Introducing Complex Numbers into Basic Growth Functions (7) : Differences between '1' and '-1' in their Hypothetic Breakdown into Complex Numbers and Application to Definite Integral of exp(t) Expanded into Infinite Series

Shimojo, Masataka
Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Faculty of Agriculture, Kyushu University

Ikeda, Kentarou
Research Fellow, Faculty of Agriculture, Kyushu University

Asano, Yoki
Employed Research Scientist, Miyazaki University

Ishiwaka, Reiko
Research Fellow, Faculty of Agriculture, Kyushu University

他

https://doi.org/10.5109/4655

出版情報：九州大学大学院農学研究院紀要. 50 (2), pp.415-420, 2005-10-01. Faculty of Agriculture, Kyushu University

J. Fac. Agr., Kyushu Univ., **50** (2), 415–420 (2005)

Introducing Complex Numbers into Basic Growth Functions – (VII) Differences between '1' and '−1' in their Hypothetic Breakdown into Complex Numbers and Application to Definite Integral of exp(t) Expanded into Infinite Series –

Masataka SHIMOJO*, Kentarou IKEDA[1], Yoki ASANO[2], Reiko ISHIWAKA[1], Hiroyuki SATO[3], Yutaka NAKANO[4], Manabu TOBISA[5], Noriko OHBA[6], Minako EGUCHI[7] and Yasuhisa MASUDA

Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Faculty of Agriculture, Kyushu University, Fukuoka 812–8581, Japan
(Received June 30, 2005 and accepted July 26, 2005)

The present study was designed to investigate differences between '1' and '−1' in their hypothetic breakdown into complex numbers and application to definite integral of exp(t) expanded into infinite series. The results obtained were as follows. Complex numbers that were left by the hypothetic breakdown of product form in the complex representation of '1' and '−1' were classified into four groups according to properties of complex numbers; plus or minus, anti–clockwise or clockwise rotation on a circumference, hypothetic right–handed or left–handed spiral. Three groups or less showing an incomplete appearance of properties of complex numbers were derived from both '1' and '−1', but four groups that showed a complete appearance of all properties of them were derived from '1' only. This difference between '1' and '−1' might be due to the presence or absence of a minus sign. Complex numbers that were left by the hypothetic breakdown of product form in the complex representation of '1+(−1)' disappeared immediately in the calculation of growth using the definite integral of exp(t) expanded into infinite series.

INTRODUCTION

Two sets of '1+(−1)' appear hypothetically when there is a calculation of growth using the definite integral of exp(t) expanded into infinite series, and each of '1' and '−1' was described as a product of eight complex numbers (Shimojo *et al.*, 2004b, d, e, f, 2005a). A representative of complex numbers is considered to be exp($i\theta$). This is also known as Euler's formula when connected to $\cos\theta + i\sin\theta$, namely $\exp(i\theta) = \cos\theta + i\sin\theta$.

[1] Research Fellow, Faculty of Agriculture, Kyushu University
[2] Employed Research Scientist, Miyazaki University, Miyazaki 889–2192, Japan
[3] Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Graduate School of Bioresource and Bioenvironmental Sciences, Kyushu University
[4] University Farm, Faculty of Agriculture, Kyushu University
[5] Faculty of Agriculture, Miyazaki University, Miyazaki 889–2192, Japan
[6] Research Student, School of Agriculture, Kyushu University
[7] Technical Official, School of Agriculture, Kyushu University
* Corresponding author (E–mail: mshimojo@agr.kyushu–u.ac.jp)

The hypothetic breakdown of product form in the complex representation '1' and '−1' leaves complex numbers. However, this breakdown process shows differences between them due to differences in pieces of complex number given a minus sign; even–numbered pieces of complex number for '1' (Shimojo *et al.*, 2005b) and odd–numbered pieces for '−1' (Shimojo *et al.*, 2004e).

The eight complex numbers constructing '1' or '−1' have some properties; (i) plus or minus (Shimojo *et al.*, 2003e, 2004b, c, d, e, f, 2005a, b), (ii) anti–clockwise or clockwise rotation on a circumference, (iii) right–handed or left–handed spiral when complex numbers are subjected to hypothetic stereographic descriptions (Yoshida, 2000; Shimojo *et al.*, 2003d, e, 2004a)

The present study was designed to investigate differences between '1' and '−1' in the hypothetic breakdown into complex numbers, followed by comparing them that were left, using their properties of plus or minus, anti–clockwise or clockwise rotation on a circumference, and hypothetic right–handed or left–handed spiral.

DIFFERENCES BETWEEN '1' AND '−1' IN THEIR HYPOTHETIC BREAKDOWN INTO COMPLEX NUMBERS

The product of eight complex numbers gives the complex representation of '1' (Shimojo *et al.*, 2003e, 2004b, c, d, e, f, 2005a, b). Giving a minus sign to each of odd–numbered pieces of complex number out of eight constructing '1' leads to the complex representation of '−1' (Shimojo *et al.*, 2004b, d, e, f, 2005a), a kind of deriving '−1' from '1'. Therefore, we will classify the eight complex numbers that construct '1' according to the following properties; plus or minus, anti–clockwise or clockwise rotation on a circumference, hypothetic right–handed or left–handed spiral.

Classification of eight complex numbers constructing '1'

The complex representation of '1' (Shimojo *et al.*, 2003e, 2004b, c, d, e, f, 2005a, b) is given by

$$1=\exp(\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta). \quad (1)$$

Properties that eight complex numbers in equation (1) show are abbreviated as follows: plus → P, minus → M, anti–clockwise rotation on a circumference → ACR, clockwise rotation on a circumference → CR, hypothetic right–handed spiral → RS, hypothetic left–handed spiral → LS.

Applying properties to each of the eight complex numbers gives
(P, ACR, RS) for $\exp(\boldsymbol{i}\theta)$, (P, CR, LS) for $\boldsymbol{i}\exp(-\boldsymbol{i}\theta)$, (P, ACR, RS) for $\boldsymbol{i}\exp(\boldsymbol{i}\theta)$, (M, CR, LS) for $-\exp(-\boldsymbol{i}\theta)$, (M, ACR, RS) for $-\exp(\boldsymbol{i}\theta)$, (M, CR, LS) for $-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)$, (M, ACR, RS) for $-\boldsymbol{i}\exp(\boldsymbol{i}\theta)$, (P, CR, LS) for $\exp(-\boldsymbol{i}\theta)$.
Thus, each of P, M, ACR, CR, RS and LS appears four times.

The hypothetic breakdown of product form in the complex representation of '1' leaves, for example, $2\exp(\boldsymbol{i}\theta)+2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)$, as shown below.

$$\exp(\boldsymbol{i}\theta)+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+(-\exp(-\boldsymbol{i}\theta))$$

$$+(-(-\exp(\boldsymbol{i}\theta)))+(-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)$$

$$=2\exp(\boldsymbol{i}\theta)+2\boldsymbol{i}\exp(-\boldsymbol{i}\theta). \quad (2)$$

Therefore, the eight complex numbers that are left are classified into four groups; (i) (P, ACR, RS): $2\exp(\boldsymbol{i}\theta)$ and $2\boldsymbol{i}\exp(\boldsymbol{i}\theta)$, (ii) (P, CR, LS): $2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)$ and $2\exp(-\boldsymbol{i}\theta)$, (iii) (M, ACR, RS): $-2\exp(\boldsymbol{i}\theta)$ and $-2\boldsymbol{i}\exp(\boldsymbol{i}\theta)$, (iv) (M, CR, LS): $-2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)$ and $-2\exp(-\boldsymbol{i}\theta)$. These four groups show the following properties. In each group, there are two sorts of complex numbers that are protected from offset effects by taking a phase shift of $\boldsymbol{i}$. There are offset effects between groups (i) and (iii), and between groups (ii) and (iv). There are opposite or inverse properties between groups (i) and (iv), and between groups (ii) and (iii). There are conjugate relationships between (i) and (ii), and between (iii) and (iv).

Differences between '1' and '–1' in their hypothetic breakdown into complex numbers

The hypothetic breakdown of product from in the complex representation of '1' leaves zero, two or four sorts of complex numbers (Shimojo *et al.*, 2005b), and that of '–1' leaves one or three sorts of complex numbers (Shimojo *et al.*, 2004e). These are given by choosing groups out of the four with excluding offset cases.

Cases of '1'

(A) Two sorts of complex numbers are given by procedures A1 and A2.
- (A1) Choosing one group to take both complex numbers
- (A2) Choosing two groups to take either of the two complex numbers from each group

(B) Four sorts of complex numbers are given by procedures B1, B2 and B3.
- (B1) Choosing two groups to take both complex numbers from each group
- (B2) Choosing one group to take both complex numbers and choosing another two groups to take either of the two complex numbers from each group
- (B3) Choosing four groups to take either of the two complex numbers from each group

Cases of '–1'

(C) One sort of complex number is given by a procedure C1.
- (C1) Choosing one group to take either of the two complex numbers

(D) Three sorts of complex numbers are given by procedures D1 and D2.
- (D1) Choosing one group to take both complex numbers and choosing another group to take either of the two complex numbers
- (D2) Choosing three groups to take either of the two complex numbers from each group

Groups that are left by hypothetic breakdown of '1' and '–1' into complex numbers

This section takes up procedures that cause groups to be left by the hypothetic breakdown of '1' and '–1' into complex numbers. This is summarized as follows.

One group that is left

This is given by the procedure A1 for '1' and the procedure C1 for '−1'.

Two groups that are left

This is given by procedures A2 and B1 for '1' and the procedure D1 for '−1'.

Three groups that are left

This is given by the procedure B2 for '1' and the procedure D2 for '−1'.

Four groups that are left

This is given by the procedure B3 for '1'.

Leaving three groups or less is found in both '1' and '−1' when there is the hypothetic breakdown of product form in the complex representation of them. This is associated with a kind of asymmetry where there is a different frequency of appearances between P and M, between ACR and RC, and between RS and LS, except that some cases for '1' show the same frequency of appearances when two groups are left. In other words, there is an incomplete appearance of properties that complex numbers show. However, the procedure B3 breaks hypothetically '1' to leave four groups of complex numbers, a kind of symmetry where there is a complete appearance of all properties of them.

The present study shows that there are eight procedures for leaving complex numbers; five for '1' and three for '−1'. Four procedures out of five for '1' and all three procedures for '−1' seem to be similar because they give an incomplete appearance of properties of complex numbers, in contrast to the remaining one procedure for '1' that gives a complete appearance of all properties of them. These similarities with one difference between '1' and '−1' might be caused by the same absolute value with presence or absence of a minus sign. The appearance of '1' from '0' requires the simultaneous appearance of '−1', namely '$0 \rightarrow 1+(-1)$'. There is an offset effect between '1' and '−1', but describing them as a product of complex numbers might show asymmetric and symmetric properties if complex numbers are liberated hypothetically from the product form.

Relationships to basic growth functions

An example of basic growth functions is given by

$$\begin{aligned}
\exp(t_2)-\exp(t_1) &= \int_{t_1}^{t_2} \exp(t)dt \\
&= \int_{t_1}^{t_2} \left(1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right) dt \\
&= \left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2} \\
&= \left[\left(1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right)-1\right]_{t_1}^{t_2} \\
&= \left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2}+\{1+(-1)\}-\{1+(-1)\}. \quad (3)
\end{aligned}$$

where it goes, however, without saying that the growth function takes the form of $\exp(r \cdot t)$ where r shows relative growth rate of forages or ruminants.

Equation (3) suggests that the invariant form of exp(t) under its definite integral requires hypothetically the appearance of two sets of '1+(−1)' from '0', followed immediately by their disappearances. This is considered to be a kind of fluctuation between '0' and '1+(−1)' that occurs hypothetically in the calculation of growth using exp(t) expanded into infinite series (Shimojo *et al.*, 2004d). Describing '1+(−1)' as the product of complex numbers and the hypothetic breakdown of it leave complex numbers, but they disappear immediately (Shimojo *et al.*, 2004d, e, 2005b). In other words, there are hypothetic pair appearances and pair disappearances of complex numbers through the fluctuation between '0' and '1+(−1)' occurring hypothetically in the calculation of growth. These hypothetic phenomena that do not influence the actual calculation might be allowed to occur.

Relationships to complex representation of some aspects of ruminant agriculture

Complex numbers that are used to describe some aspects of ruminant agriculture (Shimojo, 1998; Shimojo *et al.*, 1998a, b, 2003a, b, c, d, e, 2004a) might come from the hypothetic breakdown of product form in the complex representation of '1' and/or '−1'. Since '1' and '−1' appear from '0', complex numbers might also come from '0', if we speculate it without being afraid of making mistakes. However, this does not sound strange, because one does not ask where complex numbers come from when one uses them. One takes them out of nothing without considering or following any procedure.

Conclusions

Complex numbers that are left by the hypothetic breakdown of product form in the complex representation of '1' and '−1' are classified into four groups. There are eight procedures that leave groups of complex numbers. Three groups or less are derived from both '1' and '−1' by seven procedures, and the remaining one procedure derives four groups from '1'. This difference between '1' and '−1' might be due to the presence or absence of a minus sign. Complex numbers that are left by the hypothetic breakdown of product form in the complex representation of '1+(−1)' disappear immediately in the calculation of growth using the definite integral of exp(t) expanded into infinite series.

REFERENCES

Shimojo, M. 1998 A rough image of the relationship between light extinction and leaf inclination in plant canopy. *Proc. 8th World Conf. Anim. Prod.*, Seoul, Vol. II, 506–507

Shimojo, M., T. Bungo, Y. Imura, M. Tobisa, N. Koga, S. Tao, M. Yunus, Y. Nakano, I. Goto, M. Furuse and Y. Masuda 1998a Digestibility decrease with forage growth as interpreted using complex plane. *Proc. 8th World Conf. Anim. Prod*, Seoul, Vol. II, 514–515

Shimojo, M., T. Bungo, Y. Imura, M. Tobisa, N. Koga, S. Tao, M. Yunus, Y. Nakano, I. Goto, M. Furuse and Y. Masuda 1998b Use of complex number in the analysis of increase in dry matter indigestibility with growth of forages. *J. Fac. Agr., Kyushu Univ.*, **43**: 137–142

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003a A symbolic representation of field–forage–ruminant relationships using polar form on the complex plane. *J. Fac. Agr., Kyushu Univ.*, **47**: 359–366

Shimojo, M., Y. Asano, K. Ikeda, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003b Complex representation of field–forage–ruminant relationships using symmetric properties of Euler's formula. *J. Fac. Agr., Kyushu Univ.*, **47**: 367–372

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003c Exponential functions with base e in growth analysis and deriving them from rotations of axes of time described using Euler's formula. *J. Fac. Agr., Kyushu Univ.*, **48**: 65–69

Shimojo, M., Y. Asano, K. Ikeda, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003d A stereographic representation of Euler's formula to show spirals and topological similarities to micro–structures in ruminants and forages. *J. Fac. Agr., Kyushu Univ.*, **48**: 71–75

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003e Philosophical properties of Euler's formula in its application to symbolic representation of some aspects of ruminant agriculture. *J. Fac. Agr., Kyushu Univ.*, **48**: 77–83

Shimojo, M., Y. Asano, K. Ikeda, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano and Y. Masuda 2004a Simple descriptions of some micro– and macro–structures in ruminant agriculture. *In* "Biotechnology of Lignocellulose Degradation and Biomass Utilization", ed. by K. Ohmiya, K. Sakka, S. Karita, K. Kimura, M. Sakka and Y. Onishi, Uni Publishers Co. Ltd., Tokyo, pp. 325

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2004b Hypothetic incomplete pair disappearances of complex numbers forming '0=(–1)+1' in the application to complex representation of some aspects of ruminant agriculture. *J. Fac. Agr., Kyushu Univ.*, **49**: 61–67

Shimojo, M., K. Ikeda, R. Ishiwaka, H. Sato, Y. Asano, M. Tobisa, Y. Nakano, N. Ohba, M. Eguchi and Y. Masuda 2004c Introducing complex numbers into basic growth functions – (I) Applying complex representation of '1' to differentiation of exponential function with base e expanded into infinite series –. *J. Fac. Agr., Kyushu Univ.*, **49**: 331–335

Shimojo, M., K. Ikeda, R. Ishiwaka, H. Sato, Y. Asano, M. Tobisa, Y. Nakano, N. Ohba, M. Eguchi and Y. Masuda 2004d Introducing complex numbers into basic growth functions – (II) Applying complex representation of '(–1)+1' to definite integral of exponential function with base e expanded into infinite series –. *J. Fac. Agr., Kyushu Univ.*, **49**: 337–341

Shimojo, M., K. Ikeda, R. Ishiwaka, H. Sato, Y. Asano, M. Tobisa, Y. Nakano, N. Ohba, M. Eguchi and Y. Masuda 2004e Introducing complex numbers into basic growth functions – (III) Symmetry breakdown in complex representation of '0=(–1)+1' in definite integral of exponential function with base e expanded into infinite series –. *J. Fac. Agr., Kyushu Univ.*, **49**: 343–348

Shimojo, M., K. Ikeda, R. Ishiwaka, H. Sato, Y. Asano, M. Tobisa, Y. Nakano, N. Ohba, M. Eguchi and Y. Masuda 2004f Introducing complex numbers into basic growth functions – (IV) Hypothetic appearance of exponential function with base e from the complex representation of '0=(–1)+1' –. *J. Fac. Agr., Kyushu Univ.*, **49**: 349–353

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, M. Eguchi and Y. Masuda 2005a Introducing complex numbers into basic growth functions – (V) Hypothetic factors influencing increase in θ in $\exp(\theta)$ –. *J. Fac. Agr., Kyushu Univ.*, **50**: 135–139

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, H. Sato, Y. Nakano, M. Tobisa, N. Ohba, M. Eguchi and Y. Masuda 2005b Introducing complex numbers into basic growth functions – (VI) Hypothetic breakdown of '1' into complex numbers and application to definite integral of $\exp(t)$ expanded into infinite series – *J. Fac. Agr., Kyushu Univ.*, **50**: 407–414

Yoshida, T. 2000 Emotion for imaginary number. Tokai University Press, Tokyo. (written in Japanese)