A Comparative Study of Computational Models for Fatigue Crack Propagation

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Summary

The validity of numerical fatigue crack growth simulation models based on the Dugdale strip yield model is investigated. Newman's and authors' models are highlighted. The differences of each model are explained and the comparative study of models with the object of crack opening load is conducted. The crack opening load by authors' model gives an reasonable value compared to Newman's model, because Newman's model ignores the elastic deformation in the plastic region and cannot represent the effect of re-distribution of stresses during the cyclic loading. Besides, the availability of redefined effective stress intensity factor for the fatigue crack propagation by replacing the crack opening load with the RPG load is confirmed.

Introduction

Despite the many available fatigue life estimation methods, fatigue accidents of metal structures still occur. One of the significant reason is that most of proposed methods depreciate the role of the cyclic plasticity around a crack tip. Consideration of the cyclic plasticity ahead of a crack tip is imperative to the estimation of fatigue crack growth [1].

Some numerical fatigue crack growth simulation models based on the Dugdale strip yield model [2] were proposed in order to evaluate the fatigue crack growth considering the fatigue crack opening / closing behaviour. Newman's model [3] and authors' model [1] are highlighted as representative strip yield models in this paper.

Numerical simulation for fatigue crack propagation

The first numerical simulation model for fatigue crack propagation considering the behaviour of fatigue crack opening / closing was proposed by Newman [3]. This model enables to describe the plasticity-induced fatigue crack closure and to calculate the load level at which a crack tip becomes fully open during the cyclic loading. This model is based on the Dugdale strip yield model. Bar elements of rigid-perfectly plastic material with a flow stress (σ_0), which is defined as the average of yield stress and ultimate strength of material, are plugged into the chink corresponding to the virtual COD region in order to satisfy the condition of displacement continuity ahead of a physical crack tip. At any applied stress level, bar elements are either intact in the plastic zone ahead of a crack tip or broken in the residual plastic deformed layer in the crack wake. The broken elements can carry compressive loads only, and then only if they are in contact.

The plastic constraint factor (λ), which elevates the tensile flow stress, is applied in Dugdale model in order to give the precise value of plastic zone size and crack opening

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profile of a physical crack. The value of λ is generally given by comparing the crack opening displacement by Dugdale model with the one by elastic-plastic finite element analysis. The plastic constraint factor is also applied in Newman's numerical simulation model. At the maximum load and when the crack is fully open, the value of λ in Newman's model is set to 1.73 in the case of nominal plane strain condition and to 1.0 in the case of the plane stress condition. At the minimum loads, some elements in plastic zone and elements along crack surfaces that are in contact may yield in compression when the contact or compressive stress is equal to $-\sigma_0$ in Newman's model. In other words, the constraint factor changes during the cyclic loading. Newman explains the reason of the loss of constraint under compression as follows: the loss occurs as a result that the large stress gradient at a crack tip is greatly reduced and more uniform stress field is produced when a crack closes.

Crack opening load in Newman's model can be calculated by solving the following equation.

$$K(P_{open} - P_{min}) - K(\text{contact stress}) = 0. \tag{1}$$

The first term in the right side shows the change of the stress intensity from the minimum load (P_{min})to the crack opening load (P_{open}). The second term represents the stress intensity caused by the contact stress distribution worked on the crack closure region at the minimum load. This model is implemented into FASTRAN software [4].

The NLR model [5], which adopts the three plastic constraint factors (λ_t for tensile yielding, λ_c for compressive yielding ahead of a crack tip and λ_w for compressive yielding in the crack wake) approach, corresponds to the improved version of Newman's crack closure model. By setting $\lambda_t = \lambda_c$, The NLR model degenerates Newman's model. As far as the crack opening stress is calculated based on the NLR model, Newman's concept in equation (1) is also applied to the model. This model is implemented into NASGRO software [6].

On the other hand, authors improved Newman's model with considering the physical meaning of virtual crack opening displacement[7] and implement the proposed model into the simulation code FLARP [1]. FLARP enables to calculate not only the crack opening load, but also the RPG (Re-tensile Plastic zone Generated) load [1] at which the tensile plastic zone starts to develop ahead of a crack tip. Crack opening load in FLARP is defined as the load at which the stress over a physical crack becomes to zero under loading process and can be calculated by the linear system equations of stresses in each bar element without applying equation (1). Significant differences to Newman's model are 1) to change the constitutive relation of bar elements plugged into the virtual crack region and 2) to apply a constant plastic constraint factor (=1.04 in case of mild steels [1]). Perfect elastic-plastic material with a yield stress is adopted as the material properties in the plastic region in stead of rigid-perfectly plastic material. After loading the bar elements with uniform elastic stresses of the yield stress magnitude, they deforms elastically to accurately fit the fictitious COD obtained by Dugdale model. Inserting the segments enables to satisfy the displacement continuity ahead of a physical crack tip. Authors presume that the elastic deformation of bar element cannot be ignored in order to perform more precise numerical fatigue crack growth simulation. The reasons are described below.

1. The crack opening profile in center cracked tensile (CCT) specimen subjected to remote uniform tensile stress under plane stress condition is given in equation (2) [8].

$$
\begin{aligned}
V(x) &= \frac{8W\sigma_Y}{\pi^2 E}\sin\alpha\int_{\chi}^{\pi/2}\frac{\cos\chi}{\sqrt{1-\sin^2\alpha\sin^2\chi}}\ln\left|\frac{\sin(\chi+\phi)}{\sin(\chi-\phi)}\right|, \qquad (2)\\
\sin\chi &= \sin(\pi x/2W)/\sin\alpha,\\
\sin\phi &= \sin(\pi c/2W)/\sin\alpha = \cos(\pi\sigma/2\sigma_Y),\\
\sin\alpha &= \sin(\pi a/2W),
\end{aligned}
$$

   where

$V(x)$:	crack opening profile at x,
σ:	remoto uniform tensile stress,
$2c$:	physical crack length,
$2a$:	fictitious crack length,
$2W$:	specimen width,
σ_Y:	yield stress of the material, and
E:	Young's modulus.

   The verification of equation (2) were conducted by comparing elastic-plastic finite element analyses and the results by Dugdale model. If the material properties in plastic zone corresponds to rigid-perfectly plastic, the crack opening profile by equation (2) shows zero over fictitious crack because E is infinite in case of rigid-perfectly plastic body. This result contradicts the one by Dugdale model. Therefore, the material properties in plastic zone should not be rigid-perfectly plastic and it is clear that Newman's model contains the discrepancy concerning the material properties of bar elements plugged into the plastic region.

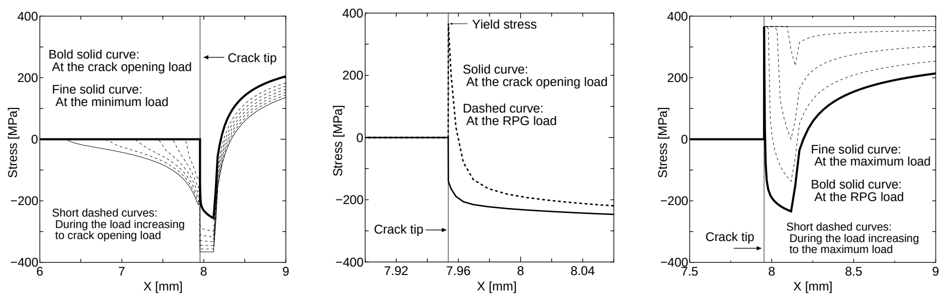
2. In order to perform the fatigue crack opening / closing simulation, the elastic behaviour under unloading and reloading must be expressed in the numerical simulation model. It follows that the bar elements plugged into the plastic region must show the elastic behaviour until the elements reach tensile / compressive yielding during the loadings. Besides, the model implemented the bar element which consist of perfect elastic-plastic material can give the stress distributions under arbitrary unloading and reloading process.

Because NLR model uses the bar element of which the stress versus strain relation is the rigid-perfectly plastic type, NLR model also contains the discrepancies mentioned above.

Comparative study of simulation models

The crack opening load is calculated by applying equation (1) in Newman's and NLR models. To the best of authors' knowledge, no report to confirm the stress distribution along a crack line at the moment of the crack opening state in the models. The verification of equation (1) to obtain the crack opening load is conducted as follows.

CCT specimen with a initial crack length ($2a$) 5mm and width ($2W$) 50mm is used as a benchmark specimen. Constant remote stress range, which the maximum stress is 118MPa and the minimum stress is 5.9MPa, is subjected to the specimen. The crack opening load, the RPG load and the stress distributions under some representative loading processes are calculated by authors' simulation code FLARP. Detailed calculation procedure is described in the reference [1].

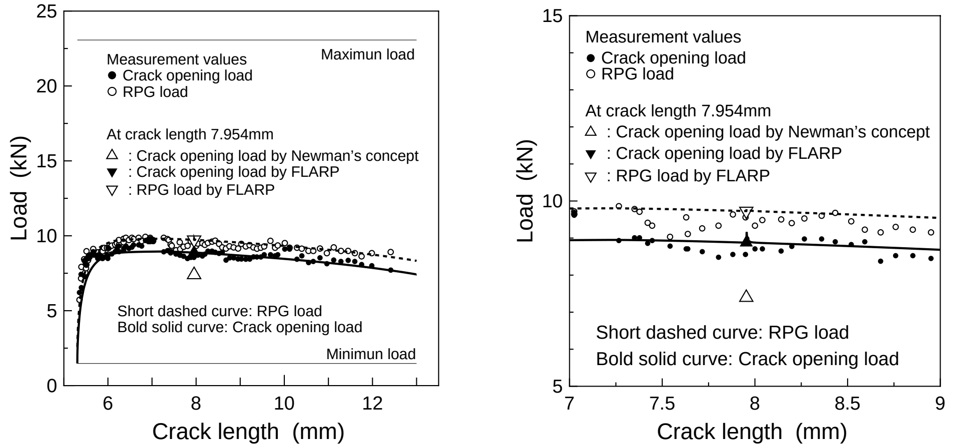


(a) (b) (c)

Figure 1: The change of stress distributions over a crack line (crack length 7.954mm)

The change of stress distributions from the minimum load to the crack opening load at a crack length of 7.954mm are shown in Figure 1(a). The contact stress works over the crack surfaces at the minimum load. The distributions at the crack opening load and the RPG load, and the ones from the RPG load to the maximum load at the same crack length are also shown in Figures 1(b) and 1(c). No stress works over the crack surfaces at the crack opening load. Crack tip stress reaches tensile yield stress at the RPG load.

The crack opening load at a crack length of 7.954mm is calculated by applying equation (1).The stress distribution at the minimum load shown in Figure 1(a) are used as a input data in equation (1). The result is plotted on Figure 2 by open triangle mark. Calculated crack opening loads and RPG loads by FLARP during the crack propagation are also drawn by solid and dashed lines on Figure 2. The crack opening load and the RPG load at a crack length 7.954mm are highlighted by plotting solid triangle and open inverse triangle marks respectively. Measured both loads with a high degree accuracy are also plotted on Figure 2 by solid and open circles respectively. Measuring method of both loads is stated in the reference [9].



(a) Overview (b) Closeup at crack length 7.954mm

Figure 2: Relationships between crack length and each load

The crack opening load by FLARP gives an reasonable value and that Newman's model gives the lower value. Newman's model cannot identify crack opening profiles and stress distributions under arbitrary loading level, because the effect of stress re-distribution during the cyclic loading cannot be considered due to the ignorance of elastic behaviour of bar elements. Therefore, the verification equation (1) cannot be conducted in case of the Newman's model. It is postulated that the ignorance of elastic behaviour of bar elements results in the low crack opening load shown in Figure 2.

It becomes clear from Figure 1(b) that the plastic work is not proceeded in the loading range from the crack opening load to the RPG load. Because the cyclic plastic work generating ahead of a crack tip consumes as the source of fatigue crack driving force, the loading range from the crack opening load to the RPG load does not contribute the fatigue crack propagation. Then, the effective stress intensity factor (ΔK_{eff}) [10] for the fatigue crack propagation should be redefined by replacing the crack opening load with the RPG load. Authors defined the effective stress intensity factor based on the RPG load (ΔK_{RPG}) and conducted the verification of ΔK_{RPG} as a useful parameter in order to describe the fatigue crack growth under arbitrary loading condition [1].

Concluding Remarks

In order to estimate fatigue crack propagation quantitatively, Newman and authors proposed numerical fatigue crack growth simulation models based on the Dugdale model [2], respectively. The crack opening load by FLARP gives an reasonable value and that Newman's model gives the lower value. The verification equation (1) cannot be conducted in case of the Newman's model, because Newman's model cannot identify crack opening profiles and stress distributions under arbitrary loading level due to the ignorance of the effect of stress re-distribution during the cyclic loading. Besides, it makes clear that the effective stress intensity factor for the fatigue crack propagation should be redefined by replacing the crack opening load with the RPG load, because the plastic work, which corresponds to the source of crack driving force, is not proceeded in the loading range from the crack opening load to the RPG load.

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