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**Paper**

# Adaptive Combination of Additive and Multiplicative Algorithms for Color Image Enhancement

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**Abstract:** Recently, Nikolova and Steidl have proposed a hue and range preserving histogram specification method which outputs a weighted average of two images generated by additive (shifting) and multiplicative (scaling) algorithms, where the weights are fixed for all pixels in an image. In this paper, we propose a method for selecting the one of the two algorithms at each pixel adaptively. Therefore, there is no need to prepare the weights for combining additive and multiplicative algorithms in the proposed method. Experimental results show that the proposed method can improve the saturation better than the convex combination method by Nikolova and Steidl. We also verify the property of saturation improvement in the proposed method theoretically.

**Keywords:** Hue-preserving color image enhancement, Gamut problem, Exact histogram equalization, Saturation improvement

## 1. Introduction

Color image enhancement is an important technique in digital image processing for human visual perception and computer vision, where color images are typically recorded in the format of RGB color channels. However, the RGB color space is not intuitive for human vision. Therefore, we often need to use the alternative intuitive color spaces such as HSV (hue, saturation, value), HSL (hue, saturation, lightness), HSB (hue, saturation, brightness), HSI (hue, saturation, intensity). Among the three attributes of color, hue, saturation and intensity, hue determines the appearance of color such as red, green and blue. Therefore, in a number of applications of color image enhancement, it is preferable to preserve hue throughout the process. The basic flow of color image enhancement procedure may proceed as follows: (1) transform a color from RGB to HSI, (2) enhance the color in a hue-preserving manner, and (3) inversely transform the enhanced color from HSI to RGB. However, such naive procedures can cause a gamut problem, i.e., the enhanced color may not exist in the RGB color cube. Naik and Murthy have proposed a scheme for hue-preserving color image enhancement without gamut problem [1]. However, as pointed out by themselves, their scheme always decreases the saturation in a common condition. To alleviate this problem, Murahira and Taguchi have proposed improved methods [2] [3], an Inoue et al. have proposed a method for maximizing the saturation while preserving the hue [4]. Recently, Nikolova and Steidl have proposed new algorithms for hue and range preserving color image enhancement [5], where a fast ordering algorithm for exact histogram specification [6] is used. Minami and Yamada have also proposed an algorithm for exact histogram equalization [7].

In their paper [5], Nikolova and Steidl combine an additive algorithm and a multiplicative algorithm convexly by introducing a weight parameter, which is fixed for all pixels in a target image. However, their convex combination method occasionally decreases the saturation because of the fixed weight parameter. In this paper, we propose a method for improving their method to alleviate the saturation deterioration. That is, the proposed method selects the additive algorithm or the multiplicative algorithm that outputs better saturation at each pixel adaptively. Experimental results show the effectiveness of the proposed method compared with the convex combination method.

The rest of this paper is organized as follows: Section 2 summarizes related algorithms proposed by Nikolova and Steidl [5] [6]. Section 3 proposes an adaptive combination method for improving saturation. Section 4 shows experimental results. Finally, Section 5 concludes this paper.

## 2. Additive and Multiplicative Algorithms for Color Image Enhancement

In this section, we briefly summarize two algorithms for color image enhancement proposed by Nikolova and Steidl [5].

Let $w \equiv (w_r, w_g, w_b)$ be an RGB color image with $M \times N$ pixels, where $w_c \in \{0, 1, \ldots, L-1\}$ for $c \in \{r, g, b\}$ are the red, green and blue channels of $w$, where $L$ denotes the number of brightness intensities, e.g., for 8-bit images we have $L = 2^8 = 256$. Then we reorder the pixels in each color channel $w_c$ columnwise into an $n$-dimensional vector,

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where $n \equiv MN$, and address each pixel by the index $i \in \mathbb{I}_n \equiv \{1, 2, \ldots, n\}$. Let

$$f(w) \equiv \frac{1}{3}(w_r + w_g + w_b) \tag{1}$$

be the intensity [8] of $w$, and let $f[i]$ be the $i$th pixel value of $f(w)$. Then we see that $f[i] \in \frac{1}{3}\{0, 1, \ldots, 3(L-1)\}$ for $i \in \mathbb{I}_n$.

Nikolova and Steidl have proposed a method for exact histogram specification (HS) [6], which transforms an image $f$ into $\hat{f}$ whose histogram matches a specified target histogram $\hat{h} \equiv (\hat{h}_1, \hat{h}_2, \ldots, \hat{h}_L)$, i.e., $\hat{h}_k = \#\{i \in \mathbb{I}_n : \hat{f}[i] = k-1\}$ for $k = 1, 2, \ldots, L$, where # stands for the cardinality. Nikolova and Steidl's HS algorithm is summarized in Algorithm 1, where $\nabla \in \mathbb{R}^{\tilde{n} \times n}$ for $\tilde{n} \equiv M(N-1) + (M-1)N = 2n - M - N$ denotes the discrete gradient operator (horizontal and vertical forward differences), whose elements are $\nabla_{l,i} = -1$ and $\nabla_{l,i+M} = 1$ for the horizontal differences, and $\nabla_{l',i} = -1$ and $\nabla_{l',i+1} = 1$ for the vertical differences. The other elements of $\nabla$ are all zeros. $\nabla^T$ denotes the transpose of $\nabla$, and

**Algorithm 1** HS Using Strict Ordering [6]

1: **Initialization:** $u^{(0)} := f$, $\beta := 0.1$, target histogram $\hat{h}$, iteration number $K := 5$, $c_0 := 0$.
2: For $k = 1, 2, \ldots, K$ compute

$$u^{(k)} := f - \eta^{-1}(\beta \nabla^T \eta(\nabla u^{(k-1)})).$$

3: Obtain the ordering $\{i_j\}_{j=1}^n$ of $\mathbb{I}_n$ from the ascending sort of the entries of $u^{(K)}$.
4: For $k = 0, \ldots, L-1$ set $c_{k+1} := c_k + \hat{h}_k$ and

$$\hat{f}[c_k + 1] := \cdots := \hat{f}[c_{k+1}] := k.$$

$$\eta(t) \equiv \frac{t}{\alpha + |t|}, \quad \eta^{-1}(y) = \frac{\alpha y}{1 - |y|}, \tag{2}$$

where $\alpha = 0.05$ is a default value.

Given a histogram-specified intensity $\hat{f}$ from the original intensity $f$ of $w$, we want to have the corresponding color image $\hat{w}$ in a hue-preserving manner. It is confirmed that $\hat{w}$ preserves the original hue of $w$ if $\hat{w} \equiv (\hat{w}_r, \hat{w}_g, \hat{w}_b)$ is expressed as follows:

$$\hat{w}_c[i] = a[i]w_c[i] + b[i], \quad c \in \{r, g, b\}, \tag{3}$$

where $a[i]$ and $b[i]$ are constants to be specified. Substituting (3) into (1), we have

$$f(\hat{w}[i]) = \frac{1}{3}\left(\hat{w}_r[i] + \hat{w}_g[i] + \hat{w}_b[i]\right) = a[i]f[i] + b[i], \tag{4}$$

which is expected to be $f(\hat{w}[i]) = \hat{f}[i]$ or $b[i] = \hat{f}[i] - a[i]f[i]$. Substituting this into (3), we have another expression of $\hat{w}_c[i]$ as follows:

$$\hat{w}_c[i] = a[i](w_c[i] - f[i]) + \hat{f}[i], \quad c \in \{r, g, b\}. \tag{5}$$

Let us define the following values:

$$M[i] \equiv \max\{w_c[i] : c \in \{r, g, b\}\},$$
$$m[i] \equiv \min\{w_c[i] : c \in \{r, g, b\}\} \tag{6}$$

for each $w[i]$, and similarly, $\hat{M}[i]$ and $\hat{m}[i]$ for each $\hat{w}[i]$. Then an upper gamut problem arises when $\hat{M}[i] > L-1$. In this case, to avoid the upper gamut problem, $a[i]$ is determined by

$$L - 1 = a[i](M[i] - f[i]) + \hat{f}[i], \tag{7}$$

so that

$$a[i] = \frac{L - 1 - \hat{f}[i]}{M[i] - f[i]}. \tag{8}$$

On the other hand, a lower gamut problem arises when $\hat{m}[i] < 0$. In this case, to avoid the lower gamut problem, $a[i]$ is determined by

$$0 = a[i](m[i] - f[i]) + \hat{f}[i], \tag{9}$$

so that

$$a[i] = \frac{\hat{f}[i]}{f[i] - m[i]}. \tag{10}$$

From (5) we have an additive transform called *shifting* for $a[i] = 1$ as $\hat{w}_c[i] = w_c[i] - f[i] + \hat{f}[i]$. On the other hand, from (3) we have a multiplicative transform called *scaling* for $b[i] = 0$ as $\hat{w}_c[i] = a[i]w_c[i]$, where $a[i]$ is chosen to avoid the gamut problem as follows: If $\hat{f}[i] \leq f[i]$, then $a[i] = \hat{f}[i]/f[i]$, which guarantees that $\hat{w}_c[i] \in [0, L-1]$ for all $i \in \mathbb{I}_n$ and $c \in \{r, g, b\}$. Nikolova and Steidl combined the shifting and scaling models as

$$\begin{aligned}\hat{w}_c[i] &= \lambda \frac{\hat{f}[i]}{f[i]} w_c[i] + (1 - \lambda)(w_c[i] - f[i] + \hat{f}[i]) \\ &= a[i](w_c[i] - f[i]) + \hat{f}[i]\end{aligned} \tag{11}$$

for $\lambda \in [0, 1]$, where

$$a[i] = \lambda \frac{\hat{f}[i]}{f[i]} + 1 - \lambda \tag{12}$$

with upper and lower gamut corrections (8) and (10) if necessary, and defined the critical points for lower and upper gamut problems as follows:

$$G_m^\lambda[i] \equiv a[i](m[i] - f[i]) + \hat{f}[i], \tag{13}$$
$$G_M^\lambda[i] \equiv a[i](M[i] - f[i]) + \hat{f}[i]. \tag{14}$$

Then the additive algorithm for computing a color-enhanced image $\hat{w}$ is given in Algorithm 2, and the multiplicative algorithm is given by Algorithm 3. Let $\hat{w}_c^+$ and $\hat{w}_c^\times$ be the output of Algorithms 2 and 3, respectively. Then Nikolova and Steidl also combined them by

$$\tilde{w}_c \equiv \lambda \hat{w}_c^\times + (1 - \lambda)\hat{w}_c^+, \tag{15}$$

**Algorithm 2** Additive Color Enhancement [5]

1: Compute the intensity $f$ of $w$ and the target intensity $\hat{f}$ using Algorithm 1.
2: For $i \in \mathbb{I}_n$ compute $M[i]$ and $m[i]$ by (6). If $f[i] = 0$, then $\hat{w}[i] = 0$. Otherwise compute $G_m^0[i] = m[i] - f[i] + \hat{f}[i]$ and $G_M^0[i] = M[i] - f[i] + \hat{f}[i]$ and for all $c \in \{r, g, b\}$:
(i) $\hat{w}_c[i] = w_c[i] - f[i] + \hat{f}[i]$ if $G_m^0[i] \geq 0$ and $G_M^0[i] \leq L-1$,
(ii) $\hat{w}_c[i] = \frac{L-1-\hat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \hat{f}[i]$ if $G_M^0[i] > L-1$,
(iii) $\hat{w}_c[i] = \frac{\hat{f}[i]}{f[i]-m[i]}(w_c[i] - f[i]) + \hat{f}[i]$ if $G_m^0[i] < 0$.

**Algorithm 3** Multiplicative Color Enhancement [5]

1: Compute the intensity $f$ of $w$ and the target intensity $\hat{f}$ using Algorithm 1.
2: For $i \in \mathbb{I}_n$ compute $M[i]$ by (6). If $f[i] = 0$, then $\hat{w}[i] = 0$. Otherwise compute $G_M^1[i] = \frac{\hat{f}[i]}{f[i]}M[i]$ and for all $c \in \{r, g, b\}$:
(i) $\hat{w}_c[i] = \frac{\hat{f}[i]}{f[i]}w_c[i]$ if $G_M^1[i] \leq L-1$,
(ii) $\hat{w}_c[i] = \frac{L-1-\hat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \hat{f}[i]$ if $G_M^1[i] > L-1$.

and showed an equivalence between $\tilde{w}_c$ and $\hat{w}_c$ given by (11).

We have the saturation of $w[i]$ in the HSI (hue, saturation and intensity) color space as follows [8]:

$$S(w[i]) = \begin{cases} 1 - \frac{m[i]}{f[i]} & \text{if} \quad f[i] > 0 \\ 0 & \text{if} \quad f[i] = 0 \end{cases}. \tag{16}$$

Then the saturation of $\tilde{w}[i]$ is given by

$$S(\tilde{w}[i]) = \begin{cases} 0 \quad \text{if } f[i] \in T_i \\ S(w[i])\left[\lambda + (1-\lambda)\frac{f[i]}{\hat{f}[i]}\right] \\ \qquad \text{if } f[i] \notin T_i,\ G_m^\lambda[i] \geq 0,\ G_M^\lambda[i] \leq L-1 \\ S(w[i])\frac{f[i]}{\hat{f}[i]}\frac{L-1-\hat{f}[i]}{M[i]-f[i]} \\ \qquad \text{if } f[i] \notin T_i,\ G_M^\lambda[i] > L-1 \\ 1 \quad \text{if } f[i] \notin T_i,\ G_m^\lambda[i] < 0, \end{cases} \tag{17}$$

where $T_i = \{m[i], M[i]\}$.

## 3. Proposed Method

In (17), the second case depends on $\lambda$, and when $\lambda = 0$, we have

$$S(\tilde{w}[i]) = S(w^+[i]) = S(w[i])\frac{f[i]}{\hat{f}[i]}. \tag{18}$$

If $\lambda = 1$, then we have $S(\tilde{w}[i]) = S(w^\times[i]) = S(w[i])$, which means that only $(1-\lambda)\hat{w}_c^+$ in (15) contributes to the modification of the saturation, which will be maximized when $\lambda = 0$. On the basis of this observation, we propose a method that switches the value of $\lambda$ adaptively to improve the saturation of each pixel in a given image. From (18), if $\hat{f}[i] < f[i]$, then we can maximize the saturation by setting $\lambda = 0$ for using the additive algorithm, otherwise we switch to the multiplicative algorithm by setting $\lambda = 1$ to prevent the saturation deterioration. This method is formally described as follows:

$$w_c^*[i] = \begin{cases} 0 \quad \text{if } f[i] = 0 \\ w_c[i] - f[i] + \hat{f}[i] \\ \qquad \text{if } f[i] \neq 0,\ \hat{f}[i] < f[i], \\ \qquad\quad G_m^0[i] \geq 0,\ G_M^0[i] \leq L-1 \\ \frac{L-1-\hat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \hat{f}[i] \\ \qquad \text{if } f[i] \neq 0,\ \hat{f}[i] < f[i],\ G_M^0[i] > L-1 \\ \frac{\hat{f}[i]}{f[i]-m[i]}(w_c[i] - f[i]) + \hat{f}[i] \\ \qquad \text{if } f[i] \neq 0,\ \hat{f}[i] < f[i],\ G_m^0[i] < 0 \\ \frac{\hat{f}[i]}{f[i]}w_c[i] \\ \qquad \text{if } f[i] \neq 0,\ \hat{f}[i] \geq f[i],\ G_M^1[i] \leq L-1 \\ \frac{L-1-\hat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \hat{f}[i] \\ \qquad \text{if } f[i] \neq 0,\ \hat{f}[i] \geq f[i],\ G_M^1[i] > L-1 \end{cases} \tag{19}$$

for $c \in \{r, g, b\}$, where the second to fourth cases correspond to the additive algorithm ($\lambda = 0$), and the fifth to sixth cases correspond to the multiplicative algorithm ($\lambda = 1$). The procedure of the proposed method is summarized in Algorithm 4.

**Algorithm 4** Proposed method

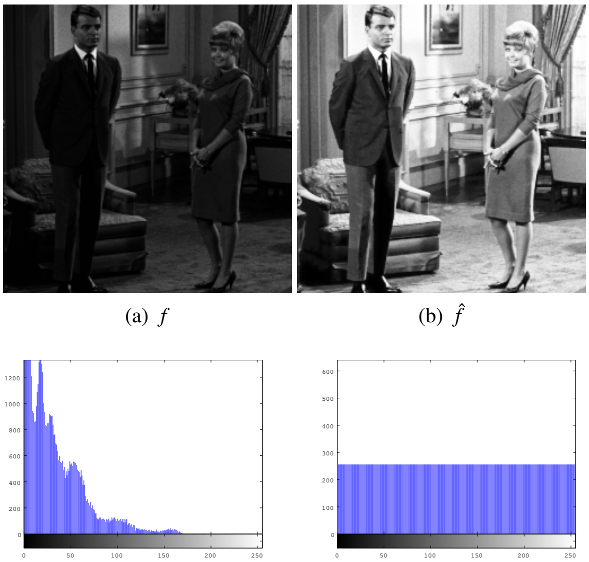
1: Compute the intensity $f$ of $w$ and the target intensity $\hat{f}$ using Algorithm 1.
2: For $i \in \mathbb{I}_n$ compute $M[i]$ and $m[i]$ by (6). If $f[i] = 0$, then $w^*[i] = 0$. Otherwise compute the following:
If $\hat{f}[i] < f[i]$, then compute $G_m^0[i] = m[i] - f[i] + \hat{f}[i]$ and $G_M^0[i] = M[i] - f[i] + \hat{f}[i]$ and for all $c \in \{r, g, b\}$:
(i) $w_c^*[i] = w_c[i] - f[i] + \hat{f}[i]$ if $G_m^0[i] \geq 0$ and $G_M^0[i] \leq L-1$,
(ii) $w_c^*[i] = \frac{L-1-\hat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \hat{f}[i]$ if $G_M^0[i] > L-1$,
(iii) $w_c^*[i] = \frac{\hat{f}[i]}{f[i]-m[i]}(w_c[i] - f[i]) + \hat{f}[i]$ if $G_m^0[i] < 0$.
Otherwise compute $G_M^1[i] = \frac{\hat{f}[i]}{f[i]}M[i]$ and for all $c \in \{r, g, b\}$:
(i) $w_c^*[i] = \frac{\hat{f}[i]}{f[i]}w_c[i]$ if $G_M^1[i] \leq L-1$,
(ii) $w_c^*[i] = \frac{L-1-\hat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \hat{f}[i]$ if $G_M^1[i] > L-1$.

The saturation of $w^*[i]$ obtained by Algorithm 4 is given by

$$S(w^*[i]) = \begin{cases} 0 \quad \text{if } f[i] \in T_i \\ S(w[i])\frac{f[i]}{\hat{f}[i]} \\ \qquad \text{if } f[i] \notin T_i,\ \hat{f}[i] < f[i], \\ \qquad\quad G_m^0[i] \geq 0,\ G_M^0[i] \leq L-1 \\ S(w[i])\frac{f[i]}{\hat{f}[i]}\frac{L-1-\hat{f}[i]}{M[i]-f[i]} \\ \qquad \text{if } \quad f[i] \notin T_i,\ \hat{f}[i] < f[i],\ G_M^0[i] > L-1 \\ 1 \quad \text{if } \quad f[i] \notin T_i,\ \hat{f}[i] < f[i],\ G_m^0[i] < 0 \\ S(w[i]) \\ \qquad \text{if } f[i] \notin T_i,\ \hat{f}[i] \geq f[i],\ G_M^1[i] \leq L-1 \\ S(w[i])\frac{f[i]}{\hat{f}[i]}\frac{L-1-\hat{f}[i]}{M[i]-f[i]} \\ \qquad \text{if } f[i] \notin T_i,\ \hat{f}[i] \geq f[i],\ G_M^1[i] > L-1. \end{cases} \tag{20}$$

In Appendix A, we show the derivation of (20).

We have a relationship between the saturation $S(\tilde{w}[i])$ in (17) and $S(w^*[i])$ in (20) as follows:



(a) $f$ (b) $\hat{f}$

(c) $h$ (d) $\hat{h}$

Figure 1: Histogram equalization by Algorithm 1: (a) original image $f$, (b) histogram-equalized image $\hat{f}$, (c) histogram $h$ of $f$ and (d) histogram $\hat{h}$ of $\hat{f}$.

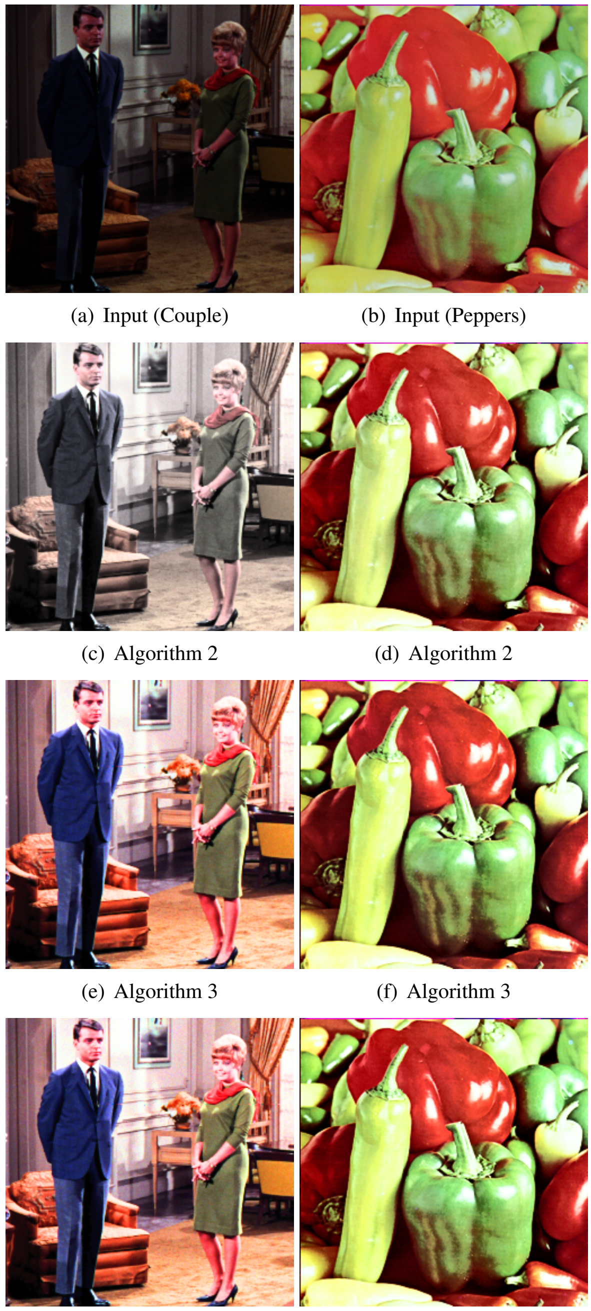
**Property 1** $S(w^*[i]) \geq S(\tilde{w}[i])$.
In Appendix B, we prove this property.

## 4. Experimental Results

In this section, we show experimental results with real images. First, we show an example of the exact histogram equalization by Algorithm 1 proposed by Nikolova and Steidl [6]. Figure 1(a) shows a grayscale image, which is enhanced by Algorithm 1 as shown in Fig. 1(b). Their histograms are shown in Figs. 1(c) and (d), respectively, where the positively skewed distribution in Fig. 1(c) (the right tail is longer) is transformed into the uniform distribution in Fig. 1(d). In the following experiments, we use the uniform distribution for the target histogram $\hat{f}$, i.e., we perform histogram equalization in color image enhancement.

Figure 2 shows the results of color image enhancement by Algorithms 1 to 4. First, the original input images in Figs. 2(a) and (b) are enhanced by Algorithm 2 as shown in Figs. 2(c) and (d), where both images have improved contrasts. Figures 2(e) and (f) show the enhanced images by Algorithm 3, where the left image (Couple) become more colorful by Algorithm 3 than the image in Fig. 2(c). On the other hand, the right image (Peppers) exhibits similar appearance to Fig. 2(d). Figures 2(g) and (h) show the results by the proposed Algorithm 4, which successfully enhances both images as well as Algorithm 3.

Figure 3 shows the results of color image enhancement by convex combinations of Algorithms 2 and 3 by Nikolova and Steidl [5] for Couple and Peppers images in the first and second rows, respectively. These images are the weighted averages of the output images of Algorithms 2 and 3 with different values of $\lambda$. We compared the average saturation per pixel between the convex combination method by Nikolova and Steidl [5] and the proposed method. Figures



(a) Input (Couple) (b) Input (Peppers)

(c) Algorithm 2 (d) Algorithm 2

(e) Algorithm 3 (f) Algorithm 3

(g) Algorithm 4 (Proposed) (h) Algorithm 4 (Proposed)

Figure 2: Comparison of Algorithms 1 to 4.

4(a) and (b) show the average saturation for Couple and Peppers images, respectively, where the vertical and horizontal axes denote the average saturation and $\lambda$, and the broken and solid lines denote the convex combination and the proposed methods, respectively. Note that the proposed method is independent of $\lambda$ as designated by the horizontal solid line. In both Figs. 4(a) and (b), the proposed method achieved higher saturation compared with the convex combination method, that supports the claim in Property 1 experimentally. As shown in Figs. 4(a) and (b), the average saturation is maximized for $\lambda = 1$ and 0, respectively, with the convex combination method. From this observation, we plot all pixels in Couple and Peppers images in Figs. 5(a)

(a) $\lambda = 0$ (b) $\lambda = 0.3$ (c) $\lambda = 0.6$ (d) $\lambda = 0.9$

(e) $\lambda = 0$ (f) $\lambda = 0.3$ (g) $\lambda = 0.6$ (h) $\lambda = 0.9$

Figure 3: Convex combination of Algorithms 2 and 3 by Nikolova and Steidl [5].

and (b), respectively, where Fig. 5(a) uses $\lambda = 1$, Fig. 5(b) uses $\lambda = 0$, and the horizontal and vertical axes denote the saturation obtained by the convex combination and the proposed methods, respectively. All pixels or points denoted by'+' are above or on the line through the points $(0, 0)$ and $(1, 1)$, that indicates that the saturation obtained by the proposed method is not smaller than that obtained by the convex combination method. These results also support the claim in Property 1.

Figure 6 shows the results of color image enhancement of photographs of various scenes by the proposed method, where the left and right columns display the input and output images. For all images, the hue is preserved, and the contrast and saturation are enhanced.

## 5. Conclusion

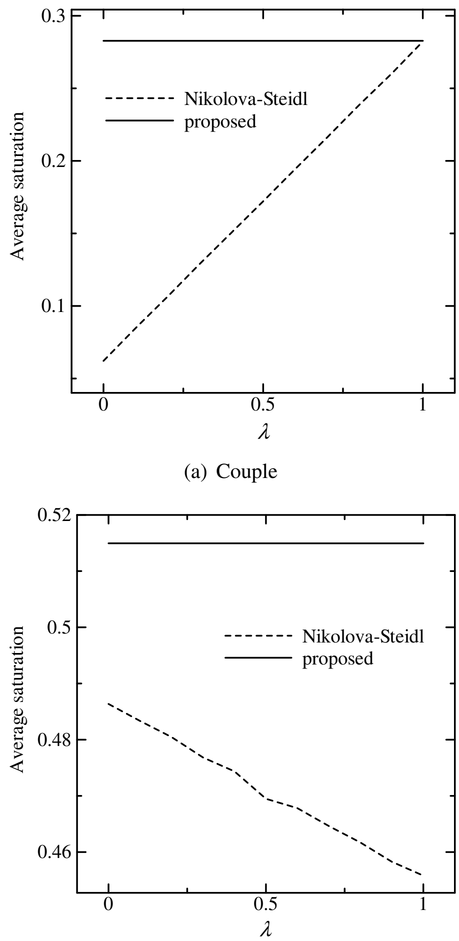
In this paper, we proposed a method for hue-preserving color image enhancement, where the additive and multiplicative algorithms by Nikolova and Steidl [5] are adaptively combined. Experimental results demonstrated that the proposed method achieves higher saturation compared with the convex combination method by Nikolova and Steidl [5]. We also showed that this property of saturation improvement is valid for any color image theoretically.

## Appendix

### A. Derivation of (20)

In this section, we derive (20), which is composed of six cases. Each case is separately described below.

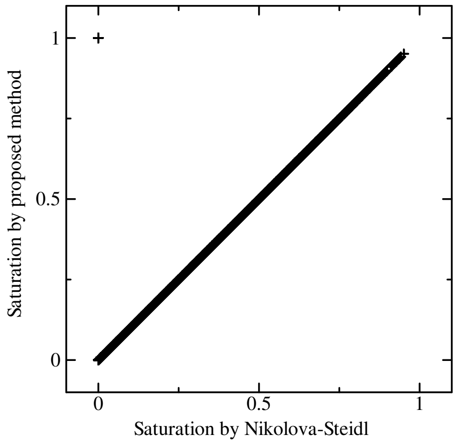
If $f[i] \in T_i$, then we have two cases as $f[i] = 0$ or $f[i] \neq 0$. If $f[i] = 0$, then, from (19), we have $w_c^*[i] = 0$ for $c \in \{r, g, b\}$, which are substituted into (16) to have $S(w^*[i]) = 0$. On the other hand, if $f[i] \neq 0$, then we have $f[i] = m[i] = M[i] = w_c[i]$ for $c \in \{r, g, b\}$,



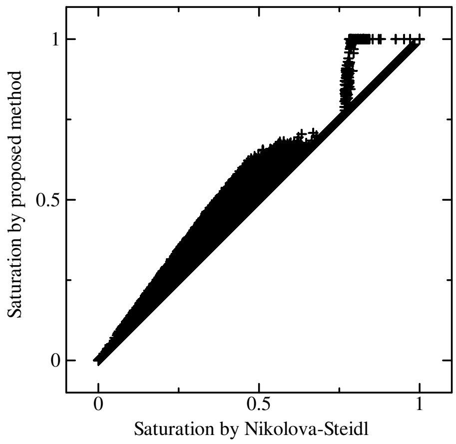

(a) Couple

(b) Peppers

Figure 4: Average saturation.




(a) Couple ($\lambda = 1$)




(b) Peppers ($\lambda = 0$)

Figure 5: All pixels on saturation plane.

and therefore, from (19), we have $w_c^*[i] = \hat{f}[i]$ for $c \in \{r, g, b\}$, which are substituted into (16) to have $S(w^*[i]) = 0$.

If $f[i] \notin T_i$, $\hat{f} < f[i]$, $G_m^0[i] \geq 0$ and $G_M^0[i] \leq L - 1$, then, from (19), we have

$$w_c^*[i] = w_c[i] - f[i] + \hat{f}[i]. \tag{21}$$

Substituting this into (16), we have

$$\begin{aligned} S(w^*[i]) &= 1 - \frac{1}{\hat{f}[i]}(m[i] - f[i] + \hat{f}[i]) \\ &= 1 - \frac{f[i]}{\hat{f}[i]}\left(\frac{m[i]}{f[i]} - 1 + \frac{\hat{f}[i]}{f[i]}\right) \\ &= \frac{f[i]}{\hat{f}[i]}\left(\frac{\hat{f}[i]}{f[i]} - \frac{m[i]}{f[i]} + 1 - \frac{\hat{f}[i]}{f[i]}\right) \\ &= \frac{f[i]}{\hat{f}[i]}\left(1 - \frac{m[i]}{f[i]}\right) \\ &= S(w[i])\frac{f[i]}{\hat{f}[i]}. \end{aligned} \tag{22}$$

If $f[i] \notin T_i$, $\hat{f}[i] < f[i]$, $G_M^0[i] > L - 1$, then, from (19), we have

$$w_c^*[i] = \frac{L - 1 - \hat{f}[i]}{M[i] - f[i]}(w_c[i] - f[i]) + \hat{f}[i]. \tag{23}$$

Figure 6: Color image enhancement results.

Substituting this into (16), we have

$$\begin{aligned} S(w^*[i]) &= 1 - \frac{1}{\hat{f}[i]}\left[\frac{L-1-\hat{f}[i]}{M[i]-f[i]}(m[i]-f[i]) + \hat{f}[i]\right] \\ &= 1 - \frac{1}{\hat{f}[i]}\frac{L-1-\hat{f}[i]}{M[i]-f[i]}(m[i]-f[i]) - 1 \\ &= \frac{f[i]}{\hat{f}[i]}\frac{L-1-\hat{f}[i]}{M[i]-f[i]}\left(1-\frac{m[i]}{f[i]}\right) \\ &= S(w[i])\frac{f[i]}{\hat{f}[i]}\frac{L-1-\hat{f}[i]}{M[i]-f[i]}. \end{aligned} \tag{24}$$

If $f[i] \notin T_i,\ \hat{f}[i] < f[i],\ G_m^0[i] < 0$, then, from (19), we have

$$w_c^*[i] = \frac{\hat{f}[i]}{f[i]-m[i]}(w_c[i]-f[i]) + \hat{f}[i]. \tag{25}$$

Substituting this into (16), we have

$$\begin{aligned} S(w^*[i]) &= 1 - \frac{1}{\hat{f}[i]}\left[\frac{\hat{f}[i]}{f[i]-m[i]}(m[i]-f[i]) + \hat{f}[i]\right] \\ &= 1 + \frac{1}{\hat{f}[i]}\frac{\hat{f}[i]}{f[i]-m[i]}(f[i]-m[i]) - 1 \\ &= 1. \end{aligned} \tag{26}$$

If $f[i] \notin T_i,\ \hat{f}[i] \geq f[i],\ G_M^1[i] \leq L-1$, then, from (19), we have

$$w_c^*[i] = \frac{\hat{f}[i]}{f[i]}w_c[i]. \tag{27}$$

Substituting this into (16), we have

$$\begin{aligned} S(w^*[i]) &= 1 - \frac{1}{\hat{f}[i]}\frac{\hat{f}[i]}{f[i]}w_c[i] \\ &= 1 - \frac{m[i]}{f[i]} \\ &= S(w[i]). \end{aligned} \tag{28}$$

If $f[i] \notin T_i,\ \hat{f}[i] \geq f[i],\ G_M^1[i] > L-1$, then, from (19), we have the same expression for $w_c^*[i]$ as the above (23). Therefore, we have $S(w^*[i]) = S(w[i])\frac{f[i]}{\hat{f}[i]}\frac{L-1-\hat{f}[i]}{M[i]-f[i]}$ again as well as (24). This completes the derivation of (20).

### B. Proof of Property 1

In this section, we prove the inequality stated in Property 1. The difference between (17) and (20) arises only when $f[i] \in T_i,\ G_m^\lambda[i] \geq 0,\ G_M^\lambda[i] \leq L-1$. Therefore, we focus on this case below.

If $\hat{f}[i] < f[i]$, then we have

$$\begin{aligned} &S(w^*[i]) - S(\tilde{w}[i]) \\ &= S(w[i])\frac{f[i]}{\hat{f}[i]} - S(w[i])\left[\lambda + (1-\lambda)\frac{f[i]}{\hat{f}[i]}\right] \\ &= S(w[i])\left(\frac{f[i]}{\hat{f}[i]} - \lambda - \frac{f[i]}{\hat{f}[i]} + \lambda\frac{f[i]}{\hat{f}[i]}\right) \\ &= S(w[i])\lambda\left(\frac{f[i]}{\hat{f}[i]} - 1\right) \\ &= S(w[i])\frac{\lambda}{\hat{f}[i]}\left(f[i]-\hat{f}[i]\right) \geq 0 \end{aligned} \tag{29}$$

On the other hand, if $\hat{f}[i] \geq f[i]$, then we have

$$\begin{aligned} &S(w^*[i]) - S(\tilde{w}[i]) \\ &= S(w[i]) - S(w[i])\left[\lambda + (1-\lambda)\frac{f[i]}{\hat{f}[i]}\right] \\ &= S(w[i])\left(1 - \lambda - \frac{f[i]}{\hat{f}[i]} + \lambda\frac{f[i]}{\hat{f}[i]}\right) \\ &= S(w[i])(1-\lambda)\left(1 - \frac{f[i]}{\hat{f}[i]}\right) \\ &= S(w[i])\frac{1-\lambda}{\hat{f}[i]}\left(\hat{f}[i]-f[i]\right) \geq 0 \end{aligned} \tag{30}$$

Both (29) and (30) conclude that $S(w^*[i]) \geq S(\tilde{w}[i])$.

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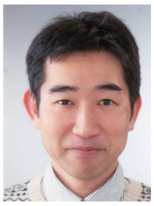

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