

# Introducing Complex Numbers into Basic Growth Functions (7) : Differences between '1' and '-1' in their Hypothetic Breakdown into Complex Numbers and Application to Definite Integral of $\exp(t)$ Expanded into Infinite Series

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**Introducing Complex Numbers into Basic Growth Functions  
– (VII) Differences between ‘1’ and ‘–1’ in their Hypothetic  
Breakdown into Complex Numbers and Application  
to Definite Integral of  $\exp(t)$  Expanded  
into Infinite Series –**

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The present study was designed to investigate differences between ‘1’ and ‘–1’ in their hypothetic breakdown into complex numbers and application to definite integral of  $\exp(t)$  expanded into infinite series. The results obtained were as follows. Complex numbers that were left by the hypothetic breakdown of product form in the complex representation of ‘1’ and ‘–1’ were classified into four groups according to properties of complex numbers; plus or minus, anti-clockwise or clockwise rotation on a circumference, hypothetic right-handed or left-handed spiral. Three groups or less showing an incomplete appearance of properties of complex numbers were derived from both ‘1’ and ‘–1’, but four groups that showed a complete appearance of all properties of them were derived from ‘1’ only. This difference between ‘1’ and ‘–1’ might be due to the presence or absence of a minus sign. Complex numbers that were left by the hypothetic breakdown of product form in the complex representation of ‘1 + (–1)’ disappeared immediately in the calculation of growth using the definite integral of  $\exp(t)$  expanded into infinite series.

## INTRODUCTION

Two sets of ‘1 + (–1)’ appear hypothetically when there is a calculation of growth using the definite integral of  $\exp(t)$  expanded into infinite series, and each of ‘1’ and ‘–1’ was described as a product of eight complex numbers (Shimojo *et al.*, 2004b, d, e, f, 2005a). A representative of complex numbers is considered to be  $\exp(i\theta)$ . This is also known as Euler’s formula when connected to  $\cos\theta + i\sin\theta$ , namely  $\exp(i\theta) = \cos\theta + i\sin\theta$ .

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The hypothetic breakdown of product form in the complex representation '1' and '-1' leaves complex numbers. However, this breakdown process shows differences between them due to differences in pieces of complex number given a minus sign; even-numbered pieces of complex number for '1' (Shimojo *et al.*, 2005b) and odd-numbered pieces for '-1' (Shimojo *et al.*, 2004e).

The eight complex numbers constructing '1' or '-1' have some properties; (i) plus or minus (Shimojo *et al.*, 2003e, 2004b, c, d, e, f, 2005a, b), (ii) anti-clockwise or clockwise rotation on a circumference, (iii) right-handed or left-handed spiral when complex numbers are subjected to hypothetic stereographic descriptions (Yoshida, 2000; Shimojo *et al.*, 2003d, e, 2004a)

The present study was designed to investigate differences between '1' and '-1' in the hypothetic breakdown into complex numbers, followed by comparing them that were left, using their properties of plus or minus, anti-clockwise or clockwise rotation on a circumference, and hypothetic right-handed or left-handed spiral.

#### DIFFERENCES BETWEEN '1' AND '-1' IN THEIR HYPOTHETIC BREAKDOWN INTO COMPLEX NUMBERS

The product of eight complex numbers gives the complex representation of '1' (Shimojo *et al.*, 2003e, 2004b, c, d, e, f, 2005a, b). Giving a minus sign to each of odd-numbered pieces of complex number out of eight constructing '1' leads to the complex representation of '-1' (Shimojo *et al.*, 2004b, d, e, f, 2005a), a kind of deriving '-1' from '1'. Therefore, we will classify the eight complex numbers that construct '1' according to the following properties; plus or minus, anti-clockwise or clockwise rotation on a circumference, hypothetic right-handed or left-handed spiral.

#### **Classification of eight complex numbers constructing '1'**

The complex representation of '1' (Shimojo *et al.*, 2003e, 2004b, c, d, e, f, 2005a, b) is given by

$$1 = \exp(i\theta) \cdot i\exp(-i\theta) \cdot i\exp(i\theta) \cdot (-\exp(-i\theta)) \\ \cdot (-\exp(i\theta)) \cdot (-i\exp(-i\theta)) \cdot (-i\exp(i\theta)) \cdot \exp(-i\theta). \quad (1)$$

Properties that eight complex numbers in equation (1) show are abbreviated as follows: plus  $\rightarrow$  P, minus  $\rightarrow$  M, anti-clockwise rotation on a circumference  $\rightarrow$  ACR, clockwise rotation on a circumference  $\rightarrow$  CR, hypothetic right-handed spiral  $\rightarrow$  RS, hypothetic left-handed spiral  $\rightarrow$  LS.

Applying properties to each of the eight complex numbers gives (P, ACR, RS) for  $\exp(i\theta)$ , (P, CR, LS) for  $i\exp(-i\theta)$ , (P, ACR, RS) for  $i\exp(i\theta)$ , (M, CR, LS) for  $-\exp(-i\theta)$ , (M, ACR, RS) for  $-\exp(i\theta)$ , (M, CR, LS) for  $-i\exp(-i\theta)$ , (M, ACR, RS) for  $-i\exp(i\theta)$ , (P, CR, LS) for  $\exp(-i\theta)$ .

Thus, each of P, M, ACR, CR, RS and LS appears four times.

The hypothetic breakdown of product form in the complex representation of '1' leaves, for example,  $2\exp(i\theta) + 2i\exp(-i\theta)$ , as shown below.

$$\exp(i\theta) + i\exp(-i\theta) + i\exp(i\theta) + (-\exp(-i\theta))$$

$$\begin{aligned}
& + (-(-\exp(i\theta))) + (-(-i\exp(-i\theta))) + (-i\exp(i\theta)) + \exp(-i\theta) \\
& = 2\exp(i\theta) + 2i\exp(-i\theta). \tag{2}
\end{aligned}$$

Therefore, the eight complex numbers that are left are classified into four groups; (i) (P, ACR, RS):  $2\exp(i\theta)$  and  $2i\exp(i\theta)$ , (ii) (P, CR, LS):  $2i\exp(-i\theta)$  and  $2\exp(-i\theta)$ , (iii) (M, ACR, RS):  $-2\exp(i\theta)$  and  $-2i\exp(i\theta)$ , (iv) (M, CR, LS):  $-2i\exp(-i\theta)$  and  $-2\exp(-i\theta)$ . These four groups show the following properties. In each group, there are two sorts of complex numbers that are protected from offset effects by taking a phase shift of  $i$ . There are offset effects between groups (i) and (iii), and between groups (ii) and (iv). There are opposite or inverse properties between groups (i) and (iv), and between groups (ii) and (iii). There are conjugate relationships between (i) and (ii), and between (iii) and (iv).

### Differences between '1' and '-1' in their hypothetic breakdown into complex numbers

The hypothetic breakdown of product from in the complex representation of '1' leaves zero, two or four sorts of complex numbers (Shimojo *et al.*, 2005b), and that of '-1' leaves one or three sorts of complex numbers (Shimojo *et al.*, 2004e). These are given by choosing groups out of the four with excluding offset cases.

#### Cases of '1'

- (A) Two sorts of complex numbers are given by procedures A1 and A2.
  - (A1) Choosing one group to take both complex numbers
  - (A2) Choosing two groups to take either of the two complex numbers from each group
- (B) Four sorts of complex numbers are given by procedures B1, B2 and B3.
  - (B1) Choosing two groups to take both complex numbers from each group
  - (B2) Choosing one group to take both complex numbers and choosing another two groups to take either of the two complex numbers from each group
  - (B3) Choosing four groups to take either of the two complex numbers from each group

#### Cases of '-1'

- (C) One sort of complex number is given by procedure C1.
  - (C1) Choosing one group to take either of the two complex numbers
- (D) Three sorts of complex numbers are given by procedures D1 and D2.
  - (D1) Choosing one group to take both complex numbers and choosing another group to take either of the two complex numbers
  - (D2) Choosing three groups to take either of the two complex numbers from each group

### Groups that are left by hypothetic breakdown of '1' and '-1' into complex numbers

This section takes up procedures that cause groups to be left by the hypothetic breakdown of '1' and '-1' into complex numbers. This is summarized as follows.

*One group that is left*

This is given by the procedure A1 for '1' and the procedure C1 for '-1'.

*Two groups that are left*

This is given by procedures A2 and B1 for '1' and the procedure D1 for '-1'.

*Three groups that are left*

This is given by the procedure B2 for '1' and the procedure D2 for '-1'.

*Four groups that are left*

This is given by the procedure B3 for '1'.

Leaving three groups or less is found in both '1' and '-1' when there is the hypothetic breakdown of product form in the complex representation of them. This is associated with a kind of asymmetry where there is a different frequency of appearances between P and M, between ACR and RC, and between RS and LS, except that some cases for '1' show the same frequency of appearances when two groups are left. In other words, there is an incomplete appearance of properties that complex numbers show. However, the procedure B3 breaks hypothetically '1' to leave four groups of complex numbers, a kind of symmetry where there is a complete appearance of all properties of them.

The present study shows that there are eight procedures for leaving complex numbers; five for '1' and three for '-1'. Four procedures out of five for '1' and all three procedures for '-1' seem to be similar because they give an incomplete appearance of properties of complex numbers, in contrast to the remaining one procedure for '1' that gives a complete appearance of all properties of them. These similarities with one difference between '1' and '-1' might be caused by the same absolute value with presence or absence of a minus sign. The appearance of '1' from '0' requires the simultaneous appearance of '-1', namely '0 → 1 + (-1)'. There is an offset effect between '1' and '-1', but describing them as a product of complex numbers might show asymmetric and symmetric properties if complex numbers are liberated hypothetically from the product form.

**Relationships to basic growth functions**

An example of basic growth functions is given by

$$\begin{aligned}
 \exp(t_2) - \exp(t_1) &= \int_{t_1}^{t_2} \exp(t) dt \\
 &= \int_{t_1}^{t_2} \left( 1 + \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right) dt \\
 &= \left[ \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right]_{t_1}^{t_2} \\
 &= \left[ \left( 1 + \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right) - 1 \right]_{t_1}^{t_2} \\
 &= \left[ \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right]_{t_1}^{t_2} + (1 + (-1)) - (1 + (-1)). \quad (3)
 \end{aligned}$$

where it goes, however, without saying that the growth function takes the form of  $\exp(r \cdot t)$  where  $r$  shows relative growth rate of forages or ruminants.

Equation (3) suggests that the invariant form of  $\exp(t)$  under its definite integral requires hypothetically the appearance of two sets of '1+(-1)' from '0', followed immediately by their disappearances. This is considered to be a kind of fluctuation between '0' and '1+(-1)' that occurs hypothetically in the calculation of growth using  $\exp(t)$  expanded into infinite series (Shimojo *et al.*, 2004d). Describing '1+(-1)' as the product of complex numbers and the hypothetic breakdown of it leave complex numbers, but they disappear immediately (Shimojo *et al.*, 2004d, e, 2005b). In other words, there are hypothetic pair appearances and pair disappearances of complex numbers through the fluctuation between '0' and '1+(-1)' occurring hypothetically in the calculation of growth. These hypothetic phenomena that do not influence the actual calculation might be allowed to occur.

### Relationships to complex representation of some aspects of ruminant agriculture

Complex numbers that are used to describe some aspects of ruminant agriculture (Shimojo, 1998; Shimojo *et al.*, 1998a, b, 2003a, b, c, d, e, 2004a) might come from the hypothetic breakdown of product form in the complex representation of '1' and/or '-1'. Since '1' and '-1' appear from '0', complex numbers might also come from '0', if we speculate it without being afraid of making mistakes. However, this does not sound strange, because one does not ask where complex numbers come from when one uses them. One takes them out of nothing without considering or following any procedure.

### Conclusions

Complex numbers that are left by the hypothetic breakdown of product form in the complex representation of '1' and '-1' are classified into four groups. There are eight procedures that leave groups of complex numbers. Three groups or less are derived from both '1' and '-1' by seven procedures, and the remaining one procedure derives four groups from '1'. This difference between '1' and '-1' might be due to the presence or absence of a minus sign. Complex numbers that are left by the hypothetic breakdown of product form in the complex representation of '1+(-1)' disappear immediately in the calculation of growth using the definite integral of  $\exp(t)$  expanded into infinite series.

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