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## ESTIMATION OF THE SHAPE EVOLUTION AND THE GROWTH HISTORY OF AN EMBEDDED CRACK BY FATIGUE LOADING

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### ABSTRACT

*Fatigue life estimation for planar cracks, e.g. part-through surface cracks or embedded cracks is very important because most of fatigue cracks found in welded built-up structures show planar crack morphologies.*

*Fatigue crack growth behaviour of an embedded crack in welded joints is investigated in this study. The estimation procedure of crack shape evolution for an embedded crack is introduced and validation of the estimation procedure of fatigue crack growth based on the numerical simulation of fatigue crack growth with EDS concept for an embedded crack is performed. The validity of the proposed shape evolution estimation method and the fatigue crack growth simulation based on the fracture mechanics approach with EDS concept are confirmed.*

### INTRODUCTION

Despite the many available fatigue life estimation methods, fatigue accidents of welded built-up structures have been occurring. Conventional fatigue life estimation for the structures is based on the combination of the hot spot stress based  $S-N$  curves and the cumulative damage rules [1]. However, this method contains the following serious weaknesses.

1. The  $S-N$  curves approach cannot give the fatigue crack growth evolution.
2. The transferability of fatigue life obtained by  $S-N$  curves to in-service structures has not established yet [2]. The relation between fatigue crack length found in in-service structures and the critical fatigue life obtained by  $S-N$  curves is not defined clearly.

On the other hand, the fatigue life predictions by the fracture mechanics approach are applied in order to improve the weaknesses of the conventional  $S-N$  curves approach. Most of fatigue life assessments by the fracture mechanics approach cannot quantitatively estimate the retardation and the acceleration of crack propagation caused by the loading history effect, because of insufficient consideration of fatigue crack opening / closing behaviour caused by the crack wake. Consideration of the cyclic plasticity ahead of a crack tip is imperative to the quantitative estimation of fatigue crack growth evolution.

Some numerical fatigue crack growth simulation models based on the Dugdale type strip yield model [3] with the stress intensity factor weight functions were proposed in order to evaluate the fatigue crack growth considering the fatigue crack opening / closing behaviour. In these methods, one-dimensional bar type elements to describe the role of crack wake over fatigue crack surfaces plugged up the chink corresponding to the virtual COD in the fictitious crack region (i.e. plastic zone ahead of a crack

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tip). The mechanical property of plugged material in the primitive model by Newman [4] corresponds to rigid-plastic material.

Toyosada et al. [5] developed Newman's model by changing the mechanical property of plugged material from the rigid-plastic to the elastic-perfectly plastic material in order to describe the elastic behaviour of material around a crack tip during the unloading and re-loading processes. Toyosada's crack growth simulation model and the effective stress intensity factor range based on RPG (Re-tensile Plastic zone Generating) load [5] allow fairly estimation of the fatigue crack growth under moderate variable amplitude loading.

These mentioned crack growth simulation can be applied for a through thickness crack and cannot be applied for part-through surface cracks or embedded cracks directly. Then, Toyosada et.al proposed the advanced numerical simulation procedure of fatigue crack growth for part-through surface cracks. They proposed the equivalent distributed stress (EDS) [6] concept, which reproduces the stress field near a crack tip of planar cracks in an infinite wide plate with a through thickness crack. The validity of EDS concept for the fatigue life estimation of part-through crack is confirmed in cases that the cracks emanating from a weld toe of out-of-plane gusset welded joints [6] and in-plane gusset welded joints [7]. In these estimations, the empirical formula [6] of part-through surface crack shape evolution and the treatment of the representative single surface crack, which replaces plural surface cracks generated at the stress concentration sites, based on shape measurements was applied.

However, the validity of this method for an embedded planer crack has not investigated yet. Then, we investigate the fatigue crack growth behaviour of an embedded crack in welded joints and the possibility of numerical simulation of an embedded fatigue crack growth in this study. Besides, we develop the reasonable estimation procedure of crack shape evolution for a planer crack and validate the estimation procedure of fatigue crack growth based on the numerical simulation with the EDS concept for an embedded crack in this study.

## ESTIMATION PROCEDURE OF SHAPE EVOLUTION OF A PLANER CRACK

Figure 1 shows the schematic illustrations of the shape evolution of a fatigue crack emanating from a planer-embedded defect. Crack front shape is approximated by the ellipsoid during an embedded and a part-through surface crack and by the parabola after penetrating plate thickness in this study. Generally, this supposition can be accepted by the observations of fracture surface caused by the fatigue.

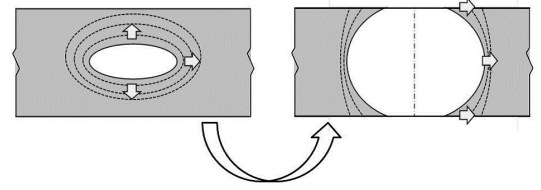


FIGURE 1. MODELLING OF THE SHAPE EVOLUTIONS

## Crack shape evolution during planer crack morphologies

Figure 2 shows a schematic illustration of the initial location and dimensions of an embedded crack existing in the plate. The

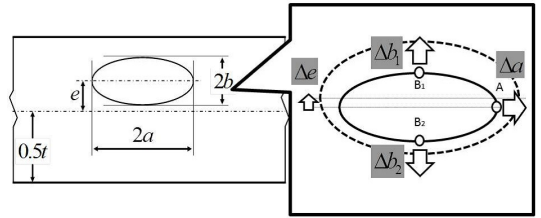


FIGURE 2. DEFINITION OF DIMENSIONS AND SHAPE EVOLUTION FOR AN EMBEDDED CRACK

shape evolution can be defined by identifying the crack length increment toward thickness and width directions. In addition, the shifting of centre of crack must be estimated because each increment of crack length is different between at B1 and at B2 in Figure 2.

By applying Paris law at points of A, B1 and B2 in Figure 2, following the systems of equations are obtained.

$$\frac{da}{dN} = C(\Delta K_A)^m, \quad (1)$$

$$\frac{db_1}{dN} = C(\Delta K_{B1})^m \text{ and} \quad (2)$$

$$\frac{db_2}{dN} = C(\Delta K_{B2})^m. \quad (3)$$

Where,

- $2a$  : crack width defined in Figure 2,
- $2b$  : crack depth defined in Figure 2,
- $N$  : number of cycles,
- $C, m$  : material constants of Paris law,
- $K_A$  : stress intensity factor at point A in Figure 2,
- $K_{B1}$  : stress intensity factor at point B1 in Figure 2 and
- $K_{B2}$  : stress intensity factor at point B2 in Figure 2.

Solving Equations (1) ~ (3) gives the crack length increments as follows.

$$da = \left( \frac{\Delta K_A}{\Delta K_{B1}} \right)^m db_1, \quad (4)$$

$$db_2 = \left( \frac{\Delta K_{B2}}{\Delta K_{B1}} \right)^m db_1 \text{ and} \quad (5)$$

$$de = (db_1 - db_2)/2. \quad (6)$$

Where,  $e$  corresponds to the eccentricity of embedded crack location defined in Figure 2.

Calculation procedures of the crack length increments are listed below.

1. Calculation of the stress intensity factors ( $K_A$ ,  $K_{B1}$  and  $K_{B2}$ ) for initial crack shape.
2. Calculation of the ratios of  $(\Delta K_A/\Delta K_{B1})^m$  and  $(\Delta K_{B2}/\Delta K_{B1})^m$ .
3. Set the increment value of crack depth ( $db_1$ ).
4. Calculation of the crack width increment ( $da$ ) and crack depth increment ( $db_2$ ) by applying Equations (4) and (5) and of the sifting increment of centre of crack ( $de$ ) by Equation (6).
5. Update the crack shape.
6. Repeat after step 2 to step 5.

Newman and Raju had proposed similar method in the reference [8]. However, the ratio of  $\Delta K_A/\Delta K_{B1}$  or  $\Delta K_A/\Delta K_{B2}$  keeps the same value obtained for initial crack shape in their method, even though the crack shape changes during the loading. On the other hand, the ratio is updating due to the crack shape evolution in our proposed method.

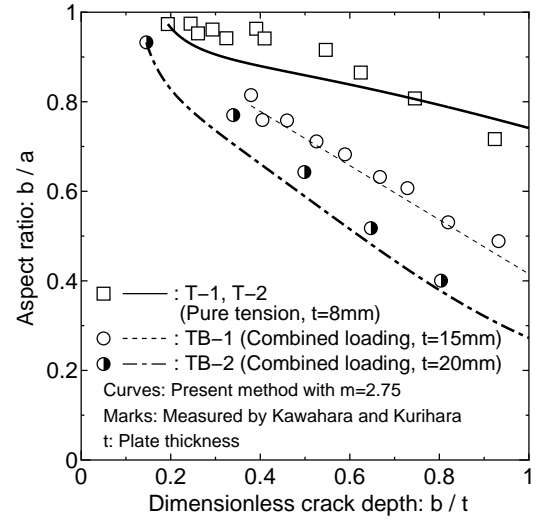
The validity of above mentioned procedure is investigated by comparing observed part-through surface crack shape evolutions in the references [9] [10]. Observed shape evolutions were obtained under i) pure tension, ii) pure bending and iii) combination of tension and bending loading conditions. Stress intensity factors for a surface crack are calculated by Raju and Newman's formula [8].

Material constant  $m$  is set to 2.75 for the comparison with the measurements by Kawahara and Kurihara [9]. This value is widely accepted for the material constant  $m$  of steels applied for welded built-up structures [11]. Material constant  $m$  is set to 2.88 for the comparison with the measurements by Putar and Schijve [10]. This value is shown in their reference [10].

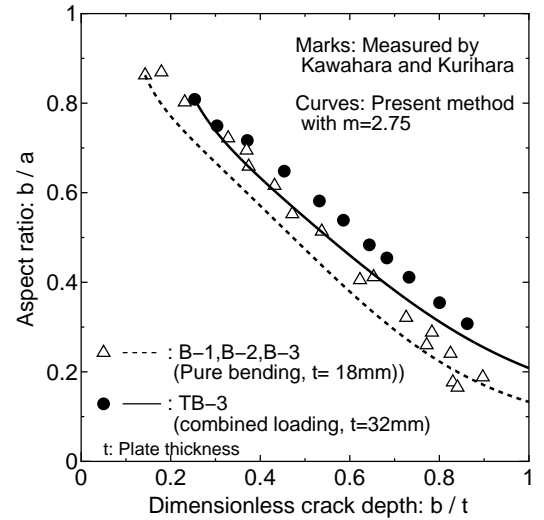
Figures 3 to 6 are the comparison of observed shape evolutions for Kawahara and Kurihara [9] and estimated ones by applying the proposed procedures.

Figure 7 is the comparison of observed shape evolutions for Putar and Schijve [10] and estimated ones.

It is concluded that our proposed method gives good estimation of the crack shape evolutions.



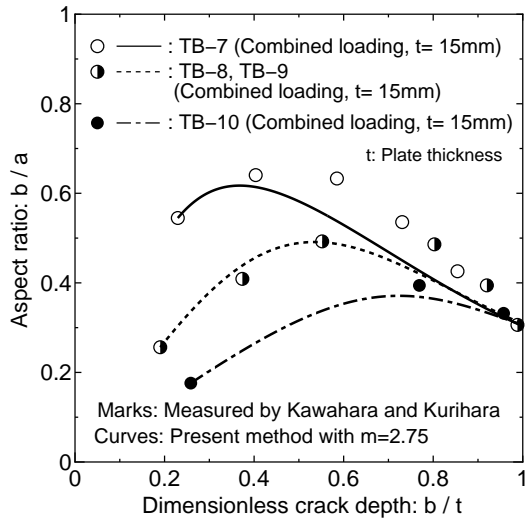
**FIGURE 3.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK SHAPE EVOLUTION FOR A SURFACE CRACK IN A PLATE UNDER PURE TENSION AND COMBINATION OF TENSION AND BENDING.



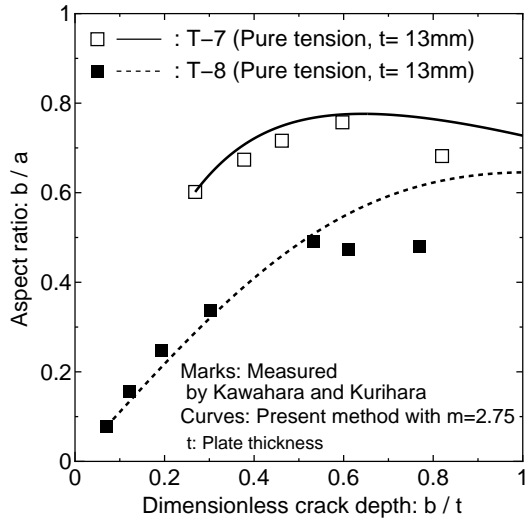
**FIGURE 4.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK SHAPE EVOLUTION FOR A SURFACE CRACK IN A PLATE UNDER PURE BENDING AND COMBINATION OF TENSION AND BENDING.

### Crack shape evolution after penetrating plate thickness

The shape of crack front edge after the penetration becomes curvature and approximated by the parabola in this study, see Figure 8. Crack length near the plate surface region dominantly grows up comparing to the inner region, and the front shape of a crack changes to the straight line following by the crack growth.



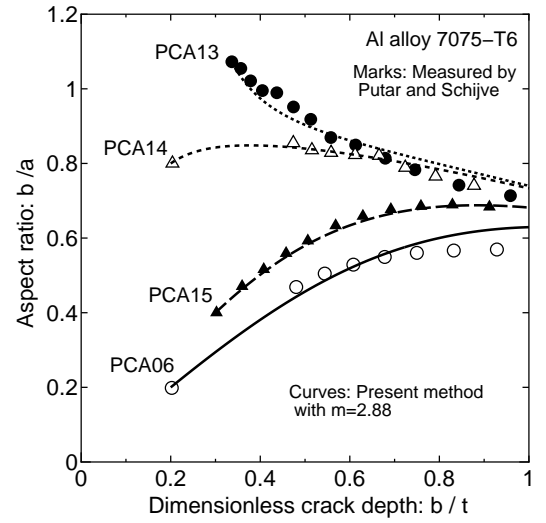
**FIGURE 5.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK SHAPE EVOLUTION FOR A SURFACE CRACK IN A PLATE UNDER COMBINATION OF TENSION AND BENDING.



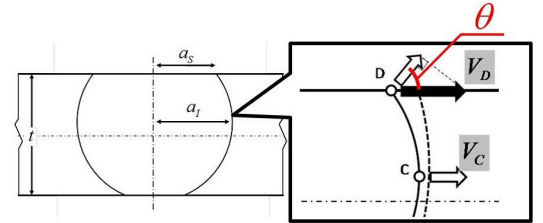
**FIGURE 6.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK SHAPE EVOLUTION FOR A SURFACE CRACK IN A PLATE UNDER PURE TENSION.

Estimation of the crack front edge evolution requires to calculate the increment of the crack length in the point C (half-length:  $a_I$ ) and point D (half-length:  $a_S$ ) shown in Figure 8. The relationship between crack propagation rates  $V_C$  at point C and  $V_D$  at point D in Figure 8 can be expressed by the following equation in case of a part-through surface crack [12].

$$V_D = V_C \cos \theta, \quad (7)$$



**FIGURE 7.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK SHAPE EVOLUTION FOR A SURFACE CRACK IN A PLATE UNDER PURE TENSION.



**FIGURE 8.** SHAPE EVOLUTION FOR A THROUGH THICKNESS CRACK

where  $\theta$  is defined in Figure 8. It is supposed that the same relation can be applied in the case shown in Figure 8, i.e. after the crack penetration. The crack propagation rates can be represented by  $V_C = da_I/dN$  at point C and  $V_D = da_S/dN$  at point D. Consequently, the ratios of increments of crack lengths at each point are obtained as follows.

$$\frac{a_I}{a_S} = \cos \theta. \quad (8)$$

Estimation procedure of a crack front shape evolution after the crack penetration can be indicated as follows.

1. Calculation of the normal direction angle ( $\theta$ ) of a crack front at point D.
2. Identification of the increment value of crack length  $da_I$ .
3. Calculation of the increment crack length  $da_S$ .
4. Update the crack front shape.

5. Repeat after step 2 to step 4.

## ESTIMATION OF SHAPE EVOLUTION AND FATIGUE CRACK GROWTH HISTORY OF AN EMBEDDED PLANER CRACK

The validity of estimation procedures of the crack shape evolution and the growth history of an embedded planer crack is investigated by comparing with measured ones [13]. Stress intensity factors for an embedded crack are calculated by applying the formula [14].

### Outline of experiments

Figure 9 shows the geometrical conditions of specimens used. Specimens contain butt welded joint at the centre position. An initial artificial defect like planer crack morphologies is introduced at the ash colour zone in Figure 9. Figures 10 are the cross section of each specimen.

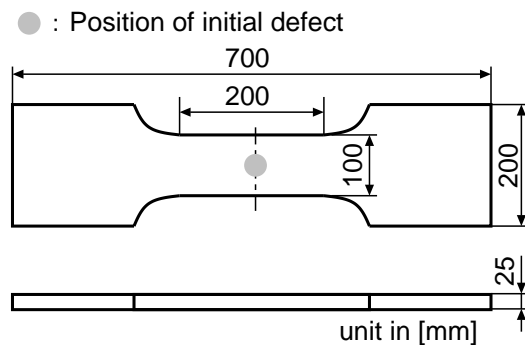


FIGURE 9. SPECIMEN CONFIGURATIONS USED

Uniform tensile loading condition is adopted in these experiments. Applied maximum stress is 200MPa for specimens A1 and A2. Applied maximum stress is 250MPa for specimen B1. Constant stress amplitudes with zero stress ratio are applied in these experiments. Crack shapes were measured by the beach mark method.

### Shape evolution of an embedded crack

The shape evolution of fatigue cracks are shown in Figures 11 to 13.

Figures 11 and 12 show comparison of experiment and calculation results for specimens A1 and A2. Figure 13 shows the comparison for specimen B1. (a) of each figure shows relationship between the dimensionless crack length and the aspect ratio of the defect during the growth. Marks in these figures indicate

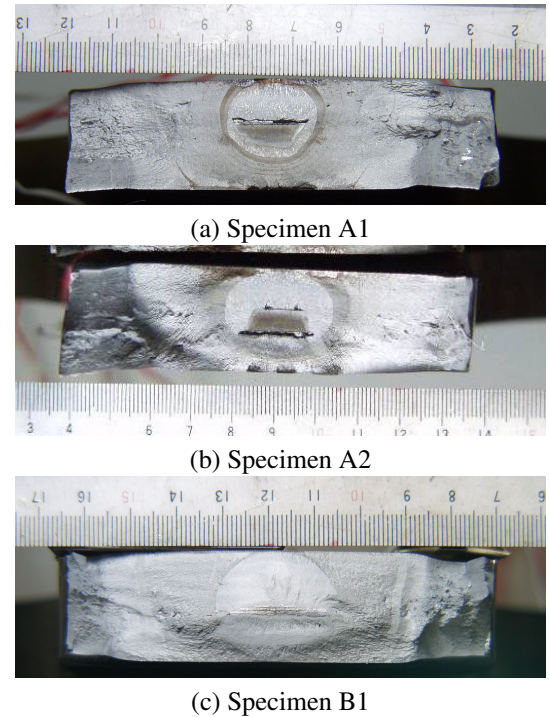


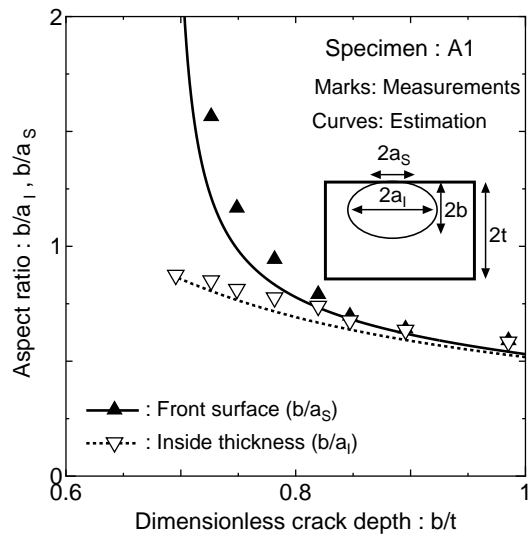
FIGURE 10. CROSSSECTION OF EACH SPECIMEN

measured results and curves indicate calculation results by applying proposed estimation method. (b) of each figure shows the crack shape evolution patterns. Dotted and solid curves indicate the measured and estimated results, respectively. Proposed method can accurately estimate the crack shape evolution for specimens A1 and B1, see Figures 11 and 13. On the other hand, measuring and calculating results show a difference for specimen A2, see Figure 12. It is confirmed that the measured crack propagation rate at crack depth point is slower than estimated one by referring Figure 12(b). The reason seems to be why the out-of-plane bending moment is induced by the eccentricity of the specimen caused by welding during the fabrication. This phenomenon is confirmed by measuring strains behaviour at specimen surfaces during the loading.

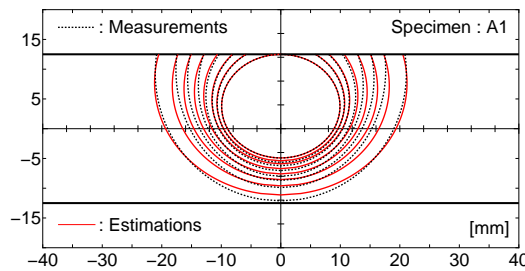
### Crack growth histories

From above results shown in Figures 11 to 13 and the equivalent distributed stress (EDS) concept, it is possible to replace an embedded crack to a through thickness crack in an infinite wide plate and to perform the fatigue crack growth simulation. Detailed treatment of EDS method is introduced in the references [6] [7].

Comparisons of the crack growth curves obtained by the numerical simulation with measurements are shown in Figures 14 and 16. Simulation code FLARP [5] is applied in these numeri-



(a) Evolution of aspect ratio



(b) Crack shape evolution patterns

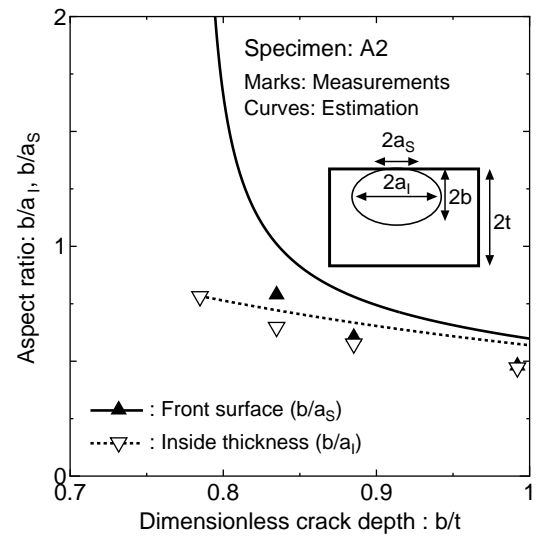
**FIGURE 11.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK SHAPE EVOLUTION (SPECIMEN A1).

cal calculations. Marks in Figures 14 and 16 indicate measured results and curves in these figures indicate numerical simulation results by FLARP.

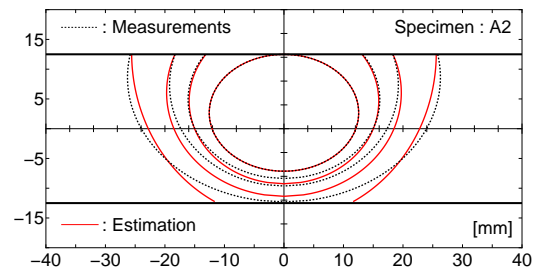
It makes clear that the application of FLARP with the EDS concept enables to estimate accurate fatigue crack growth history emanating from an embedded defect in cases of specimens A1 and B1. On the other hand, measured fatigue crack growth of width direction is faster than estimated result in case of specimen A2. Comparison of the crack shape evolution shown in Figure 12(b) indicates the existence of induced out-of-plane bending stress caused by the imperfection of specimen fabrication process. It is estimated that the secondary induced bending stress raise the fatigue crack growth speed of width direction.

## CONCLUSION

Numerical simulations of the fatigue crack shape evolution and the crack growth history from a planner defect are performed. The validity of proposed estimation method of the shape



(a) Evolution of aspect ratio



(b) Crack shape evolution patterns

**FIGURE 12.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK SHAPE EVOLUTION (SPECIMEN A2).

evolution of a planer crack and the numerical simulation of fatigue crack growth based on the fracture mechanics approach with the EDS concept are confirmed.

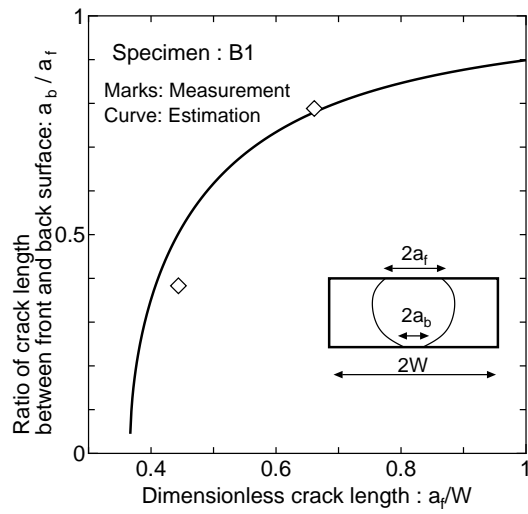
Future challenge is the consideration of loading effect for the estimation of the shape evolution and the crack growth history in the three dimensional crack problems.

## ACKNOWLEDGMENT

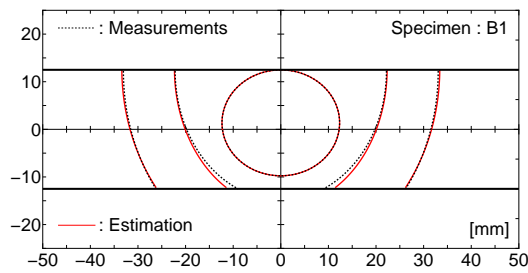
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(a) Evolution of aspect ratio

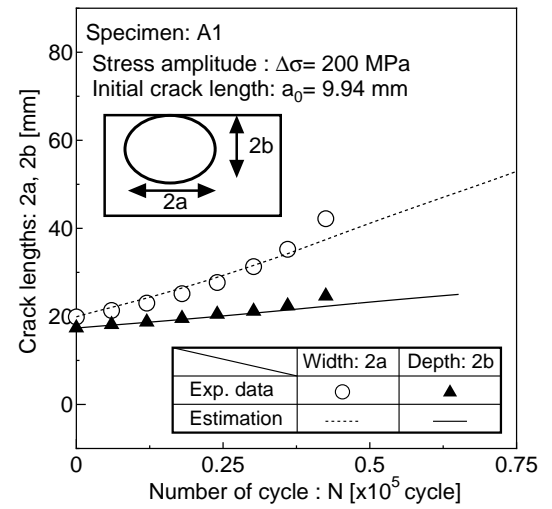


(b) Crack shape evolution patterns

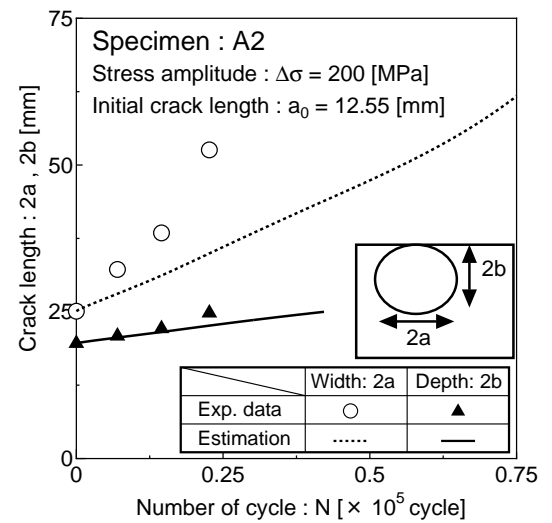
**FIGURE 13.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK SHAPE EVOLUTION (SPECIMEN B1).

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**FIGURE 14.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK GROWTH HISTORIES (SPECIMEN A1).

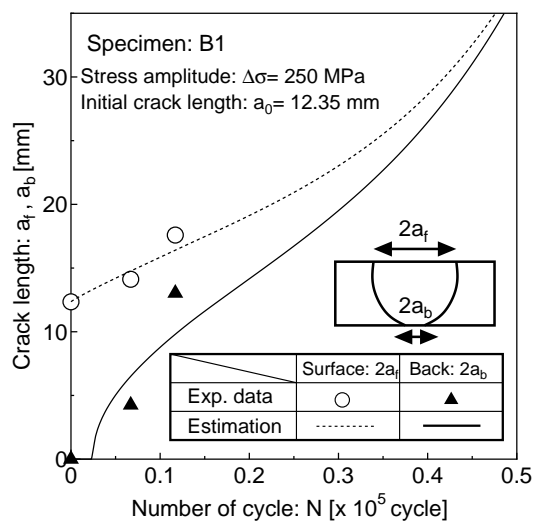


**FIGURE 15.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK GROWTH HISTORIES (SPECIMEN A2).

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**FIGURE 16.** COMPARISON OF EXPERIMENTAL AND ESTIMATED FATIGUE CRACK GROWTH HISTORIES (SPECIMEN B1).

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