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### **ABSTRACT**

Fatigue life estimation for planar cracks, e.g. part-through surface cracks or embedded cracks is very important because most of fatigue cracks found in welded built-up structures show planar crack morphologies. Although authors had proposed the estimation procedure of crack shape evolution for a planar crack based on the fracture mechanics approach, this method cannot apply if the values of stress intensity factor at the vertices of the surface crack approximated as an ellipse cannot be calculated. Then, development of the shape evolution procedure of a planar crack under the stress field with arbitrary gradient, because fatigue cracks in welded built-up structures exist near the stress concentrated region. A Practical estimation formula the shape evolution of a surface crack under stress field with the gradient is proposed in this study. This formula is established by considering the stress field under no crack condition and some former proposed formulae under uniform and pure bending stress fields. The validity of the proposed formula are confirmed by comparing some measured surface crack shape evolutions under some stress gradient conditions.

### **INTRODUCTION**

Although the fatigue crack growth curve should be given for constructions of anti-fatigue damage and for repair plans of fatigue damages, fatigue crack growth behavior does not considered in the conventional fatigue design for welded built-up steel structures. The conventional fatigue design for the structures are performed by applying the combination of S-N curves in many design codes and the cumulative damages law, i.e. Miner's law. In addition, it is not clear the transferability of fatigue life between specimens and in-service large structures.

On the other hand, fatigue crack growth curves are estimated by applying conventional fatigue crack propagation laws, e.g. Paris' law [1] or Elber's law [2], if required. However, conventional propagation laws cannot evaluate the various transient phenomena quantitatively, e.g. retardation and acceleration of crack propagation, because of insufficient consideration of fatigue crack opening / closing behavior caused by crack wake. Therefore, improved fatigue crack growth law is required in order to estimate the fatigue crack growth curves quantitatively. By considering that fatigue cracks cannot grow without the accumulation of alternating tensile / compressional plastic strain, Toyosada and Niwa [3] proposed the effective stress intensity factor range ( $\Delta K_{RPG}$ ) based on the Re-tensile Plastic zone's Generated (RPG) load, which represents the fatigue crack driving force.  $\Delta K_{RPG}$  is obtained by replacing the crack opening load with RPG load in the definition of the effective stress intensity factor range by Elber.

Toyosada et al. developed the numerical simulation code which implemented  $\Delta K_{RPG}$  criterion for the law of fatigue crack growth [4] and proposed the useful procedure of fatigue crack growth based on the fracture mechanics approach for the welded joints. The fracture mechanics approach with  $\Delta K_{RPG}$  as a key parameter is applied to their procedure [5].

Many fatigue damages in steel structures are often found in stress concentrated region, and fatigue cracks often from surface of structure. Therefore, it is important to estimate a surface crack growth quantitatively. However, there are some experimental results which show that the surface crack behavior cannot be explained by the fracture mechanics' approach mentioned above. This is partly because the behavior of fatigue crack opening / closing along the leading edge on a surface is not clear.

Therefore, Kawahara et al. proposed the practical formula for the estimation of the evolution of surface crack from a point-like crack or a wide/shallow crack under the uniform and pure bending stress field without a stress concentration [6]. And Iida et al. expanded the Kawahara et al.'s formula for the narrow/deep crack [7]. By improving these practical formulae, we propose the practical estimation formula with stress distribution under a stress concentrated region in this study.

### NOMENCLATURE

- $a$  : crack depth [mm]
- $b$  : crack length [mm]
- $w$  : ligament depth [mm]
- $a/b$  : aspect ratio
- $a_0$  : initial crack depth [mm]
- $b_0$  : initial crack length [mm]
- $x$  : normalized surface crack depth by ligament depth ( $=a/w$ )
- $\rho$  : notch radius [mm]
- $t$  : notch depth [mm]
- $\sigma$  : stress in load direction under no crack condition [MPa]
- $\Delta\sigma_m$  : in-plane tensile stress range [MPa]
- $\Delta\sigma_b$  : out-plane bending stress range [MPa]
- $\beta$  : bending stress ratio ( $=\Delta\sigma_b / (\Delta\sigma_m + \Delta\sigma_b)$ )
- $R$  : stress ratio ( $=\sigma_{min} / \sigma_{max}$ )

### FATIGUE SURFACE CRACK PROPAGATION TESTS

#### Test method

The chemical composition and mechanical properties of the tested steels are listed in Table 1. Specimens prepared for fatigue surface crack propagation tests are categorized into 2 types according to a form. Figure 1 shows configurations of each type.

TABLE 1. MECHANICAL PROPERTIES AND CHEMICAL COMPOSITION OF TESTED MATERIAL (NK GRADE KA).

a) plate thickness : 16 [mm]				
Mechanical properties				
Yield strength [MPa]		Tensile strength [MPa]		Elogation [%]
290		431		32
Chemical composition [%]				
C	Si	Mn	P	S
0.17	0.12	0.46	0.012	0.004

b) plate thickness : 25 [mm]				
Mechanical properties				
Yield strength [MPa]		Tensile strength [MPa]		Elogation [%]
279		435		31
Chemical composition [%]				
C	Si	Mn	P	S
0.14	0.23	0.86	0.018	0.005

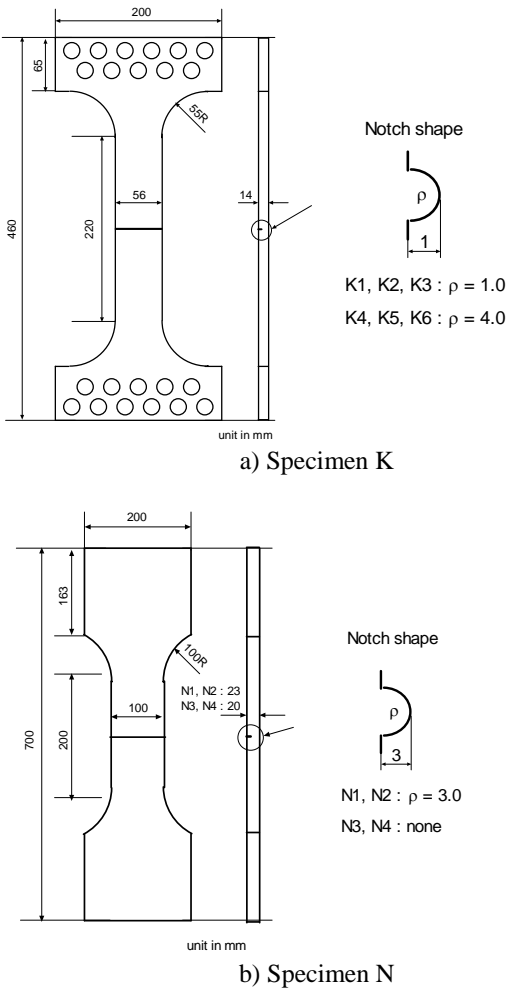


FIGURE 1. CONFIGURATION OF THE CRACKED SPECIMENS.

All specimens have each shape of initial defect that introduced by electrical discharge machining (Specimen K) or cutting (Specimen N) to obtain various shapes of initial fatigue surface crack. The specimen geometries and the applied loading conditions for each specimen are listed in Table 2.

We measured change of strain range at the back surface of specimen to know the surface crack propagation for the crack depth direction. The surface crack propagation for the crack length direction was monitored with CCD camera (Fig.2). The shape of crack was measured by the ink penetration method and the beach mark method.



FIGURE 2. SET-UP OF FATIGUE SURFACE CRACK PROPAGATION TEST UNDER TENSILE LOADING.

TABLE 2. SPECIMEN GEOMETRIES AND LOADING CONDITIONS.

Type	Specimen ID	Notch		Initial defect		Stress ratio: $R$	Stress range: $\Delta\sigma$ [MPa]	Plate thickness of tested steel [mm]
		depth [mm]	radius [mm]	Depth/ $w$	Length [mm]			
K	K1	1	1	0.04	0.5	0.4	135	16
	K2	1	1	0.04	2.5	0.4	135	16
	K3	1	1	0.04	5.0	0.4	135	16
	K4	1	4	0.08	0.5	0.05	186	16
	K5	1	4	0.04	2.5	0.05	186	16
	K6	1	4	0.04	5.0	0.05	186	16
N	N1	3	3	0.05	0.5	0.05	106	25
	N2	3	3	0.05	5.0	0.05	106	25
	N3	flat plate		0.05	0.5	0.05	200	25
	N4	flat plate		0.05	5.0	0.05	200	25

### Test results

The surface crack depth and length were measured from close-up pictures of the fatigue fracture surface (Fig. 3). Figure 4 shows that the aspect ratios of specimens categorized according to a notch shape. From Fig. 4, it can be seen that the aspect ratio of a surface crack tend to become close with growth in depth direction and that notch shape affects the aspect ratio.

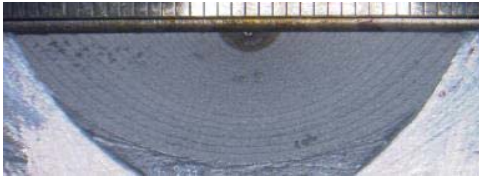


FIGURE 3. AN EXAMPLE OF OBSERVED SURFACE CRACK (K1).

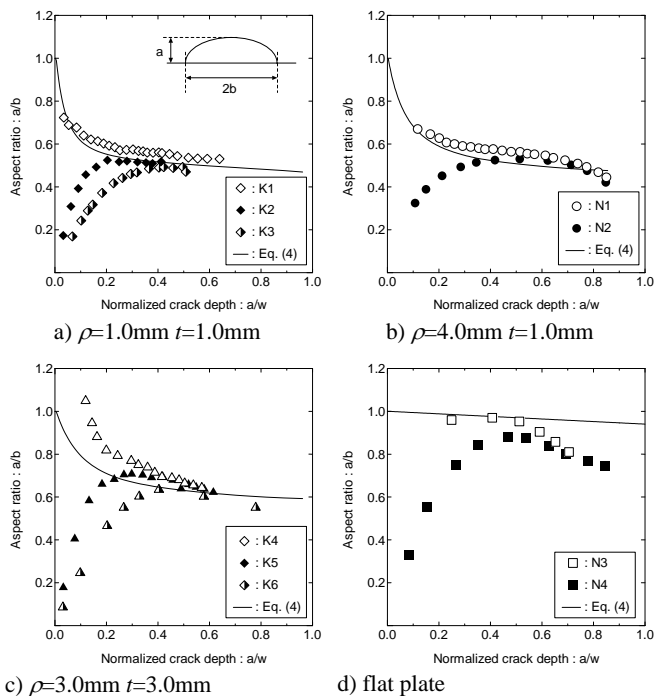


FIGURE 4. THE ASPECT RATIO OF A SURFACE CRACK.

Kawahara et al. called the surface crack growth from a point-like crack 'Balanced growth' (Solid line in Fig. 5) and indicated that the surface crack growth from a wide / shallow crack (dashed line in Fig. 5) became close to Balanced growth [6]. Iida et al. indicated that there was a similar tendency in the case that the surface crack growth from a narrow / deep crack (dashed-dotted line in Fig. 5) [7]. However, it was found from the test results with the flat plate specimen.

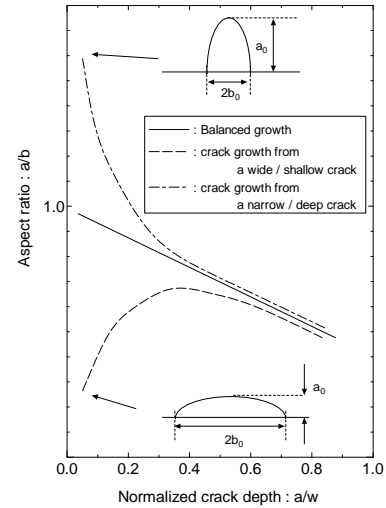


FIGURE 5. THE CONCEPTUAL RENDERING OF THE ASPECT RATIO OF A SURFACE CRACK EXISTS IN FLAT PLATE.

Figure 4 a) - c) show that the surface crack growth denotes the same tendency of Fig. 5 in notched specimen.

Therefore, in order to estimate the shape evolution of a surface crack in a stress concentrated region, it is important to examine Balanced growth in the stress concentrated region.

### BALANCED GROWTH CURVE IN THE STRESS COCENTRATED RESION

From results of four point bending fatigue tests under combination of in-plane tensile and out-plane bending load,

Kawahara et al. derived the estimation of the aspect ratio in Balanced growth. This formula consists of bending stress ratio.

$$a/b = 0.98 + 0.07\beta - (0.06 + 0.94\beta)x \quad (1)$$

Equation (1) is called 'Balanced growth curve'. However, because Eq. (1) was derived for a surface crack that exists in flat plate, this formula cannot be applied to the case that a surface crack is subjected to arbitrary stress distribution. Toyosada et al. referred Kawahara et al.'s results and derived formula of the change rate of the aspect ratio ( $=d(a/b)/dx$ ) of a surface crack that exists in a stress concentrated region [8]. However, Toyosada et al.'s formula can be only applied to the aspect ratio of a surface crack nearby notch-tip. Therefore, we examined the formula could estimate the aspect ratio of a surface crack regardless of existence of stress concentration and that can be applied to a grown surface crack.

The stress distribution is linear in Kawahara et al.'s tests. Therefore, the equation for a bending stress ratio ( $\beta$ ) and Eq. (1) can be deformed with a stress distribution ( $\sigma(x)$ ). And the change rate of aspect ratio is given by Eq. (3).

$$\frac{a}{b} = 1 - 0.06x - 0.47 \left( 1 - \frac{\sigma(x)}{\sigma(0)} \right) \quad (2)$$

$$\frac{d}{dx} \left( \frac{a}{b} \right) = -0.06 + 0.47 \frac{d}{dx} \left( \frac{\sigma(x)}{\sigma(0)} \right) \quad (3)$$

In the deviation of Eq. (2), the intercept of Eq. (1) is set as 1 for simplicity. From Eq. (2), it can be seen that aspect ratio of a surface crack is expressed by rate of the stress at the deepest point of crack to the stress at the free surface of cracked ( $x=0$ ).

Figure 6 shows that the stress distribution in section including the deepest point of a surface crack in a stress concentrated region.

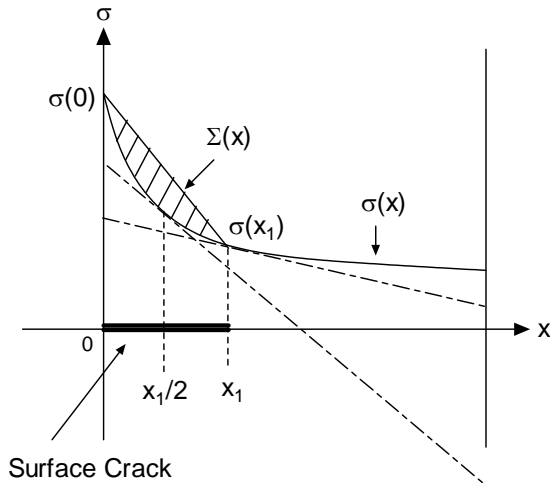


FIGURE 6. STRESS DISTRIBUTION ACTING ON CROSS SECTION CONTAINING THE DEEPEST POINT OF A SURFACE CRACK.

Equation (2) indicates that the aspect ratio of a surface crack at  $x_1$  is determined only by the stress at the deepest point of crack ( $\sigma(x_1)$ ) and that the change rate of the aspect ratio is determined only by the slope of the stress at the deepest point of crack ( $d\sigma(x_1)/dx$ ), under linear stress distribution.

However, Fig. 6 shows that in stress concentrated region, the aspect ratio is not determined only by  $\sigma(x_1)$  because the stress on a crack surface in  $0 < x < x_1$  is different from that under the linear stress distribution ( $\Sigma(x)$ ). It is also applicable for the relation between the change rate of the aspect ratio and  $d\sigma(x_1)/dx$ . Therefore, to express effect on the change rate of aspect ratio from this difference, we used the slope of stress at the midpoint of crack depth ( $d\sigma(x_1/2)/dx$ ). Assuming that  $d\sigma(x_1)/dx$  and  $d\sigma(x_1/2)/dx$  affect the change rate of aspect ratio at  $x_1$  equally, we derived equation (4) for the estimation of the change rate of the aspect ratio.

$$\frac{d}{dx} \left( \frac{a}{b} \right) = -0.06 + 0.47 \left\{ p_1 \frac{d}{dx} \left( \frac{\sigma(0.5x)}{\sigma(0)} \right) + p_2 \frac{d}{dx} \left( \frac{\sigma(x)}{\sigma(0)} \right) \right\} \quad (4)$$

where,  $p_1 = p_2 = 0.5$

Solid lines in Fig. 4 are results of the application of Eq. (4) to test results. In estimation of the aspect ratio by Eq. (4), point-like initial crack size and aspect ratio was set as  $a_0=0.03\text{mm}$  (as grain size of mild steel) and  $a_0/b_0=1$  by considering fatigue crack initiation and growth behavior discussed in Ref.[5].

$\sigma(x)$  in Eq. (4) of notched specimens were calculated by elastic finite element method under unit tensile stress and no crack condition (Fig. 7). Solid lines in Fig. 8 are results of the application of Eq. (4) to Kawahara et al.'s test results. However, even if the surface crack growth,  $\sigma(0)$  keeps the same value because the stress has no gradient for the crack length direction in this study.

Figure 4 and Fig. 8 show that Eq. (4) can estimate the aspect ratio in Balanced growth on the prudence side regardless of stress gradient for the crack depth direction. However, from Fig. 4-d) and Fig. 8 ( $\beta=0, w=8$ ), when the crack depth becomes more than half of plate thickness, the difference between the estimated results and the experimental results becomes larger. Kawahara et al. mentioned that this was because bending load was caused by the unbalance of the load according to the crack growth. On the other hand, it was partly because that stress range become large to cause a growth of the surface crack under pure tensile and no stress concentration condition. Therefore, it seems that the ligament in front of surface crack yield relatively quickly and the surface crack growth for the crack depth direction become later than that for the crack length direction.

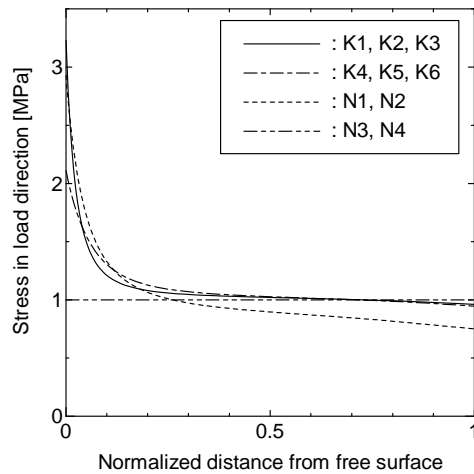


FIGURE 7. STRESS DISTRIBUTIONS ALONG CENTER AXIS OF SPECIMEN.

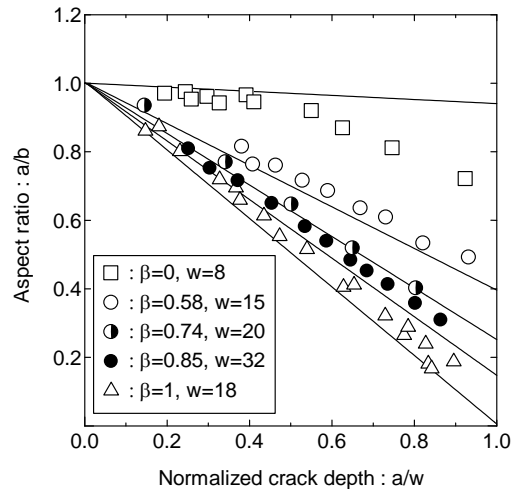


FIGURE 8. THE ASPECT RATIO OF A SURFACE CRACK IN BALANCED GROWTH.

## CONCLUDING REMARKS

Fatigue surface crack propagation tests under tensile loading were performed and the effect of the stress gradient difference on the aspect ratio of the surface crack. We presented the estimation method of shape evolution of the fatigue surface crack in Balance growth regardless of the stress gradient for the crack depth direction. Our method consists only stress distribution and not require calculation of the stress intensity factor of the surface crack to which the shape is changed according to the growth.

In the future we plan to extend this research as follows.

- 1) Improvement of our method for the case where the stress has gradient for the crack length direction. In this case, it seems that the function of stress for the crack length direction or the aspect ratio is added to right side of Eq. (4)

- 2) Improvement of our method for the estimation the aspect ratio in the early stage when the surface crack growth start from the wide / shallow initial crack.

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