APPLICABILITY EVALUATION OF THE WEIGHT FUNCTION BASED STRIP YIELD MODEL FOR AN EMBEDDED CRACK PROBLEMS

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出版情報：6, pp.107-113, 2010-02-16. The American Society of Mechanical Engineers: ASME
バージョン：

Proceedings of OMAE'09
2009 28th International Conference on Offshore Mechanics and Arctic Engineering
Honolulu, USA, May 31 - June 5, 2009

OMAE'09-79562

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ABSTRACT

Applicability evaluation of the developed weight function based strip yield model for an embedded crack by applying the slice synthesis methodology in elastic-perfect plastic bodies under monotonic uniform loading is performed. Although the weight function based strip yield model for a part-through semi-elliptical surface crack in an elastic-perfect plastic bodies under monotonic uniform loading was proposed by Daniewicz and Aveline (2000), applicable geometries of cracked bodies is limited. Their proposed strip yield model treats only a semi-elliptical surface crack in semi-infinite bodies. Besides, quantitative investigations of the applicability seem to be insufficient.

The authors proposed the improved strip yield model with slice synthesis methodology for an embedded crack, which enables to treat the finite boundary problems. By applying proposed model, the back surface effect of the crack opening behaviour and the plastic zone growth can be considered. The validity of improved strip yield model for embedded cracks is confirmed by comparing crack opening profiles under some crack geometries with elastic-plastic finite element analyses.

INTRODUCTION

Fatigue life estimation for three dimensional cracks, e.g. part-through surface flaws or embedded flaws is very important because most fatigue crack shapes found in welded built-up structures and mechanical components show three dimensional crack morphologies and these cracks are generally located in stress concentration regions. Conventional fatigue life estimation for welded built-up structures is based on the combination of the hot spot stress based $S-N$ curves and cumulative damage rules [1]. However, this method contains the following serious weaknesses.

1. $S-N$ curves based approach cannot give the fatigue crack growth history.
2. The transferability of fatigue life obtained by $S-N$ curves to in-service structures has not established yet [2]. The relation between fatigue crack length found in in-service structures and fatigue life obtained by $S-N$ curves is not defined clearly.

On the other hand, fatigue life predictions based on fracture mechanics are performed in order to overcome the weaknesses of the conventional $S-N$ curves approaches. Most of fatigue life assessments based on fracture mechanics cannot quantitatively evaluate the retardation and the acceleration of crack propagation, because of insufficient consideration of fatigue crack opening / closing behaviour caused by crack wake.

Toyosada et.al [3] proposed the procedures of fatigue crack growth simulation for three dimensional crack problems by applying numerical fatigue crack opening / closing simulation for two dimensional crack problems with the equivalent distributed stress methodology which enables the representation of the stress-strain field at a reference point of three dimensional cracks for a through thickness crack in infinite wide plate.

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Stress intensity factor of analysis objects must be given even though above mentioned method is applied. Calculation procedures of stress intensity factor for through thickness cracks under arbitrary stress distributions are generally straight-forward, while for three-dimensional cracks such as surface cracks and embedded cracks are more complicated. The slice synthesis methodology with weight function methods to calculate stress intensity factors for a three dimensional crack was developed [4] [5]. The applied method has advantages for modelling and CPU time comparing with finite element and boundary element analyses.

Although the slice synthesis methodology had already been proposed, applicability of previous methods is limited to infinite or semi-infinite cracked body problems. Therefore the back surface effect for stress intensity and crack opening behavior cannot be considered. Authors developed the slice synthesis methodology in order to consider the back surface effect for stress intensity and crack opening behaviour in elastic condition by introducing the correction factor of elastic modulus for spring slice [6].

The application of slice synthesis methodology is extending to perform the numerical calculation of a crack opening profile and plastic zone length for a semi-elliptical surface cracks in an elastic-perfectly plastic body under monotonic loading [7] [8]. However, these approaches are limited to a part-through semi-elliptical crack in semi-infinite body.

By referring published and authors achievements related to the slice synthesis methodology, it is expected that the combination of slice synthesis methodology and strip yield model enables to construct the numerical simulation of a surface or an embedded crack growth caused by fatigue.

In this study, the combination procedure of slice synthesis methodology and strip yield model is proposed in case of the cracked problem in finite body. Embedded crack morphology is highlighted as three dimensional crack shapes. By applying this procedure, it is expected that the back surface effect of crack opening and plastic zone growth behaviours of a crack in elastic-perfect plastic body can be considered. The validity of proposed strip yield model with the slice synthesis methodology for an embedded crack is confirmed by comparing crack opening profiles under some crack geometries with elastic-plastic finite element analyses.

THE SLICE SYNTHESIS METHODOLOGY

The slice synthesis methodology with weight function to calculate stress intensity factors and crack opening behaviours has potential as a usable calculation method. Calculation procedure for stress intensity factors and crack opening behaviours in elastic condition is overviewed below. The detail of this methodology was described in the reference [4] [6].

In the following description, slice synthesis methodology abbreviate to SSM in brief.

Outline of SSM in elastic body

The analysis object is an embedded elliptical crack in a finite body shown in Figure 1 and crack surface is divided by two series of orthogonal slices shown in Figure 2. The slice parallel to $y-z$ plane is called basic slice whose crack length is $2a_x$, while parallel to $z-x$ plane is spring slice and crack length is $2b_y$. Basic and spring slices are restrained by the spring whose stiffness is k_a and k_b respectively. The spring stiffness k_a enables it to have a value from zero to infinity according to the dimensional ligament area R_a. k_b also enables it to have a value from zero to infinity. That is, $R_i(i=a,b)$ tends to zero, $k_i(i=a,b)$ tends to zero, while R_i tends to infinity, k_i tends to infinity. Dimensionless ligament areas R_i are defined as follows.

$$\left.\begin{array}{l} R_a = 4t(c-b)/b^2, \\ R_b = 4t(c-b)/b^2. \end{array}\right\} \quad (1)$$

By applying the weight functions, the stress intensity factors for the basic slice and spring slice on reference point $Q(x,y)$ shown in Figure 2 are calculated as follows,

$$K_a(a_x) = \int_0^{a_x} [\sigma(x,\eta) - P(x,\eta)] w_a(R_a, a_x/t, \eta/t) d\eta \quad (2)$$

$$K_b(b_y) = \int_0^{b_y} P(\xi,y) w_b(R_b, b_y/c, \xi/c) d\xi \quad (3)$$

$w_a(R_a, a_x/t, \eta/t)$ and $w_b(R_b, b_y/c, \xi/c)$ are weight functions for basic slice and spring slice respectively. The following descrip-

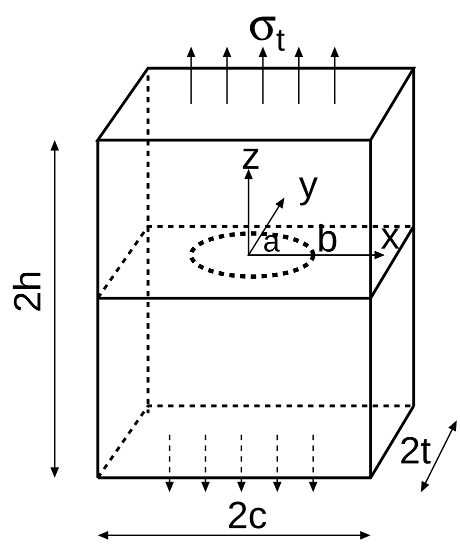


Figure 1. AN EMBEDDED ELLIPTICAL CRACK SUBJECTED TO ARBITRARY STRESS DISTRIBUTION

tion in this paper, weight functions are abbreviated to w_a and w_b in brief.

The weight functions $w_i, (i = a, b)$ are calculated by combination of two limiting conditions shown in Equations (4) and (5). R_i tends to zero, w_i tends to free boundary condition w_i^{free}, while R_i tends to infinity, w_i tends to fixed boundary condition $w_i^{\text{collinear}}$.

$$w_a = w_a^{\text{collinear}} + T(R_a)\{w_a^{\text{free}} - w_a^{\text{collinear}}\} \tag{4}$$
$$T(R_a) = 1.05^{-R_a} \tag{5}$$

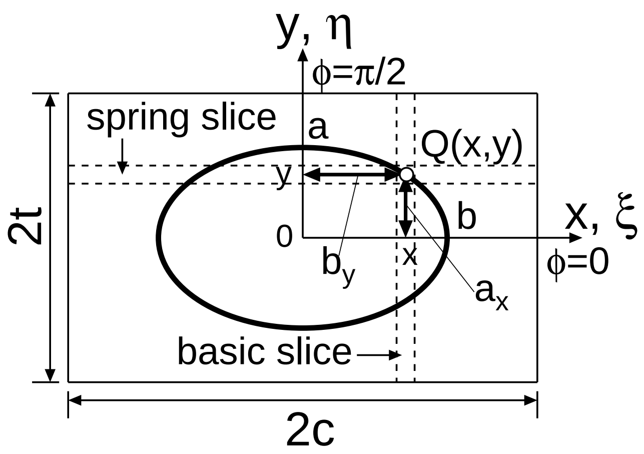


Figure 2. A COORDINATE SYSTEM FOR AN EMBEDED ELLIPTICAL CRACK AT $z = 0$

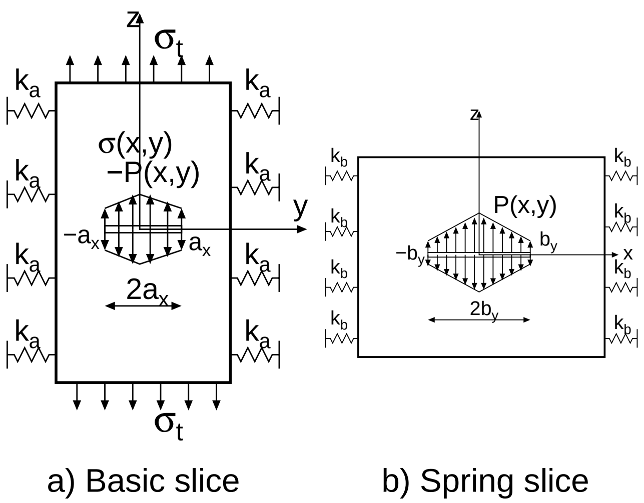


Figure 3. SCHEMATIC ILLUSTRATION OF BASIC AND SPRING SLIECES

Weight functions for the basic slice are given by Equations (7) and (7) [9] in authors researches. Weight function for spring slice is given by replacing t, a_x, and y with c, by and x in Equations (7) and (7).

$$w_a^{\text{collinear}} = \frac{2}{\sqrt{2t}}\sqrt{\tan\frac{\pi a_x}{2t}}\frac{\cos(\pi y/2t)}{\sqrt{\sin^2(\pi a_x/2t) - \sin^2(\pi y/2t)}} \tag{6}$$
$$w_a^{\text{free}} = \frac{2}{\sqrt{2t}}\{1 + 0.297\sqrt{1-(y/a_x)^2}(1-\cos(\pi a_x/2t))\} \times \left(\sqrt{\tan\frac{\pi a_x}{2t}}\Big/\sqrt{1-\left(\frac{\cos(\pi a_x/2t)}{\cos(\pi y/2t)}\right)^2}\right) \tag{7}$$

Crack opening profile along the a_x and b_y are given by,

$$V_a(x,y,a_x) = \frac{2}{E_a}\int_y^{a_x} K_a(\alpha) w_a d\alpha \tag{8}$$
$$V_b(x,y,b_y) = \frac{2}{E_b}\int_x^{b_y} K_b(\beta) w_b d\beta. \tag{9}$$

where E_a is Young's modulus and E_b is the corrected Young's modulus given by Equation (10)

$$\frac{E_S}{E_B} = \left(\frac{\Phi}{1-\nu^2} - 1\right)\frac{b}{a} \tag{10}$$

Φ and ν in Equation (10) are the complete elliptic integral of the second kind and Poisson's ratio, respectively.

At the reference point $Q(x,y)$ illustrated in Figure 2, crack opening displacement of basic slice is equal to spring slice. From Equations (8) and (9), the following relation is obtained.

$$\int_y^{a_x}\left\{\int_0^{\alpha}\sigma(x,\eta) w_a d\eta\right\} w_a d\alpha = \int_y^{a_x}\left\{\int_0^{\alpha} P(x,\eta) w_a d\eta\right\} w_a d\alpha + \frac{E_a}{E_b}\int_x^{b_y}\left\{\int_0^{\beta} P(\xi,y) w_b d\xi\right\} w_b d\beta \tag{11}$$

The spring force $P(x,y)$ is expressed as polynomial approximation.

$$P(x,y) = \sum_{i=1}^{19} e_i \left(\frac{x}{b}\right)^j \left(\frac{y}{a}\right)^l \tag{12}$$

e_i in Equation (12) is unknown coefficients. j and l in Equation (12) are selected and shown in Table 1. The same selection was

Table 1. THE VALUES OF EXPONENTS IN EQUATION (12)

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
j	0	1/3	0	1/3	1/2	0	1/2	1	0	1	2	0	2	3	0	3	4	0	4
l	0	0	1/3	1/3	0	1/2	1/2	0	1	1	0	2	2	0	3	3	0	4	4

applied in authors' previous research [6]. The other combination of polynomials was applied in the references [4] [5] [7] [8].

If some COD evaluation points are chosen more than polynomial approximation order, unknown coefficients e_i are determined by using the least-square fitting. In this study, 29 points are chosen as COD evaluation points.

Extending of SSM in elastic-perfect plastic body

Consider extending SSM in elastic-perfect plastic body such that plastic zone along the crack front may occur as illustrated in Figure 4. Using a strip yield modelling approach, the embedded crack and subsequent plastic zone are treated as a fictitious crack in an elastic body. The fictitious embedded crack is characterized using a crack depth $2a^*$ and a crack width $2b^*$ shown in Figure 4. The fictitious embedded crack shape is assumed to form concentric ellipses. Note that from a rigorous perspective, the actual shape of the fictitious crack should be a function of the stress intensity and the degree of constraint along the crack front. From the point of approximation, above mentioned modelling approach is considered adequate.

In the strip yield model, the extent of the plastic zone is identified if the material yield stress is assumed to act as a compressive cohesive stress applied within the plastic zone. Most of numerical simulations oriented the strip yield model approach, flow stress is introduced in place of yield stress or the plastic constraint factor to correct yield stress is applied in order to adjust the calculation results of crack opening behaviour and plastic zone growth.

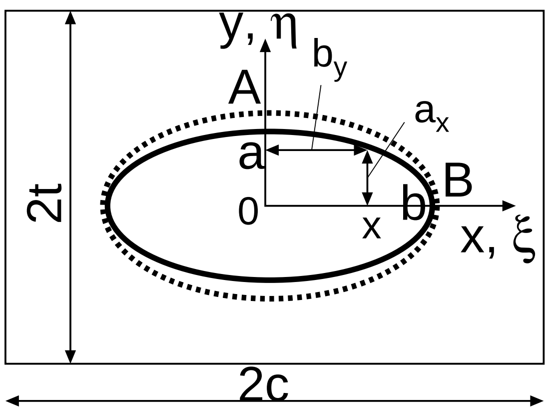


Figure 4. SCHEMATIC ILLUSTURATION OF CRACK FRONT LOCALIZED PLASTIC ZONE

On the fictitious crack front, the stress is assumed finite with the net stress intensity factor equal to zero. The slice plastic zone sizes for slices passing through x and y axes in Figure 4 can be found using the following relationships which endorse a zero net stress intensity factor at Point A($(x,y)=(0,a^*)$) and Point B($(x,y)=(b^*,0)$)

$$K_a(a^*) = \int_0^{a^*} [\sigma(0,\eta) - P(0,\eta)] w_a d\eta - \int_a^{a^*} [\sigma_Y + S(0,\eta)] w_a d\eta \tag{13}$$

$$K_b(b^*) = \int_0^{b^*} P(\xi,0) w_b d\xi - \int_b^{b^*} S(\xi,0) w_b d\xi \tag{14}$$

The additional spring force $S(x,y)$ for adjusting the cohesive force is introduced in the formulation of SSM in elastic-plastic condition. The expression of $S(x,y)$ is the same of $P(x,y)$ for applied force. $S(x,y)$ had not be applied in the previous researches [7] [8] related SSM in elastic-plastic condition. From preliminary investigations by authors, it is conducted that introducing of the additional spring force $S(x,y)$ for cohesive force seems to be unavoidable if the SSM in elastic-perfect plastic body is applied to finite geometry cracked body.

Equations (13) and (14) constitute a nonlinear system equations for the two unknown values a^* and b^* which is solved by the iterative procedure. The unknown spring force $P(x,y)$ and additional spring force $S(x,y)$ play a similar role to that of applied stress and cohesive force respectively.

The set of spring forces $P(x,y)$ and $S(x,y)$ can be determined considering the physical embedded crack and subsequent plastic zone as an fictitious crack in an elastic body.

Crack opening profiles along x and y axes for a fictitious crack (V_{a^*} and V_{b^*}) are given as follows.

$$V_{a^*} = \frac{2}{E_a} \int_y^{a_x^*} K_a(\alpha) w_a d\alpha \tag{15}$$

$$V_{b^*} = \frac{2}{E_b}\int_x^{b_y^*} K_b(\beta) w_b d\beta \tag{16}$$

From Equations (13) to (16), the following relations are established.

$$V_{a^*} = \frac{2}{E_a}\int_y^{a_x^*}\left\{\int_0^{\alpha}[\sigma(x,\eta)-P(x,\eta)]w_a d\eta - \int_{a_x}^{\alpha}[\sigma_Y + S(x,\eta)]w_a d\eta\right\} w_a d\alpha \tag{17}$$

$$V_{b^*} = \frac{2}{E_b}\int_x^{b_y^*}\left\{\int_0^{\beta}P(\xi,y)w_b d\xi + \int_{b_y}^{\beta}S(\xi,y)w_b d\xi\right\} \times w_b d\beta \tag{18}$$

Because the relation $V_{a^*} = V_{b^*}$ must be keep at the reference point (x,y), the following equation is obtained.

$$\begin{aligned}&\int_y^{a_x^*}\left\{\int_0^{\alpha}\sigma(x,\eta)w_a d\eta - \sigma_Y\int_{a_x}^{\alpha}w_a d\eta\right\} w_a d\alpha \\ =& \int_y^{a_x}\left\{\int_0^{\alpha}P(x,\eta)w_a d\eta + \int_{a_x}^{\alpha}S(x,\eta)w_a d\eta\right\} w_a d\alpha \\ &+\frac{E_a}{E_b}\int_x^{b_y}\left\{\int_0^{\beta}P(\xi,y)w_b d\xi + \int_{b_y}^{\beta}S(\xi,y)w_b d\xi\right\} \\ &\times w_b d\beta\end{aligned} \tag{19}$$

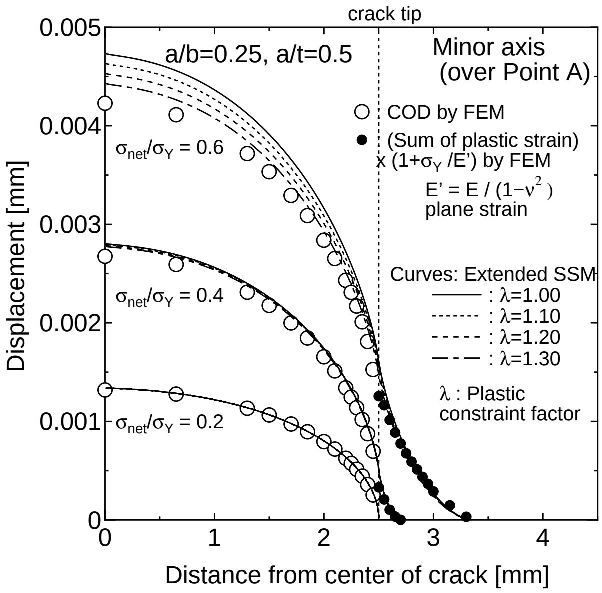
The spring forces $P(x,y)$ and $S(x,y)$ and plastic zone front a^* and b^* are determined iteratively. An initial estimate of $P(x,y)$ is obtained from a linear system equations of Equation (19) assuming no plastic deformation. (An initial $S(x,y)$ is set zero. Initial values of the plastic zone front a^* and b^* are obtained by applying the technique proposed in the reference [8].) Solving procedure is the same mentioned in the former section.

With this $P(x,y)$ and $S(x,y)$, the plastic zone front a^* and b^* are computed using Equations (13) and (14), and with these a^* and b^* and a new $P(x,y)$ and $S(x,y)$ are identified. This process is continued until $P(x,y)$ and $S(x,y)$ converged.

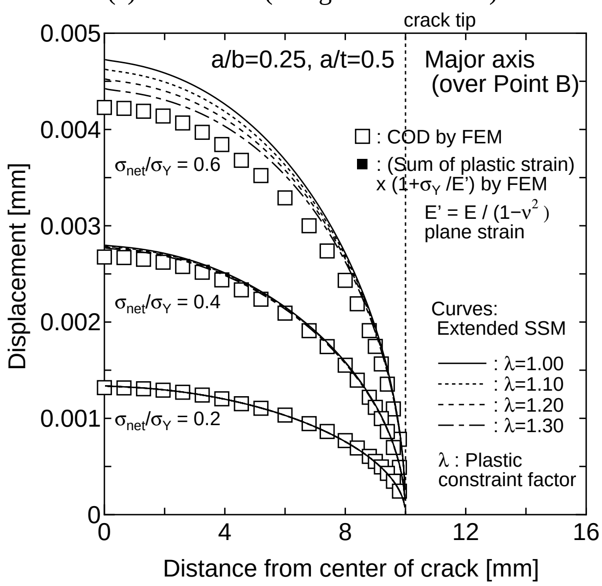
In addition, the collateral conditions related to the net stress intensity factor equal to zero at points A and B are given in the solving of a linear systems equations concerning the least square fitting to identify $P(x,y)$ and $S(x,y)$ by applying Lagrange multiplier method.

NUMERICAL INVESTIGATION OF THE APPLICABILITY OF SSM IN ELASTIC-PLASTIC CONDITION

Many cracked body with geometries are applied in order to investigate the validity of proposed extending SSM in elastic-plastic condition. In this paper, two calculation results are introduced as examples.



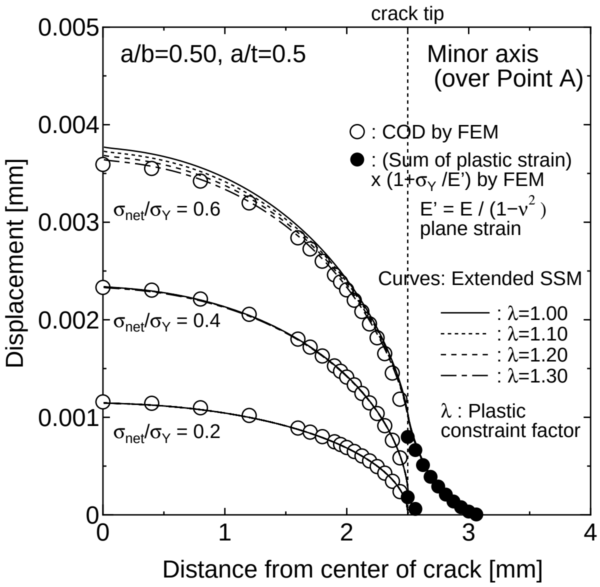
(a) Basic slice (along the minor axis)



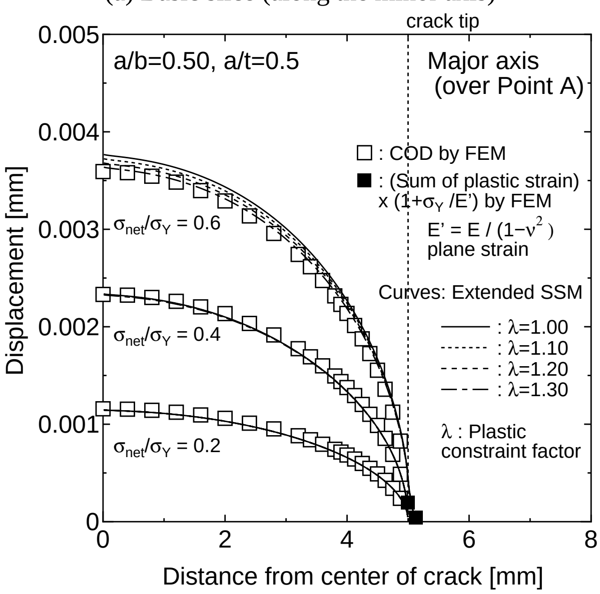
(b) Spring slice (along the major axis)

Figure 5. COMPARISON OF CALCULATED CRACK OPENING PROFILE BY APPLYING EXTENDED SSM WITH FEM (a/b=0.25, a/t=0.5)

Figures 5 and 6 shows the crack opening profiles under monotonic uniform remote loading by applying proposed extended SSM. In these numerical simulations, four value of the plastic constraint factor is applied in order to investigate the suitable value of the factor. The plastic constraint factor shown in Figures 5 and 6 should be introduced in order to adjust the calculation results of crack opening behaviour and plastic zone growth.



(a) Basic slice (along the minor axis)

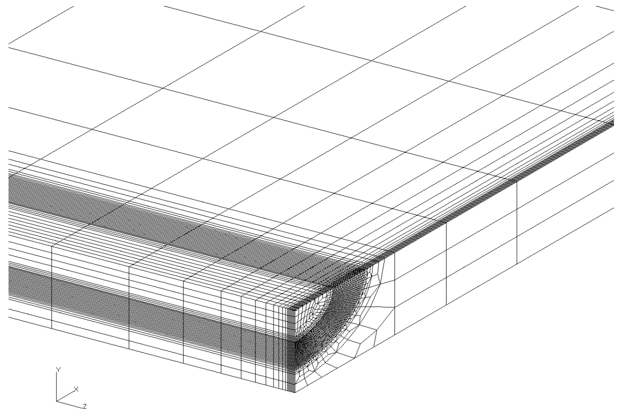


(b) Spring slice (along the major axis)

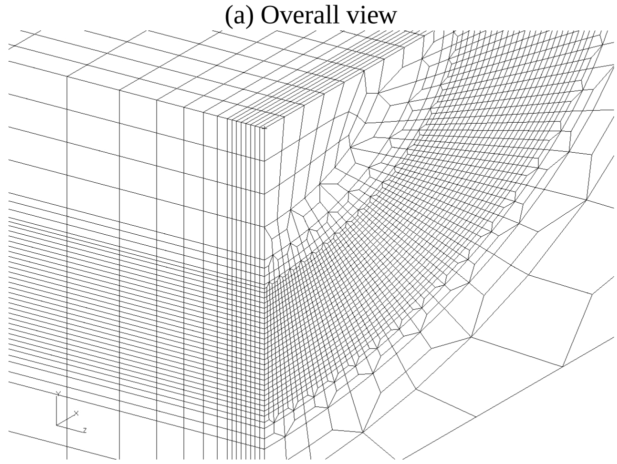
Figure 6. COMPARISON OF CALCULATED CRACK OPENING PROFILE BY APPLYING EXTENDED SSM WITH FEM (a/b=0.50, a/t=0.5)

Elastic-plastic finite element analyses are also shown in these figures as comparisons. MSC. Marc 2008r1 is used for these finite element analysis results. An example of finite element idealzation is shown in Figure 7. Minimum mesh dimensions around crack front is 0.05mm $\times$ 0.05mm $\times$ 0.06mm.

In Figure 5, aspect ratio (a/b) of an embedded crack is equal to 0.25. In Figure 6, aspect ratio (a/b) of an embedded crack is equal to 0.50. Dimensionless crack depth (a/t) is equal to 0.5 in



(a) Overall view



(b) Detaied view around a crack surface

Figure 7. AN EXAMPLE OF FINITE ELEMENT SUBDIVISION (a/b=0.50, a/t=0.5)

these figures.

Open marks in these figures are crack opening displacement by FE analyses. On the other hand, solid marks in these figures correspond to a kind of fictitious displacement in plastic zone defined as follows.

$$V^f = \left(1 + \frac{\sigma_Y}{E/(1-\nu^2)}\right) \int_{\text{plastic zone}} \varepsilon_z^p dz \qquad (20)$$

It is considered that V^f corresponds to the crack opening displacement in case of strip yield model. This consideration was conducted by numerical investigations in two dimensional cracked problems [10].

From Figures 5 and 6, it is confirmed that proposed SSM can give fairly good calculation results of crack opening profiles. The accuracy of extended SSM solution can be improved if the suitable value of plastic constraint factor is selected. Besides, the fictitious displacement defined by Equation (20) and finite element analyses is in good agreement with extended SSM solutions. This result indicates that the physical meaning fictitious crack opening displacement in strip yield model keeps three dimensional cracks.

Plastic zone growth for each axis direction show different tendency. Plastic zone develops for minor axis direction is larger than one for major axis direction. Although this tendency is different from the initial assumption in extended SSM, converged plastic front shape is almost same to finite element solutions.

From the above discussion, the applicability of extended SSM in elastic-plastic condition is verified.

CONCLUSION

Extended strip yield model for an embedded crack in finite geometrical body by applying slice synthesis methodology is proposed. By applying this model, the strip yield model for an embedded crack can be formulated and crack opening profiles and plastic zone growth can be estimated without complicated finite element analysis.

Future challenges are (i) to confirm the applicability of proposed method for a part-through surface crack problem and (ii) to implement this extended strip yield model into the numerical simulation of fatigue crack opening / closing and perform the fatigue crack growth simulation of an embedded or part-through surface crack.

ACKNOWLEDGMENT

This research fund is Grant-in-Aid for Scientific Research (B) (No. 19360395) and JSPS Fellows (No.20・1707) by Japan Society for the Promotion of Science and the research grant by Fundamental Research Developing Association for Shipbuilding and Offshore in the Shipbuilders' Association of Japan.

Gratitude is extended to Mr. Kunishige Fukudome of Graduate school of Kyushu University for finite element analyses.

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