Stress Intensity Factors for Three Dimensional Cracks by Applying Slice Synthesis Methodology

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Stress Intensity Factors for Three Dimensional Cracks by Applying Slice Synthesis Methodology

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ABSTRACT

Applicability of the slice synthesis methodology with weight function to calculate stress intensity factor for three dimensional cracks is investigated. Although slice synthesis methodology has already been proposed, quantitative investigations of the applicability seem to be insufficient. The authors show the applicable limits of the embedded cracks by comparing conventional formula based on finite element analyses. It is confirmed that the ratio of crack depth to plate thickness effects the accuracy of stress intensity factors by applying the slice synthesis methodology.

KEY WORDS: Slice synthesis methodology; Stress intensity factors; Embedded crack.

INTRODUCTION

In order to improve the structural integrity of ship and offshore structures, fatigue life estimation for three dimensional cracks is important because most fatigue cracks found in these structures are embedded and surface cracks.

The most common method for conventional fatigue life estimation is based on the combination of S-N curves with hot spot stress and cumulative damage rules (Fricke, 2002). However, this method contains some serious weaknesses as follows.

1. S-N curves method cannot give the fatigue crack growth history.
2. The transferability of fatigue life obtained by S-N curves to in-service structures has not established. That is, the relation between fatigue crack length found in in-service structures and fatigue life obtained by S-N curves is not defined clearly.

On the other hand, fatigue life predictions based on fracture mechanics are performed in order to overcome the weaknesses of the conventional S-N curves approaches. Most fatigue life assessments based on fracture mechanics cannot quantitatively evaluate the various transient phenomena under complex loading histories, e.g. retardation and acceleration of crack propagation, because of insufficient consideration of fatigue crack opening / closing behavior caused by crack wake. Toyosada et.al proposed the procedures of fatigue crack growth simulation for three dimensional crack problems by applying numerical fatigue crack opening / closing simulation for two dimensional crack problems with the equivalent distributed stress method which enables the representation of the stress-strain field at the deepest point of three dimensional cracks for a through thickness crack (Toyosada, Gotoh and Niwa, 2004). Stress intensity factor of analysis objects should be given even though above mentioned method is applied. The goal is to develop direct numerical simulation of three dimensional fatigue crack growth without the application of equivalent distributed stress.

Calculation procedures of stress intensity factor for through thickness cracks under arbitrary stress distributions are generally straight forward, while for three-dimensional cracks such as surface cracks and embedded cracks are more complicated. Because most cracks found in ships and offshore structures show embedded or surface crack morphologies and are located in stress concentration regions, it is important to calculate the stress intensity factor for three-dimensional cracks under arbitrary stress distributions in order to evaluate the fatigue strength in ships and offshore structures.

The slice synthesis methodology with weight function methods to calculate stress intensity factors for cracks in three-dimensional bodies was developed (Zhao, Wu and Yan, 1989a; Zhao, Wu and Yan, 1989b). The applied method has advantages for modeling and CPU time comparing with other numerical calculation method, e.g. finite element analyses. Although the slice synthesis methodology had already been proposed, quantitative investigations of the applicability seem to be insufficient. In this study, the authors make it clear the applicable limits by comparing conventional formulae based on finite element analyses.

THE SLICE SYNTHESIS METHODOLOGY

The slice synthesis methodology with weight function methods to calculate stress intensity factors has potential as a usable calculation method. Calculation procedure for stress intensity factors is explained below. The detail of this methodology was described in the reference (Zhao, Wu and Yan, 1989a).

The analysis object is an embedded elliptical crack shown in Fig.1 and crack surface is divided by two series of orthogonal slices shown in Fig.2. The slice corresponding to *y-z* plane is called basic slice whose crack length is $2a_x$, while corresponding to *z-x* plane is spring slice and crack length is $2b_y$. Basic slice and spring slice is restrained by the spring whose stiffness are k_a and k_b respectively. The spring stiffness k_a enables it to have a value from zero to infinity according to the dimensional ligament area R_a shown in Fig.3. k_b also enables it to have a value from zero to infinity. That is, R_i tends to zero, k_i tends to zero, while R_i tends to infinity, k_i tends to infinity. Dimensional ligament areas R_i are defined as follows,

$$R_a = 4t(c-b)/b^2, \quad R_b = 4c(t-a)/b^2 \tag{1}$$

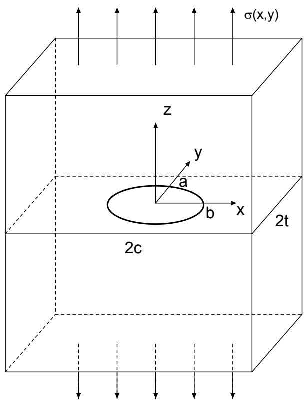

Fig.1 An embedded elliptical crack subjected to arbitrary stress distribution

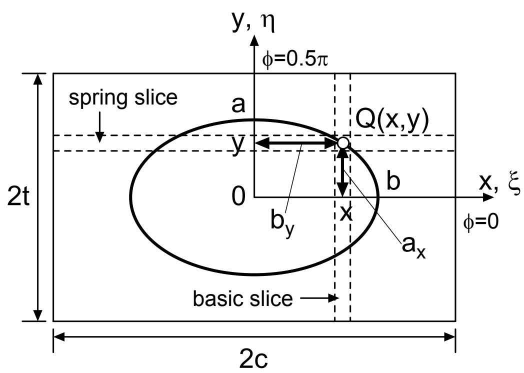

Fig.2 A coordinate system for an embedded elliptical crack

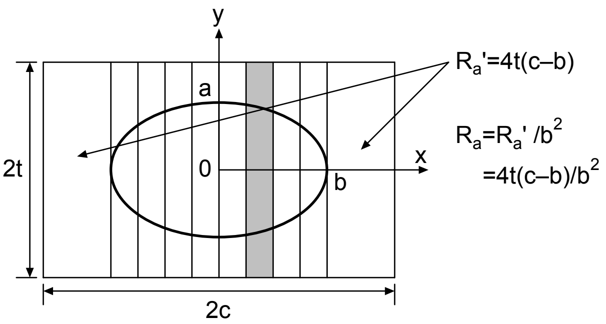

Fig.3 The dimensional ligament area R_a for basic slice

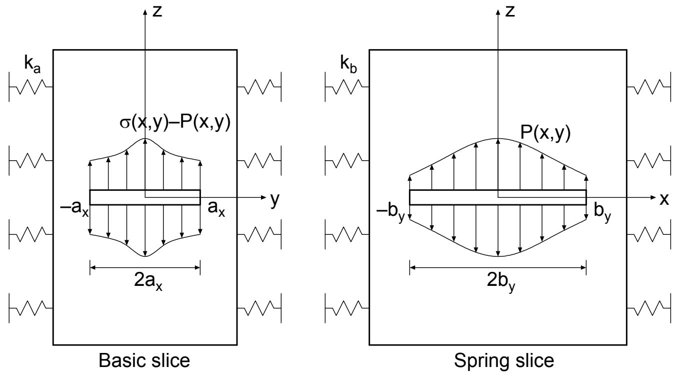

Fig. 4 The schematic illustration of basic slice and spring slice

The basic slice is subjected to arbitrary stress distribution $\sigma(x,y)$ and undetermined spring force $P(x,y)$, while the spring slice is subjected to spring force $P(x,y)$. The schematic illustration of two slices is shown in Fig.4. The spring force $P(x,y)$ is the cohesive force which has a role in fulfilling the boundary conditions toward neighboring basic slice deformation behavior. By applying the weight functions, the stress intensity factors for the basic slice and spring slice on arbitrary point $Q(x,y)$ shown in Fig.2 are calculated as follows,

$$\begin{aligned} K_a(a_x) &= \int_0^{a_x} [\sigma(x,\eta) - P(x,\eta)] w_a(R_a, a_x, t, \eta) d\eta \\ K_b(b_y) &= \int_0^{b_y} P(\xi, y) w_b(R_b, b_y, c, \xi) d\xi \end{aligned} \tag{2}$$

where w_a and w_b are weight functions for basic slice and spring slice respectively. The weight functions w_i are calculated by combination of two limiting conditions shown in Eq.(3). R_i tends to zero, w_i tends to free boundary condition w_i^{free}, while R_i tends to infinity, w_i tends to fixed boundary condition $w_i^{collinear}$.

$$\begin{aligned} w_i &= w_i^{collinear} + T(R_i)(w_i^{free} - w_i^{collinear}) \\ T(R_i) &= 1.05^{-R_i} \end{aligned} \tag{3}$$

Weight functions for the basic slice is given by Eq. (4) (Tada, Paris and Irwin, 2000),

$$\begin{aligned} w_a^{collinear} &= \frac{2}{\sqrt{2t}} \sqrt{\tan\frac{\pi a_x}{2t}} \frac{\cos(\pi y/2t)}{\sqrt{\sin^2(\pi a_x/2t) - \sin^2(\pi y/2t)}} \\ w_a^{free} &= \frac{2}{\sqrt{2t}} \left\{ 1 + 0.297\sqrt{1-(y/a_x)^2}\left(1-\cos(\pi a_x/2t)\right) \right\} \\ &\quad \times \left(\sqrt{\tan\frac{\pi a_x}{2t}} \Bigg/ \sqrt{1 - \left(\frac{\cos(\pi a_x/2t)}{\cos(\pi y/2t)}\right)^2} \right) \end{aligned} \tag{4}$$

Weight function for spring slice is given by replacing t, a_x, and y with c, b_y and x in Eq. (4). Crack opening displacement profile (COD profile) along the a_x, b_y are given by,

$$\begin{aligned} V_a(x,y,a_x) &= \frac{2}{E_a} \int_y^{a_x} K_a(\alpha) w_a(R_a, \alpha, t, y) d\alpha \\ V_b(x,y,b_y) &= \frac{2}{E_b} \int_x^{b_y} K_b(\beta) w_b(R_b, \beta, c, x) d\beta \end{aligned} \tag{5}$$

where E_a is Young's modulus and E_b is given by Eq. (6) (Zhao, Wu and Yan, 1989a),

$$E_b = \left(\frac{\Phi}{1-\nu^2} - 1 \right) \frac{b}{a} E_a, \quad for \ a/b \le 1 \tag{6}$$

Φ is the complete elliptic integral of the second kind and ν is Poisson ratio.

On the reference point $Q(x,y)$, COD of basic slice is equal to spring slice.

$$V_a(x,y,a_x) = V_b(x,y,b_y) \tag{7}$$

or,

$$\int_y^{a_x} \left\{ \int_0^{\alpha} \sigma(x,\eta) w_a(R_a,\alpha,t,\eta) d\eta \right\} w_a(R_a,\alpha,t,y) d\alpha$$
$$= \int_y^{a_x} \left\{ \int_0^{\alpha} P(x,\eta) w_a(R_a,\alpha,t,\eta) d\eta \right\} w_a(R_a,\alpha,t,y) d\alpha \tag{8}$$
$$+ \frac{E_a}{E_b} \int_x^{b_y} \left\{ \int_0^{\beta} P(\xi,y) w_b(R_b,\beta,c,\xi) d\xi \right\} w_b(R_b,\beta,c,x) d\beta$$

The spring force $P(x,y)$ is expressed as polynomial approximation.

$$P(x,y) = \sum_{i=1}^{19} e_i (x/b)^{k_i} (y/a)^{l_i} \tag{9}$$

where e_i is unknown coefficients. k_i and l_i in Eq. (9) are selected shown in Table 1. These exponents are applied in this research. The other combination of polynomials was applied in the reference (Zhao, Wu and Yan, 1989a).

Table 1. Values of exponents' k_i and l_i in Eq. (9).

i	1	2	3	4	5	6	7
k_i	0	1/3	0	1/3	1/2	0	1/2
l_i	0	0	1/3	1/3	0	1/2	1/2

i	8	9	10	11	12	13	14
k_i	1	0	1	2	0	2	3
l_i	0	1	1	0	2	2	0

i	15	16	17	18	19		
k_i	0	3	4	0	4		
l_i	3	3	0	4	4		

If some COD evaluation points are chosen more than polynomial approximation order, unknown coefficients e_i are determined by using the least-square fitting. In this study, 35 points are chosen as COD evaluation points according to the reference (Zhao, Wu and Yan, 1989a).

COMPARISON WITH CONVENTIONAL FORMULAE

In order to investigate the applicable limits of the slice synthesis methodology, comparison of stress intensity factors under uniform tension by applying the slice synthesis methodology with conventional formula developed by Newman and Raju (Newman and Raju, 1981) are performed. Total analytical cases calculated are 54, a/b = 0.1, 0.2, 0.5, 0.8, 1.0 and 2.0, a/t = 0.2, 0.5 and 0.8, b/c = 0.2, 0.4 and 0.5.

The comparison results at (a/b, b/c) = (0.2, 0.2), (0.5, 0.2), (1.0, 0.2) are shown in Figs.5, 6 and 7 as examples. In these figures, calculated results corresponding to a/t = 0.2 and 0.5 are in good agreement with the results calculated by conventional formulae based on finite element analyses. While the result corresponding to a/t = 0.8 is different from conventional formula.

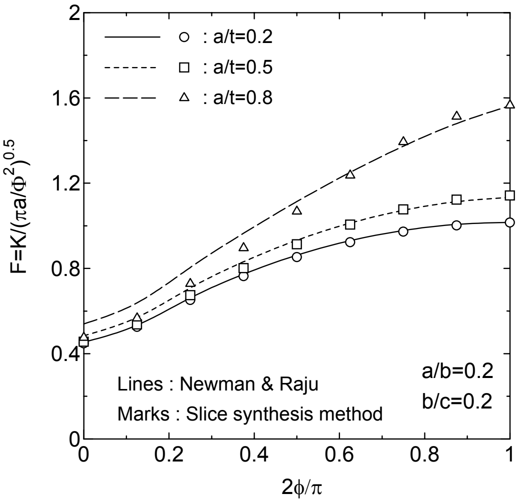

Fig.5 Comparison of boundary correction factor between the slice synthesis method and conventional formula (a/b = 0.2, b/c = 0.2)

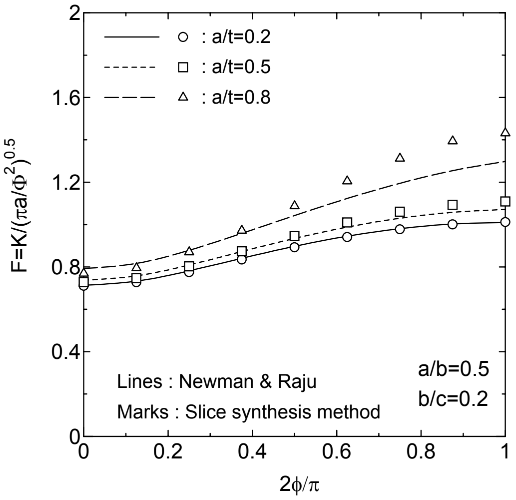

Fig.6 Comparison of boundary correction factor between the slice synthesis method and conventional formula (a/b = 0.5, b/c= 0.2)

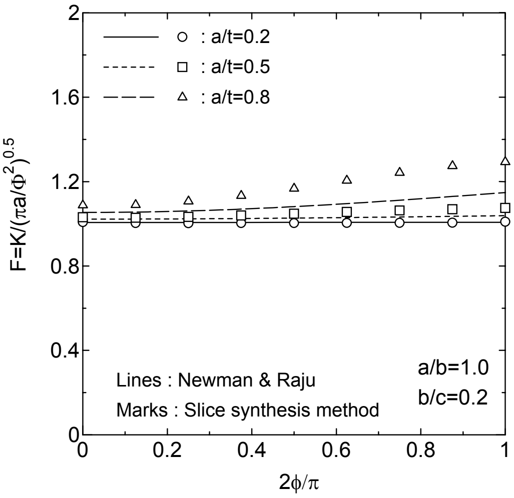

Fig.7 Comparison of boundary correction factor between the slice synthesis method and conventional formula (*a/b* = 1.0, *b/c* = 0.2)

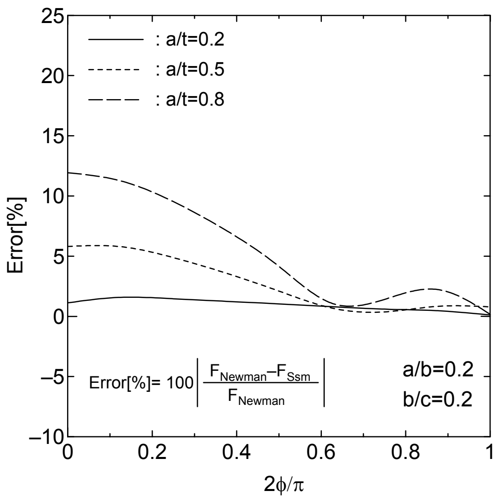

Fig.8 The difference between boundary correction factor by slice synthesis method and conventional formula (*a/b* = 0.2, *b/c* =0.2)

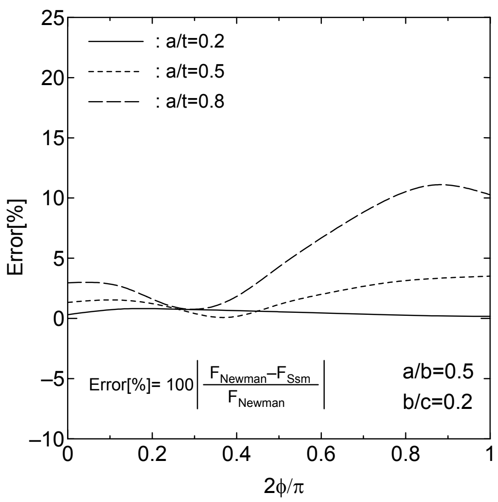

Fig.9 The difference between boundary correction factor by slice synthesis method and conventional formula (*a/b* = 0.5, *b/c* =0.2)

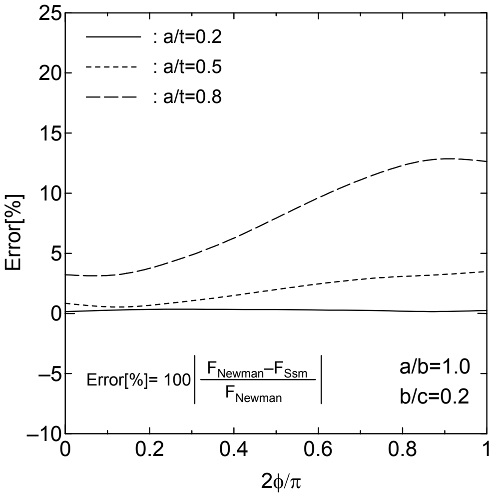

Fig.10 The difference between boundary correction factor by slice synthesis method and conventional formula (*a/b* = 1.5, *b/c* =0.2)

Figs.8, 9 and 10 show the difference between calculation results by slice synthesis methodology and ones by conventional formulae. The differences in *a/t* = 0.8 are more than 10 %.

It is considered that these differences of *a/t* = 0.8 are caused as the result of inadequate value of E_b in Eq. (6). E_b corresponds to the elastic modulus of the spring force, and derived under the condition that COD of both slices are equal to one of an elliptical crack in infinite body subjected to uniform tension. The effect of back surface on COD is not considered in the definition of E_b and the difference between slice synthesis solutions and conventional formula increases larger according to increasing of *a/t*. Hence, it is considered that the applicability of slice synthesis methodology has the limitation related *a/t*.

Table 2 shows the differences between slice synthesis methodology and conventional formula by Newman and Raju. For *a/t* = 0.2, the differences are less than 5 % at any *a/b* and *b/c.* For *a/t* = 0.5, the differences of both results show over 5% in case of *a/b* = 0.1 and 0.2. Then it is not recommended to apply the slice synthesis methodology in these cases. For *a/t* = 0.8, it is not recommended to apply this methodology because the minimum difference shown in Table 2 is larger 5% except for (*a/b*, *b/c*) = (2.0, 0.4) and (2.0, 0.5).

Table 2. Differences between slice synthesis methodology and conventional formula by Newman and Raju

a/b	0.1			0.2		
	b/c=0.2	0.4	0.5	0.2	0.4	0.5
a/t=0.2	2.0[%]	2.8	3.4	1.6	2.2	2.7
0.5	8.1	11.1	13.4	5.8	8.6	10.8
0.8	13.3	18.0	21.2	11.9	16.4	20.0
a/b	0.5			0.8		
	b/c=0.2	0.4	0.5	0.2	0.4	0.5
a/t=0.2	0.8	0.8	1.2	0.5	0.6	1.3
0.5	3.5	3.3	4.8	3.7	2.0	1.8
0.8	11.1	7.3	9.6	13.0	9.6	6.0
a/b	1.0			2.0		
	b/c=0.2	0.4	0.5	0.2	0.4	0.5
a/t=0.2	0.4	1.1	2.0	0.4	2.3	4.1
0.5	3.5	1.8	0.6	1.8	1.2	2.0
0.8	12.8	9.5	6.0	7.5	3.8	1.3

From these calculation results, recommended regions for the application of current slice synthesis methodology are conducted as follows.

- *a/t* is less than 0.2, any *a/b* is available.
- *a/t* is between 0.2 and 0.5, *a/b* is larger than 0.5.
- *a/t* is larger than 0.5, current slice synthesis methodology should not be applied.

CONCLUSIONS

In order to investigate the applicability limits of the slice synthesis methodology with weight function methods to calculate the stress intensity factors for an embedded crack, comparisons of stress intensity factors between the slice synthesis methodology and conventional formula based on finite element analyses are performed. From these calculation results, recommended regions for the application of current slice synthesis methodology are conducted.

Future challenge for the improvement of slice synthesis methodology is the modification of E_b in Eq. (6). Calibrations by applying crack opening displacement from finite element analyses considering many back surface conditions are ongoing by authors.

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