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## A SIMPLE ESTIMATION METHOD OF THE STRESS DISTRIBUTION NORMAL TO CROSS SECTION AT WELD TOE IN NON-LOAD CARRYING WELDED JOINTS

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### ABSTRACT

Many fatigue damages are occurred in the welded built-up structures designed by the hot spot stress methodology, especially near a boxing fillet weld toe. These fatigue cracks usually initiate from the toe and propagate to the plate thickness direction.

Although fatigue life is affected by the stress gradient working over crack propagation path, the effect of stress gradient in cross section is not considered in the hot spot stress methodology. Then, many attempts based on fracture mechanics for the improvement of fatigue life estimation are proposed. Whereas stress distributions along the fatigue crack path must be given in order to apply the methods based on fracture mechanics for the precise fatigue life prediction, no stress distribution along the path considering the stress concentration caused by weld toe shape is obtained in practical structural design stages because the shell elements are used in finite element analyses in the design stages.

A simple estimation method of the stress distribution normal to cross section at weld toe in non-load carrying welded joints is proposed in this paper. Calculation results of finite element analysis with shell elements and geometrical conditions (radius and flank angle of fillet weld toe and plate thickness) are used as input data for the estimation.

The validity of this method is confirmed by comparing estimation results with ones by finite element analysis with solid

elements.

### INTRODUCTION

Hot spot stress methodology with finite element analysis [1] is commonly applied in order to evaluate fatigue strength of welded built-up structures and the assessments for many types of structural elements are reported [2] [3] [4] [5]. Besides, the combination of hot spot stress  $S-N$  curves and cumulative damage rules is extensively applied to the fatigue life evaluation in the design stages of the structures. Many fatigue damages are, however, found in the structures designed by the hot spot stress methodology, especially near a boxing fillet weld toe. One of the fault of hot spot stress methodology is disregard for the stress gradient in the cross section of plate thickness. These fatigue cracks usually initiate from the toe and propagate in the cross section. An attempt to incorporate the stress gradient effect into the hot spot stress methodology was proposed [6] [4].

On the other hand, many attempts based on fracture mechanics for the improvement of fatigue life prediction are performed in order to overcome the lack of the conventional method (the combination of hot spot stress  $S-N$  curves and cumulative damage rules). In particular, authors proposed the simulation algorithm of fatigue crack growth from sound (zero initial defect) stress concentration sites for welded built-up steel structures considering loading histories and residual stress caused by welding [7] [8].

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Whereas stress distributions along the fatigue crack path must be given in order to apply fracture mechanics methods to the precise fatigue life prediction, no stress distribution along the path in the plate thickness is given in practical structural design stages because the shell elements are used in finite element analyses in the design stages.

A simple estimation method of the stress distribution normal to cross section at weld toe in non-load carrying welded joints is proposed in this paper. Calculation results of finite element analysis with shell elements and geometrical conditions (radius and flank angle of fillet weld toe and plate thickness) are used as input data for the estimation. The validity of proposed method is confirmed by comparing estimation results with ones by finite element analyses with solid elements.

In this study, non-load carrying type welded joints are highlighted as the research target.

## A SIMPLE ESTIMATION METHOD OF THE STRESS DISTRIBUTION NORMAL TO CROSS SECTION AT WELD TOE

### Requirements of Stress Distribution in Cross Section

Stress distribution in cross section  $\sigma(z)$  must satisfy the following relations.

$$\int_0^t \sigma(z) dz = P, \quad (1)$$

$$\int_0^t z \sigma(z) dz = M, \quad (2)$$

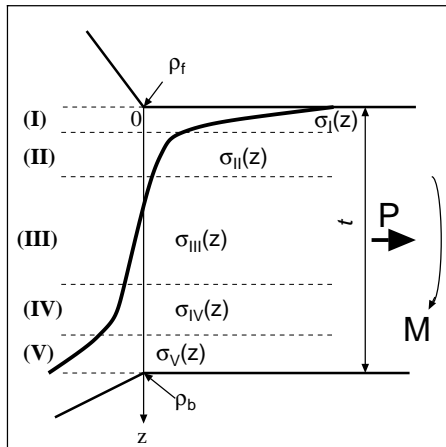


Figure 1. COORDINATE SYSTEMS AND SCHEMATIC ILLUSTRATION OF THE STRESS DISTRIBUTION NORMAL TO CROSS SECTION AT WELD TOE

where  $t$  is plate thickness,  $P$  and  $M$  are the load and the moment acting on the reference cross section respectively.

Considering linearised stress distribution over plate thickness in the finite element analysis with shell elements,  $P$  and  $M$  on the cross section can be obtained by the following relations .

$$P = t(\sigma_f + \sigma_b)/2, \quad (3)$$

$$M = t^2(2\sigma_b + \sigma_f)/6. \quad (4)$$

$\sigma_f$  and  $\sigma_b$  are stresses on front and back surfaces at the reference location obtained by finite element analysis with shell elements. Subscripts  $f$  and  $b$  means the “front” and “back” surfaces respectively. The meaning of subscripts  $f$  and  $b$  is used as the same meanings in the following formulations.

### Expression of Stress Distribution in Cross Section

Because the geometry of cross section containing weld toes can be classified into cruciform or T joint, general expression of stress distribution in cross section for cruciform and T joint, i.e. two dimensional problem, is examined at first and applicability of it to three dimensional problem is investigated subsequently.

Five regions shown in Figure 1 are established in order to formulate the stress distribution. Figure 1 corresponds to the cruciform cross section. In case of T cross section, region (III) occupies to the back surface and regions (IV) and (V) are ignored.

**Regions (I) and (V).** Because the shape of weld toe strongly affects the stress distribution in regions (I) and (V), the form of stress distribution near notch root is referred in order to represent the distribution.

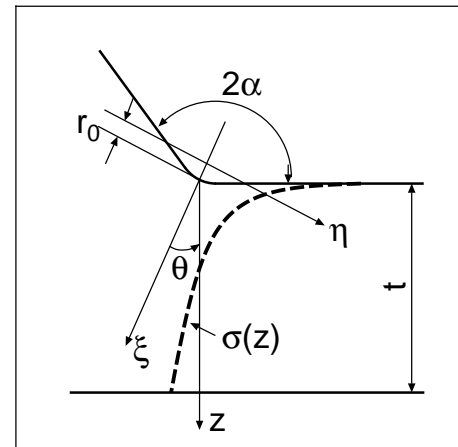


Figure 2. COORDINATE SYSTEMS FOR APPLYING EQUATIONS (7) and (8)

Although many proposed expression of stress distribution near notch root, two candidates are selected after many trials. One is proposed by Weiss [9] and the other is done by Lazzarin et al. [10]. Only the effect of weld toe radius can be considered for the distribution by applying Weiss's expression. On the other hand, the effect of weld toe radius and flank angle of fillet weld toe can be considered by applying Lazzarin's expression.

The formulation of stress distributions in each region by applying Weiss's expression is shown as follows.

$$\sigma_I = a_1 \sqrt{\rho_f / (\rho_f + 4z)} \sigma_0(z), \quad (5)$$

$$\sigma_V = a_2 \sqrt{\rho_b / (\rho_b + 4(z - t/2))} \sigma_0(z), \quad (6)$$

where  $a_1$  and  $a_2$  are unknown constants and corresponds to the stress concentration factors in each weld toe.  $\sigma_0(z)$  corresponds to the nominal stress at each location.

On the other hand, stress distributions in each region can be formulated by applying Lazzarin's expression as follows.

$$\sigma_I = a_1 \left[ \cos \theta_f \{ A_f^I (\xi + r_{0f})^{\lambda_{1f}-1} + B_f^I (\xi + r_{0f})^{\mu_{1f}-1} \} \right. \\ \left. - \sin \theta_f \{ A_f^{II} (\xi + r_{0f})^{\lambda_{2a}-1} + B_f^{II} (\xi + r_{0f})^{\mu_{2f}-1} \} \right], \quad (7)$$

$$\sigma_V = a_2 \left[ \cos \theta_b \{ A_b^I (\xi + r_{0b})^{\lambda_{1b}-1} + B_b^I (\xi + r_{0b})^{\mu_{1b}-1} \} \right. \\ \left. - \sin \theta_b \{ A_b^{II} (\xi + r_{0b})^{\lambda_{2b}-1} + B_b^{II} (\xi + r_{0b})^{\mu_{2b}-1} \} \right], \quad (8)$$

where

$$A^I = \lambda_1 \left[ (1 + \lambda_1) \cos \{ (1 - \lambda_1) \theta \} + \chi_{11} (1 - \lambda_1) \cos \{ (1 + \lambda_1) \theta \} \right], \\ A^{II} = \lambda_2 \left[ (1 + \lambda_2) \cos \{ (1 - \lambda_2) \theta \} + \chi_{21} (1 - \lambda_2) \cos \{ (1 + \lambda_2) \theta \} \right], \\ B^I = (\lambda_1 q / 4(q - 1)) r_0^{-(\mu_1 - \lambda_1)} \\ \times [\chi_{13} (1 + \mu_1) \cos \{ (1 - \mu_1) \theta \} + \chi_{12} \cos \{ (1 + \mu_1) \theta \}], \\ B^{II} = (\lambda_2 q / 4(q - 1)) r_0^{-(\mu_2 - \lambda_2)} \\ \times [\chi_{23} (1 + \mu_2) \cos \{ (1 - \mu_2) \theta \} + \chi_{22} \cos \{ (1 + \mu_2) \theta \}], \\ q = 2(\pi - \alpha) / \pi.$$

$a_1$  and  $a_2$  are the constants which can be identified by boundary conditions.  $r_0$  corresponds to the distance from the center of toe radius to the notch root. Parameters  $\lambda_1$ ,  $\mu_1$ ,  $\chi_{11}$ ,  $\chi_{12}$ ,  $\chi_{13}$ ,  $\lambda_2$ ,  $\mu_2$ ,  $\chi_{21}$ ,  $\chi_{22}$  and  $\chi_{23}$  are given as a function of flank angle ( $2\alpha$ ). The values of these constants is shown in Table 1 [10]. Coordinate systems for referring Equations (7) and (8) are illustrated in Figure 2.

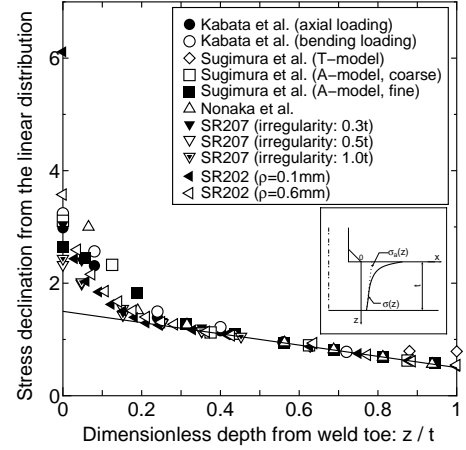


Figure 3. STRESS DECLINATION FROM THE LINEAR DISTRIBUTION

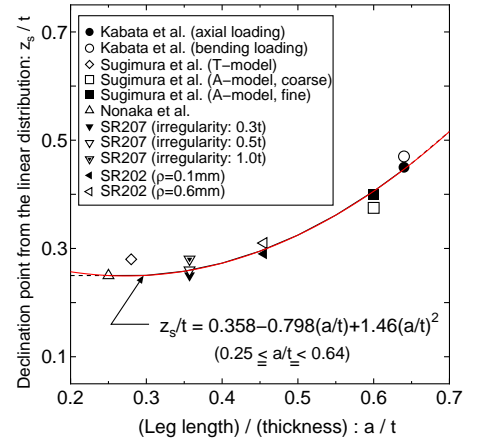


Figure 4. DECLINATION POINTS FROM THE LINEAR DISTRIBUTION

**Region (III).** In this region, stress distribution is not affected by the weld toe geometry and can be approximated as linear distribution

$$\sigma_{III}(z) = a_3 z + a_4, \quad (9)$$

where  $a_3$  and  $a_4$  are the constants which can be identified by boundary conditions.

In order to identify the range of region (III), some finite element calculation results using solid elements to calculate the stress distribution normal to cross section are investigated. Surveyed calculation models are the boxing fillet weld toe [11], structural elements with cruciform joint shape found in ship structures [12], structural element with pad plates as a pipe root support [13], cruciform joints with irregularity (0.3t, 0.5t and

Table 1. THE VALUES OF PARAMETERS IN EQUATIONS (7) AND (8)

$2\alpha[\text{rad}]$	$\lambda_1$	$\mu_1$	$\chi_{11}$	$\chi_{12}$	$\chi_{13}$	$\lambda_2$	$\mu_2$	$\chi_{21}$	$\chi_{22}$	$\chi_{23}$
0	0.5000	-0.5000	1.0000	4.0000	0.0000	0.5000	-0.5000	1.0000	-12.0000	0.0000
$\pi/6$	0.5014	-0.4561	1.0707	3.7907	0.0632	0.5982	-0.4465	0.9212	-11.3503	-0.3506
$\pi/4$	0.5050	-0.4319	1.1656	3.5721	0.0828	0.6597	-0.4118	0.8140	-10.1876	-0.4510
$\pi/3$	0.5122	-0.4057	1.3123	3.2832	0.0960	0.7309	-0.3731	0.6584	-8.3946	-0.4778
$\pi/2$	0.5448	-0.3449	1.8414	2.5057	0.1046	0.9085	-0.2882	0.2189	-2.9382	-0.2436
$2\pi/3$	0.6157	-0.2678	3.0027	1.5150	0.0871	1.1489	-0.1980	-0.3139	4.5604	0.5133
$3\pi/4$	0.6736	-0.2198	4.1530	0.9933	0.0673	1.3021	-0.1514	-0.5695	8.7371	1.1362
$5\pi/6$	0.7520	-0.1624	6.3617	0.5137	0.0413	1.4858	-0.1034	-0.7869	12.9161	1.9379

Table 2. SUMMARY OF THE INVESTIGATED FINITE ELEMENT MODELS

Authors	Type of structure	Type of loading	Number of elements in the plate	Thickness [mm]	Leg length [mm]
Kabata et al. [11]	Boxing fillet welded joint	Axial	6	12.5	8.0
		Bending	6		
Sugimura et al. [12]	Cruciform joint structure	Mixed	8	20.0	12
	T joint	3P bending	4	25.0	7.0
Nonaka et al. [13]	Lap joint	Axial	8	20.0	5.0
SR207 [14]	Cruciform joint with irregularity $0.3t, 0.5t$ and $1.0t$	Axial	10	21.0	7.5
SR202 [15]	Tubular T joint (radii of weld toe: 0.1, 0.6mm)	Axial (brace direction)	15	22.0	10.0

1.0t) [14] and T-shaped tubular joints (radii of weld toe are 0.1mm and 0.6mm) [15]. Summary of the referred finite element analysis models are shown in Table 1. Although the radius of weld toe is ignored in modelling of all of the calculations except T-shaped tubular joints, the appropriate range of region (III) is obtained from these finite element calculation results because the weld toe shape does not have an affect on the stress distribution in this region.

Figure 3 shows the investigation results. Horizontal axis means dimensionless depth from the front surface ( $z/t$ ) and vertical axis means the stress declination from the linear distribution. The stress distributions of cruciform joints [14] are investigated from front surface to the center of plate thickness. The value of vertical axis in Figure 3 is normalized by the stress on back surface. Region (III) corresponds to the range which the marks overlaps with the straight line in Figure 3.

The declination points from the linear stress distribution are investigated from Figure 3 and the approximation as a function of  $a/t$  ( $a$ : leg length of weld bead) for the points are performed. These results are shown in Figure 4. Approximation function for obtaining the declination point ( $z_s$ ) is

$$(z_s/t) = 0.358 - 0.798(a/t) + 1.46(a/t)^2, \quad (10) \\ (0.25 \leq a/t \leq 0.64).$$

Even though Equation (10) has the limit of application ( $0.25 \leq a/t \leq 0.64$ ), this equation can be applied to the estimation of stress distribution in plate thickness of actual hull and offshore structures because the ratio  $a/t$  satisfy the limit of application in Equation (10) in most of these structures.

**Regions (II) and (IV).** Formulation of the stress distribution in region (II) is accomplished by considering the continuity in the distributions in regions (I) and (III). Similar condition is required for the stress distribution in region (IV).

The expressions in case that Weiss's formula are applied in regions (I) and (V) is shown as follows.

$$\sigma_{II} = a_5 \sqrt{\rho_f / (\rho_f + 4z)} + a_6 z + a_7, \quad (11)$$

$$\sigma_{IV} = a_8 \sqrt{\rho_b / (\rho_b + 4(z - t/2))} + a_9 z + a_{10}, \quad (12)$$

where  $a_5 \sim a_{10}$  are the constants.

Similar expression to the Equations (11) and (12), which contains the same number of constants, are obtained in case that Lazzarin's representation is applied in regions (I) and (V).

### Identification of stress distribution

The stress distribution normal to cross section at weld toe can be identified by solving ten unknown constants  $a_1 \sim a_{10}$  in accordance with the following procedures.

1. Calculating the force  $P$  and moment  $M$  acting on the cross section by finite element analysis with shell elements, Equations (3) and (4).
2. Deriving the system of linear equations of unknown constants  $a_1 \sim a_{10}$  considering the following conditions.
  - (a) Equilibrium relation concerning the force ( $P$ ), which is obtained by substituting the equations related stress distribution into Equation (1).
  - (b) Equilibrium relation concerning the moment ( $M$ ), which is obtained by substituting the equations related stress distribution into Equation (2).
  - (c) Continuity conditions both stress and stress gradient at the connection points of each region.
3. Solving the systems of linear equations.
4. Identifying the coefficients  $a_1 \sim a_{10}$ .

### THE APPLICABILITY OF THE PROPOSED METHOD

Comparison of stress distributions estimated by proposed method with finite element analyses with solid elements for the validation of proposal method are performed. Two dimensional problem (Cruciform and T joints) and three dimensional problem (out of plane gusset joints containing cruciform and T cross sections) are applied as the examination objects. Case IDs and each geometries about weld toe are shown in Table 3. Common geometries of all objects are listed below.

**Plate thickness :** 10mm.

**Flank angle** ( $2\alpha$  in Figure 2): 135 degree.

**Leg length :** 5mm.

Applied remote stress distribution in main plate is linear with 2 MPa on front surface and 1 MPa on back surface.

### Cruciform and T joints

Finite element subdivisions by shell and solid elements of T joints are shown in Figure 5. One fourth of the objects are modeled considering their symmetry. Quadrilateral plane element with four nodes is used in the modelling of Figure 5 (a). Considering the use of proposed method in hull structural design stages, relatively coarse mesh size ( $0.5t \times 0.5t$ ) is applied in order to construct finite element models with shell elements. In Figure 5 (b), six-sided solid elements with eight nodes are applied for the mesh subdivisions in most part of the objects. Four-sided solid elements with four nodes and five-sided solid elements with six nodes are also applied to represent the detail of weld toe shape. Similar mesh idealizations are performed for the cruciform joints.

Comparison of stress distributions between finite element calculation results by applying solid models and estimated stress distributions by applying proposed methods are shown in Figure 6. In these figures, "Method 1", which are drawn by dash line, corresponds to the result by Weiss's type formulation of stress distribution in regions (I) and (V). On the other hand, "Method 2", which is drawn by solid line, corresponds to the result by Lazzarin's type formulation.

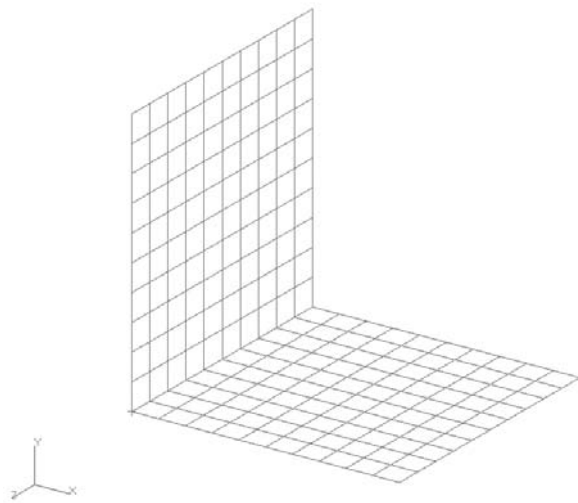
From Figure 6, it is confirmed that the proposed method enables to give fairly good estimation results and that the better estimation results are obtained by applying Lazzarin's type formulations (Equations (7) and (8)) than by Weiss's type formulations (Equations (5) and (6)).

### Out of Plane Gusset joints

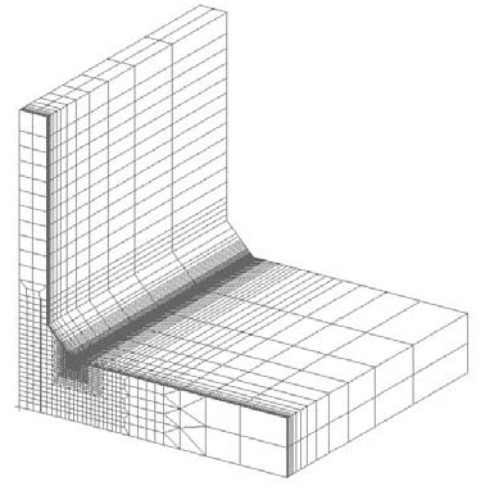
Figures 7 (a) and (b) are the finite element subdivisions of the out of plane gusset joints containing T shaped cross section. One fourth of the object is modeled by considering the geomet-

Table 3. WELD TOE GEOMETRIES OF CALCULATION OBJECTS

Case ID	Type	$\rho_f$ [mm]	$\rho_b$ [mm]
C-1	Cruciform	0.7	1.0
C-2	Cruciform	1.0	1.0
T-1	T	1.0	—
OC-1	Gusset (cruciform shape)	0.7	1.0
OC-2	Gusset (cruciform shape)	1.0	1.0
OT-1	Gusset (T shape)	1.0	—

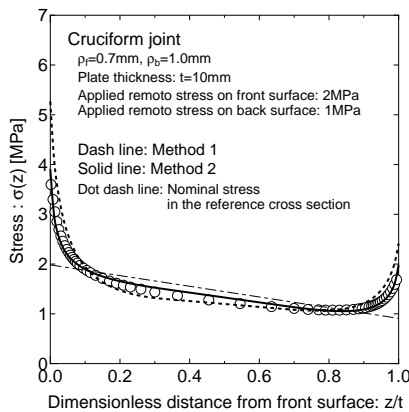


(a) Shell elements (Number of nodes: 321, Number of elements: 280, Minimum mesh size: 5mm  $\times$  5mm)

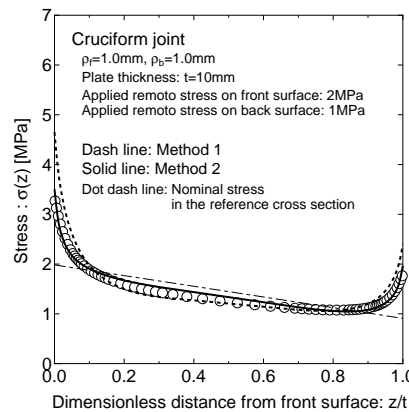


(b) Solid elements (Number of nodes: 33916, Number of elements: 31755, Minimum mesh size: 0.03mm  $\times$  0.03mm  $\times$  0.03mm)

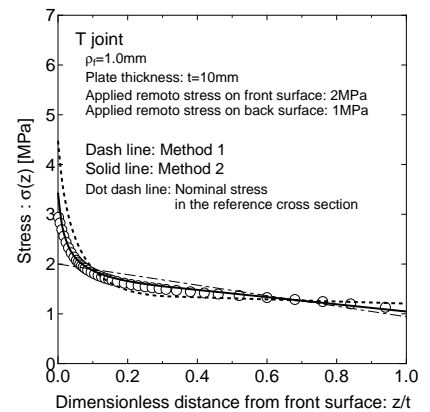
Figure 5. FINITE ELEMENT SUBDIVISIONS FOR T JOINT



(a) C-1



(b) C-2



(c) T-1

Figure 6. COMPARISON OF STRESS DISTRIBUTIONS IN THE CASE OF TWO DIMENSIONAL PROBLEMS

rical symmetry. Case IDs and geometries near weld toes are listed in Table 3. Similar mesh subdivisions are performed for the joints containing cruciform shaped cross section. Mesh size and type selection policies for each modelling are the same in the case of two dimensional problems described above.

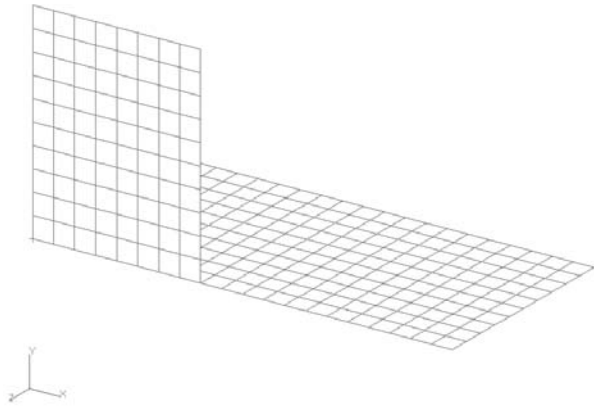
Comparison of stress distributions between finite element calculation results by solid models and estimated stress distributions by applying proposed methods are shown in Figure 8. In these figures, “Method 1” and “Method 2” are the same meaning as ones shown in Figure 6

It is confirmed once again that the proposed method enables to give fairly good estimation results and that the better estima-

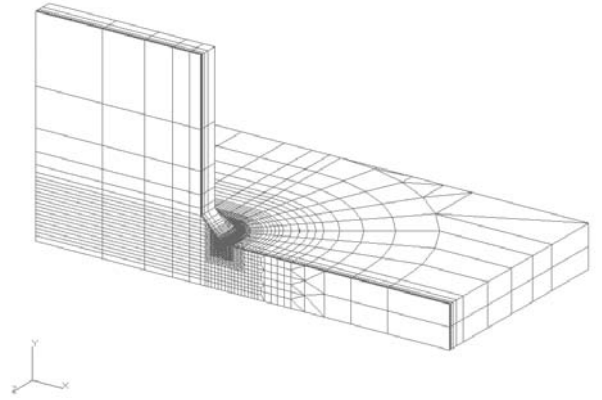
tion results is obtained by applying Lazzarin’s type formulations (Equations (7) and (8)) than by Weiss’s type formulations (Equations (5) and (6)) for the gusset joints containing cruciform type cross section.

In the case of T-shaped cross section shown in Figure 8 (c), small difference between estimated stresses and finite element analysis by solid elements are found near back surface. Such a slight difference near back surface, however, can be accepted because this region is not important to evaluate fatigue failure.

Therefore, it is concluded that the proposed method in this paper is useful to estimate the plate thickness direction of stress distribution from the finite element analysis with shell elements.

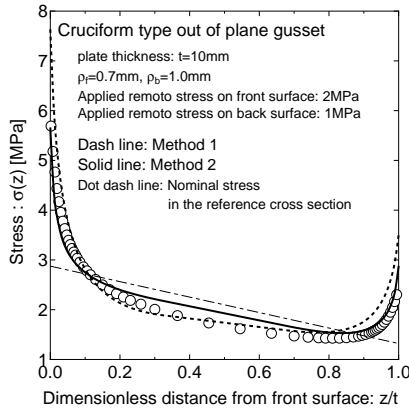


(a) Shell elements (Number of nodes: 231, Number of elements: 200, Minimum mesh size: 5mm× 5mm)

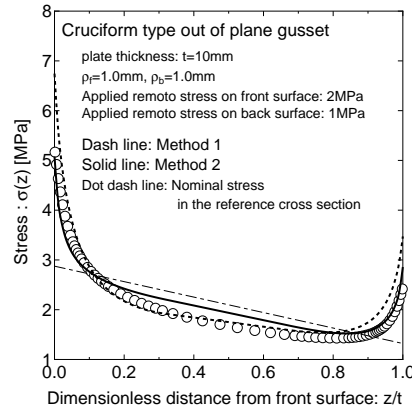


(b) Solid elements (Number of nodes: 28752, Number of elements: 26235, Minimum mesh size: 0.03mm× 0.03mm× 0.03mm)

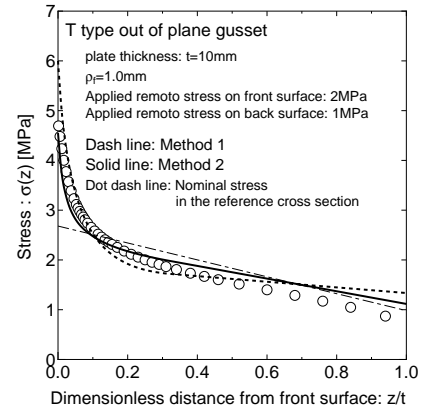
Figure 7. FINITE ELEMENT SUBDIVISIONS FOR OUT OF PLANE GUSSET JOINTS CONTAINING T-SHAPED CROSS SECTION



(a) OC-1



(b) OC-2



(c) OT-1

Figure 8. COMPARISON OF STRESS DISTRIBUTIONS IN THE CASE OF THREE DIMENSIONAL PROBLEMS

## CONCLUSION

A simple estimation method of the stress distribution normal to cross section at weld toe in non-load carrying welded joints by using calculation results of finite element analysis with shell elements and geometrical conditions is proposed in this paper. The validity of proposed method is confirmed by comparing estimation results with ones by finite element analyses with solid elements.

Although the investigation of the mesh size effect of the model idealized by shell elements remains to be solved, it is expected that the proposal method is convenient to estimate the stress distribution normal to cross section at weld toe without fine mesh modeling by applying solid elements.

## ACKNOWLEDGMENT

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