Simultaneous rejection of signals below and above the Nyquist frequency

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Simultaneous rejection of signals below and above the Nyquist frequency

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Abstract— **This paper studies the disturbance rejection problem for sampled-data control systems, where disturbance signal occurs below and above the Nyquist frequency simultaneously. Two discrete-time controllers are designed via H^∞ optimal control in two steps; at first a controller is designed to reject the low-frequency components, and then we construct the generalized plant including the first controller to design the second controller, which has the capability of rejecting the high-frequency components. In view of the well-known sampling theorem, one recognizes that any high-frequency components may be detected only as an alias in the low base band, and hence it is impossible to recover or detect such frequency components. The authors recently showed in [13] that this assumption depends crucially on the underlying analog model, and it is indeed possible to track or reject such frequency components by introducing multirate signal processing techniques. This paper aims to make this design technique applicable to the case in which the target frequencies lie both below and above the Nyquist frequency. Detailed analysis of multirate closed-loop systems are given. It is shown via examples that rejection of lower- and higher-frequency signals than the Nyquist frequency can be achieved.**

I. Introduction

This note proposes a controller design method to reject a disturbance signal in a sampled-data control system, where the disturbance occurs in multiple frequencies consisting of frequencies higher/lower than the Nyquist frequency. In some applications, we cannot necessarily take a fast enough sampling period due to various limitations. For example, in position control of hard disk drives, the sampling rate cannot be taken fast enough to cover resonance disturbances due to physical limitations [2], [15]. In such applications, it would be desirable to simultaneously reject disturbances occurring at both below and above the Nyquist frequency. The present paper attacks this problem as a continuation of our earlier work [13] where only a single frequency signal beyond the Nyquist frequency is considered.

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We first review the background the whole tracking/rejection problem in the sampled-data context where the sampling period provides only a limited resolution. In view of the well-known sampling theorem, tracking or rejecting signals beyond the Nyquist frequency may appear to be impossible. The sampling theorem (e.g, [8], [14]) tells us that if the target signal is beyond the so-called Nyquist frequency, it may appear only as an aliased signal in the base band that is below the Nyquist frequency, and cannot be recovered from such knowledge. The formula given by the sampling theorem recovers only those residing in the base band that is below the Nyquist frequency. Consequently, the Nyquist frequency is often considered as an absolute limit of controlling signals. However, the authors have recently shown in [13] that it is indeed possible to achieve such an objective provided that

- we have proper *a priori* knowledge about the plant and the tracking signal, and
- we can produce intersample input signals via upsampling.

The key ideas were that we give the controller with 1) the information of the signal frequency via a weighting function, and 2) the capability to produce such a high frequency signal via upsampling. For the controller design, H^∞ sampled-data control was used via lifting [4],[10].

Although the result may look counterintuitive, it actually does not contradict the sampling theorem. A close examination of the sampling theorem reveals that the signal recovery limitation below the Nyquist frequency arises from the very basic assumption on the analog signal model. Namely, in the sampling theorem, one assumes that the original analog signal is perfectly band limited below the Nyquist frequency. It does not say anything about the situation where this basic assumption does not hold. The result of [13] indeed shows that this limitation can be waived by invoking a suitable analog weighting and upsampling in producing control signals.

The design technique in [13], however, cannot deal with a more practical situation where there are more than one tracking/rejection signals to be tracked/rejected. In fact, except some special cases such as regulating a power supply frequency, there are generally more than one frequencies where such tracking/rejection objectives be met. For example, in the case of hard disk drives, there can be more than one disturbance signal frequencies—quite typically higher *and* lower than the Nyquist frequency. In general, it is not straightforward to address this problem by merely generalizing the approach given in [13] to cope with a weighting function with multiple peak frequencies. The reason is rooted in the very nature of sampling. If a tracking/rejection signal $\sin \omega t$, $\omega > \pi/h$ is given, then it can be detected only as an alias $\sin \omega(2\pi/h\omega)t$ by sampling. If there exist another tracking/rejection signal near this lower frequency, then the controller cannot easily distinguish the two signals, and there occurs a serious difficulty in designing a suitable controller.

In view of this observation, we aim to bypass this problem by introducing a two-step design method; 1) first we design a controller to reject signals below the Nyquist frequency, and 2) construct the generalized plant including the controller designed at the previous step to design a controller that can handle signals above the Nyquist frequency.

Let us now briefly review pertinent facts on sampled-data control. Since the introduction of lifting [4], [10], modern sampled-data control theory has established the fact that one can control and optimize the intersample behavior with a discrete-time controller; details may be found, for example, in [3], [5], [11]. Such developments usually assume that the sampling occurs at the same timing both at sensing and control. On the other hand, in the signal processing literature, multirate processing, utilizing up- and down-samplers, are known to be quite effective [9]. In particular, it allows more elaborate signal manipulation in the intersampling periods. The combination of this multirate processing and H^∞ control is fully used in [12]. The advantage of introducing multirate processing, particularly upsampling, is that it gives more freedom in handling and reconstructing intersample signals.

Multirate sampled-data control has had some history in the control literature; see, e.g., [1], [6], [7]. However, they use full information obtained by multirate sampling, and the focus is on extending the capability of control. In contrast, here once output is sampled, we do not perform further sampling so the basic sampling period is fixed. Upsampling is performed only on the side of computing the control signals.

In this framework, we propose a design method of two controllers; one is to reject low-frequency components, and the other to reject high-frequency components beyond the Nyquist frequency together with upsampler.

II. Problem Formulation

Consider the sampled-data system depicted in Fig. 1.

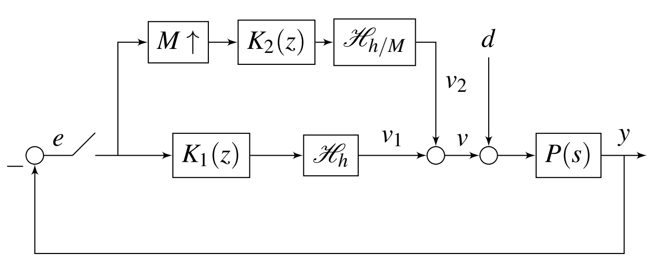

Fig. 1. Sampled feedback system

$P(s)$ is a linear, time-invariant, continuous-time plant subject to an external disturbance d. $K_1(z)$ and $K_2(z)$ are linear, time-invariant, discrete-time controllers. The error e is sampled with sampling period h, and after sampled, it is upsampled by factor M before entering the controller $K_2(z)$ to allow for a faster control processing. The action of upsampler $\uparrow M$ is given as follows:

$$(\uparrow M)(e)[kh+\ell] = \begin{cases} e[kh] & \text{if } \ell = 0 \\ 0 & \ell = h/M, \dots (M-1)h/M. \end{cases} \quad (1)$$

$\mathscr{H}_h$ and $\mathscr{H}_{h/M}$ are the zero-order holds that hold the output as constant for the period of h and h/M, respectively.

We consider the following problem:

Problem 1: *In the block diagram Fig. 1, consider the disturbance* $\sin \omega_1 t + \sin \omega_2 t$ *where* ω_1 *and* ω_2 *are lower and higher than the Nyquist frequency* π/h*, respectively. Find a discrete-time controllers* $K_1(z)$ *and* $K_2(z)$ *that suppress this disturbance* $d(t) = \sin \omega_1 t + \sin \omega_2 t$ *effectively.*

Here K_1 suppresses the disturbance in low frequency while K_2 acts on high frequency disturbances. To attack this problem, we first lift the system Fig. 1 as a state space model [5], [4], [3], [10]. We then convert this problem to an H^∞ sampled-data control problem, and show that our objective is indeed achieved.

A. State Space Description of the Lifted Multirate System

It is necessary to describe the system in Fig. 1 as a time-invariant discrete-time system with a single sampling period h. A complication arises due to the mixture of continuous-time plant $P(s)$ and also upsampler $\uparrow M$. In order to deal with these, we need to introduce both continuous-time lifting and discrete-time lifting (blocking).

Let $P(s)$ and $K(z)$ be described by the following state space equations:

$$P(s): \begin{cases} \frac{d}{dt}x_c(t) & = A_c x_c(t) + B_{c1} v(t) + B_{c2} d(t) \\ y(t) & = C_c x_c(t) \end{cases}$$

$$K_1(z): \begin{cases} x_{d1}[k+1] & = A_{d1} x_{d1}[k] + B_{d1} e[k] \\ v_1[k] & = C_{d1} x_{d1}[k] + D_{d1} e[k] \end{cases}$$

$$K_2(z): \begin{cases} x_{d2}[k+1] & = A_{d2} x_{d2}[k] + B_{d2} w_{d2}[k] \\ y_{d2}[k] & = C_{d2} x_{d2}[k] + D_{d2} w_{d2}[k]. \end{cases}$$

Here, and in what follows, we employ the convention that function values are specified as $f(t)$ with parentheses when t is a continuous variable, and as $g[k]$ when k takes on integer values.

In order to give a unified description of the equations above as a single discrete-time system, we introduce the continuous-time lifting:

$$\begin{aligned} \mathscr{L}: \quad & L^2_{loc}[0,\infty) \to \ell^2(L^2[0,h)) : x(\cdot) \mapsto \{x[k](\cdot)\}_{k=0}^{\infty}, \\ & x[k](\theta) := x(kh+\theta). \end{aligned}$$

Lifting the continuous-time plant $P(s)$ in the period h, we obtain

$$\tilde{\Sigma}_P : \begin{cases} x_c[k+1] &= e^{A_c h} x_c[k] \\ & \quad + \int_0^h e^{A_c(h-\tau)} (B_{c1} v[k](\tau) + B_{c2} d[k](\tau)) \, \mathrm{d}\tau \\ y_c[k](\theta) &= C_c e^{A_c \theta} x_c[k] \\ & \quad + \int_0^\theta C_c e^{A_c(\theta-\tau)} (B_{c1} v[k](\tau) + B_{c2} d[k](\tau)) \, \mathrm{d}\tau. \end{cases}$$

We also need to perform discrete-time lifting (i.e., blocking) for the discrete-time controller $K_2(z)$ combined with upsampler (1), because upsampler makes it time-varying in the activating timing. To remedy this, we must lift it with period h.

Proposition 2.1: When lifted with period h, the discrete-time controller $K_2(z)$ is expressible as

$$\begin{aligned} \tilde{\Sigma}_{K_2} : x_{d2}[k+1] &:= x_d(kh+h) = A_{d2}^M x_{d2}[k] + A_{d2}^{M-1} B_{d2} e[k](0) \\ &=: \overline{A_{d2}} x_{d2}[k] + \overline{B_{d2}} e[k](0) \\ y_{d2}[k] &:= \begin{bmatrix} y_{d2}(kh) \\ y_{d2}(kh+h/M) \\ \vdots \\ y_{d2}(kh+(M-1)h/M) \end{bmatrix} \\ &= \begin{bmatrix} C_{d2} \\ C_{d2}A_{d2} \\ \vdots \\ C_{d2}A_{d2}^{M-1} \end{bmatrix} x_{d2}[k] + \begin{bmatrix} D_{d2} \\ C_{d2}B_{d2} \\ \vdots \\ C_{d2}A_{d2}^{M-2}B_{d2} \end{bmatrix} e[k](0) \\ &=: \overline{C_{d2}} x_{d2}[k] + \overline{D_{d2}} e[k](0). \end{aligned}$$

Define a generalized hold function $H(\theta)$ by

$$H(\theta) := [\chi_{[0,h/M)}(\theta), \chi_{[h/M,2h/M)}(\theta), \ldots, \chi_{[(M-1)h/M,h)}(\theta)],$$

where $\chi_{[ih/M,(i+1)h/M)}(\theta)$, $i = 0, \ldots, M-1$ denotes the characteristic function of the interval $[ih/M, (i+1)h/M)$. Then the lifted input $v_2[k](\theta)$ for P can be written simply as

$$v_2[k](\theta) = H(\theta) y_{d2}[k].$$

Proof: See [13]. ■

Now define

$$B_{11}(\theta) := \int_0^\theta e^{A_c(\theta-\tau)} B_{c1} \, \mathrm{d}\tau,$$

$$B_{12}(\theta) := \int_0^\theta e^{A_c(\theta-\tau)} B_{c1} H(\tau) \, \mathrm{d}\tau,$$

$$\mathbf{B_2}(\theta) : L^2[0,h) \to L^2[0,h) : d \mapsto \int_0^\theta e^{A_c(\theta-\tau)} B_{c2} d[k](\tau) \, \mathrm{d}\tau$$

Then the lifted $\tilde{\Sigma}_{K_2}$ and $\tilde{\Sigma}_P$ are represented as

$$\begin{aligned} \tilde{\Sigma}_{K_2} : \ & x_{d2}[k+1] &&=: \overline{A_{d2}} x_{d2}[k] + \overline{B_{d2}} e[k](0) \\ & y_{d2}[k] &&=: \overline{C_{d2}} x_{d2}[k] + \overline{D_{d2}} e[k](0) \\ & v_2[k](\theta) &&=: H(\theta) y_{d2}[k] \\ \tilde{\Sigma}_P : \ & x_c[k+1] &&= e^{A_c h} x_c[k] + B_{11}(h) v_1[k] + B_{12}(h) y_{d2}[k] \\ & && \quad + \mathbf{B_2}(h) d[k] \\ & y[k](\theta) &&= C_c e^{A_c \theta} x_c[k] + C_c B_{11}(\theta) v_1[k] \\ & && \quad + C_c B_{12}(\theta) y_{d2}[k] + C_c \mathbf{B_2}(\theta) d[k]. \end{aligned}$$

Now the multirate system with upsampler $\uparrow M$ is described as a linear time-invariant system and H^∞ optimal control can be applied.

III. Design Method

We will now proceed to design discrete-time controllers $K_1(z)$ and $K_2(z)$ in Fig. 1. As noted in the introduction, designing such controllers in one step does not necessarily work in general. For example, we have considered the case $h = 1$, and the disturbance signal has frequencies at $\omega_1 = \pi/2$ and $\omega_2 = 3\pi/2$. The latter signal appears as an alias exactly at $\omega_1 = \pi/2$. This "confuses" the controller design, and there appears some large phase errors. This is clearly due to the fact that the sampled output captures only the low-frequency disturbances, and a one-step design process cannot handle such a case very well. This indicates that there should be a separation of design and roles in controllers in low and high frequency. Hence we take a two-step design process, and employ H^∞ sampled-data control at each step.

Let us first observe that our system Fig. 1 cannot be used as it is for a design block diagram for H^∞ sampled-data control, since sampling is not a bounded operator on L^2. To remedy this, we place a strictly proper filter $F(s)$ in front of the disturbance. We can use this pre-filter $F(s)$ to put weights on the target frequencies. In [13], tracking a signal beyond the Nyquist frequency was made possible by selecting $F(s)$ such that it emphasizes the target frequency and also deemphasizes the other frequency range. In the present case, the target signal contains multiple frequencies, especially below and above the Nyquist frequency simultaneously, and this induces a difficulty. As noted above, this is why we employ a two-step design procedure.

Consider the generalized plant described in Fig. 2, which is obtained by introducing the pre-filter $F(s)$ in Fig. 1. We design controllers $K_1(z)$ and $K_2(z)$ as follows:

Step 1: Assume $v_2 = 0$ in Fig. 3. Design the controller $K_1(z)$ to minimize the H^∞-norm of the transfer function from the disturbance d to the output y.

Step 2: Construct the new generalized plant including the controller $K_1(z)$ as depicted in a dashed box in Fig. 3. Design the controller $K_2(z)$ which minimizes the H^∞-norm of the transfer function from the disturbance d to the vector $[y, Rv_2]$. The weight R on the control output v_2 is selected such that it has larger gain in a high frequency range.

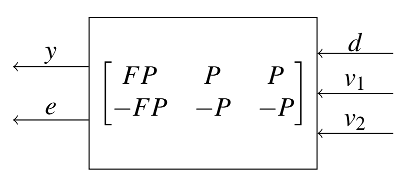

Fig. 2. Generalized plant.

IV. Examples

In this section we demonstrate the effectiveness of the present framework via two numerical examples. For both examples the upsampling factor M is set to 8.

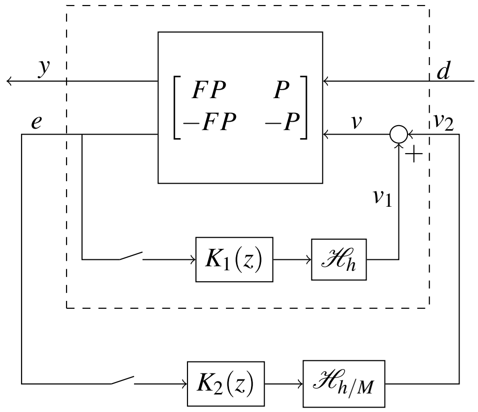

Fig. 3. Closed-loop system for the two-step design.

Example 4.1: Consider the following second-order plant

$$P(s) := \frac{1}{s^2 + 2s + 1}$$

with (normalized) sampling period $h = 1$ in Fig. 1. The Nyquist frequency is then π [rad/sec]. Suppose that we are given the disturbance signal $d(t) = \sin\omega_1 t + \sin\omega_2 t$, where $\omega_1 = \pi/2$ [rad/sec] and $\omega_2 = 3\pi/2$ [rad/sec]. Clearly, ω_1 is below the Nyquist frequency while ω_2 is above.

We take the weighting function, or the signal model, as

$$F(s) := \frac{50s}{(s^2 + 0.2s + \omega_1^2)(s^2 + 0.1s + \omega_2^2)}$$

which has clear peaks at ω_1 and ω_2, and deemphasizes the other frequencies (see Fig. 4). The weight R on the control output v_2 should be selected in such a way that the output of the controller $K_2(z)$ is not too large in high frequency (see Fig. 5). In this example, we take

$$R(s) = \frac{s + \pi}{5s + 15\pi}.$$

The resulting controllers $K_1(z)$ and $K_2(z)$ are shown in Figs. 6 and 7, respectively. The response against the disturbance $d(t) = \sin\omega_1 t + \sin\omega_2 t$ is shown in Fig. 8. The figure shows that the controllers reasonably reject the disturbance that has frequency components below and above the Nyquist frequency.

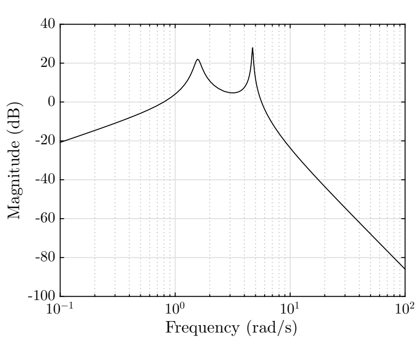

Fig. 4. Weighting function $F(s)$.

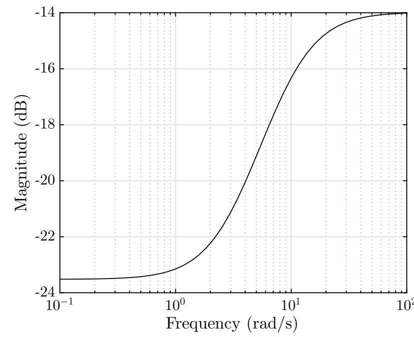

Fig. 5. Weight $R(s)$ on control output v_2.

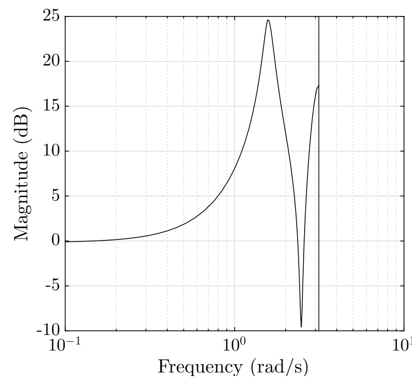

Fig. 6. Controller K_1.

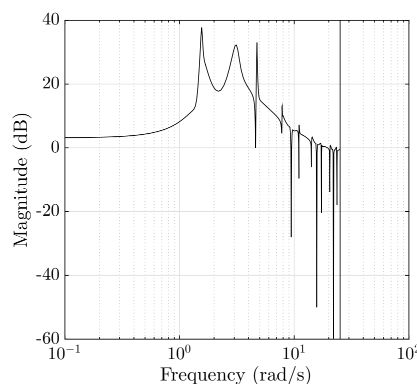

Fig. 7. Controller K_2.

The proposed method is not limited to disturbance with two frequencies and the next example verifies it.

Example 4.2: Take the same plant $P(s) := 1/(s^2 + 2s + 1)$, but with the objective of rejecting the disturbance $d(t) = \sin(\pi/4)t + \sin(\pi/2)t + \sin(3\pi/2)t$. We take a new weighting function

$$F(s) := \frac{s}{(s^2 + 0.1s + (\frac{\pi}{4})^2)(s^2 + 0.2s + (\frac{\pi}{2})^2)(s^2 + 0.2s + (\frac{3\pi}{2})^2)}$$

and a weight

$$R(s) := \frac{s + \pi}{s + 3\pi}.$$

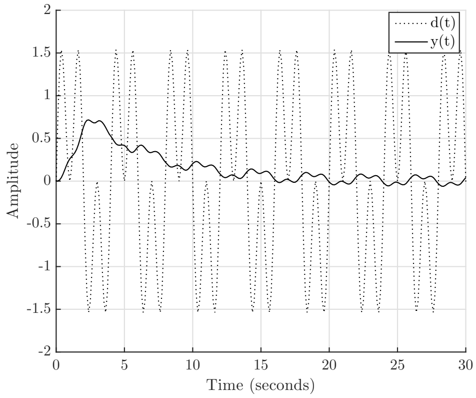

Fig. 8. System output (solid) against disturbance $\sin(\pi/2)t + \sin(3\pi/2)t$ (dotted).

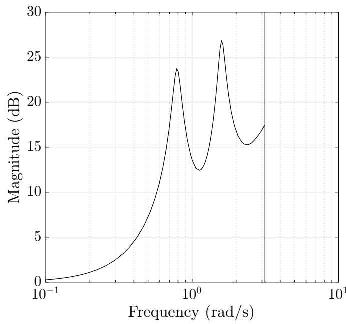

Fig. 9. Controller K_1.

The resulting controllers $K_1(z)$ and $K_2(z)$ are shown in Figs. 9 and 10, respectively. Fig. 11 shows that the two controllers effectively reject the disturbance occurring in three different frequencies, one of which being above the Nyquist frequency.

V. Conclusion

In this paper we proposed a design method to simultaneously reject disturbances occurring below and above the Nyquist frequency. The design procedure consists of two steps; the first is to design a controller to reject low-frequency disturbance, and the second is to reject the high-frequency disturbance components based on the generalized plant consisting of both the original plant and the controller obtained at the first step.

With this multiple controller construction, the obtained controller can reject disturbances occurring both at frequencies lower and higher than the Nyquist frequency.

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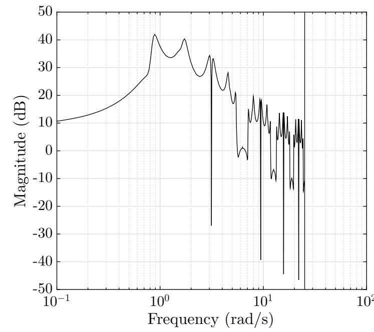

Fig. 10. Controller K_2.

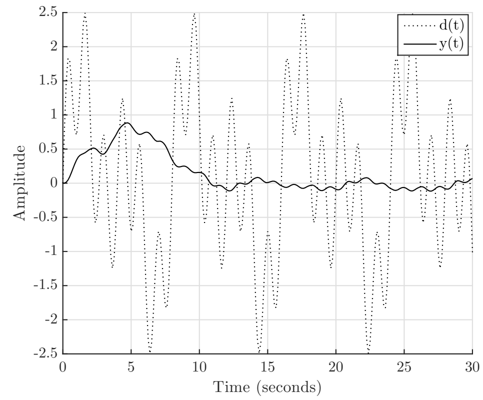

Fig. 11. System output (solid) against disturbance $\sin(\pi/4)t + \sin(\pi/2)t + \sin(3\pi/2)t$ (dotted).

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