VCG Mechanism for Multi-unit Auctions and Appearance of Information: A Subject Experiment

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VCG Mechanism for Multi-unit Auctions and Appearance of Information: A Subject Experiment

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Abstract

This paper investigates whether, in multi-unit auctions, different types of appearance of information associated with bidding generate different levels of allocative efficiency and sellers' revenue when the VCG mechanism is applied to human subject experiments of those auctions. We examine two types of appearance of information about bidders' valuations of the item given to them and the bids they are asked to submit: One type is unit valuations and the unit bids themselves and the other type is unit valuations and the unit bids multiplied by the number of units. We observed that there was no significant difference on average in either allocative efficiency or the seller's revenue between these two types of appearance of information. Rather, for each appearance of information, there was a significant difference in subjects' bidding behavior between different display types of draws of unit valuations. This behavioral difference, however, did not significantly affect allocative efficiency. The performance of the VCG mechanism is robust against display types of those draws as well as against types of appearance of information.

Keywords: multi-unit auction, VCG mechanism, subject experiment
JEL Classification: C92, D44, D82

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1 Introduction

This paper investigates whether, in multi-unit auctions, different types of appearance of information associated with bidding generate different levels of allocative efficiency and sellers' revenue when the Vickrey-Clarke-Groves (VCG) mechanism is used in the experiment. We examine two types of appearance of information about bidders' valuations of the item given to them and their bids asked to submit. One type is unit valuations and the unit bids themselves and the other type is valuations and bids, i.e., unit valuations and the unit bids multiplied by the number of units.

There are many practical examples of multi-unit auctions: oil and timber sales, flower markets, spectrum auctions, etc. It is known in theory that the VCG mechanism attains allocative efficiency but suffers from its computational intractability when the number of units of the item to be traded is large. In fact, we have not found any report that the VCG mechanism was used in practice. Many experiments of multi-unit auctions have thus focused on the price discount in discriminatory auctions (e.g., traditional treasury auctions in many countries), the demand reduction in uniform-price auctions (e.g., the treasury auction in the US), and the incentive to enter and collude in ascending clock auctions (e.g., electricity spot markets). As Klemperer (2002) noted, these practical issues might be more important than features theorists tend to be concerned with, although these auction rules do not guarantee the allocative efficiency. Accordingly, there is no paper on the experiment of the exact VCG mechanism in multi-unit auctions listed even in the latest comprehensive survey on auction experiments by Kagel and Levin (2016).

To improve the drawback of VCG, however, the approximation algorithms that reduce the computational complexity were proposed in the field of operations research (Dyer, 1984; Kothari et al., 2005). Based on the Dyer's work, Takahashi and Shigeno (2011) developed a greedy-based approximation (GBA) algorithm that is much faster than the VCG mechanism in computation time. Takahashi et al. (2018) reported that in their subject experiment where five units of an identical item were auctioned off to three bidders, VCG attained higher allocative efficiency than GBA, although there was no significant difference in seller's revenue between GBA and VCG; The average rate of efficiency was 97.37% in VCG and it was 93.65% in GBA.

The authors did not expect this result on allocative efficiency, because bidders submitted their "unit bids" confirming their "unit valuations" in the experiment. This type of appearance of information is considered as a key feature for the GBA algorithm to work well; In GBA, a bidder who submits the highest unit bid is given priority for obtaining the units of an identical item in the process of the item allocation, and thus bidders can intuitively infer how GBA allocates the item with their own bids, as compared to VCG. When this inference is difficult, bidders may underbid, which induces efficiency loss to some extent. In fact, Chen and Takeuchi (2010) reported underbidding in VCG, although it was in the experiment of combinatorial auctions. Thus, the authors expected that GBA might be even superior to VCG in allocative efficiency as well as in computation time.[1]

After observing the experimental result shown by Takahashi et al. (2018), we next investigate whether the performance of the VCG mechanism is robust against changes of appearance of information in which bidders submit "total bids" confirming their "total valuation" for each unit. This is the main research question of this paper. In theory, there should be no difference in bidders' behavior, and thus there should be no difference in both allocative efficiency and sellers' revenue.

This experiment was run in the same environment as that in Takahashi et al. (2018), except for incorporating two types of appearance of information. Our main observation is that there was no significant difference on average in either allocative efficiency or seller's revenue between those two types of appearance of information. Rather, for each appearance of information, there was a significant difference in subjects' bidding behavior between different display types of draws of unit valuations. This behavioral difference, however, did not significantly affect the difference in allocative efficiency. The performance of VCG is robust also in this sense.

In theory, the VCG mechanism has another nice property besides allocative efficiency; It induces bidders to truthfully bid their own valuations of the item for each unit, which is a dominant strategy for every bidder. In our experiment, however, the rate of approximately truth-telling bids were at most 45% in any sessions we

[1] Takahashi and Shigeno (2011) and Takahashi et al. (2018) also showed in their numerical experiments that as the number of units of the item increases, the computation time in VCG rapidly increases, whereas the increase in computation time is suppressed in GBA.

conducted. We observed that subjects underbid when unit valuations were drawn at random and shown to them as they were, although they did not necessarily do so when unit valuations were reordered in monotone non-increasing order and given to them as their unit valuations. Regardless of those differences in bidding behavior, the average rate of efficiency were more than 90% in any sessions. These results imply that in allocative efficiency the performance of VCG may be robust against bidding behavior out of theory.

Truly, it is difficult for bidders to intuitively infer how exact VCG allocates the item with their own bids, which may be one of the reasons why we have not found any report that the VCG mechanism was used in practice. According to all the above results, however, the VCG mechanism may be worth applying to practical trades when the number of units of the item to be traded is limited.

The rest of this paper is organized as follows. Section 2 describe the VCG mechanism for multiunit auctions as formally as possible. A numerical example is given in the Appendix. Section 3 describes the experimental design, and Section 4 shows the results. Section 5 closes this paper with some remarks for further research.

2 VCG Mechanism

We describe an auction rule, where a seller wishes to sell M units of an identical item and solicits bids from n buyers each of whom can purchase up to M units of the item. Let $N = \{1, ..., n\}$ be the set of buyers (bidders). For each bidder $i \in N$, denote his or her anchor values on the quantity by $\{d_i^k \mid k = 0, ..., \ell_i\}$, where $d_i^{k-1} < d_i^k$ for all k with $1 \le k \le \ell_i$, and denote his or her unit bids by $\{b_i^k \mid k = 1, ..., \ell_i\}$, where b_i^k is a buyer price in half-open range $(d_i^{k-1}, d_i^k]$ for $k = 1, ..., \ell_i$. It is assumed that $d_i^0 = 0$ and $d_i^{\ell_i} \le M$ for every bidder $i \in N$. Each bidder i has a list of his or her anchor values and unit bids, i.e., $\{d_i^k \mid k = 0, ..., \ell_i\}$ and $\{b_i^k \mid k = 1, ..., \ell_i\}$. Let $\ell = \sum_{i \in N} \ell_i$.

Define a function $B_i : \mathbb{R}_+ \to \mathbb{R}$ for each $i \in N$ by

$$B_i(y) = \begin{cases} b_i^k \cdot y & (d_i^{k-1} < y \le d_i^k, k = 1, ..., \ell_i), \\ 0 & (y = d_i^0, y > d_i^{\ell_i}). \end{cases} \tag{1}$$

The unit bids represent the gradients of this function and the anchor values stand for its discontinuous points. For each bidder $i \in N$, denote his or her unit valuations by $\{v_i^k \mid k = 1, ..., \ell_i\}$ and define another function $V_i : \mathbb{R}_+ \to \mathbb{R}$ by

$$V_i(y) = \begin{cases} v_i^k \cdot y & (d_i^{k-1} < y \le d_i^k, k = 1, ..., \ell_i), \\ 0 & (y = d_i^0, y > d_i^{\ell_i}). \end{cases} \tag{2}$$

A vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ that satisfies $\sum_{i \in N} x_i \le M$ and $x_i \ge 0$ for any $i \in N$ is called an allocation, where x_i is the units of the item assigned to bidder $i \in N$ in the allocation. An item allocation problem $(AP)_B$ is to find allocations that maximize the total amount of bids is formulated by

$$(AP)_B \left| \begin{array}{ll} \text{maximize} & \displaystyle\sum_{i \in N} B_i(x_i) \\ \text{subject to} & \displaystyle\sum_{i \in N} x_i \le M \\ & x_i \ge 0 \ \ (\forall i \in N). \end{array} \right.$$

This item allocation problem is faced with computational intractability.[2] Another problem $(AP)_V$ is formulated in the same way by

$$(AP)_V \left| \begin{array}{ll} \text{maximize} & \displaystyle\sum_{i \in N} V_i(x_i) \\ \text{subject to} & \displaystyle\sum_{i \in N} x_i \le M \\ & x_i \ge 0 \ \ (\forall i \in N), \end{array} \right.$$

to find efficient allocations that maximize the total amount of valuations. When there are two or more allocation in each of which the total amount of bids is maximized, one of those allocations is chosen at random as the solution of $(AP)_B$.

The payment scheme is as follows. Denote by $\boldsymbol{x}^*$ an optimal solution of $(AP)_B$. Let $\boldsymbol{x}^{-j}$ be an optimal solution of the following restricted item allocation problem $(AP)_B^{-j}$ with the set of bidders $N^{-j} = N \setminus \{j\}$.

$$(AP)_B^{-j} \left| \begin{array}{ll} \text{maximize} & \displaystyle\sum_{i \in N^{-j}} B_i(x_i) \\ \text{subject to} & \displaystyle\sum_{i \in N^{-j}} x_i \le M \\ & x_i \ge 0 \ \ (\forall i \in N^{-j}). \end{array} \right.$$

In the VCG mechanism, bidder j's payment p_j is determined by

[2] $(AP)_B$ is known to be $\mathcal{NP}$-hard.

$$p_j = \sum_{i \in N^{-j}} B_i(x_i^{-j}) - \sum_{i \in N^{-j}} B_i(x_i^*). \tag{3}$$

Under this payment scheme, it is the dominant strategy for each bidder to truthfully bid his or her unit valuations; Thus, the solutions of $(AP)_B$ maximize the total sum of valuations in $(AP)_V$ as well, which leads to allocative efficiency.

3 Experimental Design

This laboratory experiment is computerized, using software (cgi script) that is coded in Python. This experiment has 4 sessions and each session consists of 20 rounds in total. In each round, 5 units of a virtual item are auctioned off to 3 bidders, where for each bidder i, the number of anchor values is set as $\ell_i = 5$, and thus his or her anchor values are $d_i^0 = 0$, $d_i^1 = 1$, ..., $d_i^5 = 5$. For each bidder $i \in N$, his or her unit valuations, $\{v_i^k \mid k = 1, ..., \ell_i\}$, are independently and uniformly distributed over the integers between 1 and 200. Bids are made using non-negative integers.

In 2 out of 4 sessions, at the beginning of each round, each bidder $i \in N$ is given his or her unit valuations $\{v_i^k \mid k = 1, ..., \ell_i\}$ by the experimenter, which are privately shown only on his or her computer screen (Appearance 1). Then, each bidder i submits his or her unit bids $\{b_i^k \mid k = 1, ..., \ell_i\}$ privately to the experimenter. The computer determines the allocation of the item and bidders' payments according to $(AP)_B$ and (3). When k units of the item are allocated to bidder i, he or she receives the points in the amount $v_i^k \cdot k$ minus his or her payment. In the other 2 sessions, each bidder i is given his or her valuations $\{v_i^k \cdot k \mid k = 1, ..., \ell_i\}$ and submits his or her bids $\{b_i^k \cdot k \mid k = 1, ..., \ell_i\}$ (Appearance 2). Table 1 shows the difference in appearance of the information given to bidder i in the case of 3 units, as an example.

In each round, there is a 120-second time limit for submitting bids. If no bidder bids within the time limit, then all three bidders obtain zero points at that round. If multiple allocations attain the maximum total amount of bids, then one allocation is chosen at random. The units assigned to a bidder and his or her payment are shown to the bidder for 5 seconds at the end of each round. The cumulative points of bidders are not shown to them, and subjects were prohibited to take notes.

Table 1: Different appearance of information.

Appearance 1	2015: v_i^k shown; b_i^k bid			
# of units		1	2	3
bidder i	valuation	80×1	60×2	55×3
	bid	70×1	55×2	50×3
Appearance 2	**2017: $v_i^k \cdot k$ shown; $b_i^k \cdot k$ bid**			
# of units		1	2	3
bidder i	valuation	80	120	165
	bid	70	110	150

For each appearance of information, 2 sessions are paired in this experiment; In one session, each unit valuation of the item is drawn randomly ("at random") for each bidder and given to him or her as it is in the first 10 rounds, while in the second 10 rounds the values drawn at random are reordered in the monotone non-increasing ("descreasing") order from $k = 1$ to $k = 5$ and given to each bidder as his or her unit valuations in that order. The display types of draws are reversed between the first and second 10 rounds in the other paired session. Every subject thus bids under both display types in the same session. In the analysis, we should be careful of the effect of the order of the display types on the results.

The instruction is given to the subjects at the beginning of each session, where how the VCG mechanism works is demonstrated using an example (attached in the Appendix). The example is carefully made so that it does not imply the dominant strategy of the auction game. Subjects are informed that they will be paid according to the total points they obtain in 6 rounds (3 from the first 10 rounds and 3 from the subsequent 10 rounds) randomly selected by a computer at the end of the session they participated in with the pre-determined exchange rate in addition to the show-up fee. The exchange rate was 1 point = 1 JPY and the show-up fee was 1500 JPY. Subjects play 1 round for practice to familiarize themselves with the software, before proceeding to each set of 10 rounds in the session they participate in.

4 Results

This experiment was run at the University of Tsukuba in Japan: 2 sessions in February 2015 and 2 sessions in January 2017. The subjects were undergraduate students recruited from all over the campus, but third- or fourth-year economics majors were excluded. (Engineering majors were the largest subgroup of our subjects.) Each subject participated once in this experiment, and he or she was randomly assigned to a session. Each session involves 8 groups of 3 subjects. At the beginning of each round, all subjects were randomly re-grouped into 8 groups by a computer. Subjects are not informed of who are in the same group with. Once a subject participated in a session, he or she was prohibited from participating in any other sessions of this experiment again.

Upon arrival at the laboratory, subjects were provided with a written instruction, and then the experimenter read it around. (The complete instruction is attached in the Appendix.) Subjects could ask questions regarding the instruction by raising their hand and the experimenter gave the answers to those questions privately. Any communication among subjects was strictly prohibited; Thus, their interactions were only through the information they entered to their computers. Each session lasted about 100 minutes including the time for giving the instruction. In no case did subjects fail to make a bid within the time limit. The features of the experimental sessions are summarized in Table 2.[3]

Table 2: Features of the experimental sessions.

session no.	appearance of info.	show-up fee (JPY)	point-to-JPY ratio	# of subj.	session date	avg. point per subject
1	1	1500	1.0	24	Feb.14, 2015	475.05
2	1	1500	1.0	24	Feb.14, 2015	691.29
3	2	1500	1.0	24	Jan.26, 2017	469.42
4	2	1500	1.0	24	Jan.26, 2017	674.05

[3] A more detailed summary is available upon request.

In what follows, we analyze the data taken from the last 5 out of 10 rounds in each display type of draws in order to exclude the data that might contain outcomes of behavior chosen when subjects were still learning something. There was no case where multiple allocations attained the same maximum total amount of bids. We obtained similar results, even if we analyzed the data taken from the last 5 rounds out of the first 10 rounds and those taken from the last 5 rounds out of the second 10 rounds, separately. The data were thus merged for each display type of draws to increase our sample size. This paper follows the analysis methods used in an auction experiment by Engelmann and Grimm (2009), except using the permutation test.[4]

Let $\hat{\boldsymbol{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ be an observed allocation. The rate of efficiency is defined by

$$\frac{\sum_{i \in N} V_i(\hat{x}_i)}{\text{the optimal value of } (AP)_V}. \tag{4}$$

The rate of seller's revenue (profit) is defined by

$$\frac{\text{the total amount of observed payments}}{\text{the total amount of optimal payments}}, \tag{5}$$

where the total amount of optimal payments is represented by $\sum_{j \in N} p_j$ and p_j is calculated with (3) for each bidder $j \in N$ under the assumption that every bidder truthfully bids his or her (unit) valuations. The unit valuations were different across different rounds in different sessions, and thus we analyze those rates.

Tables 3 and 4 show the average rates of efficiency and seller's revenue, respectively, as well as their standard deviations. The sample size is 80 (5 rounds, 8 groups, 2 sessions) for each rate. The p-values for the two-sided permutation test (perm.) are also reported in those tables, where in each display type of draws the null hypothesis is that there is no difference in those averages between Appearance 1 and Appearance 2. For both rates of efficiency and seller's revenue, as is seen in Tables 3 and 4, the null hypothesis was not rejected at the 5% significance level for either display type of draws. Our main observation is thus stated as follows.

[4] Engelmann and Grimm (2009) conducted sessions in which an auction rule was used in the first 10 rounds and another auction rule was used in the second 10 rounds, and analyze the data taken from the first 10 rounds to compare the auction results with the Mann-Whitney U-test. In our analysis, we assumed that possible learning effect on subjects' bidding behavior was excluded in the first 5 rounds out of 10 rounds in each display type of draws, according to a convention.

Table 3: The rates of efficiency.

display of draws	at random		decreasing	
appearance of info.	Appearance 1	Appearance 2	Appearance 1	Appearance 2
mean	0.9306	0.9378	0.9172	0.9337
st.dev.	0.0704	0.0393	0.0621	0.0296
p-value (perm.)	0.8293		0.5567	

Table 4: The rates seller's revenue.

display of draws	at random		decreasing	
appearance of info.	Appearance 1	Appearance 2	Appearance 1	Appearance 2
mean	0.9477	0.9641	1.0564	0.8979
st.dev.	0.0932	0.0374	0.2581	0.1418
p-value (perm.)	0.6596		0.1120	

For both rates of efficiency and seller's revenue, as is seen in Tables 3 and 4, the null hypothesis was not rejected at the 5% significance level in each display types of draws. Our main observation is thus stated as follows.

Observation 1 *For each display type of draws, there was no difference on average in either allocative efficiency or seller's revenue between Appearance 1 and Appearance 2.*

Note that for both appearances of information, the standard deviation of the seller's revenue observed when displaying unit valuations in monotone non-increasing order is much larger than that for displaying the draws in random order: 0.0932 (at random) and 0.2581 (decreasing) in Appearance1, and 0.0374 (at random) and 0.1418 (decreasing) in Appearance 2. Takahashi et al. (2018) expected this result and thus chose to display values drawn at random without changing their order. We could confirm their expected result as well.

As noted at the end of Section 2, the VCG mechanism, in theory, induces allocative efficiency by providing every bidder with an incentive to submit his or her true valuations for each unit. In order to examine this feature, we counted the number of bids that are approximately truth-telling and the number of allocations that are approximately efficient. We say that a bid for a unit of the item is approximately truth-telling when it satisfies

$$\frac{|\text{unit valuation} - \text{unit bid}|}{\text{unit valuation}} \leq 0.05 \tag{6}$$

and that an allocation is approximately efficient when it satisfies

$$\text{the rate of efficiency} \geq 0.95. \tag{7}$$

Table 5 shows the observed numbers of approximately truth-telling bids and approximately efficient allocations. For each appearance of information, the sample size is 1200 (5 rounds, 24 bidders, 5 units, 2 sessions) for approximately truth-telling bids and it is 80 for approximately efficient allocations in each display type of draws. The p-values for the two-sided Fisher exact test (Fisher) are also reported, where for each appearance of information the null hypothesis is that there is no difference in number of approximately truth-telling bids (approximately efficient allocations) between the two display types of draws. For the numbers of the approximately truth-telling bids, the null hypothesis was rejected at the 5% significance level for each appearance of information, whereas for the numbers of approximately efficient allocations the null hypothesis was not rejected at the same significance level for either appearance of information. Our next observation is thus stated as follows.

Table 5: Numbers of approximately truth-telling bids and approximately efficient allocations.

	truth-telling		efficiency	
display of draws	at random	decreasing	at random	decreasing
Appearance 1	517	450	60	57
p-value (Fisher)	0.0141		0.1216	
Appearance 2	493	381	60	54
p-value (Fisher)	< 0.0001		0.3826	

For the numbers of the approximately truth-telling bids, the null hypothesis was rejected at the 5% significance level in each appearance of information, whereas for the numbers of approximately efficient allocations the null hypothesis was not rejected at the same significance level in each appearance of information. Our next observation is thus stated as follows.

Observation 2 *In each appearance of information, there was a significant difference in numbers of approximately truth-telling bids between display types of draws, but this behavioral difference did not significantly affect the difference in numbers of approximately efficient allocations between display types of draws.*

For both appearances of information and both display types of draws, the numbers of approximately truth-telling bids were less than half that of the 1200 samples, as is seen Table 5. We thus report more results on the subjects' bidding behavior. In this experiment, each unit valuation was drawn independently of the other unit valuations, and thus we here analyze the data unit by unit. If the absolute value of a unit valuation minus a unit bid falls within 5% of all those absolute values, then we dropped the data as constituing an outlier for our regression analysis.

Tables 6 and 7 show the regression results for Appearance 1 and Appearance 2, respectively. Figures 1 to 4 plot unit valuations and unit bids observed for Appearance 1 and Appearance 2. The coefficients on valuations were less than one and they are statistically significant when values were drawn at random and shown to them without reordering, regardless of the appearance of information. Some coefficients on valuations in the other display type of draws were, however, more than one and they are statistically significant. Our last observation is thus stated as follows.

Observation 3 *For both Appearance 1 and Appearance 2, subjects underbid when unit valuations were drawn at random and shown to them as they were, whereas they did not necessarily do so when values drawn at random were reordered in monotone non-increasing order and given to them as their unit valuations.*

As noted in the Introduction, Chen and Takeuchi (2010) reported subjects' underbidding when the VCG was used in their experiment, although they studied combinatorial auctions. For single-unit auctions, however, many researchers have reported that subjects overbid in a second-price auction (VCG mechanism in single-unit auctions). We observed that subjects overbid also in multi-unit auctions when

VCG was applied in the display type of draws of unit valuations that were aligned in the monotone non-increasing order.

Table 6: Regression results for Appearance 1.

	at random				
# of units	1	2	3	4	5
Constant	-5.0432	-0.1143	2.3098	-1.6640	2.2452
p-value	0.3150	0.8416	0.6750	0.7530	0.6910
Valuation	0.8925	0.9850	0.9392	0.9688	0.9634
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.6370	0.6480	0.6050	0.6580	0.6300
	descending				
# of units	1	2	3	4	5
Constant	-11.7126	-2.5721	-5.3490	1.0001	2.9950
p-value	0.3900	0.7360	0.2930	0.8530	0.3300
Valuation	0.9395	0.9528	1.0247	0.9359	0.7826
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.3660	0.5590	0.6610	0.4250	0.3250

Table 7: Regression results for Appearance 2.

	at random				
# of units	1	2	3	4	5
Constant	5.1200	-4.0728	3.0490	-1.0193	-0.5544
p-value	0.3620	0.3720	0.4750	0.8110	0.8900
Valuation	0.7741	0.9252	0.8723	0.9551	0.9505
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.4980	0.6760	0.690	0.7380	0.7480
	descending				
# of units	1	2	3	4	5
Constant	-28.4380	-31.3011	27.1133	-39.5828	-1.4110
p-value	0.6820	0.522	0.0780	0.1370	0.9080
Valuation	1.2723	1.3799	0.7797	1.8544	1.4011
p-value	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
R-squared	0.0390	0.0620	0.104	0.1100	0.1140

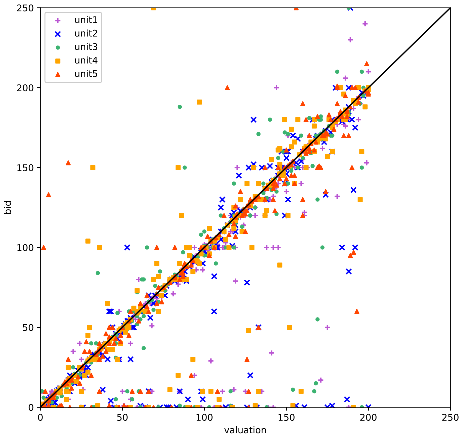

Figure 1: At random, Appearance 1.

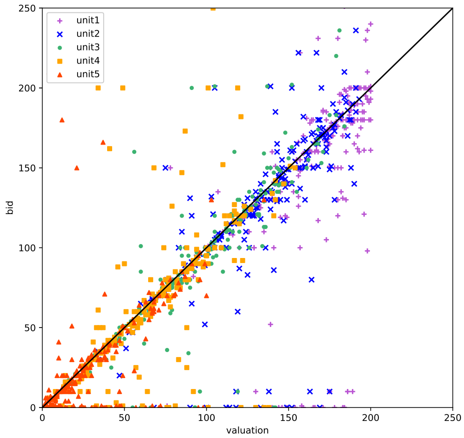

Figure 2: Descending, Appearance 1.

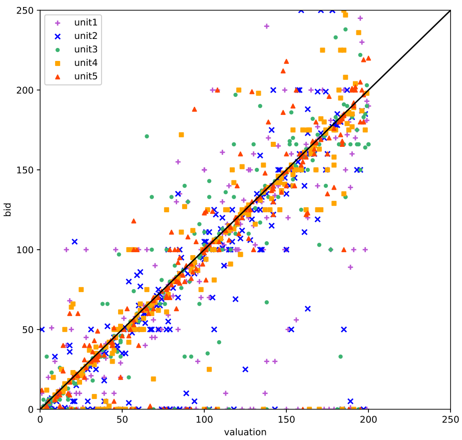

Figure 3: At random, Appearance 2.

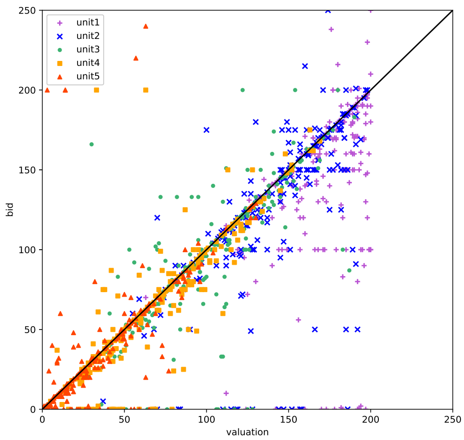

Figure 4: Descending, Appearance 2.

Observations 2 and 3 jointly imply that for each appearance of information, there was a significant difference in subjects' bidding behavior between display types of draws of unit valuations, this behavioral difference did not significantly affect the numbers of approximately efficient allocations. In this sense, the performance of the VCG mechanism is robust against display types of those draws as well.

As noted, the VCG mechanism in theory induces bidders to truthfully bid their own valuations of the item for each unit, which is a dominant strategy for every bidder. In our experiment, however, the rate of approximately truth-telling bids were less than 45% in any treatments of appearance of information and display types of draws (Table 5). We observed that subjects underbid when valuations were shown in a display type of "at random", although they did not necessarily do so when valuations were shown in a display type of "decreasing". Nevertheless, the average rate of efficiency were more than 90% in any treatments of those Table 3). These results imply that in allocative efficiency the performance of VCG may be robust against bidding behavior out of theory.

5 Final Remarks

As noted in the Introduction, there is no literature on experiments of the VCG mechanism in multi-unit auctions. Engelmann and Grimm (2009) dealt with a static Vickrey auction in an experimental environment where two units of an identical item auctioned off to two bidders, but they assumed that if each bidder places one of the two highest bids, each pays the lower bid of the other bidder. This auction rule is truly an extension of the Vickrey (second-price) auction for a single unit but is not the VCG mechanism.[5] In this paper, we confirmed that the performance of the VCG mechanism is robust against types of appearance of information and display types of draws of unit valuations.

[5] Engelmann and Grimm (2009) compared the performances of a uniform-price sealed-bid auction, a uniform-price clock auction, a discriminatory auction, a static Vickrey auction, and a dynamic Vickrey auction in the same experimental environment, including a brief but nice literature review on laboratory experiments of multi-unit auctions. Many experimental studies in multi-unit auctions were integrated in the paper. Among those studies, Kagel and Levin (2001) was a seminal paper to study the demand reduction in uniform-price auctions.

As noted in the Introduction, Takahashi et al. (2018) examined the performance of GBA only in the case of displaying draws of unit valuations to subjects without reordering them. In this paper, we studied the performance of the VCG for two display types of draws. Also for GBA, in another paper, we thus need to investigate whether different types of appearance of information associated with bidding generate different levels of allocative efficiency and sellers' revenue, because, as mentioned in the Introduction, in GBA bidders can intuitively infer how GBA allocates the item with their own bids, as compared to VCG, there was no significant difference in seller's revenue between GBA and VCG, and GBA is much faster than VCG in computation time. It was true that VCG attained higher allocative efficiency than GBA, but that the average rate of efficiency was higher than 93%.

Kagel et al. (2001) conducted an experiment in which a human bidder with a flat demand for two units competes against machine bidders each demanding a single unit, and they reported overbidding of each human bidder for both units. It is not appropriate to make a comparison with their result, but our regression analysis showed that subjects overbid for some units when each draw of unit valuations was reordered in the monotone non-increasing order. On the other hand, Chen and Takeuchi (2010) reported subjects' underbidding when the VCG was applied in combinatorial auctions. We also observed that subjects underbid when draws of unit valuations were given to subjects without reordering them. What psychological or cognitive factors induce subjects to overbid or underbid in the VCG mechanism that induces truth-telling bids as the dominant strategy? The experiment run for this paper was not designed so that we can detect those factors. This question is thus still an open question.

Notes

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Compliance with Ethical Standards

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Conflict of Interest

The authors declare that they have no conflict of interest.

Ethical Approval

All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards. This article does not contain any studies with animals performed by any of the authors.

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Appendix: Instruction[6]

Welcome to this experiment!

Thank you very much for taking the time to participate in our auction experiment. The experiment lasts for about 100 minutes, including the payment session.

At First

· Please follow the instructions given by the experiment administrators.

· Please remain silent, and do not talk to or exchange notes with other participants.

· Please do not look at what other participants are doing.

· Please do not change your position. Please do not lean in your chair.

· Please do NOT do anything other than what you are instructed to do.

· Please turn off and refrain from using your cell phones.

· Please quietly raise your hand if you have questions or need help.

[6]This is the instruction for the participants in sessions for Appearance 1 when draws of unit valuations are displayed as they are in the first 10 rounds.

In this experiment, a total of 20 auctions will be held and 5 units of a virtual item are auctioned off to 3 bidders in each auction.

Compensation

After all the 20 auctions end, a computer will randomly choose 3 auctions each from the 10 auctions in the first and second halves, that is, a total of 6 auctions. You will be compensated on the basis of the total points you earned in the 6 auctions. The final compensation will be the amount based on those points in addition to a compensation of 1500 JPY for participation.

Group Selection

At the beginning, you will be assigned an ID. Your ID will remain the same throughout the session you participate in, and it will be displayed on your computer screen. A computer will randomly make groups of 3 participants with different IDs before each auction. The participants in your group will be different in each auction and you will not be able to know who are in your group.

Please raise your hand if you have questions on the above contents.

Auction

In each auction, 5 units of an identical item are auctioned off to 3 bidders. Please bid for all units within 120 seconds. If no one in the same group bids within this time limit, all bidders in the group obtain zero points. The outcome will not be shown until the remaining time is up, even if everyone bids within the time limit.

At the beginning of each auction, each bidder is given unit valuations of the item for each unit. When the unit valuation is, e.g., 15 for 3 units, the total valuation is $15 \times 3 = 45$. You are asked to submit your unit bids for each unit. Please press the "bid" button after you fill in your unit bids on your screen. Then, a pop-up window appears and shows your total bids for each unit. If you click on the "OK" button in the pop-up window, your bids will then be sent to the server computer to compute the outcome of the auction. If you click on "cancel" button there, you can then go back to the screen to fill in your unit bids.

For each bidder, unit valuations are drawn as integers independently of those for the other bidders with equal probability between 1 and 200. Please bid in non-negative integers. The remaining time is displayed on the right upper corner of your screen. When the auction ends, the outcome is shown on your screen. The next auction will start after 5 seconds. The rule of the auction is explained next.

Please raise your hand if you have questions on the above content.

Item Allocation in the Auction

Below is an example of the auction in which 3 units of an item auctioned off to 2 bidders; In Table 8, valuations (or bids) are displayed as unit valuations (or unit bids) multiplied by the number of units. Note that the unit valuations and unit bids shown in this example do not suggest any bidding strategy in the auctions you participate in.

Table 8: Example.

# of units		1	2	3
Bidder 1	valuation	80×1	60×2	55×3
	bid	70×1	55×2	50×3
Bidder 2	valuation	40×1	70×2	65×3
	bid	40×1	60×2	65×3

The item will be allocated to bidders such that the total amount of bids is maximized as follows. Find an allocation that maximizes the total amount of bids among all possible allocations; In the example, (0, 0): 0, (1, 1): 70×1+40×1=110, (1, 0): 70×1=70, (2, 0): 55×2=110, (3, 0): 50×3=150, (0, 1): 40×1=40, (0, 2): 60×2=120, (0, 3): 65×3=195, (1, 2): 70×1+60×2=190, (2,1): 55×2+40×1 =150. Thus, this auction allocates 3 units to bidder 2. The total amount of bids is 195. When there are two or more allocation in each of which the total amount of bids is maximized, one of those allocations is chosen at random.

Payment determination in the Auction

The payments of bidders are determined as follows.

$$\begin{aligned}&\text{payment of bidder } i = \\ &\quad (\text{total amount of bids in the auction that excludes bidder } i) \\ &\qquad - (\text{total amount of bids in the auction}) \\ &\qquad + (\text{bidder } i\text{'s bid for the unit assigned to } i).\end{aligned}$$

In the example,

- payment of bidder 1 = $(65\times3) - 195 + 0 = 0$,
- payment of bidder 2 = $(50\times3) - 195 + (65\times3) = 150$.

Point

The amount of points each bidder earn is calculated as follows.

$$\begin{aligned}\text{bidder } i\text{'s points} = {} & (\text{total valuation for the units bidder } i \text{ is allocated}) \\ & - (\text{payment of bidder } i).\end{aligned}$$

In the example,

- bidder 1's points = $0 - 0 = 0$,
- bidder 2's points = $(65 \times 3) - 150 = 45$.

You will be compensated on the basis of the points you earned. The exchange rate is 1 point = 1 JPY. As mentioned, you will be compensated on the basis of the total points you earned in the 6 auctions, 3 out of the first 10 auctions and 3 out of the second 10 auctions.

Configuration

In the first 10 auctions, unit valuations are given on your computer screen and you are asked to submit unit bids there, as shown in the Example (Table 8). In the second 10 auctions, total valuations are given on your screen and you are asked to submit total bids there, as shown in Table 9.

Table 9: Another display.

# of units		1	2	3
Bidder 1	valuation	80	120	165
	bid	70	110	150
Bidder 2	valuation	40	140	195
	bid	40	120	195

Practice

At the beginning of each sequence of 10 auctions, an auction is held as a practice so that you can familiarize yourself with how to do with the computer. The points you earn in the practice auctions are not counted as those for the compensation.

Please raise your hand if you have questions.