Towards Scale Free Disturbance Suppression in Heterogeneous Mass Chains

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Towards Scale Free Disturbance Suppression in Heterogeneous Mass Chains

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***Abstract*— We study the disturbance attenuation problem in a heterogeneous mass chain where both the number of masses and the mass distribution may change. The paper studies the scalar transfer functions from the disturbance at a boundary point to a given intermass displacement. It is shown that these transfer functions can be represented in the form of composition sequences generated by Möbius transformations. The framework aims at devising a method of designing the interconnection impedances in the mass chain in a *scale-free* manner. That is, the resulting design guarantees certain performance criteria for mass chains of any length and any mass distribution. A graphical method is provided for such an interconnection design problem.**

I. Introduction

We study the propagation of disturbances in a chain of masses with passive interconnection in Fig. 1. In particular, we investigate how the transfer functions from the position of the movable point x_0 to a given intermass displacement changes as the total number of masses changes.

The problem is motivated by the prevalence of large-scale networked systems. Examples include the platooning of vehicles [9], both frequency and voltage stability problems in electrical power systems [10], [16], and flocking and consensus phenomena [1]. These applications typically involve very large numbers of subsystems, and the numbers of subsystems is often subject to change. Therefore, one problem is ensuring the ability to control a system behaviour independent of the size of the systems. This can be characterised as a *scale free* property. The advantage of a scale free method is that any design remains valid even if the number of subsystems changes.

In the case of the homogeneous mass chain, i.e., both mass m_j and impedance Z_j are all identical for $j = 1, 2, \ldots, N$ in Fig. 1, it has been shown in [18] that the H^∞-norm of these transfer functions are uniformly bounded with respect to N for certain choices of interconnection impedances. The conditions have been given with respect to a dimensionless parameter h depending on the impedance and mass. We note that this parameter h can be seen as a "generalized frequency variable" that has been introduced in [6] as a convenient notion to represent a certain class of large-scale systems. Furthermore, for the first intermass displacement, a scale free method of selecting $h(s)$ has been provided in [13] so that the supremum of the H^∞-norm over N is no greater than a prescribed value with a provable guarantee.

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It may be noted that scale free performance criteria are extremely rare. The overwhelming majority of existing scale free results, for example those based on passivity or dissipativity [17], the multivariable Nyquist criterion [8], or IQC criteria [12], have focused on the question of *robust stability*, rather than performance. It is due to the fact, at least in part, that several key performance measures relating to global behaviours of large-scale networks simply do not scale. Notable examples include the string instability [15] or network incoherence phenomena [1]. Also bidirectional control, which our problem formulation corresponds to in the context of vehicle platooning, is often subject to the inevitable string instability [2], [7] except for some specific situations where the string instability can be avoided [5]. However it appears that average or local performance measures, for example those in [1], [11], [13], [18], can be guaranteed in a scale free manner. We therefore see the results in this paper as another step towards understanding the role of scale free design, in a setting that is relevant to a wide range of application areas.

The present paper is an attempt to extend the results in [13], [18] to the heterogeneous mass chain. For this purpose, we first derive some recurrence relations in the transfer functions, which is the generalisation of the results given in [18]. In order to develop a scale free design method, we propose to select the impedance such that $Z_j(s) = h(s)/(sm_j)$ for $j = 1, 2, \ldots, N$. We then provide a graphical method to design $h(s)$ such that the H^∞-norm of the first intermass transfer function is bounded by a pre-specified value. Numerical examples suggest the worst mass distribution with respect to the infinity norm for this method when each mass is within a specific range. By designing $h(s)$ for this mass distribution, the infinity norm bound is guaranteed for any mass distribution. This means that, as long as mass is within a certain range, it can be freely added to or removed from the mass chain without violating the performance criteria.

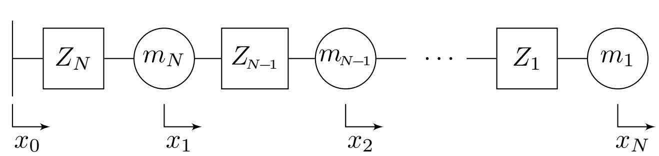


Fig. 1: Chain of N masses m_j connected by mechanical impedances Z_j and connected to a movable point x_0.

Notation

$\mathbb{C}$ and $\mathbb{Z}^+$ denote the set of complex numbers and positive integers, respectively. The composition of two functions is denoted by $f \circ g(x) = f(g(x))$. The impedance $Z(s)$ of a linear time-invariant mechanical one-port network with force-velocity pair (F, v) is defined by the ratio $v(s)/F(s)$.

II. Problem Formulation

Consider a chain of N masses m_j connected by a mechanical impedance $Z_j(s)$, $j = 1, 2, \ldots, N$, as shown in Fig. 1. Each interconnection provides an equal and opposite force on each mass and is assumed here to have negligible mass. The system is excited by a movable point $x_0(t)$ and the displacement of the ith mass from the left end is denoted by $x_i(t)$, $i = 1, 2, \ldots, N$. Assume that the initial displacements of the movable point and the mass are all zero.

Note that the index of the displacements of masses start from left while those of masses and impedances start from right. This notation becomes natural when recurrence relations are derived. To avoid confusion, we use index i from the left in the mass chain and j from the right. Hence, $j = N - i + 1$.

The equations of motion in the Laplace transformed domain are given by

$$m_{N-i+1}s^2\hat{x}_i = sZ_{N-i+1}^{-1}\left(\hat{x}_{i-1} - \hat{x}_i\right) + sZ_{N-i}^{-1}\left(\hat{x}_{i+1} - \hat{x}_i\right) \quad \text{for } i = 1, 2, \ldots, N-1,$$
$$m_1 s^2 \hat{x}_N = sZ_1^{-1}\left(\hat{x}_{N-1} - \hat{x}_N\right)$$

where $\hat{}$ denotes the Laplace transform. In matrix form this can be written as

$$\begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_N \end{pmatrix} = \begin{pmatrix} L_N & -\alpha_{N-1} & & 0 \\ -1 & L_{N-1} & \ddots & \\ & \ddots & \ddots & -\alpha_1 \\ 0 & & -1 & L_1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \hat{x}_0 \quad (1)$$
$$=: L^{-1}e_1 x_0$$

where

$$L_j = h_j + \alpha_{j-1} + 1,$$
$$h_j(s) = sZ_j(s)m_j \quad \text{for } j = 1, 2, \ldots, N, \quad (2)$$
$$\alpha_0 = 0, \quad (3)$$
$$\alpha_j = \frac{Z_{j+1}}{Z_j} = \frac{m_j}{m_{j+1}}\frac{h_{j+1}}{h_j} \quad \text{for } j = 1, 2, \ldots, N-1. \quad (4)$$

Let us consider the determinant of the matrix L, d_N. Using the Laplace expansion, we find that

$$d_N = \left(h_N + \alpha_{N-1} + 1\right)d_{N-1} - \alpha_{N-1}d_{N-2} \quad (5)$$

with $d_{-1} = 1$ and $d_0 = 1$ for $N \in \mathbb{Z}^+$.

Since $L^{-1} = \operatorname{adj} L / \det L$, (1) can be written as

$$\begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_N \end{pmatrix} = \frac{1}{d_N}\begin{pmatrix} d_{N-1} & * & \ldots & * \\ \vdots & \vdots & & \vdots \\ d_0 & * & \ldots & * \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}\hat{x}_0$$
$$= \frac{1}{d_N}\begin{pmatrix} d_{N-1} \\ \vdots \\ d_0 \end{pmatrix}\hat{x}_0.$$

Hence, the transfer functions from the disturbance x_0 to the ith intermass displacement $x_{i-1} - x_i$ in the heterogeneous mass chain of Fig. 1 are given by

$$F_N^{(i)} := \frac{d_{N-i+1} - d_{N-i}}{d_N} \quad (6)$$

for $i = 1, \ldots, N$.

III. Recurrence relations in mass chains

It is shown in this section that the transfer functions $F_N^{(i)}$ in (6) are represented as compositions of Möbius transformations. For this purpose, let us first define the following recursion with respect to i:

$$\begin{aligned} d_N^{(i)} &= (h_N + \alpha_{N-1} + 1)d_{N-1}^{(i-1)} - \alpha_{N-1}d_{N-2}^{(i-2)} \\ d_N^{(-1)} &= d_N^{(0)} = 1. \end{aligned} \quad (7)$$

Note that $d_N^{(N)}$ is equivalent to d_N defined by (5), and will be denoted as d_N in the sequel.

Theorem 1: Let $F_N^{(i)}$ be defined by (6). For any $i = 1, 2, \ldots,$ the sequence $(F_N^{(i)})_{N=i}^{\infty}$ satisfies the following recurrence relation:

$$F_N^{(i)} = \frac{d_{N-1}^{(i-2)}\alpha_{N-i}F_{N-1}^{(i)} + h_{N-i+1}}{\prod_{k=1}^{i}\alpha_{N-k}F_{N-1}^{(i)} + d_N^{(i)}} \quad (8)$$

where $F_{i-1}^{(i)} = 0$ and $h_j, \alpha_0, \alpha_j, d_N^{(i)}$ are as defined in (2), (3), (4), (7), respectively.

Sketch of proof: Define

$$\begin{aligned} &P(N, i) \\ &= (d_{N-i+1} - d_{N-i}) \\ &\quad \times \left[\prod_{k=1}^{i}\alpha_{N-k}(d_{N-i} - d_{N-i-1}) + d_{N-1}d_N^{(i)}\right] \\ &\quad - \alpha_{N-i}d_N d_{N-1}^{(i-2)}(d_{N-i} - d_{N-i-1}) - h_{N-i+1}d_{N-1}d_N. \end{aligned}$$

From (6) and (8), we see that the theorem is equivalent to $P(N, i) = 0$ for all $i \in \mathbb{Z}^+$ and $i \le N \in \mathbb{Z}^+$. The proof will follow by induction after establishing the following facts:

1) $P(N, 1) = 0$ for all $N \ge 1$.
2) $P(N, 2) = 0$ for all $N \ge 2$.
3) $P(N, i) = 0$ for any $i \ge 3, N \ge i,$ assuming $P(N, i-1) = P(N-1, i-1) = P(N-1, i-2) = 0.$

The third fact follows by showing $X(N,i)=0$ where

$$\begin{aligned} X(N,i) &:= h_{N-i+2}P(N,i) - h_{N-i+1}P(N,i-1) \\ &\quad - \alpha_{N-1}h_{N-i+2}P(N-1,i-1) \\ &\quad + \alpha_{N-1}h_{N-i+1}P(N-1,i-2). \end{aligned}$$

■

The recurrence relation (8) describes a sequence of transfer functions in the complex variable s. It can also be interpreted as compositions of Möbius transformations for a fixed $s \in \mathbb{C}$, or equivalently, fixed $h_j \in \mathbb{C}$, $j \in \mathbb{Z}^+$; writing

$$f_N^{(i)}(z) = \frac{d_{N-1}^{(i-2)}\alpha_{N-i}z + h_{N-i+1}}{\prod_{k=1}^{i}\alpha_{N-k}z + d_N^{(i)}},$$

we see that the sequence $F_N^{(i)}$ for $N = i-1,\, i,\, i+1, \ldots$ is the same as 0, $f_i^{(i)}(0)$, $f_{i+1}^{(i)} \circ f_i^{(i)}(0), \ldots,$ for given h_j, $j = 1, \ldots, N$.

In the case of homogeneous mass chains, since $Z_j(s) = Z(s)$ and $m_j = m$ for all j, (8) is simplified to

$$F_N^{(i)}(h) = \frac{d_{i-2}F_{N-1}^{(i)} + h}{F_{N-1}^{(i)} + d_i}$$

for $N = i, i+1, \ldots,$ where $F_{i-1}^{(i)} = 0$, $h(s) = sZ(s)m$ and d_i is as defined in (5). It has been shown in [18] that the H^∞-norm of these transfer functions can be uniformly bounded with respect to N for a suitable choice of h. Furthermore, for the first intermass displacement, the following theorem provides a method to select h in a scale-free manner:

Theorem 2 ([13]): If

$$h(s) = \frac{a^2 s^2}{(1-a)s^2 + 2s + 1}, \quad a > 0,$$

then

$$\sup_{N \in \mathbb{Z}^+} \|F_N^{(1)}(h(s))\|_\infty \le a.$$

We may adopt this method to the heterogeneous mass chain by making $h_j(s) = h(s)$, and hence, $Z_j(s) = h(s)/(sm_j)$ for $j = 1, \ldots, N$. Loosely speaking, by employing this interconnection design, the heterogeneous mass chain imitates a well-behaving homogeneous dynamics. This approach is studied in the rest of the paper.

IV. Heterogeneous mass chain with homogeneous dynamics

A. Stability of the interconnection

For $h_j(s) = h(s)$, the matrix L in (1) can be rewritten as $L = hI + H$ where I is the identity matrix and

$$H = \begin{pmatrix} \alpha_{N-1}+1 & -\alpha_{N-1} & 0 & \ldots & 0 \\ -1 & \alpha_{N-2}+1 & -\alpha_{N-2} & \ddots & \vdots \\ & -1 & \ddots & \ddots & 0 \\ & & \ddots & \alpha_1 + 1 & -\alpha_1 \\ 0 & & & -1 & 1 \end{pmatrix}.$$

We note that $\alpha_j = m_j/m_{j+1}$. Hence, d_N in (5) can be seen as the characteristic polynomials of H in the single variable h.

We will say that the system of Fig. 1 is stable if all poles in the transfer functions $F_N^{(i)}(h(s))$ have negative real parts in the s-domain.

Theorem 3: For $0 \not\equiv h(s)/s$ positive real, the system of Fig. 1 is stable if $h(s)$ does not take values in the interval $[-2 - 2\max_{1\le j\le N-1}\{\alpha_j\}, 0)$ for any s with $\mathrm{Re}(s) = 0$.

Proof: From (6), poles in $F_N^{(i)}(h(s))$ can only occur at an s for which $d_N(h(s)) = 0$. Note that a Gershgorin disc bound on the eigenvalues of H implies that the roots of $d_N(h)$ lie in the interval $[-2 - 2\max_{1\le j\le N-1}\{\alpha_j\}, 0]$. Also it is straightforward to check that $d_N(0) = 1\, \forall N$. The result now follows since $\mathrm{Re}(h(s)/s) > 0$ for $\mathrm{Re}(s) > 0$ from [4] (Theorem VI). ■

It may be noted that, for the homogeneous mass chain, the matrix H can be seen as the central difference approximation of $\frac{\mathrm{d}^2}{\mathrm{d}x^2}$ and the analysis tools of partial differential equations (PDEs) may be used. Approaches utilizing PDE-based analysis for a discrete chain can be found in [3], [14].

B. First intermass displacement

For the first intermass displacement ($i = 1$), the recursive relation (8) becomes

$$\begin{aligned} F_N^{(1)} &= \frac{\alpha_{N-1}F_{N-1}^{(1)} + h}{\alpha_{N-1}F_{N-1}^{(1)} + h + 1} \\ &= \frac{\frac{m_{N-1}}{m_N}F_{N-1}^{(1)} + h}{\frac{m_{N-1}}{m_N}F_{N-1}^{(1)} + h + 1}. \end{aligned} \tag{9}$$

We may observe from this expression that the distribution of masses in the chain is a critical factor for the norm bound. To see this, let us demonstrate how the H^∞-norm differs over the following different mass distribution within a certain range $[M_l, M_u]$ where $0 < M_l \le 1 \le M_u$:

CASE 1: $m_j = 1$ for $j = 1, 2, \ldots, N$ (homogeneous case)

CASE 2: $\begin{cases} m_j &= M_l \quad \text{for } j = 1,2,\ldots,N-1 \\ m_N &= M_u \end{cases}$

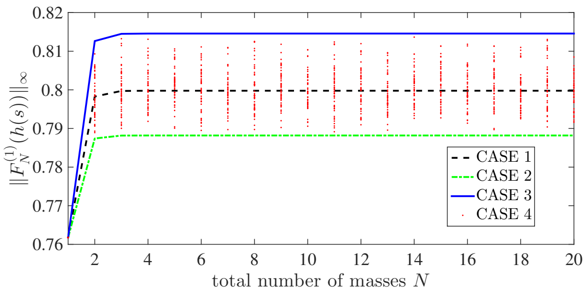
CASE 3: $\begin{cases} m_j &= M_u \quad \text{for } j = 1,2,\ldots,N-1 \\ m_N &= M_l \end{cases}$

CASE 4: $M_l \le m_j \le M_u$ for $j = 1, 2, \ldots, N$.

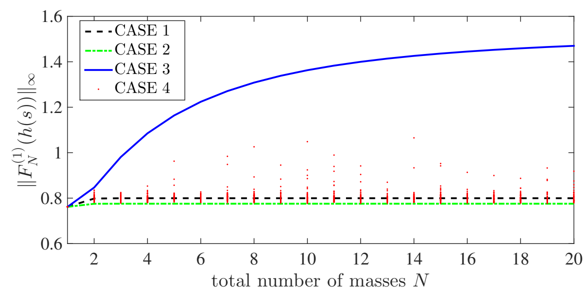
Figures 2a and 2b show $\|F_N^{(1)}(h(s))\|_\infty$ when a) $M_l = 0.8, M_u = 1.2$ and b) $M_l = 0.5, M_u = 1.5$, respectively. For both figures, 50 different mass distributions were generated in CASE 4 for each N so that m_j, $j = 1, 2, \ldots, N$, were uniformly distributed random numbers in the interval $[M_l, M_u]$. The interconnection impedances $Z_j(s)$ are selected such that

$$sZ_j(s)m_j = h(s) = \frac{0.8^2 s^2}{0.2s^2 + 2s + 1}.$$

for $j = 1, 2, \ldots, N$. This corresponds to $a = 0.8$ in Theorem 2 and indeed it is observed in Figs. 2a and 2b that $\|F_N^{(1)}((h(s))\|_\infty$ is bounded by 0.8 for all N in CASE 1, the nominal homogeneous mass chain. It may also be observed



(a) CASE 1: $m_j = 1$ for $j = 1, \ldots, N$ (homogeneous). CASE 2: $m_j = 0.8$ for $j = 1, 2, \ldots, N-1$, $m_N = 1.2$. CASE 3: $m_j = 1.2$ for $j = 1, 2, \ldots, N-1$, $m_N = 0.8$. CASE 4: $0.8 \le m_j \le 1.2$ (uniformly distributed).



(b) CASE 1: $m_j = 1$ for $j = 1, \ldots, N$ (homogeneous). CASE 2: $m_j = 0.5$ for $j = 1, \ldots, N-1$, $m_N = 1.5$. CASE 3: $m_j = 1.5$ for $j = 1, \ldots, N-1$, $m_N = 0.5$. CASE 4: $0.5 \le m_j \le 1.5$ (uniformly distributed).

Fig. 2: H^∞-norm of the first intermass displacement transfer function in a chain of N masses for different mass distributions where $sZ_j(s)m_j = \frac{0.8^2 s^2}{0.2s^2+2s+1}$.

that all the values of the H^∞-norm in CASE 4 are contained between that in CASE 2 and that in CASE 3.

Although they are just numerical evidence with limited cases, we may conjecture that CASE 3 distribution gives the largest H^∞-norm. Then, if we design the interconnection so that $\sup_{N\in\mathbb{Z}^+}\|F_N^{(1)}(h(s))\|_\infty \le \gamma$ for CASE 3 distribution, this norm bound is guaranteed in a scale-free manner for all the other mass distributions. This means that, in the context of vehicle platooning for example, any vehicle satisfying this control law for the interconnection may join or leave without violating the performance guarantee as long as the mass of the vehicle is within a prescribed range. While the verification of this conjecture is left as a topic for future work, here we demonstrate this idea via numerical results. Let us consider again the control of the mass chain where

$$m_j = 1.5 \quad \text{for } j = 1, 2, \ldots, N-1$$
$$m_N = 0.5$$

as in CASE 3 in Fig. 2b. Our task is to design $h(s)$ such that $\sup_{N\in\mathbb{Z}^+}\|F_N^{(1)}(h(s))\|_\infty \le \gamma$ for this mass distribution. For this purpose we introduce a graphical method as in Fig. 3. This contour plot shows the region of the complex values of h for which $\max_N |F_N^{(1)}(h)| \le \gamma$ with $1 \le N \le 10^5$ for a positive constant γ for the CASE 3 mass distribution. With a similar discussion given in [18], 10^5 is large enough

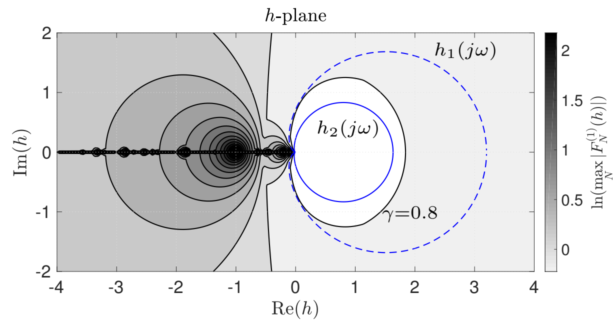


Fig. 3: Contour plot of $\max_{1\le N\le 10^5}|F_N^{(1)}(h)| = \gamma$. The blue dotted and solid curves show the Nyquist diagrams of $h_1(s) = \frac{0.8^2 s^2}{0.2s^2+2s+1}$ and $h_2(s) = \frac{0.7^2 s^2}{0.3s^2+4s+1}$, respectively.

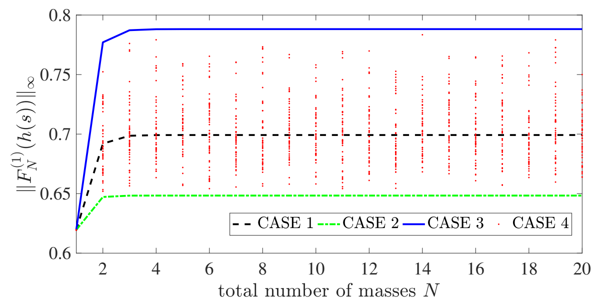


Fig. 4: H^∞-norm of the first intermass displacement transfer function in a chain of N masses for different mass distributions where $sZ_j(s)m_j = \frac{0.7^2 s^2}{0.3s^2+4s+1}$. CASE 1: $m_j = 1$ for $j = 1, \ldots, N$ (homogeneous). CASE 2: $m_j = 0.5$ for $j = 1, \ldots, N-1$, $m_N = 1.5$. CASE 3: $m_j = 1.5$ for $j = 1, \ldots, N-1$, $m_N = 0.5$. CASE 4: $0.5 \le m_j \le 1.5$ (uniformly distributed).

to accurately determine the shape of the boundary in the figure. We can shape the Nyquist diagram of $h(s)$ on this figure such that the supremum of the infinity norm is less than a prescribed value, γ. As an example, the following two Nyquist diagrams are plotted in Fig. 3:

$$h_1(s) = \frac{0.8^2 s^2}{0.2s^2+2s+1}, \quad h_2(s) = \frac{0.7^2 s^2}{0.3s^2+4s+1}.$$

We may observe that $h_1(s)$ should return γ greater than 0.8 which agrees on the result in Fig. 2b, while γ for $h_2(s)$ is less than 0.8. As we expected, not only for the CASE 3 distribution, this interconnection design gives the H^∞-norm less than 0.8 for different mass distributions as shown in Fig. 4.

V. Conclusions

The propagation of disturbances in a heterogeneous mass chain has been studied. Formulas for the transfer functions from the movable point displacement to a given intermass displacement have been derived in the form of composition sequences generated by Möbius transformations. By letting the combined dynamics of the impedance and the mass be identical, a method has been proposed that looks promising for development of scale free performance design in the heterogeneous mass chain. Topics for future work include the formal proof of the several conjectures given in this paper.

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