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Douglas–Peucker piecewise affine approximation of an optimal fuel consumption problem to apply PANOC

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Abstract: In today's world where complying with the requirements of a green economy is more and more imperative for technological progress, energy and fuel-efficient navigation is a topic of primary importance in industrial engineering. In the particular case of autonomous driving and cruise control, the inherent nonlinearity and complexity of the physical dynamics result in a highly nonconvex control problem, which becomes even more challenging if one is to further account for energy saving constraints. Leveraging on recent advancements, we propose a solution based on PANOC [19], a fast optimization solver which can cope with nonconvex problems and enjoys very low computational requirements, provided that some inner subproblems can be solved at negligible effort. In order to account for this binding requirement of the algorithm, we propose a piecewise affine approximation strategy for the fuel consumption model based on the Douglas–Peucker algorithm [7]. The effectiveness of the approach is showcased with numerical simulations on a real-time adaptive cruise control problem for fuel consumption optimization.

Keywords: Nonlinear model predictive control, fuel-efficient cruise control, Douglas–Peucker piecewise linearization

1. INTRODUCTION

In the automotive industry, fuel economy has become an important indicator for assessing the performance of road vehicles. The tightening of fuel economy standards and the continued concern about environment pollution have brought considerable challenges to the automotive industry [22]. Weather and road conditions as well as other exernal physical factors all concur in determining the fuel consumption of a vehicle. Most of these factors are however beyond human control, which singles out the driving behavior as the key aspect that can be leveraged to achieve the sought goals. In recent years, eco-driving has received great attention, helping drivers to achieve fuel-efficient driving through technological means [9]. In order to meet the control of fuel consumption in real time, model predictive control (MPC) is heavily applied in the research of related fields. For example, in [8] MPC is used to develop a fuel-optimized control algorithm for heavy-duty diesel trucks based on terrain information, and MPC and traffic data are used in [10] to calculate vehicle speed profiles to reduce fuel consumption. However, these methods need to calculate the global optimal strategy for the whole driving cycle before the vehicle operation, and are not applicable to the real-time control objectives of vehicle control at this stage. The inherent nonconvexity in the vehicle fuel consumption model has been ignored by researchers in the past studies, as this inherent nonconvexity makes it difficult to directly apply the fuel consumption term in the traditional optimization algorithm to obtain the optimal control strategy.

A successful application of an MPC strategy is inextricably linked to efficient optimization algorithms. For nonlinear and possibly nonconvex problems, classical methods such as sequential quadratic programming (SQP) and interior-point (IP) algorithms are widely used. Their high versatility and fast convergence properties, however, come at the price of expensive operation oracles, which confines their use on platforms and hardware with high computing capability. Possibly in response to this downside, recent years have witnessed a renewed interest in algorithms of splitting nature [14], as they enjoy simple iteration complexity and can handle both nonsmooth and nonconvex penalties. However, their slow convergence has long been the main hindrance against their reliable application to real-time optimization.

In recent years, considerable amount of research has been devoted to improving this aspect, see e.g. [3, 4, 13, 20, 21] and references therein. More recently, the PANOC algorithm [19] has proven a viable solution that is also amenable to cope with the limited computational and memory capability of embedded hardware and the very short sampling time prescribed by many real-time MPC applications. PANOC is based on the proximal gradient method, also known as forward-backward splitting (FBS) in a more general setting [2], which is one of the most well-known splitting algorithms. PANOC combines FBS and (limited-memory) quasi-Newton methods, and while preserving the operational simplicity of FBS it also inherits the fast convergence rates of quasi-Newton methods.

The efficiency of PANOC is however subject to the binding requirement that one of its building blocks, and specifically the so-called proximal mapping operator, see (6) below, is available in close form or can be evaluated at negligible effort. Being this not the case for the fuel-efficient adaptive cruise control problem [6], we propose a linear approximation strategy based on the Douglas–Peucker (DP) algorithm [7] that can approximate with aribtrary precision the vehicle fuel consumption model

† Hongjia Ou is the presenter of this paper.

while both preserving its nonconvexity and simplifying the computation of the proximal mapping.

1.1. Paper organization

The paper is organized as follows. Section 2 introduces the control problem at hand and frames it into a composite minimization suitable for an efficient real-time MPC solver. The key algorithmic steps for an out-of-the-box implementation are discussed in detail in Section 3, after an extensive description of the proposed Douglas–Peucker approximation. Section 4 showcases the effectiveness of the proposed methodology with numerical simulations. Section 5 concludes the paper.

2. PROBLEM SETTING

We study a cruise control problem in which the objective is to maintain a constant distance from the preceding vehicle while optimizing fuel consumption. To this end, we consider the following continuous-time nonlinear dynamical system

$$x(t) = \begin{bmatrix} s(t) \\ v(t) \\ Q(t) \\ s_{\mathrm{p}}(t) \end{bmatrix} \quad \dot{x}(t) = F(x(t), u(t)) := \begin{bmatrix} v(t) \\ a(t) \\ W(u(t)) \\ v_{\mathrm{p}}(t) \end{bmatrix},$$

where at each time instant t the state $x(t) \in \mathbb{R}^4$ comprises position $s(t)$ and velocity $v(t)$ of the controlled vehicle, total fuel $Q(t)$ consumed up to time t, and position $s_{\mathrm{p}}(t)$ of the preceding vehicle, while the input $u(t) \in \mathbb{R}$ consists of the power actuated on the vehicle. By taking into account rolling friction of the wheels and viscous resistance of the air against the vertical surface A of the vehicle (assumed flat for simplicity), the acceleration $a(t)$ is given by

$$a(t) = \frac{u(t)}{v(t)M} - \frac{\frac{1}{2}C_D \rho A v(t)^2 + \mu M g}{M},$$

where M is the mass of the vehicle, C_{D} its drag coefficient, ρ is the air density, and g the gravitational acceleration. The quantity $W(u(t)) = \dot{Q}(t)$ represents the instantaneous fuel consumption rate when a power input $u(t)$ is actuated. The control objective in a time range $[t_0, t_f]$ can thus be formulated as the following optimization problem

$$\operatorname*{minimize}_{u(t)\ t\in[t_0,t_f]} J(u) \quad \text{subject to } P_{\min} \le u(t) \le P_{\max}, \tag{1}$$

where $P_{\min}$ and $P_{\max}$ are minimum and maximum admissible power, and

$$J(u) = \int_{t_0}^{t_f} \Big(\big((s_{\mathrm{p}}(t) - s(t)) - h_{\mathrm{d}} v_{\mathrm{p}}(t)\big)^2 + \omega_0 W(u(t)) \Big) \mathrm{d}t. \tag{2}$$

Here, h_{d} is the desired time delay from the preceding vehicle, and $\omega_0 > 0$ is a model parameter for trading-off optimality of distance and fuel consumption.

2.1. Discretization and composite minimization form

We discretize the state equation of the continuous-time optimal control problem (1) with backward Euler's method into intervals of width Δt as

$$x_{k+1} - x_k - F(x_k, u_k)\Delta t = 0 \quad k = 0, \ldots, N-1, \tag{3}$$

where $x_k \in \mathbb{R}^4$ denotes the state vector at the k-th time step, and u_k the (constant) input to be actuated between the k-th and $(k+1)$-th time steps. The discretized problem (1) can thus be recast as the following nonlinear composite minimization

$$\operatorname*{minimize}_{u\in\mathbb{R}^N} \varphi(u; x_0) = f(u) + g(u), \tag{4}$$

where $u = (u_0, u_1, \ldots, u_{N-1})$ is the stacking of the input variables, and by similarly stacking the states into $\mathbb{R}^N$-vectors s, v, Q, and s_{p},

$$f(u) := \|(s_{\mathrm{p}} - s) - h_{\mathrm{d}} v_{\mathrm{p}}\|^2$$

is a smooth function, and

$$g(u) := \omega_0 \|W(u)\|_1 + \delta_{\mathcal{U}}(u) \tag{5}$$

is a nonconvex and nonsmooth function that encodes the input constraints through the indicator function $\delta_{\mathcal{U}}$, where $\mathcal{U} := [P_{\min}, P_{\max}]^N$. (The indicator function of a set $E \subseteq \mathbb{R}^n$ is $\delta_E : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ where $\delta_E(x) = 0$ if $x \in E$ and ∞ otherwise). Here, with an abuse of notation, the application of W is meant in an elementwise fashion as $\mathbb{R}^N \ni u \mapsto (W(u_0), W(u_1), \ldots, W(u_{N-1})) \in \mathbb{R}^N$. Since $W \ge 0$, this allows for a compact representation of $\sum_k W(u_k)$ through the ℓ_1-norm as in (5).

2.2. Receding-horizon control and warm-startable algorithms

Problem (4) spans a time length $N\Delta t$, and is meant to be addressed in the receding horizon fashion of MPC in real time: an optimal input sequence is computed, but only the first input is actuated. Then, the same problem is solved again by sliding the time window of one time step and by updating the initial state condition, whence the parametric dependance on the initial state x_0 as emphasized in (4). Since a new problem has to be repeatedly solved within each sampling time, it is imperative to rely on an optimization method that can provide a solution fast enough. For the purpose, we chose the optimization solver PANOC [19], which on top of the additional benefit of embeddability (which makes it suitable for a real-world onboard implementation), is also warm-startable. This poses a great advantage with respect to solvers based on interior point, since in an MPC setting all problems differ only from the initial condition x_0, and every instance can thus conveniently be initialized at the solution of the previous one, resulting in an asymptotic even faster convergence, as also observed in [16, 19].

3. A PROXIMABLE FUEL CONSUMPTION MODEL

The oracle complexity of PANOC amounts to proximal gradient evaluations, namely gradient descent steps on the smooth function f and proximal operations on the nonsmooth function g, where for a stepsize parameter $\gamma > 0$ the proximal mapping of g is given by

$$\operatorname{prox}_{\gamma g}(u) := \operatorname*{arg\,min}_{z\in\mathbb{R}^N} \Big\{ g(z) + \tfrac{1}{2\gamma}\|z - u\|^2 \Big\}. \tag{6}$$

The main hindrance against an out-of-the-box implementation of PANOC is that, for g as in (5), the latter operation is not available in close form for any fuel consumption model W. In addition, an explicit expression for W is not available in the first place, as its value has to be determined experimentally. Estimating the instantaneous fuel consumption rate W has been the subject of many works in the past decades, see e.g. [1, 5, 17] and references therein. Several models exist based on different assumptions on its form, such as quadratic models [12, 15] or concave-convex models [11]. Based on our experiments and observations, we observed that instantaneous fuel consumption profiles in terms of power inputs obey the following three characteristics:

- $W(P) = 0$ for any $P \leq 0$, as no fuel is consumed when a negative power is actuated on the vehicle, that is, when either idling or braking;
- Up to negligible errors, $W(P)$ is convex and increasing for $P > 0$;
- A discontinuity jump $\lim_{P\searrow 0} W(P) \gneqq 0 = W(0)$ occurs at $P = 0$, indicating that the total fuel consumed increases in a nondifferentiable manner from a stationary condition.

However, the proximal mapping of the estimated function W is still not available in close form. To bypass this problem and thus make full use of the potential of PANOC algorithm, we propose a solution based on the piecewise linearization offered by the Douglas–Peucker method, discussed in the following subsection.

3.1. Linear approximation with the DP algorithm

The Douglas-Peucker algorithm is a downsampling technique. Given a curve to approximate, the algorithm determines whether to keep the coordinate point by finding the farthest distance between the line formed by the start point and the end point and the original curve. Once the farthest distance exceeds a predetermined distance dimension e, the point is retained, and the original line becomes two primary line segments from the start point to the coordinate point and from the coordinate point to the end point. Once again, the two primary line segments are judged to be greater than the set distance dimension e from the original function, until all the line segments remain within the distance dimension e from the corresponding original function. Eventually, the original polynomial function can be linearly approximated as a superposition of multiple segment primary functions. Clearly, for small distance dimension e the more segments there are the more the original function can be linearly approximated with arbitrary accuracy.

Because of the discontinuity jump at $P = 0$, a linear approximation around the origin cannot satisfactorily represent the behavior of W. Nevertheless, given that $W(P)$ is the constant null function for $P \leq 0$, we construct its approximation h as follows:

$$h(P) \coloneqq \begin{cases} 0 & \text{if } P_{\min} \leq P \leq 0, \\ \tilde{h}(P) & \text{if } P > 0, \\ \infty & \text{otherwise,} \end{cases} \tag{7}$$

where $\tilde{h} : \mathbb{R} \to \mathbb{R} \cup \{\infty\}$ is a polyhedral (convex) function defined by M linear pieces with domain $[0, P_{\max}]$ obtained by means of the DP method on $W(P)$ with $P \in [0, P_{\max}]$. By playing on the parameter e, the DP method can reconstruct W with arbitrary precision; as we are about to see next, also the resulting proximal mapping is computable at negligible effort, overall resulting in algorithmic requirements of simplicity and accuracy at the same time. Therefore, we will consider $g = h$ in (4), with the usual convention of elementwise definition.

3.2. Calculation of proximal mapping

Denoting the $n + 1$ endpoints of the linear pieces of $\tilde{h}$ as (P_i, y_i), $i = 0, \dots, n$, with $P_0 = 0$ and $P_n = P_{\max}$,

$$\tilde{h}(P) = \begin{cases} y_i + m_i(P - P_i) & \text{if } P_i \leq P \leq P_{i+1}, \\ \infty & \text{if } P < P_0 \vee P > P_n, \\ i = 0, \dots, n-1, \end{cases}$$

where $m_i \coloneqq \frac{y_{i+1}-y_i}{P_{i+1}-P_i}$. Notice that convexity of $\tilde{h}$ is equivalent to having $m_i \leq m_{i+1}$ holding for all $i = 0, \dots, n-1$. Moreover, the discontinuity jump of h at 0 is $y_0 > 0$.

For any $P \in \mathbb{R}$, computing $\tilde{P} = \operatorname{prox}_{\gamma\tilde{h}}(P)$ amounts to

$$\tilde{P} = \operatorname*{arg\,min}_{w\in\mathbb{R}} \min_i \left\{ h_i(w) + \tfrac{1}{2\gamma}(w - P)^2 \right\},$$

where

$$h_i \coloneqq y_i + m_i(\,\cdot\, - P_i) + \delta_{[P_i, P_{i+1}]}\,. \tag{8}$$

Denoting $w_i \coloneqq \operatorname{prox}_{\gamma h_i}(P)$, namely,

$$w_i = \Pi_{[P_i, P_{i+1}]}(P - \gamma m_i)$$

where Π_E is the projection operator onto set E, there are two possibilities:

- either $P_i < P - \gamma m_i < P_{i+1}$ for an index i, in which case clearly $\tilde{P} = P - \gamma m_i$ (since the derivative of $h + \frac{1}{2\gamma}(\cdot - P)^2$ is zero there);
- or there exists an index $i \in \{-1, 0, \dots, n\}$ such that $P - \gamma m_i \geq P_{i+1} \geq P - \gamma m_{i+1}$, in which case $\tilde{P} = P_{i+1}$ (the first and last angular coefficients are conventionally $m_{-1} \coloneqq -\infty$ and $m_{n+1} \coloneqq \infty$).

In both cases, the index i is unique. Putting all toghether, one has that

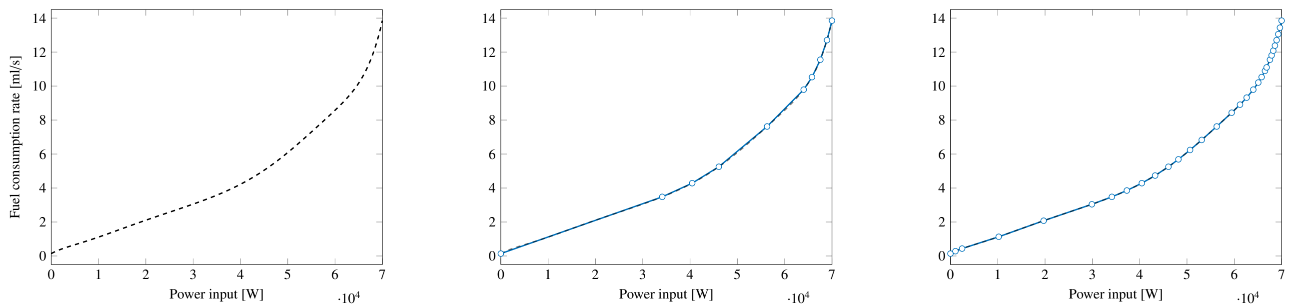
$$\tilde{P} = \min\{P_{i+1}, P - \gamma m_i\} \tag{9a}$$

$$\text{where } \; i = \operatorname*{arg\,max}_{j=0,\dots,n} \left\{ j \mid P \geq P_j + \gamma m_j \right\}. \tag{9b}$$

Now, the computation of the proximal mapping of h can be done by a simple comparison:

$$\operatorname{prox}_{\gamma h}(P) = \begin{cases} \max\{P_{\min}, P\} & \text{if } P \leq 0, \\ \tilde{P} & \text{if } P > 0 \wedge h_i(\tilde{P}) < \frac{1}{2\gamma}P^2, \\ 0 & \text{otherwise,} \end{cases}$$

where $\tilde{P}$ and i are as in (9), and h_i is as in (8).



(a) Polynomial measurement fit **(b)** DP piecewise linearization with $e = 0.1$ **(c)** DP piecewise linearization with $e = 0.01$

Fig. 1: *Generation of the fuel consumption map W. Apparently, the 9 line segments generated by the DP algorithm with accuracy* $e = 0.1$ *are enough to produce a very good approximation of the polynomial fitting of the measurements.*

4. NUMERICAL SIMULATIONS

To assess the effectiveness of the proposed method, we consider a cruise control problem as described in Section 2 where the velocity $v_{\mathrm{p}}(t)$ of the preceding vehicle is assumed known and constant $v_{\mathrm{p}} = 30$ m/s. The desired time delay between the two vehicles is set to $h_{\mathrm{d}} = 2$ s.

The gravitational acceleration is approximated as $g = 9.8$ m/s^2, and the model parameters of the control vehicle are set as follows: $M = 1400$ kg, $C_{\mathrm{D}} = 0.29$, $A = 2.6$ m^2, $\rho = 1.1841$ kg/m^3, and $\mu = 0.012$. The control inputs are constrained between a minimum value of $P_{\min} = -10^4$ W and a maximum value of $P_{\max} = 7 \cdot 10^4$ W. The various initial values of the state quantities are $s(0) = 0$ m, $v(0) = 20$ m/s, $Q(0) = 0$ ml, and $s_{\mathrm{p}}(0) = 100$ m.

The total simulation time is set to $t_{\mathrm{f}} = 300$ s, the sampling time interval to $\Delta t = 0.1$ s, and the time window of each MPC problem to 3 s, resulting in 3000 many MPC problem instances with discrete-time horizon of $N = 30$.

The instantaneous fuel consumption W was generated by polynomial interpolation of measurements using the `polyfit` function in Matlab. To review the number of polynomial fits, we solved the instantaneous fuel consumption graph for a vehicle traveling at a fixed speed and observed that the instantaneous fuel consumption graph remained virtually unchanged when the number of polynomial fits was greater than 9. We therefore opted for a 9th-order polynomial approximation; the result is given in Figure 1a. Figures 1b and 1c show the results obtained by setting different distance dimensions $e = 0.1$ and $e = 0.01$ as accuracy parameters for the DP approximation, which respectively correspond to piecewise linearization with 9 and 30 line segments. Apparently, the few line segments produced with accuracy $e = 0.1$ are enough to well approximate the polynomial fit.

For the choice of distance dimension of the DP algorithm, we also compared three cases when the distance dimension was 0.1, 0.01 and 0.001, respectively, to see whether the increase in accuracy would be computationally burdensome. PANOC proved successful in all trials, and the final cost was subject to negligible differences within a 0.5% relative range in optimality.

A more important modeling factor is the weight ω_0, responsible for the trade-off between control stability and fuel efficiency. The value was fine tuned among different choices, and from our experiments the choice $\omega_0 = 1$ was considered the most satisfactory in the performance.

4.1. Simulation results

The results of the simulation are shown in Figure 2. From the state diagram, it can be learned that the speed of the control vehicle is finally maintained in a similar speed range as that of the preceding vehicle, and the distance between vehicles is succesfully kept around the target distance. Our results confirm the pulse-and-glide phenomenon of fuel-efficient control [11]: to keep fuel consumption low, instead of maintaining one speed continuously, the vehicle is kept in a control mode that constantly switches between acceleration and deceleration, initially staying within the two speed intervals in the form of bang-bang control [18]. A per-stage cost term $\omega_1(u_k - u_{k-1})^2$ can be also included in function f to account for abrupt speed variations in the solution, so as to trade-off a fuel-efficient and a smooth cruise.

Each problem is solved largely within sampling time,

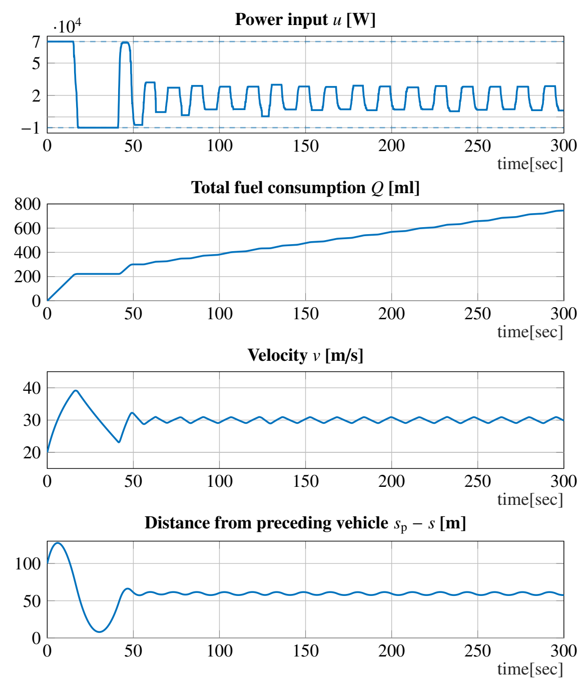


Fig. 2: *Simulation results when the distance dimension e is 0.1*

taking about 0.007 s on average. Thanks to the cheap iteration oracle typical of proximal algorithms, the method is well suited for an embedded implementation. This strikes a tangible advantage of our approach if compared with other techniques, such as based on SQP or IP methods, which perform well if run on powerful processing units but are penalized on low capacity platforms. Moreover, our approach well copes with problems that are nonsmooth and nonconvex at the same time, a scenario that precludes the use of other embedded-friendly methods whose applicability is bound to either smooth or convex problems, such as (sub)gradient methods. We should also remark that an ad hoc implementation should drastically further reduce computation time.

Regarding the computation of the proximal mapping, we compared our DP approach against a numerical solution of the subproblem with the original 9^{th}-order polynomial, both with a Newton method and by solving a polynomial equation involving the derivative. Our method was on average twice as fast than the Newton method, and 10 times as fast than the one with the polynomial equation, with an absolute and relative error of about 10^{-4}.

5. CONCLUSION

We applied nonlinear model predictive control to address a cruise control problem in a fuel efficient regime. PANOC algorithm was chosen as optimization solver for its fast solving time and low computational requirements. Because of the complicated nonsmooth part of the problem at hand, however, PANOC was not readily applicable. To account for this issue, we proposed a piecewise linearization technique based on the Douglas–Peucker method that, while not sacrificing optimality for it very closely approximates the objective, it drastically simplifies the optimization oracle. The application of PANOC for the cruise control problem proved successful in all our simulations. Given the low memory and computational requirements of the proposed method in combination with the PANOC algorithm, future work may consider real-world implementations on on-board CPUs.

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