, the present cost D_O (Total deterioration cost for OW) is

$$D_O = D_{O1} + D_{O2} + D_{O3} \qquad \textbf{......(49)}$$

For the time period t_1 to T, the present value Sh_O (Shortage cost for OW) is

$$Sh_O = -c_4\int_{t_1}^{T} I_O(t)e^{-rt}dt$$

$$Sh_O = c_4(a + b\mu)\left[(1 - \delta T)\left(\frac{(T^2 + t_1^2)}{2} - t_1 T - \frac{rT^3}{3} + \frac{rt_1T^2}{2} - \frac{rt_1^3}{6}\right) + \frac{\delta}{2}\left(\frac{(T^3 + 2t_1^3)}{3} - t_1^2 T - \frac{r(T^4 + t_1^4)}{4} + \frac{rt_1^2T^2}{2}\right)\right] \qquad \textbf{......(50)}$$

Due to partial backlogging, the present value L_S (Lost sales cost) for the time period t_1 to T is

$$L_S = c_5\int_{t_1}^{T}\left(1 - e^{-\delta(T-t)}\right)(a + b\mu)e^{-rt}dt$$

$$L_S = c_5\delta(a + b\mu)\left[\frac{(T^2 + (1 + rT)t_1^2)}{2} - \frac{r(T^3 + 2t_1^3)}{6} - t_1 T\right] \qquad \textbf{......(51)}$$

The current cost of total inventory per unit of time can be calculated as follows:

$$T_{C2}(t_1, T) = \frac{1}{T}[A_0 + H_R + H_O + D_R + D_O + Sh_O + L_S] \qquad \textbf{......(52)}$$

Case III: $\mu \le x_1 \le t_1$

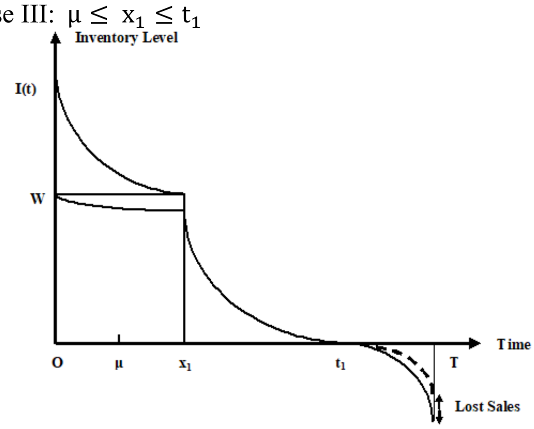


Fig. 3

The following equations (1) to (4) are defined as follows in this case:

$$\frac{dI_R(t)}{dt} + \alpha t.I_R(t) = -(a + bt), 0 \le t \le \mu \qquad \textbf{......(53)}$$

$$\frac{dI_R(t)}{dt} + \alpha t.I_R(t) = -(a + b\mu), \mu \le t \le x_1 \qquad \textbf{......(54)}$$

By using B. C. (boundary condition) $I_R(x_1) = 0$

$$\frac{dI_O(t)}{dt} + \beta t.I_O(t) = 0, \quad 0 \le t \le \mu \qquad \textbf{......(55)}$$

By using I. C. (initial condition) $I_O(0) = W$

$$\frac{dI_O(t)}{dt} + \beta t.I_O(t) = 0, \quad \mu \le t \le x_1 \qquad \textbf{......(56)}$$

$$\frac{dI_O(t)}{dt} + \beta t.I_O(t) = -(a + b\mu), x_1 \le t \le t_1 \qquad \textbf{......(57)}$$

By using B. C. (boundary condition) $I_O(t_1) = 0$

$$\frac{dI_O(t)}{dt} = -e^{-\delta(T-t)}(a + b\mu), t_1 \le t \le T \qquad \textbf{......(58)}$$

By using B. C. (boundary condition) $I_O(t_1) = 0$

The solution to the above mentioned equations are as follows

$$I_R(t) = \left[(a + b\mu)\left(x_1 + \frac{\alpha x_1^3}{6}\right) - \frac{b\mu^2}{2} - \frac{\alpha b\mu^4}{24} - \left(at + \frac{bt^2}{2} + \frac{\alpha at^3}{6} + \frac{\alpha bt^4}{8}\right)\right]e^{-\alpha t^2/2}, 0 \le t \le \mu \qquad \textbf{......(59)}$$

$$I_R(t) = \left[(a + b\mu)\left((x_1 - t) + \frac{\alpha}{6}(x_1^3 - t^3)\right)\right]e^{-\alpha t^2/2}, \mu \le t \le x_1 \qquad \textbf{......(60)}$$

$$I_O(t) = We^{-\beta t^2/2}, 0 \le t \le \mu \qquad \textbf{......(61)}$$

$$I_O(t) = \left[W + (a + b\mu)\left((\mu - t) + \frac{\beta}{6}(\mu^3 - t^3)\right)\right]e^{-\beta t^2/2}, \mu \le t \le x_1 \qquad \textbf{......(62)}$$

$$I_O(t) = \left[(a + b\mu)\left((t_1 - t) + \frac{\beta}{6}(t_1^3 - t^3)\right)\right]e^{-\beta t^2/2}, x_1 \le t \le t_1 \qquad \textbf{......(63)}$$

$$I_O(t) = -(a + b\mu)\left[(1 - \delta T)(t - t_1) + \frac{\delta}{2}(t^2 - t_1^2)\right], t_1 \le t \le T \qquad \textbf{......(64)}$$

For the time period 0 to μ, the present cost H_{R1} (Holding cost for RW) is

$$H_{R1} = c_1\int_0^{\mu} I_R(t)e^{-rt}dt$$

$$H_{R1} = c_1\left[\left((a + b\mu)\left(x_1 + \frac{\alpha x_1^3}{6}\right) - \frac{b\mu^2}{2} - \frac{\alpha b\mu^4}{24}\right)\left(\mu - r\left(\frac{\mu^2}{2} - \frac{\alpha\mu^4}{8}\right) + \frac{\alpha\mu^3}{6}\right) - \frac{a\mu^2}{2} - \frac{b\mu^3}{6} - \frac{a\alpha\mu^4}{24} - \frac{b\alpha\mu^5}{40} + r\left(\frac{a\mu^3}{6} + \frac{b\mu^4}{8} - \frac{a\alpha\mu^5}{15} - \frac{a\alpha\mu^6}{48} - \frac{a\alpha^2\mu^7}{84} - \frac{b\alpha^2\mu^8}{128}\right) + \frac{\alpha}{2}\left(\frac{a\mu^4}{4} + \frac{b\mu^5}{10} + \frac{a\alpha\mu^6}{36} + \frac{b\alpha\mu^7}{56}\right)\right] \qquad \textbf{......(65)}$$

For the time period μ to x_1, the present cost H_{R2} (Holding cost for RW) is

$$H_{R2} = c_1 \int_{\mu}^{x_1} I_R(t)e^{-rt}dt$$

$$H_{R2} = c_1(a+b\mu)\left[\left(t_1+\frac{\alpha t_1^3}{6}\right)\left((x_1-\mu) - r\left(\frac{(x_1^2-\mu^2)}{2} - \frac{\alpha(x_1^4-\mu^4)}{8}\right) - \frac{\alpha(x_1^3-\mu^3)}{3}\right) + r\left(\frac{(x_1^3-\mu^3)}{3} - \frac{\alpha(x_1^5-\mu^5)}{15} - \frac{\alpha^2(x_1^7-\mu^7)}{84}\right) + \frac{\alpha}{2}\left(\frac{(x_1^4-\mu^4)}{4} + \frac{\alpha(x_1^6-\mu^6)}{36}\right) - \frac{a(x_1^2-\mu^2)}{2} - \frac{\alpha(x_1^4-\mu^4)}{24}\right] \quad \textbf{......(66)}$$

Now, for the time period 0 to x_1, current cost H_R (Total holding cost for RW) is

$$H_R = H_{R1} + H_{R2} \quad \textbf{......(67)}$$

For the time period 0 to μ, the present cost H_{O1} (Holding cost for OW) is

$$H_{O1} = c_2 \int_0^{\mu} I_O(t)e^{-rt}dt$$

$$H_{O1} = W\left[c_2\mu + (\eta_2 - c_2 r)\frac{\mu^2}{2} - \left(\eta_2 r + \frac{c_2\beta}{2}\right)\frac{\mu^3}{3} - (\eta_2\beta + c_2\beta r)\frac{\mu^4}{8} + \frac{\eta_2\beta r\mu^5}{10}\right] \quad \textbf{......(68)}$$

For the time period μ to x_1, the present cost H_{O2} (Holding cost for OW) is

$$H_{O2} = c_2 \int_{\mu}^{x_1} I_O(t)e^{-rt}dt$$

$$H_{O2} = c_2\left[\left(W + (a+b\mu)\left(t_1+\frac{\beta t_1^3}{6}\right)\right)\left((x_1-\mu) - r\left(\frac{(x_1^2-\mu^2)}{2} - \frac{\beta(x_1^4-\mu^4)}{8}\right) - \frac{\beta(x_1^3-\mu^3)}{6}\right) + (a+b\mu)\left(r\left(\frac{(x_1^3-\mu^3)}{3} - \frac{\beta(x_1^5-\mu^5)}{15} - \frac{\beta^2(x_1^7-\mu^7)}{84}\right) + \frac{\beta}{2}\left(\frac{(x_1^4-\mu^4)}{4} + \frac{\beta(x_1^6-\mu^6)}{36}\right) - \frac{a(x_1^2-\mu^2)}{2} - \frac{\beta(x_1^4-\mu^4)}{24}\right)\right] \textbf{......(69)}$$

For the time period x_1 to t_1 the present cost H_{O3} (Holding cost for OW) is

$$H_{O3} = \int_{x_1}^{t_1} I_O(t)e^{-rt}dt$$

$$H_{O3} = c_2(a+b\mu)\left[\left(t_1+\frac{\beta t_1^3}{6}\right)\left((t_1-x_1) - r\left(\frac{(t_1^2-{x_1}^2)}{2} - \frac{\beta(t_1^4-{x_1}^4)}{8}\right) - \frac{\beta(t_1^3-{x_1}^3)}{6}\right) + r\left(\frac{(t_1^3-{x_1}^3)}{3} - \frac{\beta(t_1^5-{x_1}^5)}{15} - \frac{\beta^2(t_1^7-{x_1}^7)}{84}\right) + \frac{\beta}{2}\left(\frac{(t_1^4-{x_1}^4)}{4} + \frac{\beta(t_1^6-{x_1}^6)}{36}\right) - \frac{a(t_1^2-{x_1}^2)}{2} - \frac{\beta(t_1^4-{x_1}^4)}{24}\right] \quad \textbf{......(70)}$$

Now, for the time period 0 to t_1, current cost H_O (Total holding cost for OW) is

$$H_O = H_{O1} + H_{O2} + H_{O3} \quad \textbf{......(71)}$$

For the time interval 0 to μ, the present value D_{R1} (Deterioration cost for RW) is

$$D_{R1} = c_3 \int_0^{\mu} \alpha t I_R(t)e^{-rt}dt$$

$$D_{R1} = c_3\alpha\left[\left((a+b\mu)\left(x_1+\frac{\alpha {x_1}^3}{6}\right) - \frac{b\mu^2}{2} - \frac{\alpha b\mu^4}{24}\right)\left(\frac{\mu^2}{2} - \frac{r\mu^3}{3} - \frac{\alpha\mu^4}{8} + \frac{\alpha r\mu^5}{10}\right) - \frac{a\mu^3}{3} + (2ar-b)\frac{\mu^4}{8} + (3br - 2a\alpha)\frac{\mu^5}{30} + \left(\frac{b}{48} - \frac{ar}{18}\right)\alpha\mu^6 + \left(\frac{a\alpha}{84} - \frac{br}{56}\right)\alpha\mu^7 + \alpha^2\left(\frac{b}{256} - \frac{ar}{96}\right)\mu^8 - \left(\frac{\alpha^2 rb}{144}\right)x_1^9\right] \quad \textbf{......(72)}$$

For the time interval μ to x_1, the present value D_{R2} (Deterioration cost for RW) is

$$D_{R2} = c_3 \int_{\mu}^{x_1} \alpha t I_R(t)e^{-rt}dt$$

$$D_{R2} = c_3\alpha(a+b\mu)\left[\left(x_1+\frac{\alpha x_1^3}{6}\right)\left(\frac{({x_1}^2-\mu^2)}{2} - \frac{r({x_1}^3-\mu^3)}{3} - \frac{\alpha({x_1}^4-\mu^4)}{8} + \frac{\alpha r({x_1}^5-\mu^5)}{10}\right) - \frac{({x_1}^3-\mu^3)}{3} + \frac{r({x_1}^4-\mu^4)}{4} + \frac{\alpha({x_1}^5-\mu^5)}{15} - \frac{\alpha r({x_1}^6-\mu^6)}{18} + \frac{\alpha^2({x_1}^7-\mu^7)}{84} - \frac{\alpha^2 r({x_1}^8-\mu^8)}{96}\right] \quad \textbf{......(73)}$$

Now, for the time period 0 to x_1, the present cost D_R (Total deterioration cost for RW) is

$$D_R = D_{R1} + D_{R2} \quad \textbf{......(74)}$$

For the time interval 0 to μ, the present value D_{O1} (Deterioration cost for OW) is

$$D_{O1} = c_3 \int_0^{\mu} \beta t I_O(t)e^{-rt}dt$$

$$D_{O1} = c_3 W\beta\left[\frac{\mu^2}{2} - \frac{r\mu^3}{3} - \frac{\beta\mu^4}{8} + \frac{\beta r\mu^5}{10}\right] \quad \textbf{......(75)}$$

For the time interval μ to x_1, the present value D_{O2} (Deterioration cost for OW) is

$$D_{O2} = c_3 \int_{\mu}^{x_1} \beta t I_O(t)e^{-rt}dt$$

$$D_{O2} = c_3\beta\left[\left(W + (a+b\mu)\left(\mu+\frac{\beta\mu^3}{6}\right)\right)\left(\frac{({x_1}^2-\mu^2)}{2} - \frac{r({x_1}^3-\mu^3)}{3} - \frac{\beta({x_1}^4-\mu^4)}{8} + \frac{\beta r({x_1}^5-\mu^5)}{10}\right) - (a+\right.$$

$$b\mu)\left(\frac{(x_1{}^3-\mu^3)}{3}-\frac{r(x_1{}^4-\mu^4)}{4}-\frac{\beta(x_1{}^5-\mu^5)}{15}+\frac{\beta r(x_1{}^6-\mu^6)}{18}-\frac{\beta^2(x_1{}^7-\mu^7)}{84}+\frac{\beta^2 r(x_1{}^8-\mu^8)}{96}\right)\Bigg] \quad \textbf{......(76)}$$

For the time interval x_1 to t_1, the present value D_{O3} (Deterioration cost for OW) is

$$D_{O3}=c_3\int_{x_1}^{t_1}\beta t I_O(t)e^{-rt}dt$$

$$D_{O3}=c_3\beta(a+b\mu)\left[\left(t_1+\frac{\beta t_1^3}{6}\right)\left(\frac{(t_1{}^2-x_1{}^2)}{2}-\frac{r(t_1{}^3-x_1{}^3)}{3}-\frac{\beta(t_1{}^4-x_1{}^4)}{8}+\frac{\beta r(t_1{}^5-x_1{}^5)}{10}\right)-\frac{(t_1{}^3-x_1{}^3)}{3}+\frac{r(t_1{}^4-x_1{}^4)}{4}+\frac{\beta(t_1{}^5-x_1{}^5)}{15}-\frac{\beta r(t_1{}^6-x_1{}^6)}{18}+\frac{\beta^2(t_1{}^7-x_1{}^7)}{84}-\frac{\beta^2 r(t_1{}^8-x_1{}^8)}{96}\right]$$

......(77)

Now, for the time period 0 to t_1, the present cost D_O (Total deterioration cost for OW) is

$$D_O=D_{O1}+D_{O2}+D_{O3} \quad \textbf{......(78)}$$

For the time period t_1 to T, the present value Sh_O (Shortage cost for OW) is

$$Sh_O=-c_4\int_{t_1}^{T}I_O(t)e^{-rt}dt$$

$$Sh_O=c_4(a+b\mu)\left[(1-\delta T)\left(\frac{(T^2+t_1^2)}{2}-t_1T-\frac{rT^3}{3}+\frac{rt_1T^2}{2}-\frac{rt_1{}^3}{6}\right)+\frac{\delta}{2}\left(\frac{(T^3+2t_1^3)}{3}-t_1^2T-\frac{r(T^4+t_1^4)}{4}+\frac{rt_1^2T^2}{2}\right)\right]$$

......(79)

Due to partial backlogging, the present value L_S (Lost sales cost) for the time interval t_1 to T is

$$L_S=c_5\int_{t_1}^{T}\left(1-e^{-\delta(T-t)}\right)(a+b\mu)e^{-rt}dt$$

$$L_S=c_5\delta(a+b\mu)\left[\frac{(T^2+(1+rT)t_1^2)}{2}-\frac{r(T^3+2t_1^3)}{6}-t_1T\right]$$

......(80)

The current cost of total inventory per unit of time can be calculated as follows:

$$T_{C3}(t_1,T)=\frac{1}{T}[A_O+H_R+H_O+D_R+D_O+Sh_O+L_S]$$

......(81)

Now, the current cost of total Inventory per unit time is

$$T_C(t_1,T)=min\begin{cases}T_{C1}(t_1,T), x_1\le t_1\le\mu\\ T_{C2}(t_1,T), x_1\le\mu\le t_1\\ T_{C3}(t_1,T), \mu\le x_1\le t_1\end{cases} \quad \textbf{......(82)}$$

Our primary goal is to reduce the total cost function $T_C(t_1,T)$. The necessary conditions for minimizing total inventory costs are

$$\frac{\partial T_{Ci}(t_1,T)}{\partial T}=0 \text{ and } \frac{\partial T_{Ci}(t_1,T)}{\partial t_1}=0, \text{ where i = 1, 2, 3}$$

......(83)

By using the software MATHEMATICA-5.2, Equation (83) and (82) are used to calculate the optimal values t_1* and T* as well as the optimal value $T_C(t_1,T)$ of the total inventory cost. On the other hand, the optimal values t_1* and T*, satisfy the necessary conditions for lowering the cost of total inventory $T_C(t_1,T)$ given as

$$\left.\frac{\partial^2 T_C(t_1,T)}{\partial t_1^2}\right|_{(t_1^*,T^*)}>0, \left.\frac{\partial^2 T_C(t_1,T)}{\partial T^2}\right|_{(t_1^*,T^*)}>0 \text{ and}$$

$$\left.\left[\frac{\partial^2 T_C(t_1,T)}{\partial t_1^2}\frac{\partial^2 T_C(t_1,T)}{\partial T^2}-\left(\frac{\partial^2 T_C(t_1,T)}{\partial t_1\partial T}\right)^2\right]\right|_{(t_1^*,T^*)}>0$$

4. Numerical Example

Example: Let us assume r = 0.1, A = 600, W = 100, a = 175, b = 2, x_1 = 0.8, c_1 = 1.7, η_1 = 0.05, c_2 = 1.5, α = 0.01, η_2 = 0.06, $\beta = 0.02$, μ = 0.95, $c_3 = 1.3$, c_4 = 3, c_5 = 6, δ = 0.55.

We determine the optimal values of $T_C(t_1,T)$, t_1* and T* using the software Mathematica-5.2 with the above input data:

$T_C(t_1,T)$= 3421.65, t_1* = 1.374, T* = 1.719 and Fig. 4 shows the total cost function's convexity.

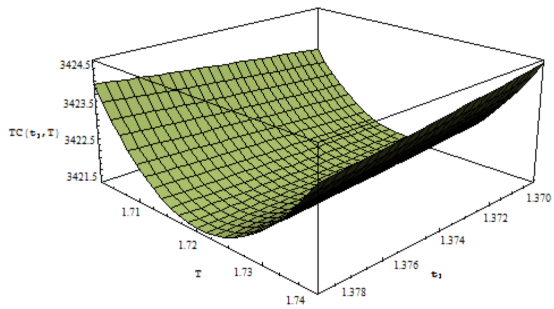


Fig. 4

5. Sensitivity Analysis

In light of the numerical example presented above, a sensitivity analysis is conducted. We executed a sensitivity analysis by modifying the parameters a, b, α, β and r by -25%, -50%, +25% and +50% respectively. With respect to these changes, the remaining parameters have their original values. The associated optimal values of $t_1{}^0$, T^0 and $T_C{}^0(t_1,T)$ are calculated. The PCI (percentage cost increase) is

$$PCI=\frac{T_C{}^0(t_1,T)-T_C{}^*(t_1,T)}{T_C{}^*(t_1,T)}\times 100\%$$

The PCI of the parameters is numerically and graphically shown below:

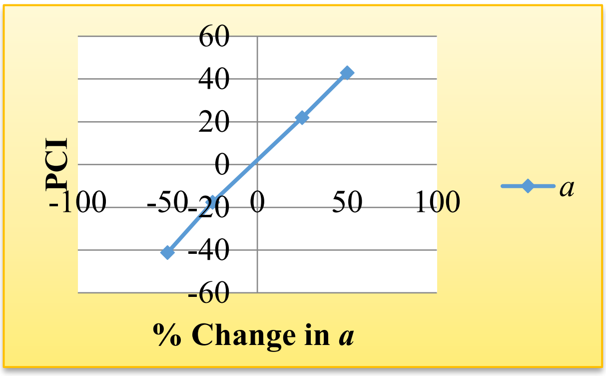
Table 5.1: For the % change in parameter ‘*a*’

a	**t₁**	**T**	$T_C(t_1, T)$	**PCI %**
-50	1.395	1.702	2012.20	-41.1921
-25	1.384	1.712	2714.84	-17.6570
+25	1.367	1.733	4170.79	21.8941
+50	1.353	1.746	4890.40	42.9252

Table 5.3: For the % change in parameter ‘*α*’

α	**t₁**	**T**	$T_C(t_1, T)$	**PCI %**
-50	1.382	1.705	3366.28	-1.6182
-25	1.378	1.713	3393.65	-0.8183
+25	1.371	1.727	3450.39	0.8400
+50	1.367	1.735	3477.82	1.6416

Graph 5.1: For the % change in parameter ‘*a*’



Graph 5.3: For the % change in parameter ‘*α*’

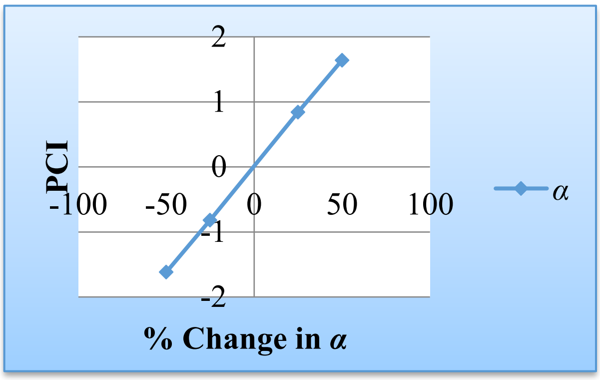


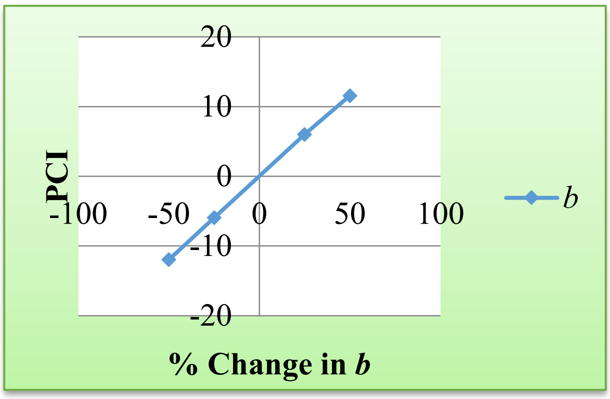
Table 5.2: For the % change in parameter ‘*b*’

b	**t₁**	**T**	$T_C(t_1, T)$	**PCI %**
-50	1.387	1.703	3012.40	-11.9606
-25	1.381	1.710	3217.89	-5.9550
+25	1.368	1.726	3626.98	6.0009
+50	1.361	1.735	3817.47	11.5681

Table 5.4: For the % change in parameter ‘*β*’

β	**t₁**	**T**	$T_C(t_1, T)$	**PCI %**
-50	1.380	1.705	3376.28	-1.3260
-25	1.377	1.712	3399.65	-0.6430
+25	1.370	1.729	3444.40	0.6649
+50	1.366	1.737	3468.22	1.3610

Graph 5.2: For the % change in parameter ‘*b*’



Graph 5.4: For the % change in parameter ‘*β*’

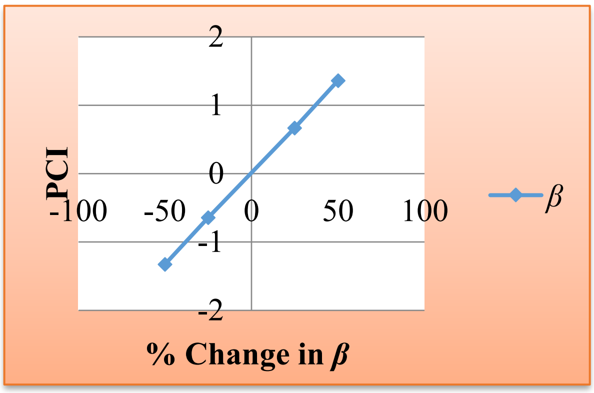
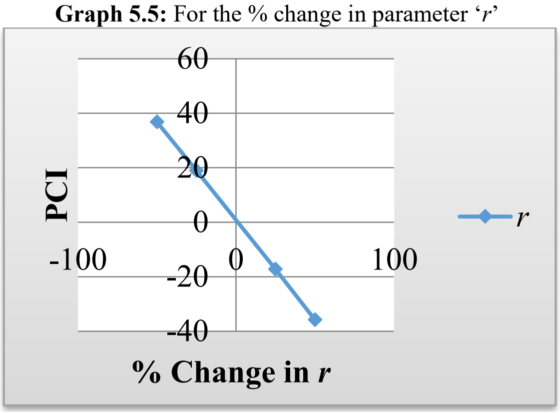


Table 5.5: For the % change in parameter '*r*'

r	t_1	T	$T_C(t_1, T)$	PCI %
-50	1.456	1.814	4681.18	36.8106
-25	1.413	1.773	4071.37	18.9885
+25	1.332	1.675	2835.33	-17.1356
+50	1.292	1.623	2196.66	-35.8011

Graph 5.5: For the % change in parameter '*r*'



We found that when

1. Parameter '*a*' increase, the optimal value of t_1* reduces while the optimal value of T* and the average total cost $T_C(t_1, T)$ of the inventory system increase.
2. Parameter '*b*' increase, the optimal value of t_1* reduces while the optimal value of T* and the average total cost $T_C(t_1, T)$ of the inventory system increase.
3. Parameter 'α' increase, the optimal value of t_1* reduces while the optimal value of T* and the average total cost $T_C(t_1, T)$ of the inventory system increase.
4. Parameter 'β' increase, the optimal value of t_1* reduces while the optimal value of T* and the average total cost $T_C(t_1, T)$ of the inventory system increase
5. Parameter 'r' increase, the optimal value of t_1* and T* and the average total cost $T_C(t_1, T)$ of the inventory system declines.

Thus we can say that, the behavior of the parameters is according to the realism.

6. Conclusion

We created a partially backlogged inventory model for a two-storage system in this paper. A warehouse (OW) with limited storage and a leased warehouse (RW) with limitless storage is considered. Holding costs and depreciation costs differ from OW and RW due to different storage areas. Inventory costs (including catch costs and depreciation costs) in RW were considered higher than those in OW. In order to reduce inventory costs, it would be economical for firms to keep assets in OW before RW, and to clear items in RW before OW. The stock is transferred from RW to OW in accordance with the bulk issuance law. Most of the researchers till now have ignored the effects of deterioration in both the warehouses or had considered a constant rate of deterioration. But in the present study deterioration taken with a time varying decay rate. The consumption rate of the product is taken as a ramp-type function which is more realistic in real life. Shortages partially backlogged and the effect of inflation is also being considered. To demonstrate the model, a numerical example is provided. The model is solved for different system parameters and the optimal solution is selected from amongst the available solutions. Finally, a graphical sensitivity analysis was carried out to demonstrate the impact of systemic changes on the average overall cost. The further research can be done with fuzzy surroundings and trade-credit facility. It is also applicable for other items where the demand is dependent linearly with time.

Acknowledgements

Not Applicable.

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