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NUMERICAL SIMULATION OF FATIGUE CRACK GROWTH BASED ON STRIP YIELD MODEL CONSIDERING WORK HARDENING OF MATERIALS

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ABSTRACT
This paper presents the improved numerical simulation of fatigue crack growth considering the crack opening / closing behaviour based on the strip yield model with the stress intensity factor weight function.

The mechanical property in the primitive model corresponds to rigid-plastic material and is replaced to the elastic - perfectly plastic material in order to describe the elastic behaviour of material around a crack tip during the unloading process.

However, the simulation model based on the elastic - perfect plastic material gives poor growth estimations under rapidly changing of loading histories, e.g. the spike loading. The possibility is pointed out that insufficient considerations of work hardening effect of materials lead the excess crack closure in the numerical simulations.

Authors propose the improved numerical simulation fatigue crack growth considering the work hardening effect of materials in this paper. Comparison of proposed simulation results with previous ones and with measured results confirms the primacy of proposed method over previous ones.

INTRODUCTION
Despite the many available fatigue life estimation methods, fatigue accidents of welded built-up structures have been occurring. Conventional fatigue life estimation for the structures is based on the combination of the hot spot stress based S–N curves and the cumulative damage rules [1]. However, this method contains the following serious weaknesses.

1. S–N curves based approach cannot give the fatigue crack growth history.
2. The transferability of fatigue life obtained by S–N curves to in-service structures has not established yet [2]. The relation between fatigue crack length found in in-service structures and fatigue life obtained by S–N curves is not defined clearly.

On the other hand, fatigue life predictions based on fracture mechanics are applied in order to improve the weaknesses of the conventional S–N curves approaches. Most of fatigue life assessments based on fracture mechanics cannot quantitatively evaluate the retardation and the acceleration of crack propagation, because of insufficient consideration of fatigue crack opening / closing behaviour caused by the crack wake. Consideration of the cyclic plasticity ahead of a crack tip is imperative to the estimation of fatigue crack growth quantitatively [5].

Some numerical fatigue crack growth simulation models based on the Dugdale type strip yield model [3] with the stress intensity factor weight function were proposed in order to evaluate the fatigue crack growth considering the fatigue crack opening / closing behaviour. In these methods, one-dimensional bar elements to describe the role of crack wake over fatigue crack surfaces plugged up the chink corresponding to the virtual COD in the plastic zone. The mechanical property of plugged material in the primitive model by Newman [4] corresponds to rigid-plastic

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Toyosada et al. [5] developed Newman’s model by changing the mechanical property of plugged material from the rigid-plastic to the elastic-perfectly plastic material in order to describe the elastic behaviour of material around a crack tip during the unloading process. Toyosada’s crack growth simulation model and the effective stress intensity factor range based on RPG (Re-tensile Plastic zone Generating) load [5] allow fairly estimation of the fatigue crack growth history under moderate variable amplitude loading.

However, his simulation method gives poor growth estimations under rapidly changing of loading histories, e.g. spike loading. The possibility is pointed out that insufficient considerations of work hardening effect of materials lead the overestimation of crack wake evolution and the excess crack closure in the numerical simulations.

Authors proposed the improved numerical simulation model of fatigue crack growth based on Toyosada’s model. Elastic-plastic with work hardening material is applied as a mechanical property in the improved simulation. Numerical simulations of the fatigue crack growth under many types of loading are performed in order to investigate the validity of our proposed model.

**NUMERICAL SIMULATION MODEL OF FATIGUE CRACK GROWTH BASED ON THE STRIP YIELD MODEL**

**General aspect of previous numerical simulation model**

General aspects of the numerical simulation model of fatigue crack growth based on the strip yield model with the stress intensity factor weight function is explained briefly according to the reference [5].

The condition of the displacement continuity ahead of the physical crack tip is not satisfied in the primitive strip yield model. To eliminate this deficiency Toyosada et al. postulate that the chink corresponding to the virtual COD in the plastic zone ahead of a physical crack tip should be plugged up by a small segment shown in Figure 1. The physical meaning of the virtual COD is investigated in the reference [6]. After loading the segment with uniform elastic stresses of the yield stress magnitude under an appropriate triaxial constraint condition, it deforms elastically to accurately fit the fictitious COD in the plastic zone of strip yield model, as indicated by the solid line in Figure 1 (b). Inserting the segments enables to satisfy the displacement continuity and improves the model performance compared to the original concept of the strip yield model.

The advantage of the Toyosada’s approach can be proved by considering Newman’s crack closure model [4] for which the segment material is assumed to be rigid-plastic. In Newman’s model, the crack opening load is given by

\[ K(P_{op} - P_{min}) - K(\text{contact stress}) = 0, \]

where

- \( P_{op} \): crack opening load,
- \( P_{min} \): applied minimum load and
- \( K(x) \): stress intensity factor at load \( x \).

The first term in the right side shows the change of the stress intensity from the minimum load to the crack opening load. The second term represents the stress intensity caused by the contact stress distribution worked on the crack closure region at the minimum load. Although equation (1) appears correctly at the first sight, \( P_{op} \) by this equation gives lower value than actual one, because release of the contact stress acts to proceed the compressive plastic zone and shrinking COD [7]. More load increment is necessary to release the contact stress completely. Newman assumed a very high plastic constraint factor (even larger than 3) for getting large \( P_{op} \), because his model gives smaller \( P_{op} \) than measured one [8]. Moreover, in posterior works [9] Newman changes a value of plastic constraint factor in response to fatigue crack propagation rates in order to coincide the computed results with experimental data.

If the current tensile plastic zone extends beyond the previous plastic zone, the original strip yield model should be satisfied. A layer of the residual tensile deformations, which is commonly known as the crack wake, generated by the previous loading is on fatigue crack surfaces. The thickness of the layer \( L_j \) is assumed to equal length of the segment after removing the applied stress, because the perfect elastic-plastic material of the layer is assumed.

\[ L_j = V_j/(1 + \lambda \sigma_y/E'), \]

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where

\[ E' = \begin{cases} E & \text{ (plane stress condition) } \\ E/(1 - v^2) & \text{ (plane strain condition) } \end{cases} \]

\[ V_j \]: fictitious COD at \( x_j \) in a plastic zone which is beyond the previous plastic zone at maximum load,

\[ L_j \]: thickness of the plasticity elongated layer at \( x_j \),

\[ \sigma_Y \]: material yield strength,

\[ \lambda \]: plastic constraint factor,

\[ E \]: Young’s modulus,

\[ v \]: Poisson’s ratio.

If the present plastic zone is embedded in the previous plastic zone, the superposition principle shown in Figure 2 is satisfied based on the strip yield model and the following relations are introduced.

\[ V_j = P \sum_{i=1}^{n} s_i F(x_j, x_i, a^*) - \sum_{i=1}^{n} \sigma_i F(x_j, x_i, a^*) + \sum_{i=1}^{n} \sigma_i^R F(x_j, x_i, a^*), \tag{3} \]

where

\[ V_j \]: COD at \( x_j \) when external load is \( P \),

\[ P \]: magnitude of the external load,

\[ s_i \]: applied stress at \( x_i \) per unit external load,

\[ a^* \]: length of the fictitious crack (tip of the tensile plastic zone),

\[ F(x_j, x_i, a^*) \]: COD at \( x_j \) when a uniform unit stress acts between \( B_i \) and \( B_{i+1} \) on the crack surfaces, \( x_i = (B_i + B_{i+1})/2 \),

\[ \sigma_i^R \]: pre-existing residual stress at \( x_i \),

\[ \sigma_j \]: working stress at \( x_j \) along the crack line.

If an element remains elastic, the following relation should be satisfied in the fictitious crack region and the crack closure region.

\[ V_j = (1 + \sigma_j/E')L_j. \tag{4} \]

By substituting Equation (3) into Equation (4), \( \sigma_j \) can be obtained by solving the resulting linear system of equations through an iterative method with the following constraints:

For the region ahead of the crack tip,

- if \( \sigma_j < -\lambda \sigma_Y \), then \( \sigma_j = -\lambda \sigma_Y \)
- and
- if \( \sigma_j > \lambda \sigma_Y \), then \( \sigma_j = \lambda \sigma_Y \).

For the plastic wake zone,

- if \( \sigma_j > 0 \), then \( \sigma_j = 0 \)
- and
- if \( \sigma_j < -\lambda \sigma_Y \), then \( \sigma_j = -\lambda \sigma_Y \).

From the solution of \( \sigma_j \), COD is obtained by Equation (3) at \( P_{\text{min}} \) or \( P_{\text{max}} \) if the tensile plastic zone is inside the previous tensile plastic zone. The RPG load is obtained when the stress in the bar element adjacent to a crack tip reaches the yield stress during the loading reversal. The crack opening load is obtained when the stress in the bar element adjacent to a crack tip is equal to zero during the loading reversal.

The plastic constraint factor (\( \lambda \)) for mild steel was identified 1.04 on the basis of the experimental results reported in the literature [10]. The plate thickness of the specimens mentioned above is 6mm, which corresponds to the plane stress condition.

**Considering the work hardening effect**

Previous numerical simulation of fatigue crack growth cannot consider the work hardening effect of materials, because the mechanical property of bar element arranged in the fictitious crack zone, i.e. the plastic zone ahead of a crack tip, and the crack wake over crack surfaces corresponds to the rigid plastic or the elastic - perfect plastic materials. Although the plastic constraint factor (\( \lambda \)) mentioned above is applied to the simulations in order to give more precise crack growth behaviour, the accuracy of estimated fatigue crack growth history is insufficient under variable loading conditions. Then, authors consider that the work hardening effect of materials should be considered in the numerical simulation of fatigue crack growth.

Improvements of the numerical simulation are listed below.

1. The maximum allowable stress (\( \sigma_{00}^{(\text{max})} \)) of each bar element arranged in the fictitious crack zone and the crack wake is replaced by referring the accumulating plastic strain (\( \varepsilon_p^{(k)} \)) at each reference point. The accumulating plastic strain in this

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Figure 2. APPLICATION OF THE SUPERPOSITION PRINCIPLE TO COMPUTED COD IN THE PLASTIC ZONE EMBEDDED IN THE PREVIOUS PLASTIC ZONE

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simulation model is defined as follows.

\[ \varepsilon_p^{(k)} = \sum_{l=1}^{k-1} \Delta \varepsilon_p^l \]  

(5)

\[ \Delta \varepsilon_p^{(k)} = \bar{OB} + \bar{AB} \]  

(6)

\[ \bar{OB} \text{ and } \bar{AB} \] are shown in Figure 3 (b). That is, \( \sigma_0^{\max} \) is as a function of \( \varepsilon_p^{(k)} \).

2. The plastic strain of bar elements arranged in the fictitious crack region and the crack wake is defined as follows.

\[ \Delta \varepsilon_p^{(k)}(x_j) = L_{j}^{\max(k)} + (L_{j}^{\max(k)} - L_{j}^{\min(k)}) \]  

(7)

\( L_j \) is the gauge length of bar element at \( x_j \) and defined by Equation (2). \( L_j \) corresponds to the condensed plastic strain at the reference point \( x_j \) along the normal direction to the crack line [6].

3. Upper limit of allowable stress is equal to the tensile strength of materials (\( \sigma_T \)).

4. Isotropic hardening rule is applied. Bauschinger effect is ignored.

5. The stress versus strain curve is given by the multi linear function which is schematically shown in Figure 3.

**COMPARISON PROPOSED SIMULATION METHOD WITH PREVIOUS ONE AND EXPERIMENTS**

Fatigue crack growth simulations based on improved formulation mentioned above are performed. Applied loading histories are a) two blocks loading (step-down in maximum load) and b) spike overload loading shown in Figures 4 and 5 respectively.

These simulation results are compared with the results by previous simulation code FLARP [5] and with the measured results. Fatigue crack propagation law based on RPG (Re-tensile Plastic zone Generating) load criterion are applied in both improved and previous numerical simulations. Center cracked tensile (CCT) specimen made of mild steel is applied. Yield strength of the material is 352 MPa. The specimen configuration used is shown in Figure 6. Detailed explanations of the fatigue crack propagation law based on RPG criterion, the previous simulation code FLARP and the referenced fatigue crack measuring results are explained in the reference [5].

Figures 7 and 8 are the comparison results of fatigue crack
growth curves under block loading conditions. Solid curves show the crack growth curve by the improved numerical simulation proposed in this study. Dashed curves show the crack growth curve by the previous simulation code FLARP and open circle marks show the measured fatigue crack length.

Figures 9 and 10 are the comparison results of fatigue crack growth curves under spike over loading conditions. Solid curves, dashed curves open circle marks in these figures represent the same meaning in Figures 7 and 8.

Overall, the numerical simulation considering the work hardening effect of materials gives less tendency of the crack growth retardation caused by variable loading amplitude as compared with the previous simulation which applied mechanical property of material is the elastic - perfect plastic one. Fatigue crack propagates in the plastic zone generated ahead of a crack tip and the zone is taken in crack surfaces as the crack wake. The crack wake has a significant contribution to the crack closure. Considering the work hardening effect of materials causes less plastic deformation in the fictitious crack region and the crack wake comparing with elastic - perfect plastic materials. As a result, occurrence of crack closure in the work hardening materials is less than one in the elastic - perfect plastic materials and the fatigue crack growth in the work hardening materials is faster than one in the elastic - perfect plastic materials.

Figures 7 ~ 10 conduct the primacy of improved simulation method over previous one. Therefore, consideration of the work hardening of materials is imperative factor in order to describe the plastic induced fatigue crack closure behaviour.

**CONCLUSION**

Improved numerical simulation of the fatigue crack growth based on the strip yield model with the weight function is performed. The mechanical property in the previous model corresponds to the elastic - perfectly plastic material is updated to the work hardening materials. Comparison of improved numerical simulation results with previous ones and with experimental results confirms the primacy of proposed method over previous ones.

Future challenges are as follows.

1. Incorporations of the kinematic hardening rule and the Bauschinger effect of mechanical properties of materials.
2. Establishment of the numerical simulation of fatigue crack growth for a part through surface crack or an embedded plane crack considering the crack closure.
REFERENCES


ACKNOWLEDGMENT

This research fund is Grant-in-Aid for Young Scientists (S) (No. 21676007) by Japan Society for the Promotion of Science.

Gratitude is extended to Dr. Yukinobu NAGATA, former doctor course student in Kyushu University and Mr. Koji KADOWAKI, former master course student in Kyushu University for this research.