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Introducing Complex Numbers into Basic Growth Functions – (VIII) A Series of Hypotheses for Appearance of exp(θ) from Complex Representation of '0=1+(−1)' under the Concept of Symmetry –

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Exponential functions with base e are often used for the analysis and estimation of some aspects in ruminant agriculture. The present study was designed to investigate a series of hypotheses influencing the appearance of exp(θ) from '0' using the concept of symmetry. The results obtained were as follows. There was a hypothetic appearance of '1+(−1)' from '0' if there were concepts of function, indefinite integral and symmetry. Then, '1+(−1)' was described using the product of complex numbers, in which this equality held for even when the variable took any value. Therefore, the variable θ in exp($\pm i\theta$) was replaced with $\mp i\theta$. This changed exp($i\theta$) and exp($-i\theta$) into exp($\pm\theta$) and exp($\mp\theta$), respectively, a transformation of variables from imaginary numbers into real numbers where the equality with '1+(−1)' was conserved in the new equation. The hypothetic breakdown of product from in the new equation left exp($\pm\theta$), for example. The property that the new equation held for all θs might give a hypothetic case where θ took values in ascending order, namely an increase in θ. If this hypothetic property was inherited to exp($\pm\theta$) that was left, then the growth phenomenon described using the definite integral of exp(θ) might be allowed to occur with an increase in θ, but this speculation will require further investigation.

INTRODUCTION

Investigating growth of both forages and ruminants is of great importance to ruminant agriculture. Exponential function with base e, exp(t) where t is time, is often used for basic growth analysis of forages and ruminants. It is, however, too difficult to speculate how exp(t) appears (Shimojo *et al.*, 2003c, 2004a). Therefore, exp(θ) where θ is just a

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variable has been taken up in order to investigate, under hypotheses, its appearance from exp ($\boldsymbol{i}\theta$) that was given by the breakdown of product form in the complex representation of '1' and '−1' on condition that they came from '0' (Shimojo *et al.*, 2004f, 2005a). This kind of study is considered to be a product of imagination with a lot of trials and errors, and examining properties of '0 → 1+(−1)', exp($\boldsymbol{i}\theta$), and exp(θ) with an increase in θ requires a concept of symmetry.

The present study was designed to investigate a series of hypotheses influencing the appearance of exp(θ) from the complex representation of '0=1+(−1)' using the concept of symmetry.

HYPOTHETIC APPEARACNES OF exp($\boldsymbol{i}\theta$) AND exp(θ) FROM '0=1+(−1)' UNDER THE CONCEPT OF SYMMETRY

Hypothetic appearance of '1+(−1)' from '0'

Broadly speaking at the risk of making mistakes, there might be a hypothetic appearance of '1+(−1)' from '0' if there are concepts of function, indefinite integral and symmetry (Shimojo *et al.*, 2005a). This hypothesis might be based on some beautiful mathematical phenomena as follows, where some basic numbers and state (e, $\boldsymbol{i}$, π, 0, −1, 1, ∞, etc.) are related simply. Thus,

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e,\quad \exp(\boldsymbol{i}\pi)+1=0,\quad \exp(\boldsymbol{i}\pi)=-1,\quad \exp(\boldsymbol{i}\pi)=\sum_{n=0}^{\infty}\frac{(\boldsymbol{i}\pi)^n}{n!},$$

$$z=\cos\left(\frac{2k\pi}{n}\right)+\boldsymbol{i}\sin\left(\frac{2k\pi}{n}\right)\quad (k=0,1,\cdots n-1), \tag{1}$$

where e=Napierian constant, $\boldsymbol{i}$=imaginary unit, π=circular constant, z=nth root of '1', '1'=a divisor that is common to all prime numbers.

Complex representation of '1+(−1)' and hypothetic appearance of exp($\boldsymbol{i}\theta$) from it

At the back of phenomena (1), there is exp($\boldsymbol{i}\theta$)=cosθ+$\boldsymbol{i}$sinθ, a function that is called Euler's formula. It was shown in some reports (Shimojo *et al.*, 2004b, c, d, e, f, 2005a, b, c) that '1+(−1)' was described using the product of complex numbers. Thus, for example,

$$\begin{aligned}1+(-1)=&\exp(\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))\\&\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta)\\&+\exp(\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))\\&\cdot(-(-\exp(\boldsymbol{i}\theta)))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta).\end{aligned} \tag{2}$$

Equation (2), which holds for all θs, shows naturally that a factorization of '1+(−1)' using complex numbers contains exp($\boldsymbol{i}\theta$) and its phase shifts.

Shimojo *et al.* (2004b, c, d, e, f, 2005a, b, c) showed that the hypothetic breakdown of product form in the complex representation of '1+(−1)' left complex numbers according to the way of breakdown procedures (Shimojo *et al.*, 2004e, 2005b, c). It is, however, not known whether or not the hypothetic breakdown of product form requires something like force or energy. Anyway, the following is given by this procedure when applied to equation (2). Thus,

$$\begin{aligned}&\exp(\boldsymbol{i}\theta)+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+(-\exp(-\boldsymbol{i}\theta))\\&+(-\exp(\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)\\&+\exp(\boldsymbol{i}\theta)+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+(-\exp(-\boldsymbol{i}\theta))\\&+(-(-\exp(\boldsymbol{i}\theta)))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)\\&=2\exp(\boldsymbol{i}\theta).\end{aligned} \tag{3}$$

With the increase in θ, there is an anti-clockwise rotation on a circumference with radius '1' for $\exp(\boldsymbol{i}\theta)$ and a clockwise rotation for $\exp(-\boldsymbol{i}\theta)$ when it is left. Hatamura (2004) and Ueno (2005) consider Euler's formula, $\exp(\boldsymbol{i}\theta)=\cos\theta+\boldsymbol{i}\sin\theta$, to be a kind of software that might compress a spiral into a circle. Yoshida (2000) showed a three-dimensional description of Euler's formula to give a spiral, a hypothetic decompression of $\exp(\boldsymbol{i}\theta)$ using the coordinate $(\theta, \cos\theta, \boldsymbol{i}\sin\theta)$. Subsequent trials (Shimojo *et al.*, 2003d, e, 2004a) showed that a stereographic description of coordinate $(\theta, \cos\theta, \boldsymbol{i}\sin\theta)$ associated with $\exp(\boldsymbol{i}\theta)$ gave a right-handed spiral and that of coordinate $(\theta, \cos\theta, -\boldsymbol{i}\sin\theta)$ associated with $\exp(-\boldsymbol{i}\theta)$ gave a left-handed spiral. A double-spiral structure and a triple-spiral structure were also given by combining spirals under phase shifts, provided that coordinate axes were removed in order to avoid the interference of waves (Shimojo *et al.*, 2003d, 2004a). Therefore, the complex representation of '$0=1+(-1)$' and the hypothetic appearance of $\exp(\pm\boldsymbol{i}\theta)$ from it might be associated with some basic properties of natural phenomena including those observed in ruminant agriculture.

In addition, there are some other reports where complex numbers were applied to rough descriptions of some aspects of ruminant agriculture; light receiving in forage canopy (Shimojo, 1998), digestibility changes with forage growth (Shimojo *et al.*, 1998a, b), cycling of matter in field–forage–ruminant relationships (Shimojo *et al.*, 2003a, b).

Hypothetic appearance of exp(θ) from exp(iθ)

In our previous reports, $\exp(t)$ (Shimojo *et al.*, 2003c, 2004a) or $\exp(\theta)$ (Shimojo *et al.*, 2004f, 2005a) was given by $\pi/2$ clockwise rotation [namely, $\times(-\boldsymbol{i})$] of axis in $\exp(\boldsymbol{i}t)$ or $\exp(\boldsymbol{i}\theta)$. Thus,

$$\exp(\pm\boldsymbol{i}\cdot(\boldsymbol{i}\theta))=\exp(\mp\theta), \tag{4}$$

where both upper or both lower signs should be chosen in the double signs.

The present trial will take up the property that equation (2) holds for all θs, an approach that is different from the axis rotation in equation (4). Thus, replacing every θ in equation (2) with $\mp\boldsymbol{i}\theta$ gives

$$1+(-1)=\exp(\pm\theta)\cdot\boldsymbol{i}\exp(\mp\theta)\cdot\boldsymbol{i}\exp(\pm\theta)\cdot(-\exp(\mp\theta))$$

$$\cdot(-\exp(\pm\theta))\cdot(-\boldsymbol{i}\exp(\mp\theta))\cdot(-\boldsymbol{i}\exp(\pm\theta))\cdot\exp(\mp\theta)$$
$$+\exp(\pm\theta)\cdot\boldsymbol{i}\exp(\mp\theta)\cdot\boldsymbol{i}\exp(\pm\theta)\cdot(-\exp(\mp\theta))$$
$$\cdot(-(-\exp(\pm\theta)))\cdot(-\boldsymbol{i}\exp(\mp\theta))\cdot(-\boldsymbol{i}\exp(\pm\theta))\cdot\exp(\mp\theta) \quad (5)$$

where both upper or both lower signs should be chosen in the double signs.
There are differences between equations (2) and (5) in their components, but the equality with '1 + (-1)' is conserved, a kind of symmetry for the replacement of variables.

The hypothetic breakdown of product form in equation (5) gives

$$\exp(\pm\theta)+\boldsymbol{i}\exp(\mp\theta)+\boldsymbol{i}\exp(\pm\theta)+(-\exp(\mp\theta))$$
$$+(-\exp(\pm\theta))+(-\boldsymbol{i}\exp(\mp\theta))+(-\boldsymbol{i}\exp(\pm\theta))+\exp(\mp\theta)$$
$$+\exp(\pm\theta)+\boldsymbol{i}\exp(\mp\theta)+\boldsymbol{i}\exp(\pm\theta)+(-\exp(\mp\theta))$$
$$+(-(-\exp(\pm\theta)))+(-\boldsymbol{i}\exp(\mp\theta))+(-\boldsymbol{i}\exp(\pm\theta))+\exp(\mp\theta)$$
$$=2\exp(\pm\theta). \quad (6)$$

This suggests that the appearance of $\exp(\pm\theta)$ from equation (2) is related to the following two hypotheses; (i) a kind of symmetry where the complex representation of '1 + (-1)' holds for replacing θ with $\mp\boldsymbol{i}\theta$, (ii) an occurrence of breakdown of the product form.

Hypothetic increase in θ in $\exp(\pm\theta)$

Equation (5) holds for all θs, a kind of global symmetry. This property, if we are not afraid of making mistakes, might give a hypothetic case where all θs in equation (5) take values in ascending order, namely an increase in θ that is shown as θ_1, θ_2, θ_3, $\cdots$, for example. If this property is inherited from equation (5) to equation (6), then there is a hypothetic increase in θ in $\exp(\pm\theta)$, a phenomenon of growth for exp (θ) or that of decline for $\exp(-\theta)$.

Another hypothesis that causes an increase in θ might be given by the procedure that offsets differences in θs. This is explained as follows. If all θs in equation (5) show different values one another, then the product of them is not equal to '1 + (-1)', a kind of symmetry breakdown. Thus,

$$\exp(\pm\theta_1)\cdot\boldsymbol{i}\exp(\mp\theta_2)\cdot\boldsymbol{i}\exp(\pm\theta_3)\cdot(-\exp(\mp\theta_4))$$
$$\cdot(-\exp(\pm\theta_5))\cdot(-\boldsymbol{i}\exp(\mp\theta_6))\cdot(-\boldsymbol{i}\exp(\pm\theta_7))\cdot\exp(\mp\theta_8)$$
$$+\exp(\pm\theta_9)\cdot\boldsymbol{i}\exp(\mp\theta_{10})\cdot\boldsymbol{i}\exp(\pm\theta_{11})\cdot(-\exp(\mp\theta_{12}))$$
$$\cdot(-(-\exp(\pm\theta_{13})))\cdot(-\boldsymbol{i}\exp(\mp\theta_{14}))\cdot(-\boldsymbol{i}\exp(\pm\theta_{15}))\cdot\exp(\mp\theta_{16})$$
$$\neq 1+(-1), \quad (7)$$

where both upper or both lower signs should be chosen in the double signs.

This will require, therefore, offset effects in order to recover the equality with '1 + (-1)'. Thus, for example,

$$\prod_{k=1}^{2n+1}\Big[\exp(\pm\theta_k)\cdot\boldsymbol{i}\exp(\mp\theta_k)\cdot\boldsymbol{i}\exp(\pm\theta_k)\cdot(-\exp(\mp\theta_k))$$
$$\cdot(-\exp(\pm\theta_k))\cdot(-\boldsymbol{i}\exp(\mp\theta_k))\cdot(-\boldsymbol{i}\exp(\pm\theta_k))\cdot\exp(\mp\theta_k)\Big]$$
$$+\prod_{k=1}^{2n+1}\Big[\exp(\pm\theta_k)\cdot\boldsymbol{i}\exp(\mp\theta_k)\cdot\boldsymbol{i}\exp(\pm\theta_k)\cdot(-\exp(\mp\theta_k))$$
$$\cdot(-(-\exp(\pm\theta_k)))\cdot(-\boldsymbol{i}\exp(\mp\theta_k))\cdot(-\boldsymbol{i}\exp(\pm\theta_k))\cdot\exp(\mp\theta_k)\Big]$$
$$=1+(-1), \tag{8}$$

where both upper or both lower signs should be chosen in the double signs, and k is running from '1' to '$2n+1$' (odd numbers) in order to conserve the value of '−1'.
Equation (8) shows a recovery from the inequality with '1+(-1)' that is shown in (7), a procedure that conserves a kind of local symmetry. This might give another hypothesis for an increase in θ due to the increase in θ from '1' to '$2n+1$'. If this property is conserved even when there is a hypothetic breakdown of product form in equation (8), then

$$\sum_{k=1}^{2n+1} 2\exp(\pm\theta_k), \tag{9}$$

where there is a hypothetic increase in θ in $\exp(\pm\theta)$.

What is left from the hypothetic breakdown of product form in the complex representation of '1+(-1)' is not predicted, namely the remainder (9) is not always obtained. If the following is the remainder,

$$\sum_{k=1}^{2n+1}(-2\exp(\mp\theta_k)), \tag{10}$$

then all phenomena occur in the world of negative numbers.

Definite integral of exp(θ)

The hypothetic increase in θ in $\exp(\theta)$ will lead to the calculation of definite integral of $\exp(\theta)$. Thus,

$$\int_{\theta_1}^{\theta_2}\exp(\theta)d\theta=\big[\exp(\theta)\big]_{\theta_1}^{\theta_2}$$
$$=\exp(\theta_2)-\exp(\theta_1). \tag{11}$$

Equation (11) shows a kind of growth calculation. It goes, however, without saying that the growth function takes the form of $\exp(r\cdot t)$ where r shows relative growth rate of forages or ruminants.

Pair appearances and pair disappearances of complex numbers with their opposites in the hypothetic fluctuation between '0' and '1+(−1)' in definite integral of exp(θ)

As shown by Shimojo *et al.* (2004d, e, f, 2005b, c), two sets of '1+(−1)' will appear hypothetically in the definite integral of $\exp(\theta)$ expanded into infinite series. Thus,

$$\int_{\theta_1}^{\theta_2} \exp(\theta)d\theta = \int_{\theta_1}^{\theta_2} \left(1+\frac{\theta}{1!}+\frac{\theta^2}{2!}+\cdots+\frac{\theta^n}{n!}+\cdots\right)d\theta$$

$$= \left[\frac{\theta}{1!}+\frac{\theta^2}{2!}+\cdots+\frac{\theta^n}{n!}+\cdots\right]_{\theta_1}^{\theta_2}$$

$$= \left[\left(1+\frac{\theta}{1!}+\frac{\theta^2}{2!}+\cdots+\frac{\theta^n}{n!}+\cdots\right)+(-1)\right]_{\theta_1}^{\theta_2}$$

$$= \left[\frac{\theta}{1!}+\frac{\theta^2}{2!}+\cdots+\frac{\theta^n}{n!}+\cdots\right]_{\theta_1}^{\theta_2}+\{1+(-1)\}-\{1+(-1)\}. \quad (12)$$

The hypothetic breakdown of product form in the complex representation of '1+(−1)' in equation (12) gives the following, for example,

$$\int_{\theta_1}^{\theta_2} \exp(\theta)d\theta = \left[\frac{\theta}{1!}+\frac{\theta^2}{2!}+\cdots+\frac{\theta^n}{n!}+\cdots\right]_{\theta_1}^{\theta_2}+\{1+(-1)\}-\{1+(-1)\}$$

$$= \left[\frac{\theta}{1!}+\frac{\theta^2}{2!}+\cdots+\frac{\theta^n}{n!}+\cdots\right]_{\theta_1}^{\theta_2}$$

$$+\{(2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)-2\exp(-\boldsymbol{i}\theta))+(2\exp(\boldsymbol{i}\theta))\}$$

$$-\{(2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)-2\exp(-\boldsymbol{i}\theta))+(2\exp(\boldsymbol{i}\theta))\}$$

$$= \left[\frac{\theta}{1!}+\frac{\theta^2}{2!}+\cdots+\frac{\theta^n}{n!}+\cdots\right]_{\theta_1}^{\theta_2}. \quad (13)$$

Equation (13) suggests that the growth phenomenon described by $\exp(\theta)$ with an increase in θ is accompanied hypothetically by pair appearances and pair disappearances of complex numbers with their opposites, provided that they are related to the hypothetic fluctuation between '0' and '1+(−1)' occurring in the definite integral of $\exp(\theta)$ expanded into infinite series.

Similarities and differences

It is known that $\exp(\boldsymbol{i}\theta)$ does not take the value of '0' but is related to '0' through '0=1+(−1)' when it is described using the product of complex numbers. Broadly speaking at the risk of making mistakes, '0' might be filled with complex numbers. However, this is not seen because of offset effects between complex numbers. Thus,

$$0=\prod_{k=1}^{2n+1}\Big[\exp(\boldsymbol{i}\theta_k)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta_k)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta_k)\cdot(-\exp(-\boldsymbol{i}\theta_k))$$
$$\cdot(-\exp(\boldsymbol{i}\theta_k))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta_k))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta_k))\cdot\exp(-\boldsymbol{i}\theta_k)\Big]$$
$$+\prod_{k=1}^{2n+1}\Big[\exp(\boldsymbol{i}\theta_k)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta_k)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta_k)\cdot(-\exp(-\boldsymbol{i}\theta_k))$$
$$\cdot(-(-\exp(\boldsymbol{i}\theta_k)))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta_k))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta_k))\cdot\exp(-\boldsymbol{i}\theta_k)\Big]. \quad (14)$$

The hypothetic breakdown of product form in equation (14) might allow $\exp(\boldsymbol{i}\theta_k)$ to appear, which is then followed by $\exp(\theta_k)$ that might also be allowed to appear hypothetically according to equations (8) and (9). Thus,

$$0 \xrightarrow{\text{hypothetic appearance}} \sum_{k=1}^{2n+1} 2\exp(\boldsymbol{i}\theta_k) \xrightarrow{\text{hypothetic appearance}} \sum_{k=1}^{2n+1} 2\exp(\theta_k) \qquad (15)$$

The function $\exp(\boldsymbol{i}\theta_k)$ shows a similarity for changes in θ_k, because the coordinate ($\cos\theta_k$, $\boldsymbol{i}\sin\theta_k$) is on a circumference with radius '1' for every θ_k. This suggested similarity is, however, broken in $\exp(\theta_k)$ to allow it to show an increase as θ_k increases. In other words, similarities in the world of complex numbers might break to give differences in the world of real numbers, a kind of liberation from the restriction.

The coordinate ($\cos\theta_k$, $\boldsymbol{i}\sin\theta_k$) of function $\exp(\boldsymbol{i}\theta_k)$ is on a circumference with radius '1' when θ_k takes very small and large values such as Planck's constant (h) and Einstein's energy–mass equivalence ($E=mc^2$), and even when infinitely large and small values are taken. Planck's constant is considered to be one of the factors influencing the rate of denaturation of protein under heating of milk (Ito, 2004). Einstein's energy–mass equivalence was taken up by Brody (1945a) who, in the explanation of energetics in farm animals fed feeds, wrote the principle of conservation of energy and matter that are not separable due to different expressions or measures of the same thing. It goes, however, without saying that the energetics in farm animals is related to the combustion energy of feeds (Brody, 1945a; Kleiber, 1987).

Now, let us imagine, at the risk of making mistakes, what will occur if θ in $\exp(\boldsymbol{i}\theta)$ is replaced by $E=mc^2$ that is also considered to be a beautiful equation. Thus,

$$\exp(\boldsymbol{i}\cdot mc^2)=\cos(E)+\boldsymbol{i}\sin(E). \qquad (16)$$

Taking natural logarithm of both sides of equation (16) gives

$$\log_e(\exp(\boldsymbol{i}\cdot mc^2))=\log_e(\cos(E)+\boldsymbol{i}\sin(E)). \qquad (17)$$

Then,

$$\boldsymbol{i}\cdot mc^2=\log_e(\cos(E)+\boldsymbol{i}\sin(E)),$$

namely

$$mc^2=-\boldsymbol{i}(\log_e(\cos(E)+\boldsymbol{i}\sin(E))). \qquad (18)$$

Equation (18) is also equal to

$$mc^2=\boldsymbol{i}(\log_e(\cos(E)-\boldsymbol{i}\sin(E))). \qquad (19)$$

In equation (18), the left–hand side shows the substance in the world of real numbers, and the right–hand side shows a process that the energy is liberated from the world of complex numbers by being multiplied by $-\boldsymbol{i}$, a kind of $\pi/2$ clockwise rotation. There are also equations that are mathematically symmetric to equations (18) and (19), an exchange between mc^2 and E. Thus,

$$E=\mp\boldsymbol{i}(\log_e(\cos(mc^2)\pm\boldsymbol{i}\sin(mc^2))), \qquad (20)$$

where both upper or both lower signs should be chosen in the double signs.

This hypothetic procedure might be applied to Planck's constant, though it is ridiculous due to the existence of h in both sides of the following:

$$h = \mp \boldsymbol{i}(\log_e(\cos(h) \pm \boldsymbol{i}\sin(h))), \tag{21}$$

where both upper or both lower signs should be chosen in the double signs.
This shows h not only in the real world but also in the world of complex numbers with a procedure to be liberated from it.

There is a very large difference in the value between $E=mc^2$ and h in the real world. However, both coordinates ($\cos(E)$, $\boldsymbol{i}\sin(E)$) and ($\cos(h)$, $\boldsymbol{i}\sin(h)$) are on the circumference with radius '1' on the complex plane, except for the hypothesis that E shows a large number of rotations on the circumference if there is a parallel relationship between the size of values and the number of rotations.

Examples of exponential functions with base *e* used in the area of ruminant agriculture

This section takes up some examples of exponential functions with base e that are used in the area of ruminant agriculture.
(i) Basic growth analysis of forages and ruminants

$$W = W_0 \cdot \exp(\mathrm{RGR} \cdot t), \tag{22}$$

where W_0=weight at time t_0, W=weight at time t, RGR=relative growth rate.
There are many references, for example; studies by Watson (1952), Radford (1967) and Hunt (1990) for forages, those by Brody (1945b) and Parks (1982) for ruminants.
(ii) Light interception by canopy leaves of forages

$$I_j = 100 \cdot \exp(-K \cdot F_j), \tag{23}$$

where F_j=cumulative leaf area index from the top to the jth layer of leaves, 100=relative light intensity above the canopy, I_j=relative light intensity below the jth layer of leaves, K=light extinction coefficient of the canopy.
Equation (23) was reported first by Monsi und Saeki (1953).
(iii) Degradability of protein or dry matter in the rumen

$$P = a + b \cdot (1 - \exp(-c \cdot t)), \tag{24}$$

where P=the degradation of protein or dry matter after time t, a=rapidly soluble fraction, b=degradable fraction, c=degradation rate of b.
This method using nylon bags suspended in the rumen was reported first by Ørskov and McDonald (1979) for protein degradation, and then applied to the degradation of dry matter (Ørskov, 1989).

These equations analyze some basic aspects of ruminant agriculture; how forages receive solar radiation for the photosynthesis [equation (23)], how rumen microbes degrade forages for the ruminant nutrition [equation (24)], and how forages and ruminants grow for the ruminant agriculture [equation (22)].

Conclusions and suggestions from the present study

We attempted to investigate how $\exp(\theta)$ appeared from '0' using the concept of symmetry under a series of hypotheses, but we recognize that it is too difficult to even approach it. Exponential functions with base e are considered to be only tools of convenience to analyze and estimate the production of forages and ruminants. However, these functions might exist at the back of various aspects of ruminant agriculture in order to give substantial significance to them as well as to describe them, if we would speculate about it without being afraid of making mistakes.

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