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The Significance of Plastic Zone Growth under Cyclic Loading and Crack Opening/Closing Model in Fatigue Crack Propagation

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Abstract Fatigue cracks remain closed at lower loading level during a part of load cycle even though a tension-to-tension loading is applied. The crack closure plays a role to obstruct the generation and growth of compressive plastic zone during unloading. Cyclic plastic work, which corresponds to an irreversible energy consumed in a cracked body is generated ahead of a crack, is required as a fatigue crack driving force. The amout of cyclic plasticity is reduced by a crack closure. The crack opening/closing model based on the Dugdale model under arbitrary stress distributions for a through thickness straight crack is proposed and the fatigue crack growth under various loadings is investigated.

Overview of the Fatigue Crack Propagation Law Based on Crack Closure

Fatigue cracks remain closed during a part of load cycles because the fatigue crack propagates in the residual tensile plastic deformed zone ahead of a crack tip. The effective stress intensity factor range based on an opening was proposed as a fatigue crack propagation parameter by Elber[1]. Fig. 1 shows the schematic illustration of history of plastic deformation and a working stress along a crack line in one loading cycle during fatigue crack propagation. The plastic work is not proceeded in the loading range from a crack opening load (P_{op}) to a Re-tensile Plastic zone's Generated load (P_{RPG}) [2]. The measurement method of P_{RPG} is introduced in Refs.[3] and [4].

There is no stress singularity due to a crack at P_{op}, because the compressive (contact) stresses in the crack closure region release just before P_{op}, as shown in Fig. 1(b). Stress at the crack tip increases very rapidly just after the loading because of very large stress concentration caused by the crack. When stress at the crack tip reaches yield stress, the stress distribution ahead of the crack tip shows an concave as shown in Fig. 1(c). More applied loading is required in order to develop the tensile plastic zone ahead of a crack tip. In other word, P_{RPG} is higher than P_{op} with a significance difference, because plastic behavior appears just after the situation in which a certain area with the order of grain size (but not at a point) is satisfied with yield condition as shown in Fig. 2. Sugeta's presentation for Ref. [5] at a Japanese committee that his coworkers observed no movement of dislocations at P_{op} by means of an atomic force microscopy supports above discussion.

P_{op} approaches P_{min} and P_{RPG} approaches P_{max} as a crack growth stops under a stepwise decreasing amplitude loading while P_{max} keeps a constant value, because the crack tends to remain open and shows no cyclic plasticity at the final stage [2]. The threshold does not appear in the relation between $\Delta K_{\mathrm{RPG}}(=\left(P_{\mathrm{max}}-P_{\mathrm{RPG}}\right)\sqrt{\pi a}\,f$ (f : magnification factor, a : crack length) and the fatigue crack propagation rate. The equation of fatigue crack propagation can be described as follows.

$$da/dN = C\left(\Delta K_{\mathrm{RPG}}\right)^m , \tag{1}$$

where C, m: material constants.

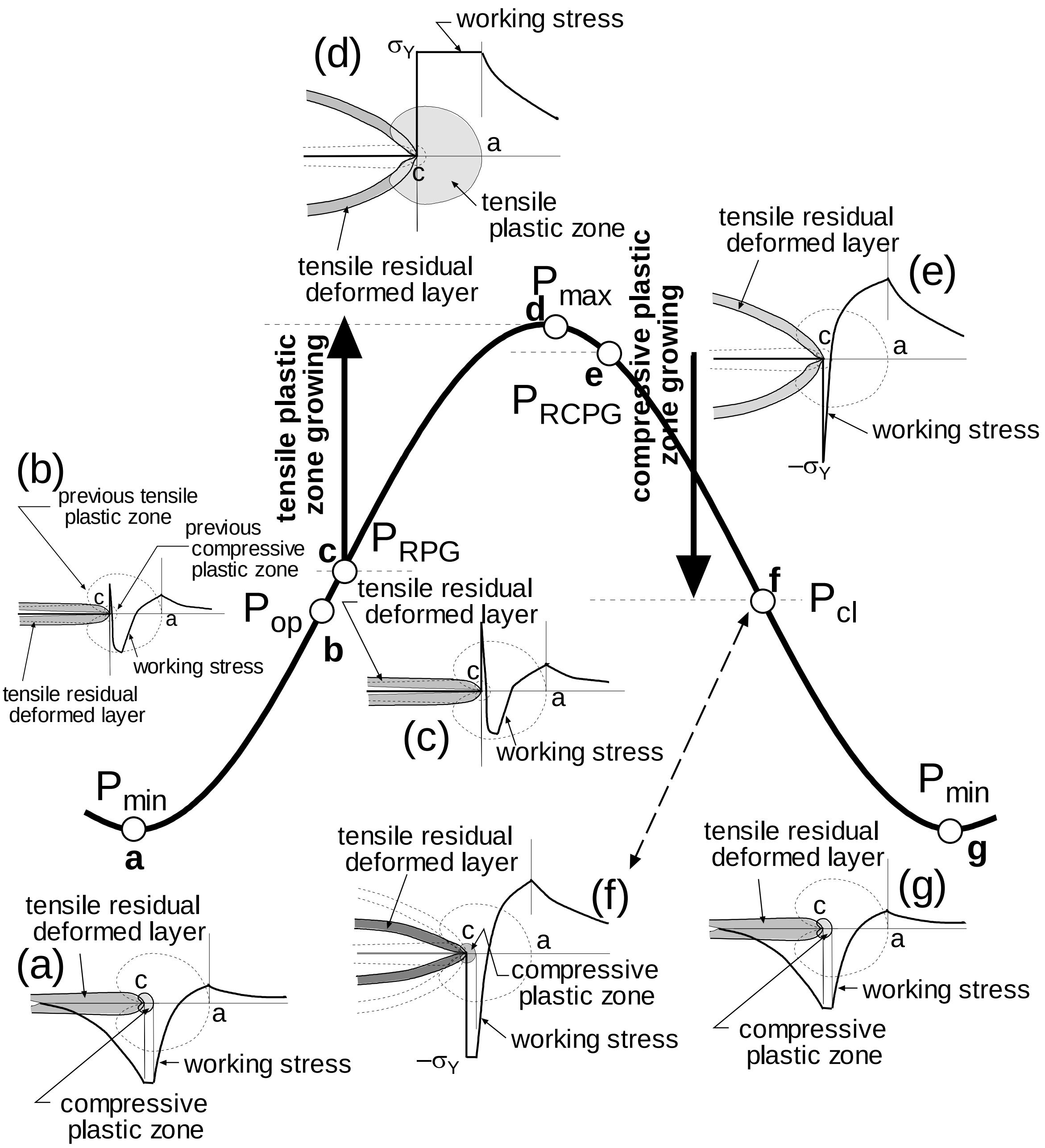


Fig. 1 Plastic deformation and the working stress distributions along the crack line in one loading cycle during the fatigue crack propagation

Crack opening/closing model

In order to investigate the meaning of COD for the fictitious crack in the Dugdale model, elastic-plastic FEM analyses for a center cracked specimen were performed. Constitutive relation in the material corresponds to a bi-linear stress-strain curve with a second modulus of E/100. The distribution of plastic strain component parpendicular to a crack line ($\varepsilon_y^{\mathrm{p}}(x)$) at some

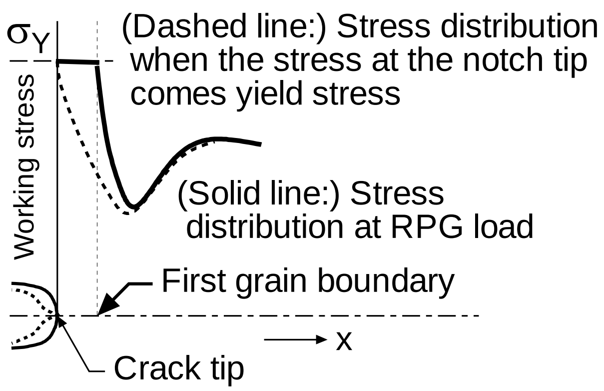


Fig.2 Stress distributions near a crack tip at RPG load and at notch tip yielding

positions ahead of the crack tip by FEM analyses under plane stress condition was obtained. Fig. 3 shows an example of the relation between fictitious CODs by the Dugdale model and the intergral of plastic strain with working stress of yielding in the plastic zone ($V_{\mathrm{FEM}}(x)$) defined by Eq. 2.

$$V_{\mathrm{FEM}}(x) = (1+\lambda\sigma_{\mathrm{Y}}/E)\int_{\Gamma}\varepsilon_y^{\mathrm{P}}(x)dy = (1+\lambda\sigma_{\mathrm{Y}}/E)L(x)$$

(2)

where

Γ : existing area of plastic strain at x,

λ : plastic constraint factor,

σ_{Y} : yield strength.

The Dugdale model gives a reasonable COD for the real crack region in case that λ is equal to 1.02. From the result of Fig. 3, it can be interpreted that a perfect elastic-plastic segment with the length of $L(x)$ erupts along the crack line in the fictitious crack zone just after $P_{\max}$.

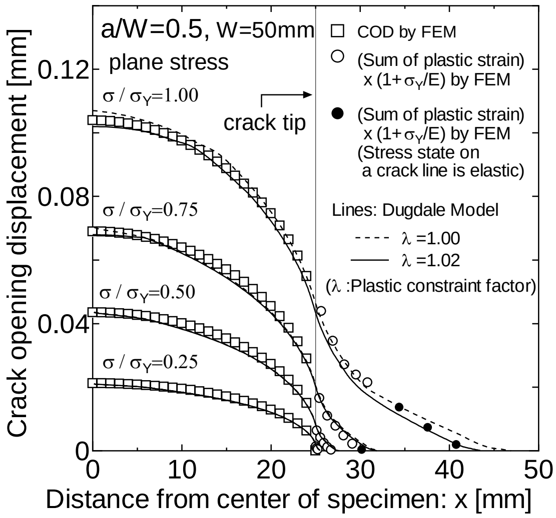


Fig. 3 Comparison between COD at fictitious crack and summation of plastic strain in plastic zone ahead of a crack tip

Now let us consider the problem of a crack with a length of a, which is along to x axis, subjected to splitting force p at x on crack surfaces and gives the stress intensity factor as follows.

$$K = pg(x,a). \tag{3}$$

Considering the problem that normal stress ($S(x)$) caused by a unit external force and residual stress ($R(x)$) over x axis act on the body without a crack, the following equation should hold in the Dugdale model in order to diminish stress singularity at the fictitious crack tip.

$$\int_0^a \left[P_{\max}S(x)+R(x)\right]g(x,a)dx - \lambda\sigma_{\mathrm{Y}}\int_c^a g(x,a)dx = 0, \tag{4}$$

where c : real crack length,

a : hypothetical crack length (tip of the tensile plastic zone),

$P_{\max}$: maximum load,

λ : plastic constrained factor.

COD at x_j is represented in Eq. 5 by the discretization.

$$V(x_j) = \sum_{i=1}^{n}(P_{\max}S_i + R_i)F(x_j, B_i, B_{i+1}, a) - \lambda\sigma_{\mathrm{Y}}\sum_{i=k}^{n}F(x_j, B_i, B_{i+1}, a), \tag{5}$$

where $F(x_j, B_i, B_{i+1}, a)$ corresponds to COD at x_j for the crack with a length of a when uniform unit stress acts between B_i and B_{i+1} over crack surfaces, $B_1 = 0$, $B_k = c$ and $B_{n+1} = a$. S_i and R_i are the discretized value of $S(x)$ and $R(x)$ respectively. $F(x_j, B_i, B_{i+1}, a)$ is formulated in Eq. 6.

$$F(x_j, B_i, B_{i+1}, a) = \frac{2}{E'}\int_0^a g(x_j,a)da\int_{B_i}^{B_{i+1}} g(x,a)dx \tag{6}$$

Taking $x_j = (B_j + B_{j+1})/2$ for j from k to n and $B_{k+1} = x_0 = c + d/2$ (d: grain size), the length of a segment at x_j just after $P_{\max}$ is given by

$$L(x_j) = V(x_j)/(1+\lambda\sigma_{\mathrm{Y}}/E') \qquad (\text{ for } j = k \text{ to } n\ ). \tag{7}$$

The segment is divided into n bar elements along the crack direction for simplicity. COD at x_j during unloading process is represented in Eq. 8 by the principle of superposition.

$$V(x_j)=\sum_{i=1}^{n}(P\,S_i+R_i)F(x_j,B_i,B_{i+1},a)-\sum_{i=1}^{n}\sigma_i F(x_j,B_i,B_{i+1},a). \tag{8}$$

Length of individual bar element becomes

$$V(x_j)=L(x_j)(1+\sigma_j/E'), \tag{9}$$

if the working stress σ_j remains elastic. Above bar elements exists not only in the fictitious crack part but also in the real crack part, which were in the previous fictitious crack. Eq. 10 should hold in an elastic zone except the crack opening region.

$$L(x_j)(1+\sigma_j/E')=\sum_{i=1}^{n}(P\,S_i+R_i)F(x_j,B_i,B_{i+1},a)-\sum_{i=1}^{n}\sigma_i F(x_j,B_i,B_{i+1},a). \tag{10}$$

σ_j can be calculated using Eq. 9 under the condition that $|\sigma_j|$ does not exceed in $\lambda\sigma_Y$ and σ_j is not greater than zero in a real crack part. By substituting the solutions of σ_j into Eq. 8, crack opening profile at $P_{\min}$ is obtained.

If contact stresses do not work in the crack wake at $P_{\min}$, the compressive plastic zone grows more and then the COD decreases as shown in case C in Fig. 4. The distributions of stresses and COD $\tilde{V}(x_j)$ after a crack extension at $P_{\min}$ associated with an entire release of the contact stress in the crack closure region are also shown by dotted lines as case B in Fig. 4.

If a fatigue crack propagates on unloading, compressive stresses in the crack extension region will release just after the extension. The thickness of plastic elongated layer in the region at the moment of crack extension might be less than $\hat{V}(x_j)$ and greater than $\tilde{V}(x_j)$. If a crack propagates during loading, the tensile stresses in the crack extention region will release just after crack growth.

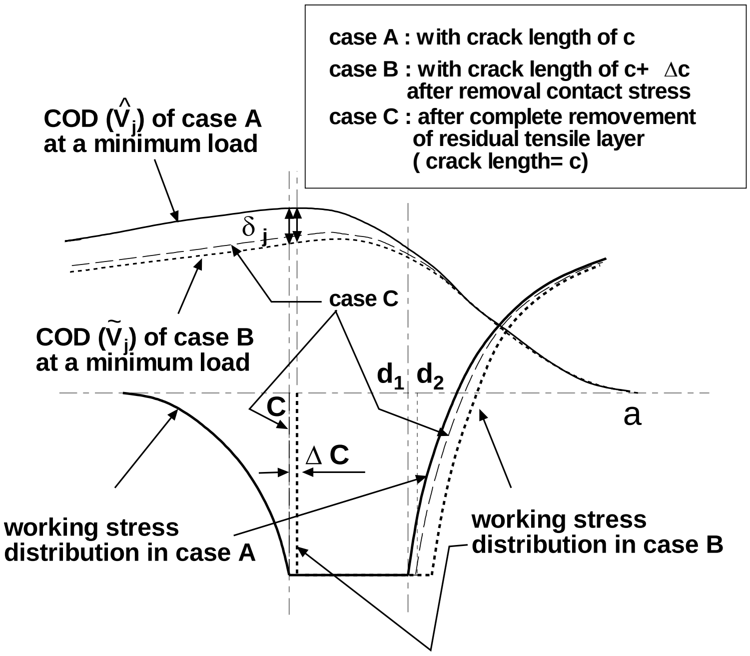


Fig. 4 Plastic shrinkage at the crack extension (in case C, no crack extension)

The release ratio of contact stresses depends on a timing of crack extension during a cyclic loading. The same holds true for the rate of the plastic shrinkage ($\delta(x_j)$, see Fig. 4). The amount of shrinkage is defined as $\kappa\delta(x)$ and the coefficient κ can be represented as a function of cumulative plastic strain in the crack extension region. The value of $L(x_j)$ is changed by Eq. 11 if x_j locates in the compressive plastic zone at $P_{\min}$.

$$L(x_j)=V(x_j)/(1-\lambda\sigma_Y/E'). \tag{11}$$

Stress distribution (σ_j) at $P_{\min}$ is calculated by substituting $L(x_j)$ except Δc region and $L(x_j)$ consdering the plastic shrinkage in Δc region at $P_{\max}$ into Eq. 10. $V(x_j)$ at $P_{\min}$ is obtaind following by the same treatment mentioned above.

P_{RPG} can also be calculated by considering that the elastic plastic boundary becomes x_0 $(=c+d/2)$ and $V(x_0)$ becomes $L(x_0)(1+\lambda\sigma_Y/E')$ when tensile plastic zone starts to grow under the loading process where $L(x_j)$ corresponds to the length of bar element at x_j just after the minimum load.

A certain crack growth Δc will be given at once on calculation for block loading as a matter of convenience. Fatigue crack growth must occur within the overlap region between the compressive and the tensile plastic zones, because cyclic plasticity appears only in this region. It is recognized that Δc with 5% of the overlap length gives a smooth contact stress distribution [6].

Detailed discussion about the numerial simulation of fatigue crack growth mentioned in this section and the plastic shrinkage of bar element are explained in Ref. [6].

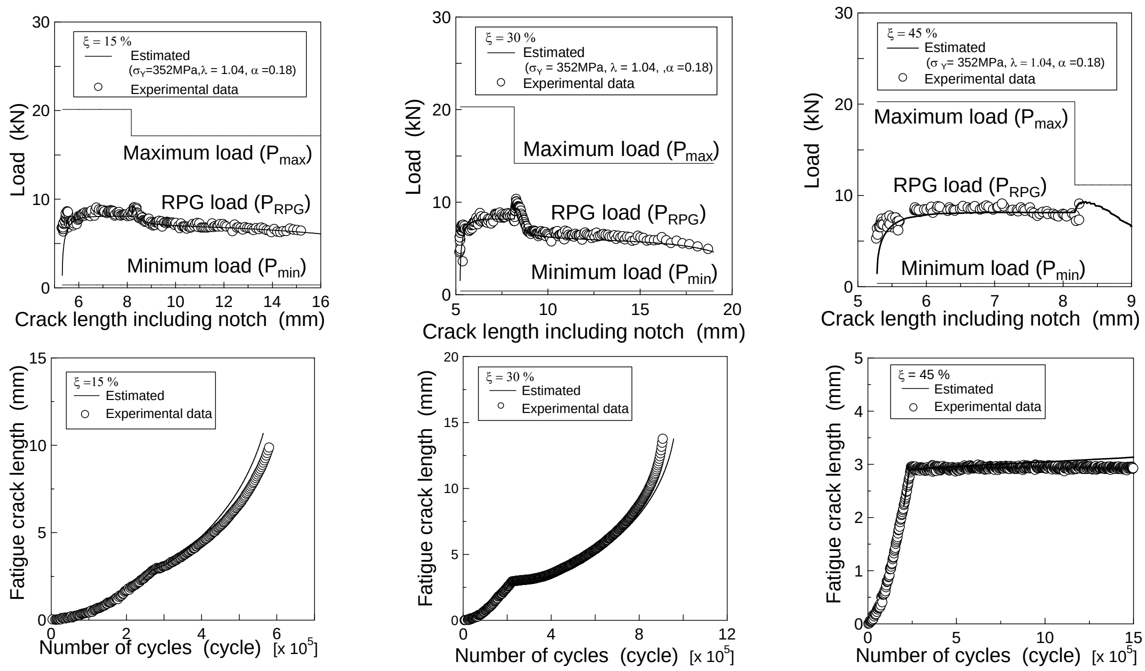
Numerical Simulation of Fatigue Crack under Various Loading Conditions

Many factors for fatigue performance were investigated by applying experiments and numerical simulations [7]. Loading history for a fatigue is explained in this section. Block loadings and storm simulating loadings are applied as examples. Numerical simulation results of an effect of spike loading, stress ratio and pre-existing residual stress on fatigue crack growth are introduced in Ref. [7].

Fig. 5 shows a change of RPG load and fatigue crack growth curves in case of block loading with the step-down of the maximum load. Center cracked specimens are used in these experiments. α value in Fig. 5 relates κ value, which was determined by a constant amplitude loading with the stress ratio of zero. RPG load increases just after the step-down of the maximum load. This is because the crack closure region at a minimum load extends due to decrement of COD just after a maximum load, even though the thickness of residual tensile layers does not change so much. The crack closure region extends over crack surfaces at the minimum load and then the RPG load increases. The retardation effect increases with increasing a reduction ratio of a maximum load (ξ) which is defined by Eq. 12.

$$\xi = \left(P_{\mathrm{p}} - P_{\mathrm{c}}\right) / P_{\mathrm{c}}\,, \tag{12}$$

where P_{p} is load before dropping and P_{c} is load after dropping.When ξ is equal to 0.45, the fatigue crack almost stops, as shown in Fig. 5(c). The numerical simulations give valid results in all cases. Detailed discussion is explained in Ref. [7].



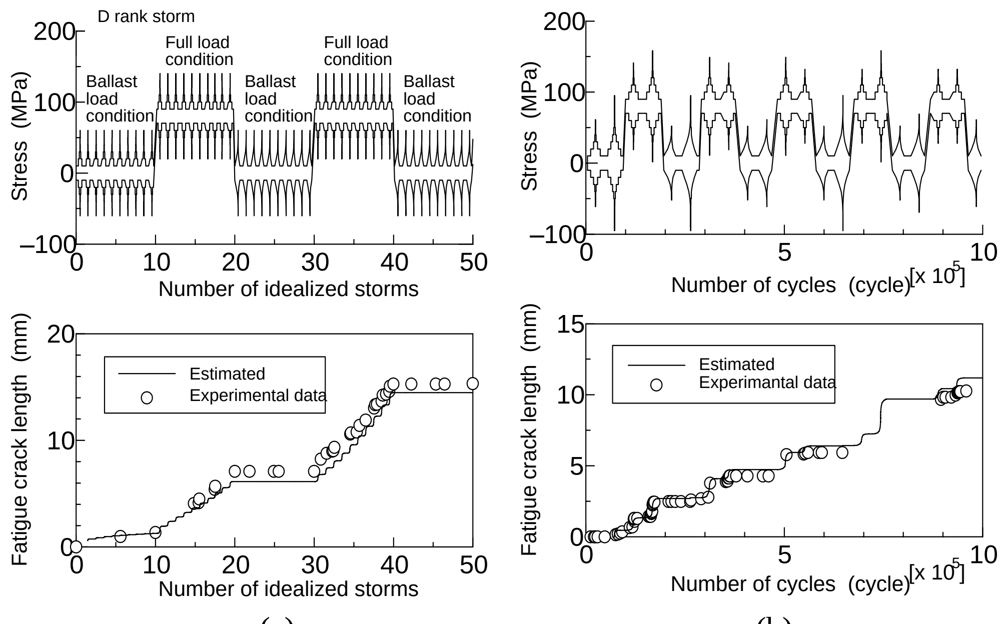
(a) ξ =15% (b) ξ =30% (c) ξ =45%

Fig. 5 The change of RPG loads and crack growth after a step-down in maximum load decrease

Fig. 6 shows the observed and the estimated crack growth curves under a storm-simulating loading [8], which corresponds to applied load to ships encountering during voyages. Two types of storm loading histories are supposed. Again good agreement between the estimated and observed results is achieved.

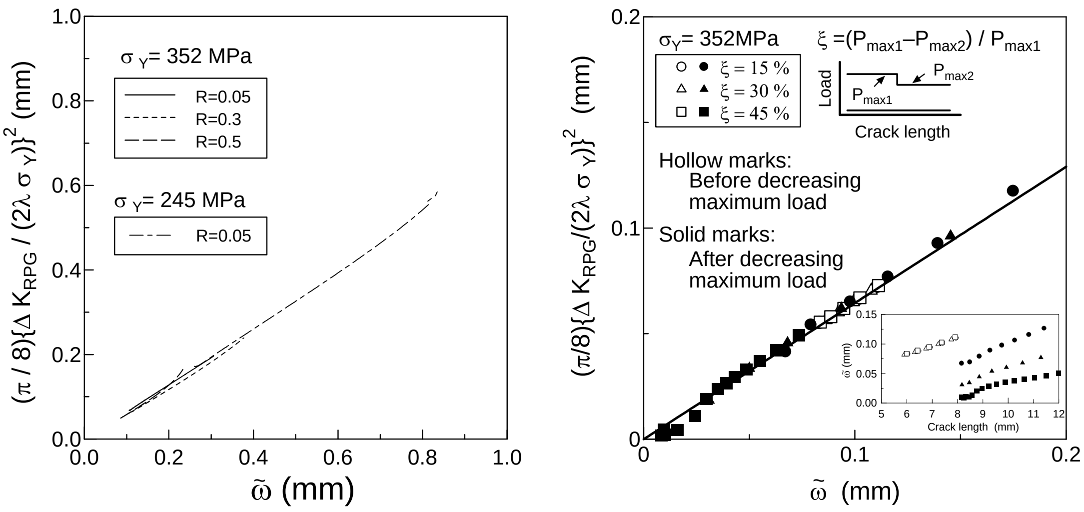
Fig. 7 shows a relation between ΔK_{RPG} and a size of overlapping region of tensile plastic zone at $P_{\max}$ and a compressive plastic zone at $P_{\min}$ in a given loading cycle ($\tilde{\omega}$) for various loading conditions. From these results, the following relation is obtained.

$$\tilde{\omega} = (1.55\pi/8)(\Delta K_{\mathrm{RPG}}/2\lambda\sigma_Y)^2 . \qquad (13)$$



(a) (b)

Fig. 6 Fatigue growth curves under two storm-simulation loads histories



(a) Under constant amplitude load

(b) In case of decreasing maximum load

Fig. 7 Relationship between the $\tilde{\omega}$ parameter and the effective stress intensity factor range based on the RPG load

The shape of plastic region around a fatigue crack tip is affected by applied loading history. An example of the relation between loading history and plastic zone shape is shown in the center of Fig. 8. Two types of the plastic region, which are identified as case (I) and (II), are observed under cyclic loading. Case (I) appears in the ordinary loading condition. Case (II) can appear just after enormous decrease of the loading amplitude. The difference of plastic region should be considered in order to evaluate the fatigue strength under variable loading, because both types of region can appear randomly.

Fatigue crack cannot propagate when $\tilde{\omega}$ is zero, because the plastic work is only consumed in the overlapping region of the plastic zones. Threshold behavior of a crack growth under arbitrary loading conditions is automatically represented by ΔK_{RPG} without introducing ΔK_{th} or $(\Delta K_{\mathrm{eff}})_{\mathrm{th}}$.

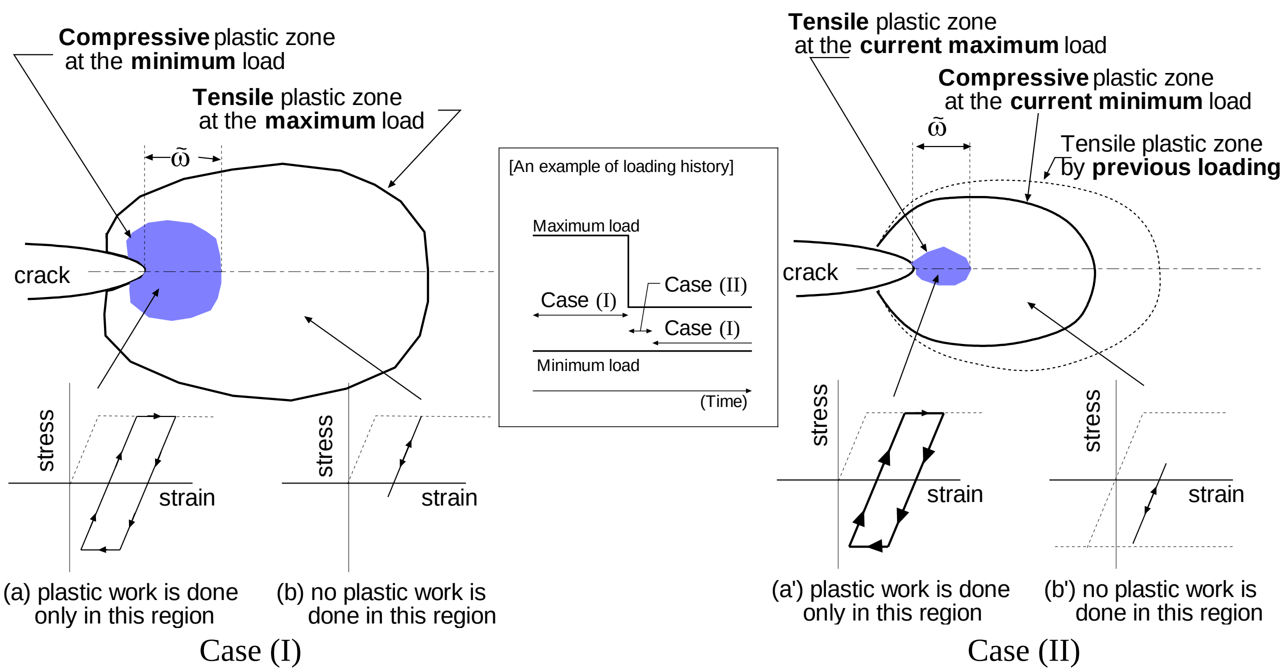


Fig. 8 Illustration of the physical meaning of $\tilde{\omega}$

Concluding Remarks

A fatigue crack closure model to predict fatigue crack growth under an arbitrary uni-axial stress field is introduced in this paper. The basic assumption behind this concept is that fatigue crack growth in a fatigue cycle starts at an RPG load at which the tensile plastic zone ahead of the crack tip starts to develop. The effective stress intensity factor range is redefined by replacing the crack opening load by Elber with the RPG load. Crack driving force parameter with physical meaning based on the loading range from RPG load to maximum load is termed ΔK_{RPG}.

Unlike the traditional effective stress intensity factor range based on a crack opening level, the ΔK_{RPG} can account for a threshold behavior. The crack growth rate versus ΔK_{RPG} data can be described within the full range of crack growth, including a threshold region. The model has been shown to give accurate crack growth predictions under various loading conditions.

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