An Experimental Study of VCG Mechanism for Multi-unit Auctions: Competing with Machine Bidders

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An Experimental Study of VCG Mechanism for Multi-unit Auctions: Competing with Machine Bidders

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Abstract

This paper complements the main experimental result reported in Takahashi et al. (2019) in order to understand subjects' bidding behavior under the VCG mechanism more deeply. In this experiment, there are two types of appearance of information about bidders' valuations of the item given to them and the bids they are asked to submit: One is unit valuations and the unit bids themselves (Appearance 1) and the other is unit valuations and the unit bids multiplied by the number of units (Appearance 2). When subjects compete with truth-telling machine bidders in multi-unit auctions, we confirmed that in Appearance 1, they choose truth-telling bids more frequently, and thus efficient allocations are observed more frequently, as compared to the situation where they compete with human bidders. This result suggests a possibility that in Appearance 1 subjects learn their dominant strategy not by practicing with other subjects but by practicing with machine bidders in experiments for multi-unit auctions, although the item allocation and payment determination under the VCG mechanism is never intuitively understandable to subjects.

Keywords: multi-unit auction, VCG mechanism, experiment, human and machine bidders, truth-telling bids

JEL Classification: C92, D44, D82

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1 Introduction

In theory, Vickrey-Clarke-Groves mechanism (VCG) attains allocative efficiency by inducing bidders to represent their true valuation for each unit of the item auctioned off. However, the VCG suffers from its computational intractability when the number of units of the item to be traded is large. This drawback is one of the reasons why we have not found any report that VCG was used in practice, although there are many practical examples of multi-unit auctions: oil and timber sales, flower markets, spectrum auctions, etc. Thus, the approximation algorithms that reduce the computational complexity were proposed by Dyer (1984) and Kothari et al. (2005). Based on the Dyer's work, Takahashi and Shigeno (2011) developed a greedy-based approximation (GBA) algorithm that is much faster than VCG in computation time. VCG has another drawback; it is difficult for bidders to intuitively infer how it allocates the item with their own bids. In the GBA, however, a bidder who submits the highest unit bid is given priority for obtaining the units of the item, and thus bidders can infer how it allocates the item with their own bids more easily.

In order to confirm the practical performance of GBA, Takahashi et al. (2018) conducted a subject experiment where five units of an identical item were auctioned off to three bidders. They reported that there was no significant difference in seller's revenue between VCG and GBA, but VCG attained higher allocative efficiency than GBA; on average, the efficiency rate was 97.37% in VCG and it was 93.65% in GBA. These efficiency rates in VCG were remarkably higher than expected, even though it is difficult for subjects to intuitively understand how the VCG works. There are, unfortunately, few papers on the experiments of the VCG for multi-unit auctions, and thus we cannot compare our result with others in different sessions. The efficiency rates in VCG were, however, not higher in experiments for other types of auction (Kagel and Levin, 2016), as compared to our observation. What factors generate higher allocative efficiency in the VCG for multi-unit auctions?

To seek for the answer to this question, Takahashi et al. (2019) next investigated whether the performance of the VCG is robust against changes of appearance of information in which bidders submit "total bids" confirming their "total valuation" for each unit. Their main result is that there was no significant difference on average in either allocative efficiency or seller's revenue between those two types of appearance of information. Rather, for each appearance of information, there was a significant difference in subjects' bidding behavior between different display types of draws of unit valuations. The average rates of efficiency were, again, more than 90% in any sessions. The rates of 95% approximately truth-telling bids were, however, 31.8-43.1% in those sessions, which implies that many efficient allocations were not generated even by approximately truth-telling bids. Do we not need to induce bidders to report their true preferences in order to attain the allocative efficiency?

When subjects compete with human bidders, it may be difficult for them to realize that truth-telling is their dominant strategy, because they do not necessarily report their true preferences. Let us confirm this point introducing truth-telling machine bidders.

Main Hypothesis: When subjects compete with truth-telling machine bidders in multi-unit auctions, if the VCG mechanism induces subjects to choose truth-telling bids more frequently, and efficient allocations are observed more frequently, as compared to the situation where they compete with human bidders.

We test Main Hypothesis under two types of appearance of information about bidders' valuations of the item given to them and the bids they are asked to submit: One is unit valuations and the unit bids themselves (Appearance 1) and the other is unit valuations and the unit bids multiplied by the number of units (Appearance 2), Takahashi et al. (2019) reported that the VCG generated no significant difference in the allocative efficiency between Appearances 1 and 2. In which appearance is the allocative efficiency attained more frequently?

We had the following observations. (1) In seller's revenue there was no significant difference on average but in allocative efficiency there was a significant difference on average between Appearance 1 and Appearance 2. (2) Approximately truth-telling bids were more frequently chosen by subjects and approximately efficient allocations were more frequently observed in Appearance 1 than in Appearance 2. (3) When opponents are truth-telling machine bidders in Appearance 1, subjects chose approximately truth-telling bids more frequently and approximately efficient allocations realized more frequently, as compared to the case where opponents were human bidders. There were no such differences in Appearance 2. Thus, Main Hypothesis was confirmed in Appearance 1. This is our main result.

The main result suggests a possibility that in Appearance 1 subjects learn their dominant strategy not by practicing with other subjects but by practicing with machine bidders in experiments for multi-unit auctions, although the item allocation and payment determination under the VCG mechanism is never intuitively understandable to subjects.

The rest of this paper is organized as follows. Section 2 formally describes the VCG mechanism for multiunit auctions. A numerical example of the mechanism is given in Appendix 2. Section 3 describes the experimental design, and Section 4 shows the results. Section 5 closes this paper with some remarks.

2 VCG Mechanism

From a computational aspect, the VCG mechanism is formulated in the following way. There is a seller who wishes to sell $M(<\infty)$ units of an identical item and solicits bids from $n(<\infty)$ buyers each of whom can purchase up to M units of the item. Let $\mathcal{N}=\{1,...,n\}$ be the set of all buyers (bidders). For each bidder $i\in\mathcal{N}$, denote his or her anchor values on the quantity by $\{d_i^k \mid k=0,...,\ell_i\}$, where $d_i^{k-1}<d_i^k$ for all k with $1\leq k\leq \ell_i$, and denote his or her unit bids by $\{b_i^k \mid k=1,...,\ell_i\}$, where b_i^k is a buyer price in half-open range $(d_i^{k-1},d_i^k]$ for $k=1,...,\ell_i$. For every bidder $i\in\mathcal{N}$, $d_i^0=0$ and $d_i^{\ell_i}\leq M$. Let $\ell=\sum_{i\in\mathcal{N}}\ell_i$.

Define a function $B_i:\mathbb{R}_+\to\mathbb{R}$ for each $i\in\mathcal{N}$ by

$$B_i(y)=\begin{cases} b_i^k\cdot y & (d_i^{k-1}<y\leq d_i^k, k=1,...,\ell_i),\\ 0 & (y=d_i^0, y>d_i^{\ell_i}).\end{cases} \tag{1}$$

The unit bids represent the gradients of this function and the anchor values stand for its discontinuous points. For each bidder $i\in\mathcal{N}$, denote his or her unit valuations by $\{v_i^k \mid k=1,...,\ell_i\}$ and define another function $V_i:\mathbb{R}_+\to\mathbb{R}$ by

$$V_i(y)=\begin{cases} v_i^k\cdot y & (d_i^{k-1}<y\leq d_i^k, k=1,...,\ell_i),\\ 0 & (y=d_i^0, y>d_i^{\ell_i}).\end{cases} \tag{2}$$

A vector $\boldsymbol{x}=(x_1,x_2,\ldots,x_n)$ that satisfies $\sum_{i\in\mathcal{N}}x_i\leq M$ and $x_i\geq 0$ for any $i\in\mathcal{N}$ is called an allocation, where x_i is the units of the item assigned to bidder $i\in\mathcal{N}$ in the allocation. An item allocation problem $(AP)_B$ is to find allocations that maximize the total amount of bids is formulated by

$$
(AP)_B \left| \begin{array}{ll} \text{maximize} & \displaystyle\sum_{i \in \mathcal{N}} B_i(x_i) \\ \text{subject to} & \displaystyle\sum_{i \in \mathcal{N}} x_i \leq M \\ & x_i \geq 0 \;\; (\forall i \in \mathcal{N}). \end{array} \right.
$$

This item allocation problem is faced with computational intractability, i.e., $(AP)_B$ is known to be $\mathcal{NP}$-hard. Another problem $(AP)_V$ is formulated in the same way by

$$
(AP)_V \left| \begin{array}{ll} \text{maximize} & \displaystyle\sum_{i \in \mathcal{N}} V_i(x_i) \\ \text{subject to} & \displaystyle\sum_{i \in \mathcal{N}} x_i \leq M \\ & x_i \geq 0 \;\; (\forall i \in \mathcal{N}), \end{array} \right.
$$

to find efficient allocations that maximize the total amount of valuations. When there are two or more allocation in each of which the total amount of bids is maximized, one of those allocations is chosen at random as the solution of $(AP)_B$.

The payment scheme is as follows. Denote by $\boldsymbol{x}^*$ an optimal solution of $(AP)_B$. Let $\boldsymbol{x}^{-j}$ be an optimal solution of the following restricted item allocation problem $(AP)_B^{-j}$ with the set of bidders $\mathcal{N}^{-j} = \mathcal{N} \setminus \{j\}$.

$$
(AP)_B^{-j} \left| \begin{array}{ll} \text{maximize} & \displaystyle\sum_{i \in \mathcal{N}^{-j}} B_i(x_i) \\ \text{subject to} & \displaystyle\sum_{i \in \mathcal{N}^{-j}} x_i \leq M \\ & x_i \geq 0 \;\; (\forall i \in \mathcal{N}^{-j}). \end{array} \right.
$$

In the VCG mechanism, bidder j's payment p_j is determined by

$$
p_j = \sum_{i \in \mathcal{N}^{-j}} B_i(x_i^{-j}) - \sum_{i \in \mathcal{N}^{-j}} B_i(x_i^*). \tag{3}
$$

Under this payment scheme, it is the dominant strategy for each bidder to truthfully bid his or her unit valuations; Thus, the solutions of $(AP)_B$ maximize the total sum of valuations in $(AP)_V$ as well, which leads to allocative efficiency.

3 Experimental Design

3.1 Basic Setup

The basic setup of this experiment is the same as that of Takahashi et al. (2019). This experiment is computerized, using software (cgi script) that is coded in Python. We conducted 4 sessions. Each session consists of 20 rounds. In each round, 5 units of a virtual item are auctioned off to 3 bidders, where for each bidder i, the number of anchor values is set as $\ell_i = 5$, and thus his or her anchor values are $d_i^0 = 0$, $d_i^1 = 1$, ..., $d_i^5 = 5$. For each bidder $i \in \mathcal{N}$, his or her unit valuations, $\{v_i^k \mid k = 1, ..., \ell_i\}$, are independently and uniformly distributed over the integers between 1 and 200. Bids are made using non-negative integers.

In 2 out of 4 sessions, at the beginning of each round, each bidder $i \in \mathcal{N}$ is given his or her unit valuations $\{v_i^k \mid k = 1, ..., \ell_i\}$ by the experimenter, which are privately shown only on his or her computer screen (Appearance 1). Then, each bidder i submits his or her unit bids $\{b_i^k \mid k = 1, ..., \ell_i\}$ privately to the experimenter. The computer determines the allocation of the item and bidders' payments according to $(AP)_B$ and (3). In the other 2 sessions, each bidder i is given his or her valuations $\{v_i^k \cdot k \mid k = 1, ..., \ell_i\}$ privately and submits his or her bids $\{b_i^k \cdot k \mid k = 1, ..., \ell_i\}$ (Appearance 2). When k units of the item are allocated to bidder i, he or she receives the points in the amount $v_i^k \cdot k$ minus his or her payment, which is shown to bidder i through his or her computer screen in both Appearances 1 and 2. Table 1 shows the difference in appearance of the information given to bidder i in the case of 5 units, as an example.

In each round, there is a 120-second time limit for submitting bids. If no bidder bids within the time limit, then all three bidders obtain zero points at that round. If multiple allocations attain the maximum total amount of bids, then one allocation is chosen at random. The units assigned to a bidder and his or her payment are shown to the bidder for 5 seconds at the end of each round. The cumulative points of bidders are not shown to them, and subjects were prohibited to take notes.

This paper follows the paired setting used in an auction experiment conducted by Engelmann and Grimm (2009). For each appearance of information, 2 sessions are paired in this experiment; In one session, each unit valuation of the item is drawn "at random" for each

Table 1: Different appearance of information.

Appearance 1	v_i^k shown; b_i^k bid					
# of units		1	2	3	4	5
bidder i	valuation	80×1	60×2	55×3	43×4	77×5
	bid	70×1	55×2	50×3	43×4	72×5
Appearance 2	$v_i^k \cdot k$ shown; $b_i^k \cdot k$ bid					
# of units		1	2	3	4	5
bidder i	valuation	80	120	165	172	385
	bid	70	110	150	172	360

bidder and given to him or her as it is in the first 10 rounds, while in the second 10 rounds the values drawn at random are reordered in the monotone non-increasing ("decreasing") order from $k = 1$ to $k = 5$ and given to each bidder as his or her unit valuations in that order. The display types of draws are reversed between the first and second 10 rounds in the other paired session. Every subject thus bids under both display types in the same session. In the analysis, we should be careful of the effect of the order of the display types on the results.

The instruction is given to the subjects at the beginning of each session, where how the VCG mechanism works is demonstrated using an example (attached in Appendix 2). The example is carefully made so that it does not imply the dominant strategy of the auction game. Subjects are informed that they will be paid according to the total points they obtain in 6 rounds (3 from the first 10 rounds and 3 from the subsequent 10 rounds) randomly selected by a computer at the end of the session they participated in with the pre-determined exchange rate in addition to the show-up fee. The exchange rate was 1 point = 1 JPY and the show-up fee was 1500 JPY. Subjects play 1 round for practice to familiarize themselves with the software, before proceeding to each set of 10 rounds in the session they participate in.

3.2 Session Details

This experiment was run at the University of Tsukuba in Japan, and 4 sessions were conducted in January 2018. The subjects were undergraduate students recruited from all over the campus, but economics majors were excluded. Engineering majors were the largest subgroup of our subjects. In total, 23 male students and 9 female students participated in this experiment. There was no subjects who had participated in the sessions for multi-unit auctions conducted in 2015 and 2017. Subjects were randomly assigned to a session. Each session involves 8 groups of 3 bidders, one of whom is a human bidder. Subjects were not informed of truth-telling bids made by machine bidders. The units assigned to machine bidders as well as their payments and points were not shown to subjects' monitor.

At the beginning of each session, each subject was randomly assigned to a group. In a companion experiment which was run in 2015 and 2017 (Takahashi et al., 2019), each session involves 8 groups of 3 subjects. At the beginning of each round, all subjects were randomly re-grouped into 8 groups by a computer. Subjects are not informed of who are in the same group with.

Upon arrival at the laboratory, subjects were provided with a written instruction, and then the experimenter read it around.[1] In all sessions, the same experimenter read the instruction and provided simple review questions to make subjects reconfirm how the experiment proceeds. Subjects could ask questions about the instruction as well as the review questions and the experimenter gave the answers to those questions privately.

Any communication among subjects was strictly prohibited; their interactions were only through the information shown on their computer monitors. Each session lasted about 100 minutes including the time for giving the instruction. In no case did subjects fail to make a bid within the time limit. The features of the experimental sessions are summarized in Table 2.

4 Results

We analyze the data taken from the last 5 out of 10 rounds in each display type of draws in order to exclude the data that might contain outcomes of behavior chosen when subjects were still learning something on bidding strategies. There were no multiple allocations with the same maximum total amount of bids. Even if we analyzed the data taken from the last 5 rounds out of the first 10 rounds and those taken from the last 5 rounds out of the second

[1]The complete instruction is attached in Appendix 2.

Table 2: Features of the experimental sessions.

session no.	appearance of info.	display of draws	show-up fee (JPY)	point-to-JPY ratio	# of subj.	session date	avg. point per subject
1	2	at random	1500	1.0	8	Jan.22	496.54
1	2	descending					1127.89
2	2	descending	1500	1.0	8	Jan.22	876.87
2	2	at random					346.65
3	1	at random	1500	1.0	8	Jan.23	600.12
3	1	descending					1285.73
4	1	descending	1500	1.0	8	Jan.23	1306.58
4	1	at random					527.61

Note: The display type of draws used in the first 10 rounds is listed first in each session with the same session number. For a companion experiment, at the university of Tsukuba, 2 sessions for Appearance 1 were conducted in February 2015 and 2 sessions for Appearance 2 were conducted in January 2017, in each session of which 24 subjects participated.

10 rounds separately, we obtained similar results as far as the allocative efficiency, seller's revenue, and the number of approximately truth-telling bids. The data were thus merged for each display type of draws to increase our sample size. We do not consider bidding behavior for different subjects' attributes (gender, age, and academic major), because the sample size is not large enough for that purpose.

Let $\hat{\boldsymbol{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ be an observed allocation. The rate of efficiency is defined by

$$\frac{\sum_{i \in \mathcal{N}} V_i(\hat{x}_i)}{\text{the optimal value of } (AP)_V}. \tag{4}$$

The rate of seller's revenue (profit) is defined by

$$\frac{\text{the total amount of observed payments}}{\text{the total amount of optimal payments}}, \tag{5}$$

where the total amount of optimal payments is represented by $\sum_{j \in \mathcal{N}} p_j$ and p_j is calculated with (3) for each bidder $j \in \mathcal{N}$ under the assumption that every bidder truthfully bids his or her (unit) valuations. The unit valuations were different across different rounds in different sessions, and thus we analyze those rates.

Tables 3 and 4 show the average rates of efficiency and seller's revenue for each auction (appearance of information and display type of draws) as well as their standard deviations. The sample size is 80 (5 rounds, 8 groups, 2 sessions) for each auction, which is the same as the sample size in Takahashi et al. (2019). The p-values for the two-sided permutation test (perm.) are also reported in those tables, where in each display type of draws the null hypothesis is that there is no difference in those averages between Appearance 1 and Appearance 2, which is rejected at the 5% significance level.

Table 3: The rates of efficiency.

display of draws	at random		decreasing	
appearance of info.	Appearance 1	Appearance 2	Appearance 1	Appearance 2
mean	0.9892	0.9351	0.9987	0.9765
st.dev.	0.0204	0.0601	0.0022	0.0251
p-value (perm.)	**0.0108**		**0.0016**	

Note: The p-values for the two-sided permutation test are listed in this table. The null hypothesis is rejected at the 5% significance level.

Table 4: The rates seller's revenue.

display of draws	at random		decreasing	
appearance of info.	Appearance 1	Appearance 2	Appearance 1	Appearance 2
mean	0.9966	1.0014	1.0101	1.0668
st.dev.	0.0125	0.0693	0.0383	0.1487
p-value (perm.)	0.8311		0.2875	

Note: The p-values for the two-sided permutation test are listed in this table. The null hypothesis is rejected at the 5% significance level.

Observation 1 *(1) Efficient allocations were on average observed significantly more frequently between Appearance 1 and Appearance 2 for each display type of draws. (2) In seller's revenue there was no significant difference on average between Appearance 1 and Appearance 2 for each display type of draws.*

In the case where the auctions were conducted by 3 human bidders, Observation 1 in Takahashi et al. (2019) states that for there was no difference on average in either allocative efficiency or seller ⊠'s revenue between Appearance 1 and Appearance 2 for each display type of draws. At that time, the efficiency rates in Appearance 1 were 0.9306 (at random) and 0,9172 (decreasing), whereas in Appearance 2 they were 0.9378 (at random) and 0.9337. Thus, when we introduced 2 machine bidders, the efficiency rates were improved in Appearance 1.

The VCG mechanism in theory induces bidders to truthfully bid their own valuations of the item for each unit, which is a dominant strategy for every bidder. Figure 1 - Figure 8 in Appendix 1 depict bid plots observed in each session and show that those bids were apparently nearer to the truth-telling bids in Appearance 1 than in Appearance 2. This point is a major factor with which we obtained Observation 1 (1), in contrast to the results with 3 human bidders. We thus next proceed to counting the number of bids that are approximately truth-telling and the number of allocations that are approximately efficient. We say that a bid for a unit of the item is approximately truth-telling when it satisfies

$$\frac{|\text{unit valuation} - \text{unit bid}|}{\text{unit valuation}} \leq 0.05 \tag{6}$$

and that an allocation is approximately efficient when it satisfies

$$\text{the rate of efficiency} \geq 0.95. \tag{7}$$

Table 5 presents the numbers of approximately truth-telling bids and approximately efficient allocations observed in the last 5 periods. For each appearance of information, the sample size is 400 (5 rounds, 8 bidders, 5 units, 2 sessions) for approximately truth-telling bids and it is 80 (5 rounds, 8 auctions, 2 sessions) for approximately efficient allocations in each display type of draws. The p-values for the one-sided Fisher exact test (Fisher) are also reported, where for each display type of draws the null hypothesis is that the number of approximately truth-telling bids (approximately efficient allocations) in Appearance 1 is less than or equal to the number of those bids in Appearance 2, which is rejected at the 5% significance level.

Observation 2 *For each display of draws, approximately truth-telling bids were chosen by subjects significantly more frequently and approximately efficient allocations were observed significantly more frequently in Appearance 1 than in Appearance 2.*

Table 5: Numbers of approximately truth-telling bids and approximately efficient allocations.

	truth-telling		efficiency	
appearance of info.	Appearance 1	Appearance 2	Appearance 1	Appearance 2
at random	232	148	78	63
p-value (Fisher)	$<$ **0.0001**		**0.0002**	
descending	214	119	74	60
p-value (Fisher)	$<$ **0.0001**		**0.0023**	

Note: The p-values for the one-sided Fisher exact test are listed in this table. The null hypothesis is rejected at the 5% significance level.

By Observation 2, we expect that in Appearance 1, when subjects compete with truth-telling machine bidders, they choose their dominant strategy more frequently, as compared to the case where they compete with human bidders.

Table 6 presents the observed numbers of approximately truth-telling bids with those numbers observed in 2015 and 2017 (Takahashi et al., 2019). For each appearance of information, the sample size of the data taken in 2015 and 2017 is 1200 (5 rounds, 24 bidders, 5 units, 2 sessions) in each display type of draws. The p-values for the two-sided Fisher exact test (Fisher) are also reported, where for each display of draws the null hypothesis is that there is no difference in number of approximately truth-telling bids between the two types of appearance of information.

Table 6: Numbers of approximately truth-telling bids

display of draws	at random		descending	
opponent bidders	vs. machines	vs. subjects	vs. machines	vs. subjects
Appearance 1	232/400	517/1200	214/400	450/1200
p-value (Fisher)	$<$ **0.0001**		$<$ **0.0001**	
Appearance 2	148/400	493/1200	119/400	381/1200
p-value (Fisher)	0.1576		0.4933	

Note: The p-values for the two-sided Fisher exact test are listed in this table. The null hypothesis is rejected at the 5% significance level. In Appearance 1, the p-values for the one-sided test are also less than 0.001. The sample size is 400 for this experiment and it is 1200 for a companion experiment, respectively.

Table 7 shows the observed numbers of approximately efficient allocations with those numbers observed in 2015 and 2017 (Takahashi et al., 2019). For each appearance of information, the sample size is 80 (5 rounds, 8 auctions, 2 sessions) in each display type of draws. The p-values for the two-sided Fisher exact test (Fisher) are also reported, where for each display type of draws the null hypothesis is that there is no difference in number of approximately efficient allocations between the two types of appearance of information.

Table 7: Numbers of approximately efficient allocations

display of draws	at random		descending	
opponent bidders	vs. machines	vs. subjects	vs. machines	vs. subjects
Appearance 1	78	60	74	57
p-value (Fisher)	$<$ **0.0001**		**0.0008**	
Appearance 2	63	60	60	54
p-value (Fisher)	0.7080		0.3826	

Note: The p-values for the two-sided Fisher exact test are listed in this table. The null hypothesis is rejected at the 5% significance level. In Appearance 1, the p-values for the one-sided test are also less than 0.001.

Observation 3 *When opponents are truth-telling machine bidders, for each display type of draws in Appearance 1, subjects chose approximately truth-telling bids more frequently and approximately efficient allocations realized more frequently, as compared to the case where opponents were human bidders. There were no such differences for each display type of draws in Appearance 2.*

Observation 3 may imply that subjects are confused by behavior of human bidders. We eventually reached the following result.

Main Result: Main Hypothesis was confirmed when unit valuations are shown to subjects and their unit bids are submitted in multi-unit auctions.

Main Result suggests a possibility that in Appearance 1 subjects learn their dominant strategy by practicing with machine bidders in experiments for multi-unit auctions, although the item allocation and payment determination under the VCG mechanism is never intuitively understandable to subjects.

5 Final Remarks

In economic experiments, the instructors should not suggest desirable actions to subjects. The purpose of those experiments are, in many cases, to verify various theoretical results or to find some behavioral features of subjects. If subjects received such an instruction in an experiment, the purpose of the experiment would then be changed to testing whether they could believe that the suggested actions were the best ones for themselves. In some cases, however, it is difficult for subjects to infer the outcomes of their own choice of behavior, when, in particular, those experiments are conducted for verifying the theoretical results derived in game theory and mechanism design. In fact, for example, It is difficult for subjects to intuitively understand the item allocation and payment determination under the VCG mechanism. What we showed by Result 1 is that there is a type of appearance of information provided to subjects in which they can learn their dominant strategy, not by practicing with other subjects but by practicing with machine bidders for some periods.

Figures 1 to 8 plot unit valuations and unit bids observed in our sessions. The data were taken from the last 5 out of 10 rounds in each display type of draws. Tables 8 and 11 show the regression results for the corresponding sessions. As is seen in Table 9, there was a subject who took an extraordinary biding behavior in Appearance 2. In Appearance 1, some coefficients on valuations were less than one but others were more than one, regardless of the display types of draws. Thus, we did not have clear evidence for subjects' overbidding or underbidding when subjects compete with truth-telling machine bidders.

When subjects compete with human bidders, on the other hand, Observation 3 in Takahashi et al. (2019) states that for both Appearance 1 and Appearance 2, subjects underbid when unit valuations were drawn at random and shown to them as they were, whereas they did not necessarily do so when values drawn at random were reordered in monotone non-increasing order and given to them as their unit valuations.

Kagel et al. (2001) conducted an experiment in which a human bidder with a flat demand for two units competes against machine bidders each demanding a single unit, and they reported overbidding of each human bidder for both units. It is not appropriate to make a comparison with their result, but our regression analysis showed that subjects overbid for some units when each draw of unit valuations was reordered in the monotone non-increasing order. As noted above, on the other hand, we sometimes observed that subjects' underbidding. What are the major factors that induce subjects to overbid or underbid? As was mentioned, there are few papers on the experiments of the VCG in multi-unit auctions.[2] We need to further investigate how the VCG mechanism work in multi-unit auctions.

Finally, we need to reconfirm the same result we observed at different experimental sites to ensure the robustness. This paper simply provided some preparatins to investigate how the VCG mechanism work in multi-unit auctions for the future research.

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Compliance with Ethical Standards

Conflict of Interest

The authors declare that they have no conflict of interest.

Ethical Approval

All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards. This article does not contain any studies with animals performed by any of the authors.

[2] In a situation where two units of an item auctioned off to two bidders, Engelmann and Grimm (2009) compared the performances of a uniform-price sealed-bid auction, a uniform-price clock auction, a discriminatory auction, a static Vickrey auction, and a dynamic Vickrey auction, including a literature review on laboratory experiments of multi-unit auctions. Among those studies, Kagel and Levin (2001) was a seminal paper to study the demand reduction in uniform-price auctions. Thereafter, experiments for multi-unit auctions have been focused on the demand reduction under uniform-price.

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Appendix 1: Bid Plots and Regression Results

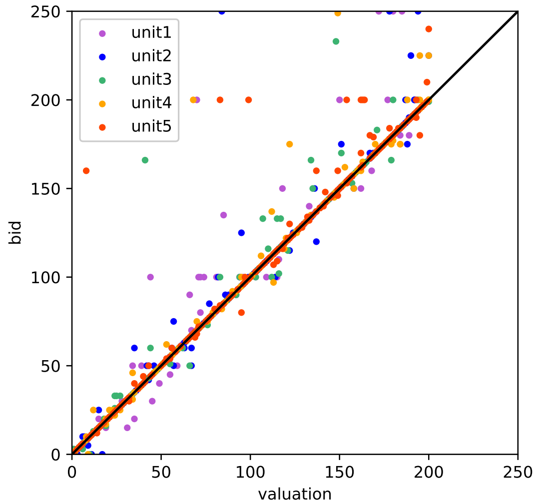

Figure 1: Appearance 2, session 1, At Random.

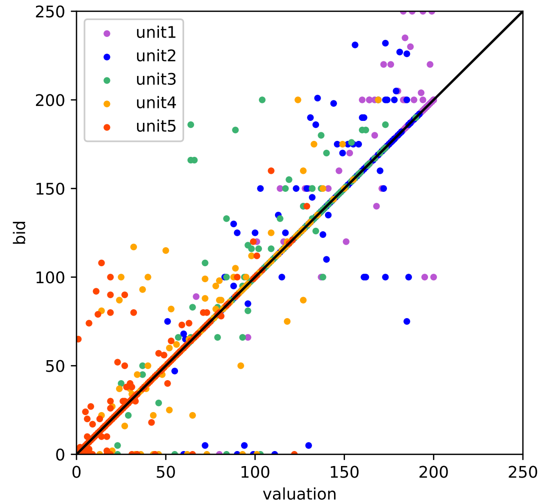

Figure 2: Appearance 2, session 1, Descending.

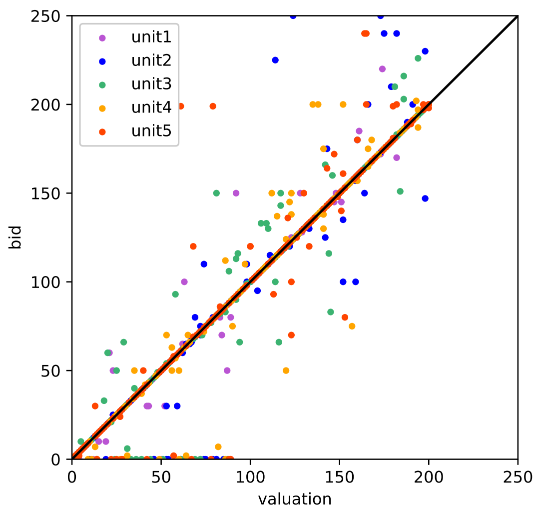

Figure 3: Appearance 2, session 2, At Random.

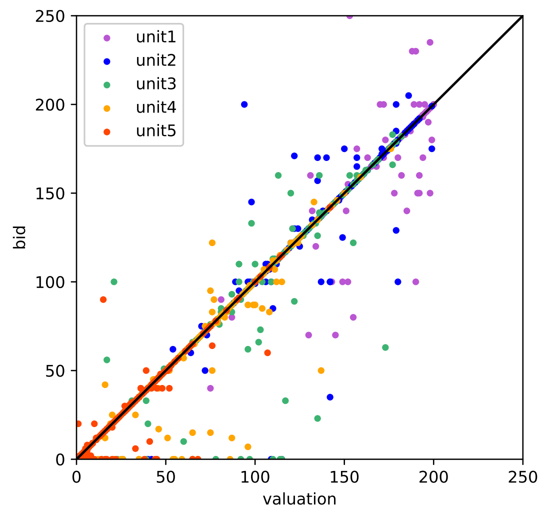

Figure 4: Appearance 2, session 2, Descending.

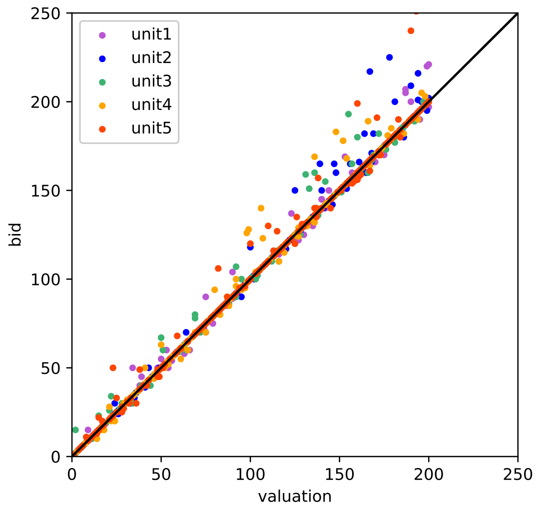

Figure 5: Appearance 1, session 3, At Random.

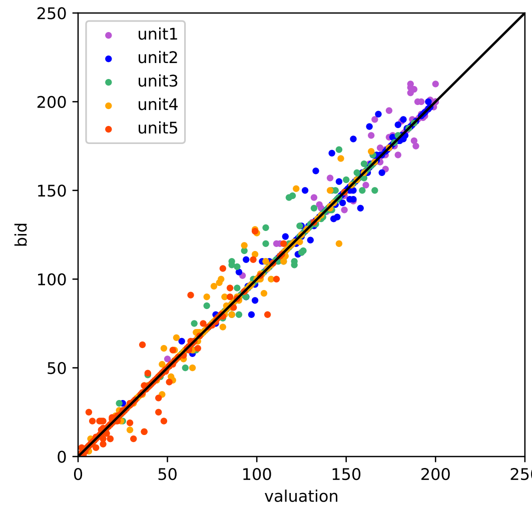

Figure 6: Appearance 1, session 3, Descending.

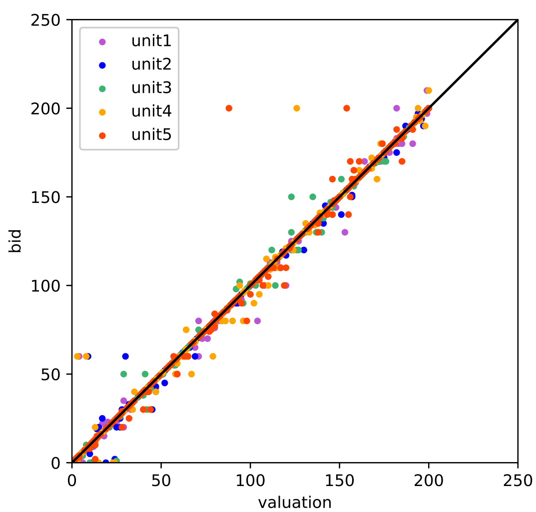

Figure 7: Appearance 1, session 4, At Random.

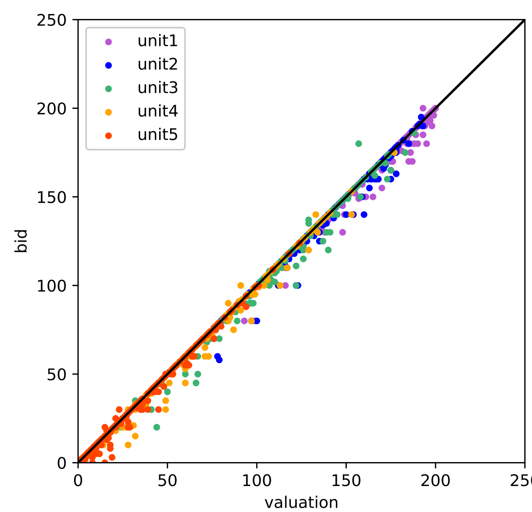

Figure 8: Appearance 1, session 4, Descending.

Table 8: Regression results for Appearance 2: session 1.

	at random				
# of units	1	2	3	4	5
Constant	8.6487	0.2704	-0.0477	-1.5924	-0.5665
p-value	0.5317	0.7787	0.9759	0.5838	0.9143
Valuation	1.0346	1.0111	1.0176	1.0385	1.0495
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.3695	0.9903	0.9796	0.9368	0.8387
	descending				
# of units	1	2	3	4	5
Constant	-2.5882	-8.5249	2.7530	1.2758	6.2880
p-value	0.5444	0.2024	0.4701	0.7029	0.0135
Valuation	1.0423	1.0776	1.0109	1.0064	0.8667
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.9270	0.7970	0.8678	0.7935	0.6270

Table 9: Regression results for Appearance 2: session 2.

	at random				
# of units	1	2	3	4	5
Constant	58.7651	-4.4447	23.4051	10.8658	-21904.1247
p-value	0.1089	0.4023	0.0952	0.7727	0.4814
Valuation	0.8618	1.0680	0.8375	1.1039	437.3332
p-value	0.0173	< 0.0001	< 0.0001	0.0015	0.1079
R-squared	0.0471	0.8091	0.2779	0.0825	0.0218
	descending				
# of units	1	2	3	4	5
Constant	-3.2232	0.2276	0.7252	-2.7964	0.0934
p-value	0.5080	0.9608	0.8708	0.3046	0.9458
Valuation	1.0097	0.9994	0.9800	1.0067	0.9176
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.9027	0.8715	0.8168	0.8725	0.8642

Table 10: Regression results for Appearance 1: session 3.

	at random				
# of units	1	2	3	4	5
Constant	0.1097	-0.8554	1.1903	0.4402	-0.4954
p-value	0.8428	0.4930	0.1573	0.6737	0.6792
Valuation	1.0049	1.0276	1.0002	1.0081	1.0168
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.9969	0.9867	0.9934	0.9903	0.9878
	descending				
# of units	1	2	3	4	5
Constant	0.4552	0.0595	0.2821	-1.5299	-0.1532
p-value	0.6877	0.9598	0.7719	0.1242	0.8355
Valuation	1.0018	1.0026	0.9980	1.0313	1.0040
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.9943	0.9910	0.9900	0.9818	0.9616

Table 11: Regression results for Appearance 1: session 4.

	at random				
# of units	1	2	3	4	5
Constant	0.9578	0.8311	5.8840	1.7431	0.6277
p-value	0.3321	0.4292	0.0593	0.3495	0.7243
Valuation	0.9924	0.9901	0.9592	0.9955	0.9992
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.9914	0.9903	0.9173	0.9684	0.9724
	descending				
# of units	1	2	3	4	5
Constant	-1.8971	-1.2454	-1.4327	-0.3842	-0.8211
p-value	0.1071	0.2531	0.0849	0.5481	0.0376
Valuation	1.0023	0.9987	1.0009	0.9905	0.9963
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.9940	0.9921	0.9924	0.9902	0.9882

Appendix 2: Instruction[3]

Welcome to this experiment!

Thank you very much for taking the time to participate in our auction experiment. The experiment lasts for about 100 minutes, including the payment session.

At First

· Please follow the instructions given by the experiment administrators.

· Please remain silent, and do not talk to or exchange notes with other participants.

· Please do not look at what other participants are doing.

· Please do not change your position. Please do not lean in your chair.

· Please do NOT do anything other than what you are instructed to do.

· Please turn off and refrain from using your cell phones.

· Please quietly raise your hand if you have questions or need help.

In this experiment, a total of 20 auctions will be held and 5 units of a virtual item are auctioned off to 3 bidders in each auction.

Compensation

After all the 20 auctions end, a computer will randomly choose 3 auctions each from the 10 auctions in the first and second halves, that is, a total of 6 auctions. You will be compensated on the basis of the total points you earned in the 6 auctions. The final compensation will be the amount based on those points in addition to a compensation of 1500 JPY for participation.

Group Selection

At the beginning, you will be assigned an ID. Your ID will remain the same throughout the session you participate in, and it will be displayed on your computer screen. You will be matched with two machine bidders. During the experiment this matching will be fixed.

[3] This is the instruction for the participants in sessions for Appearance 1.

Please raise your hand if you have questions on the above contents.

Auction

In each auction, 5 units of an identical item are auctioned off to 3 bidders; one is you, and the other two are machine bidders. The bidding strategy of each machine bidder is programmed in advance. Please bid for all units within 120 seconds. If no one in the same group bids within this time limit, all bidders in the group obtain zero points. The outcome will not be shown until the remaining time is up, even if everyone bids within the time limit.

At the beginning of each auction, each bidder is given unit valuations of the item for each unit. When the unit valuation is, e.g., 15 for 3 units, the total valuation is $15\times3=45$. You are asked to submit your unit bids for each unit. Please press the "bid" button after you fill in your unit bids on your screen. Then, a pop-up window appears and shows your total bids for each unit. If you click on the "OK" button in the pop-up window, your bids will then be sent to the server computer to compute the outcome of the auction. If you click on "cancel" button there, you can then go back to the screen to fill in your unit bids.

For each bidder, unit valuations are drawn as integers independently of those for the other bidders with equal probability between 1 and 200. Please bid in non-negative integers. The remaining time is displayed on the right upper corner of your screen. When the auction ends, the outcome is shown on your screen. The next auction will start after 5 seconds. The rule of the auction is explained next.

Please raise your hand if you have questions on the above content.

Item Allocation in the Auction

Below is an example of the auction in which 3 units of an item auctioned off to 2 bidders; In Table 12, valuations (or bids) are displayed as unit valuations (or unit bids) multiplied by the number of units.[4] Note that the unit valuations and unit bids shown in this example do not suggest any bidding strategy in the auctions you participate in.

[4] In the instruction, we explained how the VCG mechanism allocate the item with this example for three units, in order to reduce the influence of the numerical values on the behavior of our subjects in the auctions for five units.

Table 12: Example.

# of units		1	2	3
Bidder 1	valuation	80×1	60×2	55×3
	bid	70×1	55×2	50×3
Bidder 2	valuation	40×1	70×2	65×3
	bid	40×1	60×2	65×3

The item will be allocated to bidders such that the total amount of bids is maximized as follows. Find an allocation that maximizes the total amount of bids among all possible allocations; In the example, (0, 0): 0, (1, 1): 70×1+40×1=110, (1, 0): 70×1=70, (2, 0): 55×2=110, (3, 0): 50×3=150, (0, 1): 40×1=40, (0, 2): 6×2=120, (0, 3): 65×3=195, (1, 2): 70×1+60×2=190, (2,1): 55×2+40×1 =150. Thus, this auction allocates 3 units to bidder 2. The total amount of bids is 195. When there are two or more allocation in each of which the total amount of bids is maximized, one of those allocations is chosen at random.

Payment determination in the Auction

The payments of bidders are determined as follows.

payment of bidder i = (total amount of bids in the auction that excludes bidder i)
− (total amount of bids in the auction) + (bidder i's bid for the unit assigned to i).

In the example,

- payment of bidder $1 = (65 \times 3) - 195 + 0 = 0$,
- payment of bidder $2 = (50 \times 3) - 195 + (65 \times 3) = 150$.

Point

The amount of points each bidder earn is calculated as follows.

bidder i's points = (total valuation for the units bidder i is allocated)
− (payment of bidder i).

In the example,

- bidder 1's points $= 0 - 0 = 0$,
- bidder 2's points $= (65 \times 3) - 150 = 45$.

You will be compensated on the basis of the points you earned. The exchange rate is 1 point = 1 JPY. As mentioned, you will be compensated on the basis of the total points you earned in the 6 auctions, 3 out of the first 10 auctions and 3 out of the second 10 auctions.

Configuration

In the first 10 auctions, unit valuations are given on your computer screen and you are asked to submit unit bids there, as shown in the Example (Table 12). In the second 10 auctions, total valuations are given on your screen and you are asked to submit total bids there, as shown in Table 13.

Table 13: Another display.

# of units		1	2	3
Bidder 1	valuation	80	120	165
	bid	70	110	150
Bidder 2	valuation	40	140	195
	bid	40	120	195

Practice

At the beginning of each sequence of 10 auctions, an auction is held as a practice so that you can familiarize yourself with how to do with the computer. The points you earn in the practice auctions are not counted as those for the compensation.

Please raise your hand if you have questions.