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Unsteady Streamtube Model for Pulse Performance of Bipropellant Thrusters

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We propose a theoretical model for predicting the impulse bit of bipropellant thrusters under pulse-firing operations. The present theoretical model, which considers the non-uniformity of the mixture ratios created inside the thrust chamber, extends the streamtube approach limited to the prediction of the steady performance. In pulse-firing operation, the fuel or the oxidizer alone can be injected in isolation due to the mismatched injection timing prior to the rated injection, which leads to the deterioration of performance compared with the steady operation. The present approach, the unsteady streamtube model, successfully implements a time-dependent streamtube structure inside the thrust chamber, allowing for the prediction of the impulse bit as a straightforward function of the injection conditions. Three different pulse-firing tests using distinct hypergolic propellants demonstrate the validity of this model, typically reproducing the notable trend of deterioration in the impulse bit in the short-pulsed mode. We also examine the time-averaged specific impulse and mass flow rate to improve the impulse bit during short-pulsed operations.

Nomenclature

c^*	=	characteristic exhaust velocity, m/s
C_f	=	thrust coefficient
D	=	nozzle diameter, m
F	=	thrust, N
F_c	=	film ratio $[\dot{m}_{fc}/(\dot{m}_f + \dot{m}_{fc})]$
F_R	=	flow-rate ratio
g	=	gravity acceleration (9.8m/s ²)
I_B	=	impulse bit, N · s
I_{sp}	=	specific impulse, s

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$\dot{m}$	=	mass flow rate, kg/s
M_R	=	mixture ratio $[\dot{m}_o/(\dot{m}_f + \dot{m}_{fc})]$
N	=	nozzle number ratio of fuel to oxidizer
$\dot{q}$	=	spray flux, kg/s · m ²
t	=	time, s
T	=	time for injection phase, s
V	=	propellant injection velocity, m/s
Y	=	absolute center of gravity for spray width, m
α	=	injection matching rate
Λ	=	non-dimensional injection condition
ρ	=	density, kg/m ³
$\langle X \rangle$	=	time-averaged value of X
Subscripts		
f	=	main fuel
fc	=	film fuel
i	=	local value
o	=	oxidizer

I. Introduction

PROPULSION systems that deliver large thrusts and high specific impulses are in demand for space missions, and they have been successfully developed[1]. Bipropellant thrusters in the thrust range of ten to several hundreds of newtons play important roles in attitude control and orbital transfer, as well as in planetary landing and takeoff[2–6]. This is because they achieve higher thrust levels than electric thrusters and higher specific impulses than monopropellant thrusters. The propellant employed can be a conventional hypergolic combination of a storable fuel and an oxidizer, and less-toxic green propellants are also available[7–12]. Hypergolic propellants can not only produce continuous thrust in a certain period of time in steady operation, but they can also accumulate impulse by repeated short-pulsed operations, which is necessary for accurate control of satellites[13].

To extend the lifetimes of satellites, a high specific impulse is desired for both the steady and pulsed modes of a thruster. The sequential phenomena inside a bipropellant thruster involve the injection of fuel and oxidizer jet streams, spray formation by unlike impingement of the jets[14, 15], mixing in the spray, condensed-phase reactions, droplet evaporation, gas-phase reactions, and production of combustion gas[16–18]. Simultaneously, the film-cooling technique is used to protect the chamber wall from hot combustion gas by the latent heat of the liquid fuel film spreading on the

inner wall, and eventually self-decomposing[19–21]. The complex multi-phase thermo-fluid dynamics of these processes have necessitated time and cost consuming combustion tests for performance measurements. Recently, Fujii et al. successfully visualized the sequential events throughout a pulsed mode in a practical thruster using monomethylhydrazine (MMH) and a mixture of nitrogen tetroxide with 3 % nitric oxide (MON3) as propellants[22]. The results indicate that the injection timing can be different (mismatched) between the fuel and oxidizer jets. Tanaka et al. conducted combustion tests of continuous and pulsed operations in a 10 N ceramic thruster[23]. Bangalore et al. performed unsteady measurements of the combustion pressure and impulse bit during the pulsed mode in a thruster fabricated by additive manufacturing[24], which enables the use of small dribble volumes to minimize the injection delay after the valves open. Past combustion tests have found that the impulse bit can start deteriorating even when firing with pulses as long as 100 ms, and it can be less than 50 % of the steady operation with short pulses of 10 ms duration[23–25], making theoretical performance predictions more complex.

As a theoretical framework mainly devoted to performance prediction for steady operation, Rupe identified an important non-dimensional number, the so called Rupe factor[26, 27], to design unlike doublet injectors achieving the best performance, which is still effectively used to develop thrusters. Elverum and Morey extends the application to unlike triplet injectors[28]. Wrobel proposed a stratification approach, also known as the conventional streamtube model, to consider the non-uniform distributions of combustion gas inside a thrust chamber[29]; once the non-uniformity is known, the performance can be obtained. For the prediction of performance, however, one problem that remains is to identify the non-uniformity in a quantitative sense before conducting combustion tests[30].

Following on from those studies, the present authors clarified a non-dimensional number governing the non-uniform mixture ratio occurring inside a thrust chamber, successfully formulating c^* and I_{sp} for any mixture ratio as a direct function of injection conditions during steady operation[31, 32]. As a next step, it was hoped that a physics-based theoretical model useful for predicting pulse performance could be established. Therefore, here, we propose a new theoretical model for predicting the pulse performance of the impulse bit and the specific impulse of bipropellant thrusters by further extending the streamtube model, which is limited to calculating the steady performance. The validity of our model is evidenced by comparisons with several pulse-firing tests. To the best of the authors' knowledge, no previous research has provided a prediction model for pulse performance as a straightforward function of injection conditions.

The remainder of this paper is structured as follows; the theoretical model at each intermediate step is formulated in Sec. II; the results and discussion, including the model validation, are provided in Sec. III; and the conclusions are summarized in Sec. IV.

II. Modeling

We first explain the steady streamtube model, which was developed to predict the performance during steady operation, followed by a description of the new unsteady streamtube model for pulsed operation.

A. Steady Model[32]

A schematic illustration of an unlike doublet injector is shown in Fig. 1. The oxidizer and fuel jets are ejected from nozzles with diameters of D_o and D_f , respectively, aligned along the x axis and impinging at the coordinate origin at a mutual angle of θ . The spray extends in the y direction with spreading toward the z axis. The spray flux $\dot{q}$ across the x - y plane provides the absolute value of the center of gravity along the y direction, such that $Y_o = (\int \dot{q}_o |y| dx dy) / (\int \dot{q}_o dx dy)$ and $Y_f = (\int \dot{q}_f |y| dx dy) / (\int \dot{q}_f dx dy)$ for the oxidizer and fuel, respectively. Y_o and Y_f represent the half width of the spray. We formulate the non-dimensional injection conditions of Λ in a unified expression for an unlike doublet ($N = 1$), a fuel-oxidizer-fuel triplet ($N = 2$), and an oxidizer-fuel-oxidizer triplet ($N = 0.5$), corresponding to the spray width ratio in a phenomenological sense as follows[31].

$$\Lambda \equiv \sqrt{N \frac{\rho_f V_f^2 D_f}{\rho_o V_o^2 D_o}} = \frac{Y_o}{Y_f} \quad (1)$$

We note that Λ^2 at $N = 1$ is equivalent to the Rupe factor[26, 27]. Λ is, however, consistently applicable to unlike doublet and triplet injectors with the clear physical meaning.

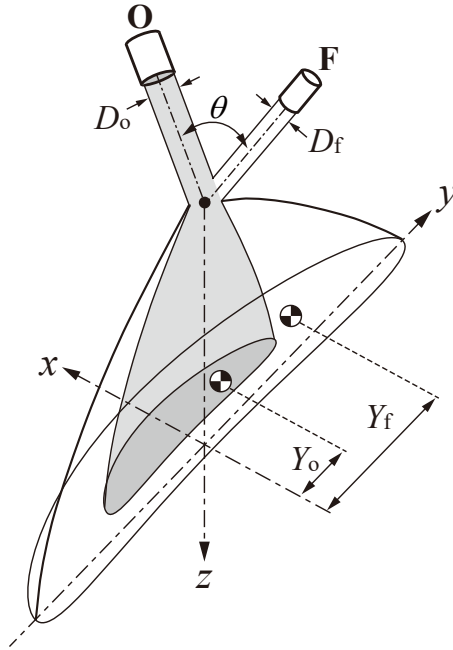


Fig. 1 Schematic of an unlike doublet injector.

We establish the streamtube model depicted in Fig. 2, allowing non-uniformity in the local mixture ratio inside the thrust chamber for calculating the steady performance. The oxidizer and fuel sprays mix in the streamtube of "Main-1", the oxidizer or fuel spray exists in "Main-2", and the film cooling fuel flows in "Film". The local mixture ratio of M_{Ri} and flow-rate ratio F_{Ri} in each streamtube are given as direct functions of the injection conditions of M_R , F_c , and Λ , as summarized in Table 1[32]. We use $M_R = 100$ as an alternative value of $M_R = \infty$. Using the local performance of X_i obtained by a streamtube at a mixture ratio of M_{Ri} , such that $X_i(M_{Ri})$, we deduce the overall performance of $X = \sum \{X_i(M_{Ri})F_{Ri}\}$ weighted by the flow-rate ratio. The performance properties of the characteristic velocity, c^* , and specific impulse, I_{sp} , at M_R are given as follows:

$$c^*(M_R) = \sum \{c_i^*(M_{Ri})F_{Ri}\} \quad (2)$$

$$I_{sp}(M_R) = \sum \frac{c_i^*(M_{Ri})C_{fi}F_{Ri}}{g} \quad (3)$$

Here, $c_i^*(M_{Ri})$ is calculated by chemical equilibrium analysis[33], and a thrust coefficient at each streamtube of C_{fi} is

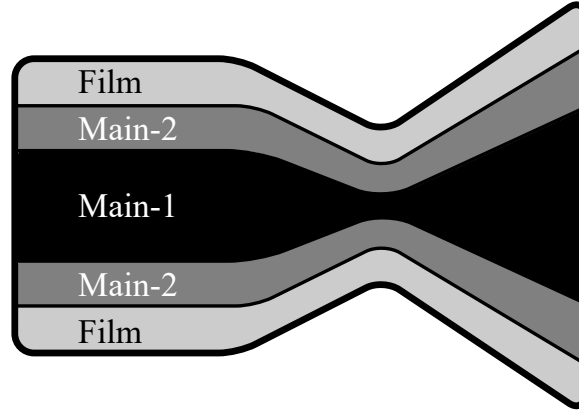


Fig. 2 Streamtube model.

Table 1 Ratios of local mixture and flow rate in streamtube.

Condition	Streamtube	M_{Ri}	F_{Ri}
$\Lambda \leq 1$	Main-1	$\frac{M_R}{\Lambda(1-F_c)}$	$\frac{M_R + \Lambda(1-F_c)}{M_R + 1}$
	Main-2	0	$\frac{(1-\Lambda)(1-F_c)}{M_R + 1}$
	Film	0	$\frac{F_c}{M_R + 1}$
$\Lambda > 1$	Main-1	$\frac{M_R}{\Lambda(1-F_c)}$	$\frac{M_R/\Lambda + 1 - F_c}{M_R + 1}$
	Main-2	∞	$\frac{(1-\Lambda^{-1})M_R}{M_R + 1}$
	Film	0	$\frac{F_c}{M_R + 1}$

provided as a function of M_{Ri} , nozzle expansion ratio, and nozzle exit pressure. The maximum c^* and I_{sp} values are achievable at a Λ of unity, in which the spray widths of the fuel and oxidizer are equivalent at the corresponding mixture ratio of M_R^* , derived from Eq.(1).

$$M_R^* = \frac{1 - F_c}{\sqrt{N}} \left(\frac{\rho_o}{\rho_f} \right)^{0.5} \left(\frac{D_o}{D_f} \right)^{1.5} \quad (4)$$

The validity of the streamtube model for steady performance was well tested in practical thrusters[32].

B. Unsteady Model

In the pulsed operation of a bipropellant thruster, the injection mixture ratio can vary depending on the time after the valves open[22, 25]. Here, we newly propose an unsteady streamtube model, in which the mixture ratio and streamtube structure change according to the progression of time. An overview of the unsteady streamtube model of a single injection sequence is shown in Fig. 3, including the time-variant visualization images, along with the mass flow rate, mixture ratio, characteristic exhaust velocity, and thrust. Here, the fuel is assumed to be injected earlier than the oxidizer, as observed by Fujii et al.[22], while we note that the same equations can be derived for the case of oxidizer injection prior to fuel injection. It is assumed that the propellant injection events start and stop instantaneously. The signals to control the ON/OFF states of the valves are generally simultaneous for the fuel and oxidizer flow paths, reasonably allowing identical injection times for the fuel and oxidizer (see also the Appendix). We define that the injection of fuel begins at $t = 0$ s, the oxidizer begins injection at T_1 , the fuel injection ends at T_2 , and the oxidizer injection completes at T_3 in a quasi-steady manner. This means that: in the period $0 \leq t < T_1$, only the fuel is injected into the combustion chamber with a mass flow rate of $\dot{m}_f + \dot{m}_{fc} = (1 - F_c)^{-1} \dot{m}_f$ including the film fuel; in the period $T_1 \leq t \leq T_2$, both of the fuel and oxidizer are injected with $(1 - F_c)^{-1} \dot{m}_f + \dot{m}_o$; and in the period $T_2 < t \leq T_3$, only the oxidizer is injected with $\dot{m}_o$, wherein $\dot{m}_f$ and $\dot{m}_o$ coincide with the nominal flow rates. The instantaneous mixture ratio and characteristic velocity change accordingly. By integrating the instantaneous thrust of F , an important pulse performance index, impulse bit I_B , is obtained.

$$I_B \equiv \int_0^{T_3} F dt = \int_0^{T_3} (\dot{m} c^* C_f) dt \quad (5)$$

Here, $\dot{m}$, c^* , and C_f are the values of the propellant mass flow rate, characteristic velocity, and thrust coefficient, respectively, at each injection phase. We calculate the time-variant c^* at a corresponding mixture ratio based on the steady streamtube model. The injection phase in $T_1 \leq t \leq T_2$ dominantly contributes to I_B ; thus, we introduce the injection matching rate, α , defined as follows.

$$\alpha = \frac{T_2 - T_1}{T_3} \quad (6)$$

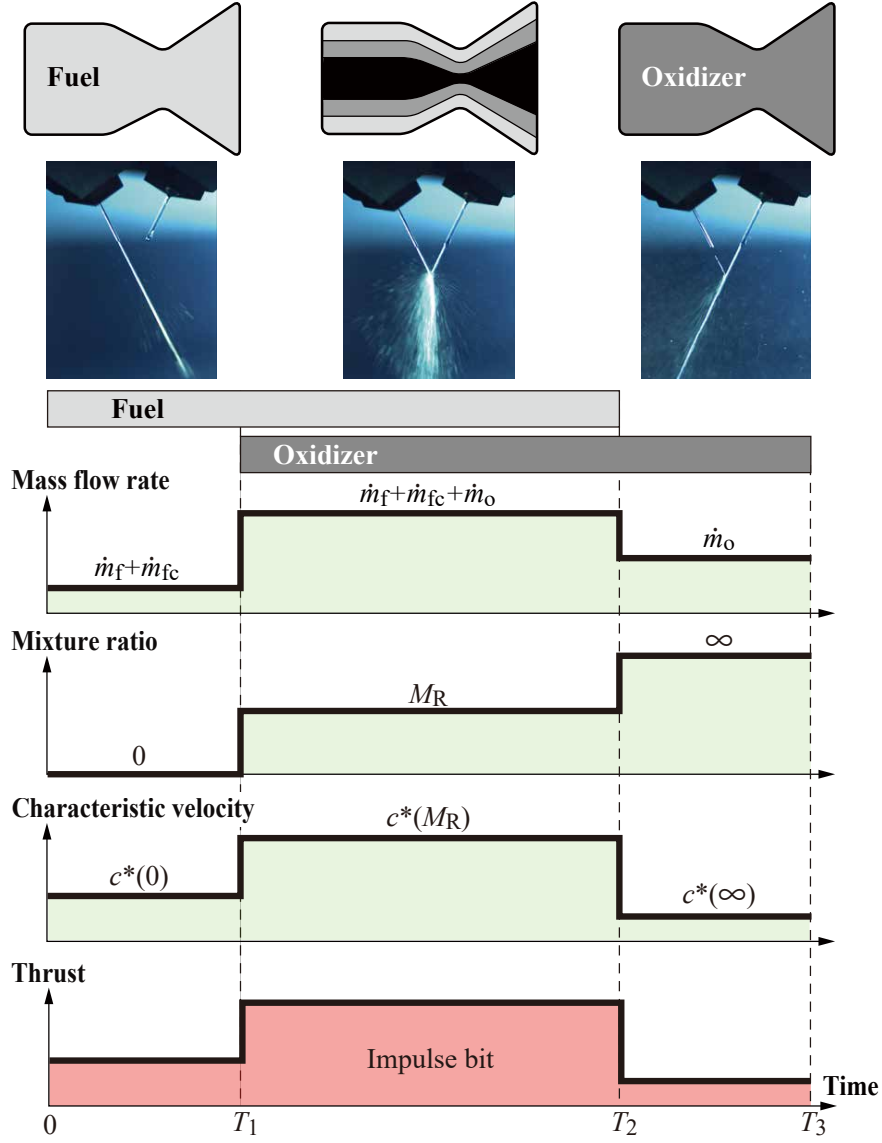


Fig. 3 Unsteady streamtube model.

The maximum I_B value is realized at an α of unity equivalent with the steady injection. I_B decreases with decreasing α , and I_B is thus clearly a function of α . In addition to the time-integrated performance, we evaluate time-averaged parameters during the period $0 \leq t \leq T_3$. The simple assumption of $T_1 = T_3 - T_2$ leads to $T_1/T_3 = 0.5(1 - \alpha)$ and $T_2/T_3 = 0.5(1 + \alpha)$. Then, we obtain the time-averaged mass flow rate and time-averaged characteristic velocity.

$$\langle \dot{m} \rangle \equiv \frac{\int_0^{T_3} \dot{m} dt}{T_3} = (1 + \alpha) \frac{(1 - F_c)^{-1} \dot{m}_f + \dot{m}_o}{2} \quad (7)$$

$$\langle c^* \rangle = \alpha c^*(M_R) + (1 - \alpha) \frac{c^*(0) + c^*(\infty)}{2} \quad (8)$$

Combining I_B with $\langle \dot{m} \rangle$, the time-averaged specific impulse can be calculated.

$$\langle I_{sp} \rangle \equiv \frac{\int_0^{T_3} F dt}{g \int_0^{T_3} \dot{m} dt} = \frac{I_B}{g \langle \dot{m} \rangle T_3} \quad (9)$$

III. Results and Discussion

A. Validity of Unsteady Streamtube Model

To validate the proposed unsteady streamtube model, we examine past pulse-firing tests. Table 2 lists the combustion conditions in three test cases. The employed propellants were MMH for the fuel and MON3 for the oxidizer in Case 1, MMH and nitrogen tetroxide (NTO) in Case 2, and hydrazine blended with 20 % monomethylhydrazine (M20) and MON3 in Case 3. The given values of F , M_R , and I_{sp} are nominal values for steady injection of the propellants. Since important design parameters, e.g., throat area, D_o , and D_f , are confidential, it is necessary to conjecture some values for calculation of the performance indexes. The nominal values of F , I_{sp} , and M_R can be used to deduce the nominal mass flow rate following $\dot{m} = F/(g I_{sp})$ and the propellant flow rate as $\dot{m}_f = (1 - F_c)(1 + M_R)^{-1} \dot{m}$ and $\dot{m}_o = (1 + M_R^{-1})^{-1} \dot{m}$. It is plausible to consider M_R in Table 2 as being equivalent to M_R^* ; Λ is then unity for the nominal injection. The film ratio and thrust coefficient are consistently set as $F_c = 0.3$ and $C_f = 1.8$ as general values.

The net propellant injection time in the combustion tests can be affected by many parameters[22, 25], including the valve ON/OFF times, valve response, manifold temperature, and number of pulses, in which the contributions of these effects are difficult to quantify. However, the valve ON time should be a primary factor determining the net injection time; thus, we use the valve ON time declared for the test conditions, which coincides with the net propellant injection time of T_3 . Other relevant effects are included in α , which is a fitting parameter. Figure 4 shows the experimental results of I_B against T_3 in Case 1 alongside the theoretical results from the unsteady streamtube model with varying α values. At long pulses of $T_3 \geq 10^{-1}$ s, the experimental results for I_B are clearly proportional to T_3 and correspond to the theoretical results with $\alpha \approx 1$, indicating that a long pulse is equivalent to steady injection. Thus, we confirm the validity of the unsteady streamtube model for its consistent applicability to long-pulsed operation. In contrast, the experimental results for I_B deteriorate at short pulses of $T_3 \approx 10^{-2}$ s, corresponding to the theoretical result for $\alpha = 0.5$. Assuming that the injection delay of T_1 is irrespective of T_3 , we estimate $T_1 = 2.6$ ms for Case 1 by using $\alpha = 0.5$ at $T_3 \approx 10^{-2}$ s.

Table 2 Combustion test conditions.

	Case 1 [23]	Case 2 [25]	Case 3 [24]
Propellants	MMH/MON3	MMH/NTO	M20/MON3
F [N]	10	22	45
M_R	1.60	1.65	1.31
I_{sp} [s]	295	295	290

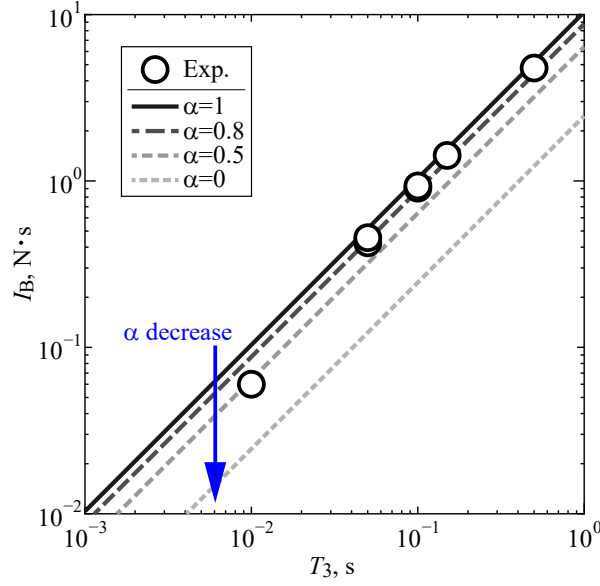


Fig. 4 Variation of I_B against T_3 with changing α in Case 1.

Then, we recalculate I_B with a fixed delay time of $T_1 = 2.6$ ms, as shown in Fig. 5(a). The theoretical result based on the unsteady streamtube model reproduces the combustion test results in a wide range of T_3 across almost two orders of magnitude, providing convincing evidence of the validity of the present modeling framework. This good agreement also indicates that the constant-delay assumption is reasonable for all conditions of T_3 . The injection delay is insignificant for long-pulsed combustion with $\alpha \approx 1$, while the delay clearly decreases I_B for short pulses when compared with the steady assumption of $I_B = FT_3$, as plotted by the dashed line. The minimum injection time is imposed by the total delay time, depicted as $T_3 > 2T_1$.

The identical theoretical strategy applied to Case 1 leads to $T_1 = 5.1$ ms in Case 2 and $T_1 = 0.82$ ms in Case 3 by fitting α to the combustion test results at the shortest pulse. Figures 5(b)(c) demonstrate the validity of the theoretical model in both cases. In Case 2, I_B steeply decreases at $T_3 \approx 10^{-2}$ s, in which the fuel and oxidizer jets are sequentially injected. Since the thruster in Case 3 was fabricated by additive manufacturing[24], which enables a fast response time, T_1 is remarkably short and the deterioration of I_B is suppressed. As noted, there are many factors leading to the decrease of I_B in the short pulse mode; the essential reason for this decrease, however, is mismatch between the fuel and oxidizer injection timings.

The established validity of the unsteady streamtube model allows for investigation of the mean performance indexes. In the combustion tests, $\langle I_{sp} \rangle$ is provided in addition to I_B . Following Eq.(9), $\langle \dot{m} \rangle$ can be calculated using T_3 . For the theoretical analysis, we use a given T_3 and the estimated T_1 to calculate $\alpha (= 1 - 2T_1/T_3)$ for every T_3 . Substituting α , $\dot{m}_o$, and $\dot{m}_f$ into Eq.(7), $\langle \dot{m} \rangle$ is obtained. The theoretical $\langle I_{sp} \rangle$ value is deduced from Eq.(9). Figure 6(a) shows $\langle I_{sp} \rangle$ and $\langle \dot{m} \rangle$ against T_3 in Case 1. We confirm that the experimental results of $\langle I_{sp} \rangle$ slightly deteriorate according to

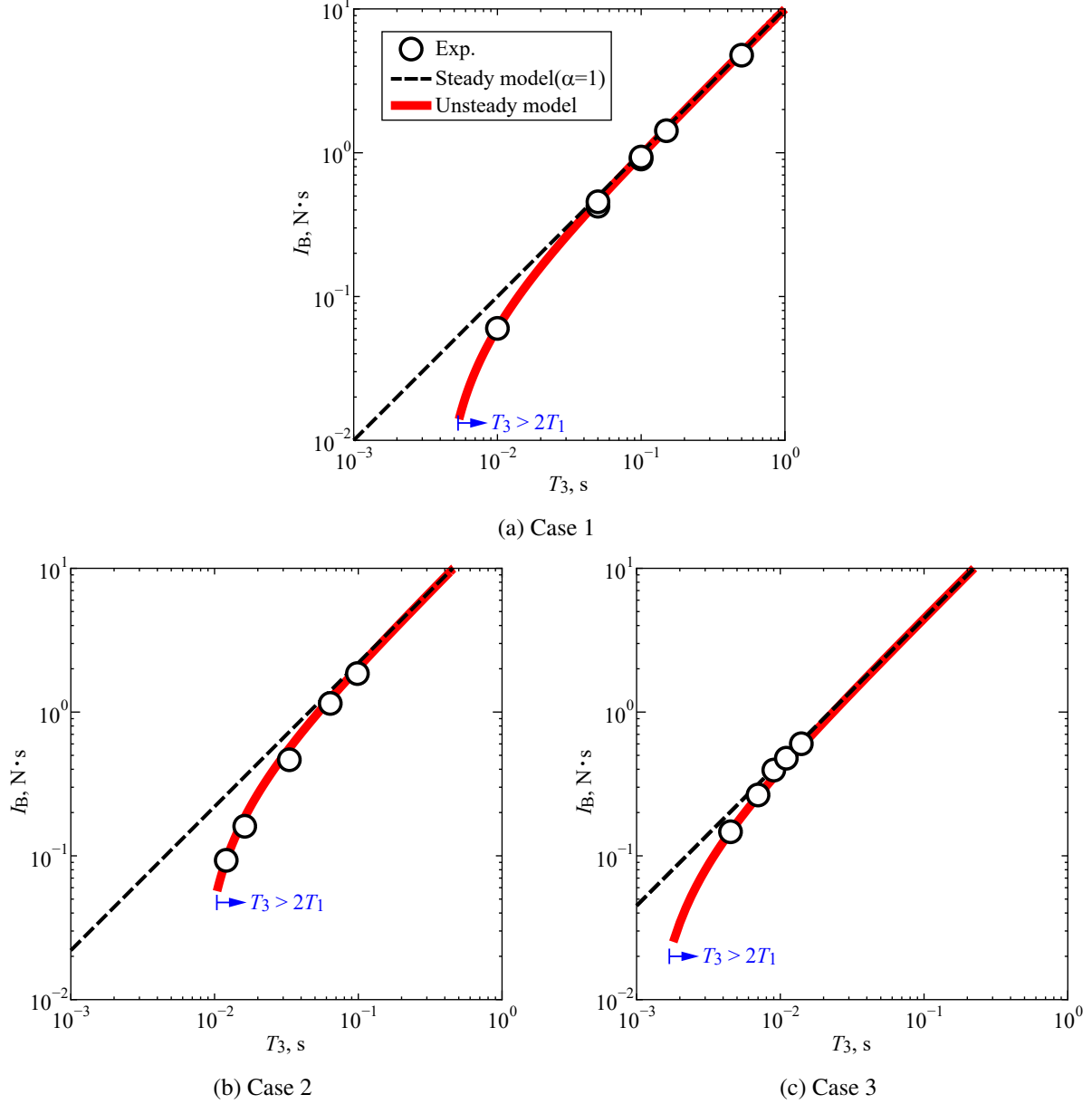


Fig. 5 Experimental and theoretical results of I_B against T_3 : (a) $T_1 = 2.6$ ms, (b) $T_1 = 5.1$ ms, and (c) $T_1 = 0.82$ ms for unsteady model.

the reduction of $\langle \dot{m} \rangle$ in the short pulse condition. These trends can be captured by the unsteady streamtube model by deducing small α values, corresponding to the short pulse condition. Here in Case 1, $\langle \dot{m} \rangle$ is smaller than in long pulse mode, and no overshoot of $\langle \dot{m} \rangle$ is visible for a short pulse. Conversely, strong overshoot of $\langle \dot{m} \rangle$ is evident in Case 2, as shown in Fig. 6(b), which was also confirmed experimentally by Lecourt and d’Herbigny[34] using the same propellants. Vapor pressure of the prior-injected oxidizer may mitigate the hydraulic resistance inside the fuel feed line[25, 34], possibly leading to the overshooting flow rate and the rapid deterioration of $\langle I_{sp} \rangle$ at $T_3 \leq 10^{-1}$ s. Because it does not consider the effects of these propagating pressure waves, the present streamtube model does not reproduce the overshoot

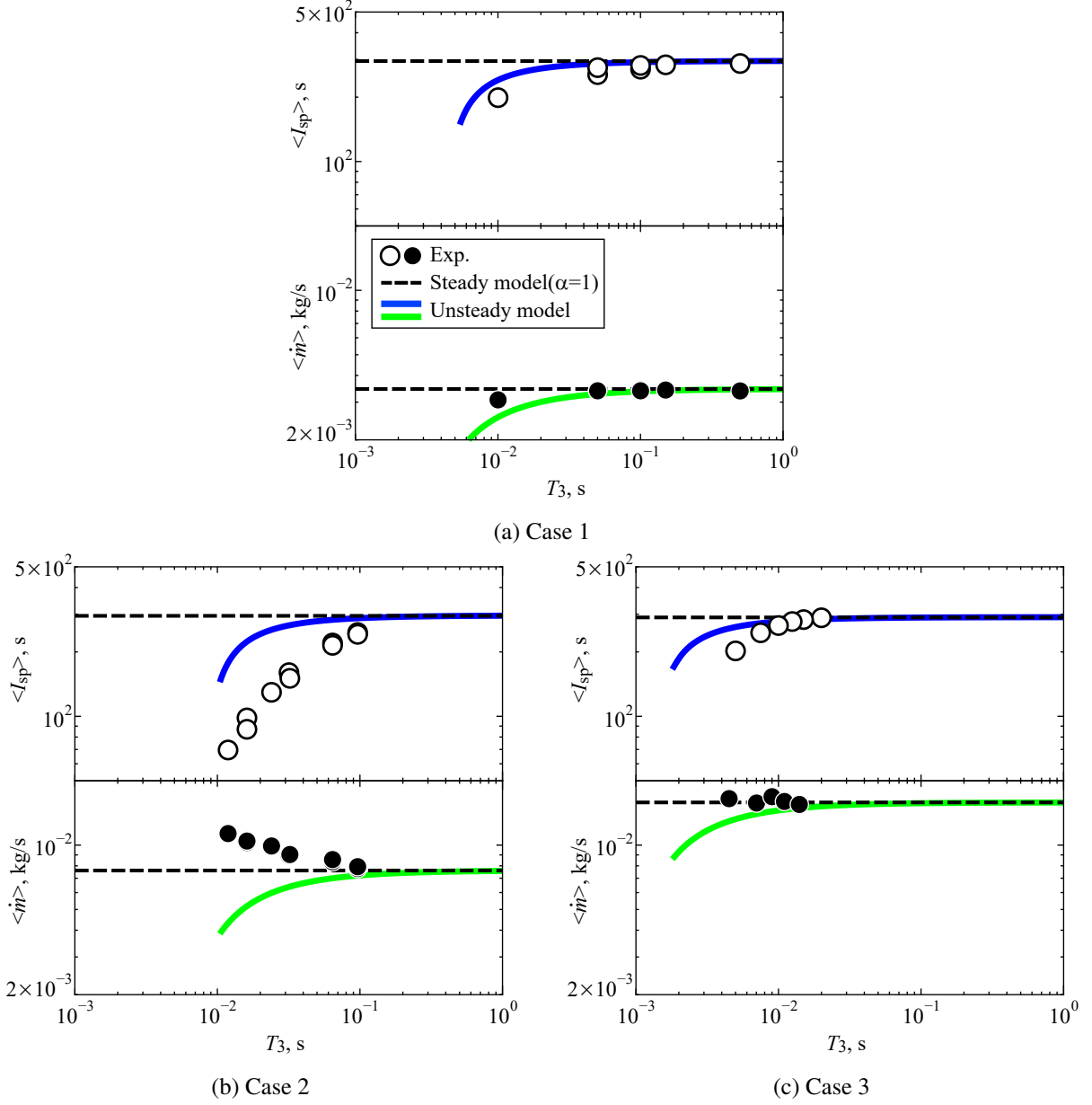


Fig. 6 Time-averaged I_{sp} and $\dot{m}$: (a) $T_1 = 2.6$ ms, (b) $T_1 = 5.1$ ms, and (c) $T_1 = 0.82$ ms for unsteady model.

of $\langle \dot{m} \rangle$. In Case 3, $\langle \dot{m} \rangle$ is almost equivalent to the steady value, and the deterioration of $\langle I_{sp} \rangle$ is moderate, corresponding to the theoretical prediction. As demonstrated by Driscoll et al.[25], a small dribble volume yields higher $\langle I_{sp} \rangle$ at the short impulse conditions, attributed to minimizing both T_1 and residual propellants inside the dribble volume.

B. Effect of Injection Mismatch

To improve the thruster design for pulsed operation, it is beneficial to quantitatively understand the sensitivity of the pulse performance as a function of T_1 . We calculate the Case 1 condition with T_1 values from 0 to 10 ms, in which the combustion test corresponds to $T_1 = 2.6$ ms. Figure 7(a) demonstrates that deterioration of I_B becomes visible

at longer T_3 as T_1 increases. The minimum pulse mode at $T_3 \gg 10T_1$ can operate with no significant deterioration of I_B , avoiding propellant loss; thus, the thruster should be designed to achieve $T_1 < 1$ ms for the operation of $T_3 \approx 10$ ms, and it is possible to be realized by additive manufacturing fabrication as demonstrated by Bangalore et al.[24]. For each T_1 , the minimum I_B is naturally achieved at $\alpha = 0$ or $T_3 = 2T_1$, leading to the minimum $I_B = 0.5\{(1 - F_c)^{-1}\dot{m}_f c^*(0) + \dot{m}_o c^*(\infty)\}C_f T_3 \propto T_3$ following Eq.(5). The minimum I_B corresponds to approximately 20 % of the steady performance ($T_1 = 0$) at the same T_3 . We also notice in Fig. 7(b) that the time-averaged values of $\langle I_{sp} \rangle$ and $\langle \dot{m} \rangle$ decrease with increasing T_1 . The minimum values of $\langle I_{sp} \rangle$ and $\langle \dot{m} \rangle$ are constant at $\alpha = 0$, irrespective of T_1 , as confirmed in Eqs.(7) and (8).

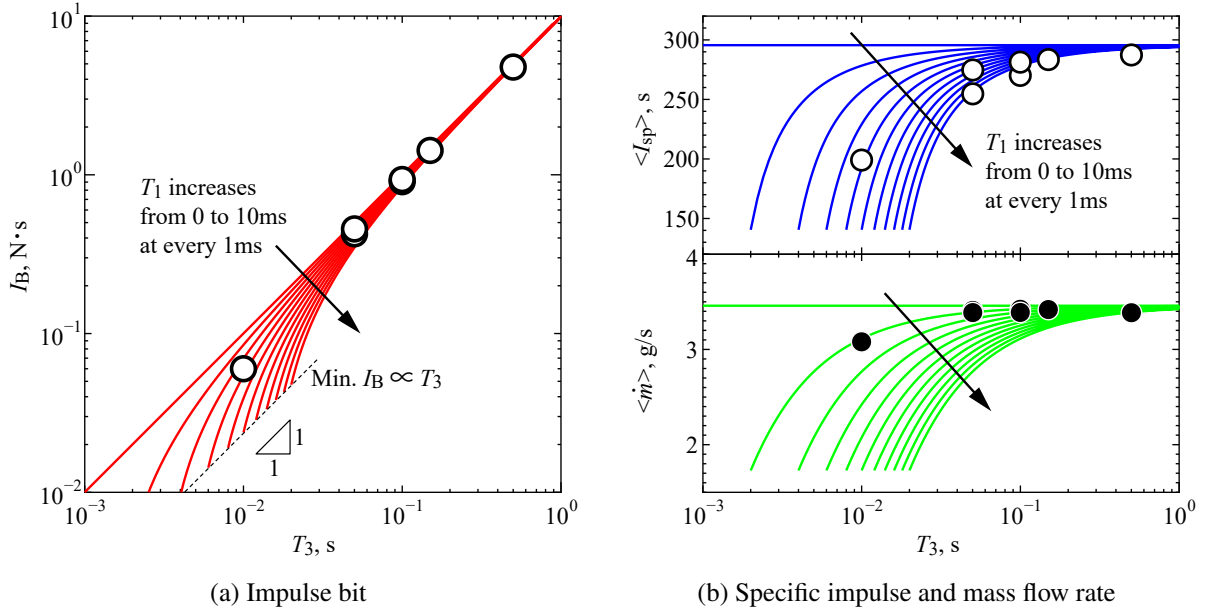


Fig. 7 Effects of T_1 on I_B , $\langle I_{sp} \rangle$, and $\langle \dot{m} \rangle$ in Case 1. The plots show the combustion test results from Ref.[23], and the lines depict the calculated results from the unsteady streamtube model with T_1 values from 0 to 10 ms in increments of 1 ms.

C. Effect of Dribble Volume Conditions

In the combustion test series in Case 2, two distinct conditions for the propellant dribble volume were examined[25]. The dribble volume of both the fuel and oxidizer is empty before injection in the "Dry" condition, as shown in Fig. 5(b), and the oxidizer dribble volume is empty while the fuel dribble volume is full in the "Wet" condition. The fill time of the dribble volume in the experiment was 2.6 ms for the oxidizer and 7.7 ms for the fuel, indicating that the oxidizer was ejected earlier than the fuel in Case 2. Their difference is $T_1 = 5.1$ ms, exactly coinciding with the delay estimated by the present model in Sec. III.A. The combustion test results in Fig. 8 show that the Wet condition achieves higher performance than the Dry condition. The present unsteady streamtube model reproduces both experimental results by deducing the respective T_1 values. We estimate $T_1 = 2.6$ ms for the Wet condition, which is identical to the experimental

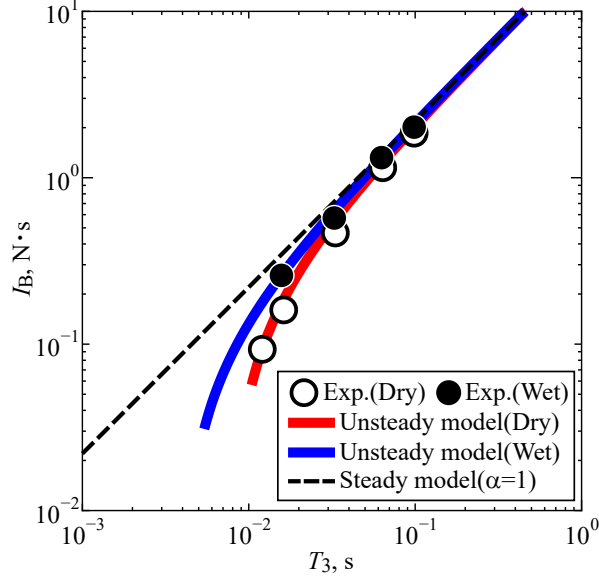


Fig. 8 Comparison of I_B at dry and wet conditions in Case 2. For the unsteady model, $T_1 = 5.1$ ms in the dry condition and $T_1 = 2.6$ ms in the wet condition.

condition of the fill time of the oxidizer. The validity of the present model is again demonstrated to predict the pulse performance at different dribble volume conditions and to inversely calculate the practical injection delay under pulse-firing operations. It is widely known that increasing the duration of the pulse OFF time enhances propellant evaporation, being the Dry condition, and causes the deterioration of I_B [24, 25]. The demonstrated agreement between the experimental and theoretical results in Fig. 8 suggests that the present model has the potential to consider the effects of the pulse OFF time and duty ratio.

IV. Conclusion

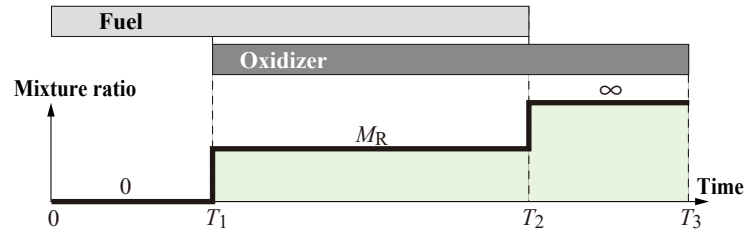
To predict the pulse performance of bipropellant thrusters, we performed theoretical investigations leading to a new formulation for the impulse bit. In practical pulse-firing operations, there are many factors that can cause mismatched injection, in which either the fuel or oxidizer is injected alone before both are supplied to the combustion chamber together. We implement the effect of mismatched injection as a delay of injection, allowing for the development of an unsteady streamtube model, in which the streamtube structures inside the thrust chamber change depending on the time-variant injection conditions. The injection delay can be estimated by fitting the theoretical result to an impulse bit obtained at the shortest pulse-firing test. Once this delay is determined, the unsteady streamtube model can be used to calculate the impulse bit for any injection time period. The results of three different pulse-firing tests using distinct hypergolic propellants confirm the validity of the present modeling framework, consistently reproducing both the deterioration of the impulse bit during short-pulsed operation and the matching of the steady operation impulse during long-pulsed operation. The time-averaged specific impulse and propellant mass flow rate in a single pulse can also be

predicted as long as the overshoot of the mass flow rate is not significant. With an injection duration ten times longer than the delay between the injections of the fuel and oxidizer, the deterioration of the impulse bit becomes insignificant.

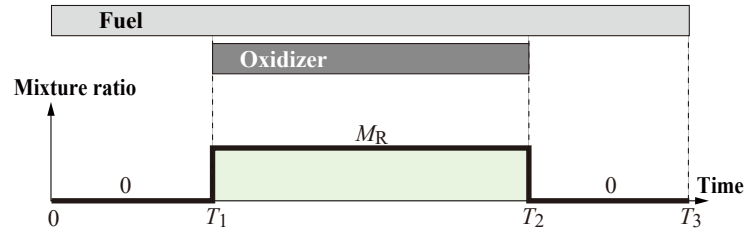
The present unsteady streamtube model is expected to contribute to the design of high-performance thrusters, even in pulse-firing mode. In addition, it can enable the advance prediction of the satellite motions by combining with guidance control simulations.

Appendix: Non-Equivalent Injection Time

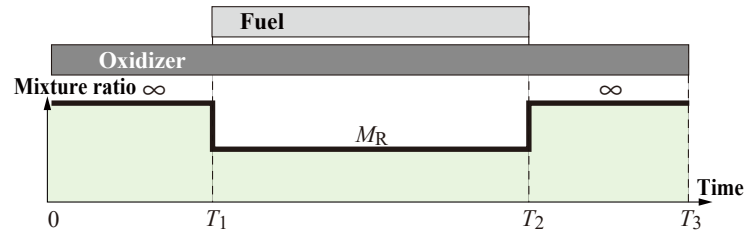
We have assumed an identical injection period for the fuel and oxidizer, as shown in Fig. 3. Here, we examine three distinct cases of injection phase to further demonstrate the validity of the present unsteady streamtube model. Type 1 in Fig. 9(a) is the present model, wherein the mismatched injection time is T_1 and the injection period is equivalent for the fuel and oxidizer. We consistently assume $T_1 = T_3 - T_2$. In Type 2, the fuel injection time is T_3 , while the oxidizer injection time is shorter and given by $T_2 - T_1$. Contrary, in Type 3, the oxidizer injection time is longer than the fuel injection time. The injection mixture ratio changes depending on the injection phase. We deduce $\dot{m}_f$ and $\dot{m}_o$ in the



(a) Type 1



(b) Type 2



(c) Type 3

Fig. 9 Three different injection sequences.

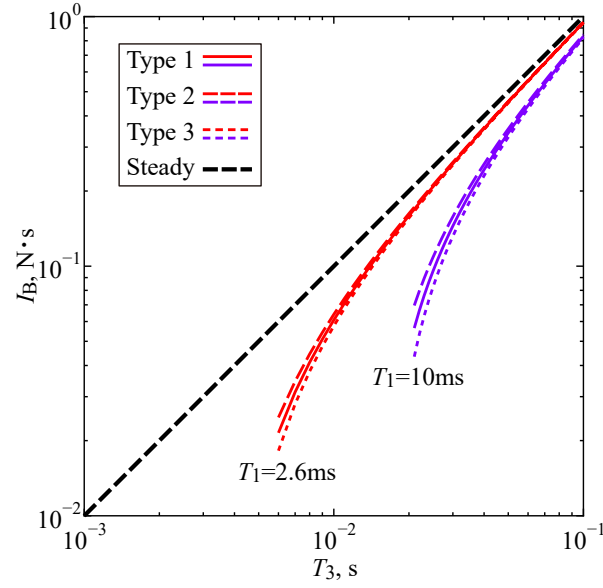


Fig. 10 Comparison of I_B for different injection types.

firing conditions of Case 1 for Type 1, as denoted in Sec. III, and the same values are applied to Types 2 and 3. I_B is calculated by following Eq.(5) at $C_f = 1.8$. Figure 10 shows the analysis results at $T_1 = 2.6$ and 10 ms. We confirm that results of the three injection types are almost equivalent, demonstrating that the Type 1 (present) unsteady streamtube model is robust. The agreement among them also indicates that the matching injection in $T_1 \leq T \leq T_2$ is dominant for I_B . We notice that I_B in Type 2 is slightly larger than I_B in Types 1 and 3, attributed to $c^*(0) = 1313$ m/s being larger than $c^*(100) = 461$ m/s at the mismatched injection phase. In practical thruster operation, the ON/OFF signals are simultaneous for the fuel and oxidizer valves; thus, the assumption of equivalent injection times in the model is reasonable.

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