# Introducing Complex Numbers into Basic Growth Functions (6) : Hypothetic Breakdown of '1' into Complex Numbers and Application to Definite Integral of exp(t) Expanded into Infinite Series

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# Introducing Complex Numbers into Basic Growth Functions – (VI) Hypothetic Breakdown of '1' into Complex Numbers and Application to Definite Integral of exp($t$) Expanded into Infinite Series –

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The definite integral of exp($t$) expanded into infinite series included two sets of '1+(–1)' in the calculation of growth, and each of '1' and '–1' was described using the product of eight complex numbers. The present study was designed under hypotheses to leave complex numbers by the breakdown of product form in the complex representation of '1', where a minus sign was given to each of even–numbered pieces of complex number out of eight in order to conserve the value of '1'. This was followed by inserting them into the calculation of growth using exp($t$). The results obtained were as follows. The value of '1' was constructed by the product of complex numbers that were different in part from those used in the primary description, which was caused by giving a minus sign to each of even–numbered (2, 4, 6, 8) pieces of complex number. The hypothetic breakdown of product form in the complex representation of '1' left zero, two or four sorts of complex numbers. The complex numbers were left in more pieces when a minus sign was given to each of the four pieces of complex number than when given to each of the two, six or eight pieces. Inserting complex numbers that were left into exp($t$) did not influence the calculation of growth, suggesting that the definite integral of exp($t$) expanded into infinite series was accompanied hypothetically by pair appearances and pair disappearances of complex numbers with their opposites.

## INTRODUCTION

There are some attempts where the complex number [$\exp(\boldsymbol{i}\theta)=\cos\theta+\boldsymbol{i}\sin\theta$] was applied to descriptions of some aspects of ruminant agriculture; (1) a rough relationship between light extinction and leaf inclination in forage canopy (Shimojo, 1998), (2)

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digestibility changes with forage growth expressed on the complex plane (Shimojo *et al.*, 1998a, b), (3) rotation properties of $\exp(\boldsymbol{i}\theta)$ for describing the cycling of matter in field–forage–ruminant relationships (Shimojo *et al.*, 2003a, b), (4) rotating the axis of $\exp(\boldsymbol{i}t)$ to give $\exp(t)$ used for growth functions (Shimojo *et al.*, 2003c), (5) hypothetic spiral properties of $\exp(\boldsymbol{i}\theta)$ similar to micro–structures in living things (Shimojo *et al.*, 2003d), (6) philosophy in the use of complex representation of ruminant agriculture (Shimojo *et al.*, 2003e), (7) a hypothetic way to derive complex numbers from the complex representation of '$0=(-1)+1$' where both '$-1$' and '1' were described using the product of eight complex numbers (Shimojo *et al.*, 2004b), and (8) introducing complex numbers into the differentiation and definite integral of $\exp(t)$ expanded into infinite series (Shimojo *et al.*, 2004c, d, e, f, 2005). Although these reports include many trials and errors, the complex number that is known as Euler's formula seems to have a potential, through speculations under hypotheses, for giving rough images of some aspects of ruminant agriculture (Shimojo *et al.*, 2004a).

Two reports (Shimojo *et al.*, 2004d, e) showed that the definite integral of $\exp(t)$ expanded into infinite series included two sets of '$(-1)+1$'. The complex representation of '$-1$' was given more attention than that of '1', because the hypothetic breakdown of product form in the eight complex numbers constructing '$-1$' left odd–numbered pieces of complex number (Shimojo *et al.*, 2004b, d, e, f, 2005). At first we did not consider that complex numbers were left in the case of '1' (Shimojo *et al.*, 2003e, 2004b, c, d, e, f, 2005), but we have noticed that there might be ways to leave complex numbers from the breakdown of complex representation of '1'. This is explained as follows. Giving a minus sign to each of even–numbered pieces of complex number does not influence the value of '1', and the hypothetic breakdown of product form might be expected to leave complex numbers.

This study was designed under hypotheses to leave complex numbers by the breakdown of product form in the complex representation of '1', followed by inserting them into the calculation of growth using the definite integral of $\exp(t)$ expanded into infinite series.

## HYPOTHETIC BREAKDOWN OF COMPLEX REPRESENTATION OF '1' TO LEAVE COMPLEX NUMBERS

### Giving a minus sign to each of even–numbered pieces of complex number out of eight constructing '1'

The complex representation of '1' (Shimojo *et al.*, 2003e, 2004b, c, d, e, f, 2005) is given by

$$1=\exp(\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta). \quad (1)$$

Equation (1) is composed of four different pairs of a complex number and its opposite, and in addition, eight complex numbers are related through phase shifts. This is the primary description that forms one group, namely ${}_8C_0=1$, where no complex number is given an additional minus sign.

*Giving a minus sign to each of the two complex numbers out of eight*

The combination of two from eight is given by

$$ {}_8C_2 = 28. \tag{2} $$

The 28 cases are divided into two groups; (i) the group composed of a complex number from each of the two different pairs, (ii) the group composed of a pair of complex number and its opposite.

(i) The group composed of a complex number from each of the two different pairs

The number of cases in group (i) is given by

$$ ({}_4C_2)\cdot({}_2C_1)\cdot({}_2C_1) = 24. \tag{3} $$

An example is given by

$$ \begin{aligned} 1 = &(-\exp(\boldsymbol{i}\theta))\cdot \boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot \boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta)) \\ &\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta). \end{aligned} \tag{4} $$

The hypothetic breakdown of product form in (4) gives

$$ \begin{aligned} &(-\exp(\boldsymbol{i}\theta)) + \boldsymbol{i}\exp(-\boldsymbol{i}\theta) + \boldsymbol{i}\exp(\boldsymbol{i}\theta) + (-\exp(-\boldsymbol{i}\theta)) \\ &+(-\exp(\boldsymbol{i}\theta)) + (-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))) + (-\boldsymbol{i}\exp(\boldsymbol{i}\theta)) + \exp(-\boldsymbol{i}\theta) \\ &= -2\exp(\boldsymbol{i}\theta) + 2\boldsymbol{i}\exp(-\boldsymbol{i}\theta). \end{aligned} \tag{5} $$

(ii) The group composed of a pair of complex number and its opposite

The number of cases in group (ii) is given by

$$ {}_4C_1 = 4. \tag{6} $$

An example is given by

$$ \begin{aligned} 1 = &\exp(\boldsymbol{i}\theta)\cdot \boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-\exp(-\boldsymbol{i}\theta)) \\ &\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta)))\cdot\exp(-\boldsymbol{i}\theta). \end{aligned} \tag{7} $$

The hypothetic breakdown of product form in (7) gives

$$ \begin{aligned} &\exp(\boldsymbol{i}\theta) + \boldsymbol{i}\exp(-\boldsymbol{i}\theta) + (-\boldsymbol{i}\exp(\boldsymbol{i}\theta)) + (-\exp(-\boldsymbol{i}\theta)) \\ &+(-\exp(\boldsymbol{i}\theta)) + (-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)) + (-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))) + \exp(-\boldsymbol{i}\theta) \\ &= 0. \end{aligned} \tag{8} $$

Equation (8) shows that choosing a pair does not make changes due to the exchange between plus and minus signs [$\boldsymbol{i}\exp(\boldsymbol{i}\theta) \rightarrow -\boldsymbol{i}\exp(\boldsymbol{i}\theta)$; $-\boldsymbol{i}\exp(\boldsymbol{i}\theta) \rightarrow \boldsymbol{i}\exp(\boldsymbol{i}\theta)$], namely nothing occurs.

*Giving a minus sign to each of the four complex numbers out of eight*

The combination of four from eight is given by

$$ {}_8C_4 = 70. \tag{9} $$

The 70 cases are divided into three groups; (i) the group composed of a complex number from each of the four different pairs, (ii) the group composed of a pair and a complex number from each of the two different pairs out of the remaining three, (iii) the group composed of two different pairs.

(i) The group composed of a complex number from each of the four different pairs

The number of cases in group (i) is given by

$$({}_2C_1)\cdot({}_2C_1)\cdot({}_2C_1)\cdot({}_2C_1)=16. \tag{10}$$

An example is given by

$$1=(-\exp(\boldsymbol{i}\theta))\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-\exp(-\boldsymbol{i}\theta))$$
$$\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-\exp(-\boldsymbol{i}\theta)). \tag{11}$$

The hypothetic breakdown of product form in (11) gives

$$-2\exp(\boldsymbol{i}\theta)+2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)-2\boldsymbol{i}\exp(\boldsymbol{i}\theta)-2\exp(-\boldsymbol{i}\theta). \tag{12}$$

(ii) The group composed of a pair and a complex number from each of the two different pairs out of the remaining three

The number of cases in group (ii) is given by

$$({}_4C_1)\cdot({}_3C_2)\cdot({}_2C_1)\cdot({}_2C_1)=48. \tag{13}$$

An example is given by

$$1=(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))$$
$$\cdot(-(-\exp(\boldsymbol{i}\theta)))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta)))\cdot\exp(-\boldsymbol{i}\theta). \tag{14}$$

The hypothetic breakdown of product form in (14) gives

$$-2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+2\boldsymbol{i}\exp(\boldsymbol{i}\theta). \tag{15}$$

(iii) The group composed of two different pairs

The number of cases in group (iii) is given by

$${}_4C_2=6. \tag{16}$$

An example is given by

$$1=(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))$$
$$\cdot(-(-\exp(\boldsymbol{i}\theta)))\cdot(-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta). \tag{17}$$

The hypothetic breakdown of product form in (17) gives

$$0. \tag{18}$$

Choosing two different pairs causes the exchange between plus and minus signs in each pair, resulting in that nothing occurs.

*Giving a minus sign to each of the six complex numbers out of eight*

The combination of six from eight is given by

$$_8C_6=28. \quad (19)$$

The 28 cases are divided into two groups; (i) the group composed of two different pairs and a complex number from each of the remaining two different pairs, (ii) the group composed of three different pairs.

(i) The group composed of two different pairs and a complex number from each of the remaining two different pairs

The number of cases in group (i) is given by

$$({}_4C_2)\cdot({}_2C_1)\cdot({}_2C_1)=24. \quad (20)$$

An example is given by

$$1=(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-\exp(-\boldsymbol{i}\theta))$$
$$\cdot(-(-\exp(\boldsymbol{i}\theta)))\cdot(-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-\exp(-\boldsymbol{i}\theta)). \quad (21)$$

The hypothetic breakdown of product form in (21) gives

$$-2\boldsymbol{i}\exp(\boldsymbol{i}\theta)-2\exp(-\boldsymbol{i}\theta). \quad (22)$$

(ii) The group composed of three different pairs

The number of cases in group (ii) is given by

$$_4C_3=4. \quad (23)$$

An example is given by

$$1=(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-\exp(-\boldsymbol{i}\theta))$$
$$\cdot(-(-\exp(\boldsymbol{i}\theta)))\cdot(-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)))\cdot(-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta)))\cdot\exp(-\boldsymbol{i}\theta). \quad (24)$$

The hypothetic breakdown of product form in (24) gives

$$0. \quad (25)$$

This shows that choosing three different pairs causes the exchange between plus and minus signs in each pair, namely nothing occurs.

*Giving a minus sign to each of the eight complex numbers out of eight*

The combination of eight from eight is given by

$$_8C_8=1. \quad (26)$$

This case is given by taking four different pairs,

$$_4C_4=1. \quad (27)$$

Therefore,

$$1=(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-(-\exp(-\boldsymbol{i}\theta)))$$
$$\cdot(-(-\exp(\boldsymbol{i}\theta)))\cdot(-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)))\cdot(-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta)))\cdot(-\exp(-\boldsymbol{i}\theta)). \quad (28)$$

The hypothetic breakdown of product form in (28) gives

$$0. \tag{29}$$

Choosing four different pairs causes the exchange between plus and minus signs in each pair, resulting in that nothing occurs.

## COMBINING HYPOTHETIC BREAKDOWN OF '1' INTO COMPLEX NUMBERS WITH DEFINITE INTEGRAL OF exp($t$) EXPANDED INTO INFINITE SERIES

### Definite integral of exp($t$) expanded into infinite series

As shown by Shimojo *et al.* (2004d, e), the definite integral of exp($t$) expanded into infinite series is given by

$$\begin{aligned}\exp(t_2)\text{-}\exp(t_1) &= \int_{t_1}^{t_2} \exp(t)dt\\ &= \int_{t_1}^{t_2}\left(1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right)dt\\ &= \left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2}\\ &= \left[\left(1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right)\text{-}1\right]_{t_1}^{t_2}\\ &= \left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2}+\{1+(-1)\}-\{1+(-1)\}.\end{aligned} \tag{30}$$

Equation (30) gives a kind of growth calculation. It goes, however, without saying that the growth function takes the form of exp($r \cdot t$) where $r$ shows relative growth rate of forages or ruminants.

### Combining equation (30) with hypothetic breakdown of '1' into complex numbers

This is, for example, given by inserting equation (22) into equation (30). Thus,

$$\begin{aligned}\exp(t_2)\text{-}\exp(t_1) &= \int_{t_1}^{t_2} \exp(t)\, dt\\ &= \left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2}+\{(-2\boldsymbol{i}\exp(\boldsymbol{i}\theta)\text{-}2\exp(-\boldsymbol{i}\theta))+(-1)\}\\ &\quad -\{(-2\boldsymbol{i}\exp(\boldsymbol{i}\theta)\text{-}2\exp(-\boldsymbol{i}\theta))+(-1)\}\\ &= \left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2}+(-2\boldsymbol{i}\exp(\boldsymbol{i}\theta)\text{-}2\exp(-\boldsymbol{i}\theta))\\ &\quad +(2\boldsymbol{i}\exp(\boldsymbol{i}\theta)+2\exp(-\boldsymbol{i}\theta))\\ &= \left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2}.\end{aligned} \tag{31}$$

Equation (31) shows that the insertion of equation (22) coming hypothetically from '1' into equation (30) does not influence the calculation.

Equations (31) suggests that the calculation of growth using the definite integral of $\exp(t)$ expanded into infinite series is accompanied hypothetically by pair appearances and pair disappearances of complex numbers with their opposites. The way of hypothetic breakdown into complex numbers for '1' is different from that for '−1' (Shimojo *et al.*, 2004d). This difference and related problems will be taken up in our following report in the same issue.

**Conclusions and suggestions from the present study**

Giving a minus sign to each of even–numbered pieces (2, 4, 6, 8) of complex number out of eight constructing '1' does not influence the value of '1'. The hypothetic breakdown of product form in the complex representation of '1' leaves zero, two or four sorts of complex numbers. The complex numbers are left in more pieces when a minus sign is given to each of the four pieces of complex number than when given to each of the two, six or eight pieces. Inserting complex numbers that are left into $\exp(t)$ does not influence the calculation of growth, suggesting that the definite integral of $\exp(t)$ expanded into infinite series is accompanied hypothetically by pair appearances and pair disappearances of complex numbers with their opposites.

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