Introducing Complex Numbers into Basic Growth Functions (3) : Symmetry Breakdown in Complex REpresentation of '0=(-1) + 1' int Definite Integral of Exponential Function with Base e Expanded into Infinite Series

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Introducing Complex Numbers into Basic Growth Functions – (III) Symmetry Breakdown in Complex Representation of '0=(−1)+1' in Definite Integral of Exponential Function with Base *e* Expanded into Infinite Series –

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The present study was designed to investigate various types of symmetry breakdown that left complex numbers by giving a minus sign to each of the odd–numbered pieces of complex number out of eight constructing '1'. The results obtained were as follows. Giving a minus sign to each of the odd–numbered (1, 3, 5, 7) pieces of complex number out of eight changed '1' into '−1'. The hypothetic breakdown of multiplication form that connected complex numbers to construct '−1' left one or three sorts of complex numbers. The symmetry breakdown, which was given by the breakdown of multiplication form, was larger when a minus sign was given to each of the three and five pieces of complex number than when given to one piece and each of the seven pieces out of eight. The larger symmetry breakdown left more pieces of complex number.

INTRODUCTION

In reports by Shimojo *et al.* (2004a, c) who applied the complex representation of '(−1)+1' to the definite integral of $\exp(t)$ expanded into infinite series, '−1' was obtained by giving a minus sign to one of the eight complex numbers constructing '1'. The hypothetic breakdown of multiplication form connecting the eight complex numbers constructing '−1' showed incomplete pair disappearances of them to leave a sort of complex number (Shimojo *et al.*, 2004a, c). This is in contrast to complete pair disappearances of complex numbers constructing '1' (Shimojo *et al.*, 2003, 2004b, c). The complex representation of '−1' reported by Shimojo *et al.* (2004a, c) seems to be an example of

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minimum breakdown of symmetry in terms of the number of pieces of complex number that is left by the hypothetic breakdown of multiplication form. There are cases where giving a minus sign to each of the three, five or seven complex numbers out of eight will be involved in obtaining '−1'.

This third study was designed to investigate various types of symmetry breakdown leaving complex numbers by giving a minus sign to each of the odd-numbered pieces of complex number out of eight constructing '1'.

SYMMETRY BREAKDOWN IN COMPLEX REPRESENTATION OF '-1'

Definite integral of exp(t) expanded into infinite series

As shown by the preceding report (Shimojo *et al.*, 2004c), the definite integral of exp(t) expanded into infinite series is given by

$$\int_{t_1}^{t_2} \exp(t)dt = \left[\frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right]_{t_1}^{t_2}$$

$$= \left[\frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right]_{t_1}^{t_2} + \{(-1)+1\} - \{(-1)+1\}. \tag{1}$$

Complex representation of '−1' by giving a minus sign to odd-numbered pieces of complex number out of eight constructing '1'

The complex representation of '1' (Shimojo *et al.*, 2003, 2004a, b, c) is given by

$$1 = \exp(\boldsymbol{i}\theta) \cdot \boldsymbol{i}\exp(-\boldsymbol{i}\theta) \cdot \boldsymbol{i}\exp(\boldsymbol{i}\theta) \cdot (-\exp(-\boldsymbol{i}\theta))$$
$$\cdot (-\exp(\boldsymbol{i}\theta)) \cdot (-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)) \cdot (-\boldsymbol{i}\exp(\boldsymbol{i}\theta)) \cdot \exp(-\boldsymbol{i}\theta). \tag{2}$$

Equation (2) is composed of four different pairs of a complex number and its opposite.

Giving a minus sign to one complex number out of eight

Giving a minus sign to one of the eight complex numbers changes '1' into '−1'. Thus, the combination of eight complex numbers taken one at a time is given by

$${}_8C_1 = \frac{{}_8P_1}{1!} = \frac{8!}{(1!)\cdot(7!)} = 8. \tag{3}$$

An example of this case is given by

$$-1 = (-\exp(\boldsymbol{i}\theta)) \cdot \boldsymbol{i}\exp(-\boldsymbol{i}\theta) \cdot \boldsymbol{i}\exp(\boldsymbol{i}\theta) \cdot (-\exp(-\boldsymbol{i}\theta))$$
$$\cdot (-\exp(\boldsymbol{i}\theta)) \cdot (-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)) \cdot (-\boldsymbol{i}\exp(\boldsymbol{i}\theta)) \cdot \exp(-\boldsymbol{i}\theta). \tag{4}$$

The hypothetic breakdown of multiplication form in (4) gives

$$(-\exp(\boldsymbol{i}\theta)) + \boldsymbol{i}\exp(-\boldsymbol{i}\theta) + \boldsymbol{i}\exp(\boldsymbol{i}\theta) + (-\exp(-\boldsymbol{i}\theta))$$
$$+ (-\exp(\boldsymbol{i}\theta)) + (-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)) + (-\boldsymbol{i}\exp(\boldsymbol{i}\theta)) + \exp(-\boldsymbol{i}\theta)$$
$$= -2\exp(\boldsymbol{i}\theta). \tag{5}$$

Giving a minus sign to each of the three complex numbers out of eight

Giving a minus sign to each of the three complex numbers changes '1' into '−1'. Thus, the combination of eight complex numbers taken three at a time is given by

$$ {}_8C_3=\frac{{}_8P_3}{3!}=\frac{8!}{(3!)\cdot(5!)}=56. \tag{6}$$

The 56 cases are divided into two groups; (i) the group including a complex number from each of the three different pairs, (ii) the group including a pair and a complex number from the other six complex numbers.

(i) The group including a complex number from each of the three different pairs

Three different pairs are obtained from the combination of four taken three at a time, and a complex number from each selected pair is obtained from the combination of two taken one at a time. Therefore, the total number of cases in this group is given by

$$({}_4C_3)\cdot({}_2C_1)\cdot({}_2C_1)\cdot({}_2C_1)=32. \tag{7}$$

An example of this group is given by

$$\begin{aligned}-1&=(-\exp(\boldsymbol{i}\theta))\cdot \boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-\exp(-\boldsymbol{i}\theta))\\ &\quad\cdot(-\exp(\boldsymbol{i}\theta))\cdot\{-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\}\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta).\end{aligned} \tag{8}$$

The hypothetic breakdown of multiplication form in (8) gives

$$\begin{aligned}&(-\exp(\boldsymbol{i}\theta))+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+(-\exp(-\boldsymbol{i}\theta))\\ &\quad+(-\exp(\boldsymbol{i}\theta))+\{-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\}+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)\\ &=-2\exp(\boldsymbol{i}\theta)+2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)-2\boldsymbol{i}\exp(\boldsymbol{i}\theta).\end{aligned} \tag{9}$$

(ii) The group including a pair and a complex number from the other six complex numbers

A pair is obtained from the combination of four taken one at a time. A complex number is obtained from the combination of six taken one at a time. Therefore, the total number of cases in this group is given by

$$({}_4C_1)\cdot({}_6C_1)=24. \tag{10}$$

An example of this case is given by

$$\begin{aligned}-1&=(-\exp(\boldsymbol{i}\theta))\cdot \boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot \boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))\\ &\quad\cdot\{-(-\exp(\boldsymbol{i}\theta))\}\cdot\{-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\}\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta).\end{aligned} \tag{11}$$

The hypothetic breakdown of multiplication form in (11) gives

$$\begin{aligned}&(-\exp(\boldsymbol{i}\theta))+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+(-\exp(-\boldsymbol{i}\theta))\\ &\quad+\{-(-\exp(\boldsymbol{i}\theta))\}+\{-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\}+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)\\ &=2\boldsymbol{i}\exp(-\boldsymbol{i}\theta).\end{aligned} \tag{12}$$

Equation (12) shows that selecting a pair does not make changes due to the exchange between plus and minus signs [$\exp(\boldsymbol{i}\theta)\rightarrow-\exp(\boldsymbol{i}\theta)$; $-\exp(\boldsymbol{i}\theta)\rightarrow\exp(\boldsymbol{i}\theta)$]. This is, therefore, the same case as that of giving a minus sign to one of the eight complex numbers.

Giving a minus sign to each of the five complex numbers out of eight

Giving a minus sign to each of the five complex numbers changes '1' into '−1'. Thus, the combination of eight complex numbers taken five at a time is given by

$$ {}_8C_5 = \frac{{}_8P_5}{5!} = \frac{8!}{(5!)\cdot(3!)} = 56. \tag{13}$$

The 56 cases are divided into two groups; (i) the group including a pair and a complex number from each of the other three different pairs, (ii) the group including two pairs and a complex number from the other four complex numbers.

(i) The group including a pair and a complex number from each of the other three different pairs

A pair is obtained from the combination of four taken one at a time. A complex number from each pair is obtained from the combination of two taken one at a time for the other three different pairs. Therefore, the total number of cases in this group is given by

$$({}_4C_1)\cdot({}_2C_1)\cdot({}_2C_1)\cdot({}_2C_1)=32. \tag{14}$$

An example of this case is given by

$$\begin{aligned}-1 &= (-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot\{-(-\exp(-\boldsymbol{i}\theta))\}\\ &\quad\cdot\{-(-\exp(\boldsymbol{i}\theta))\}\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot\{-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\}\cdot\exp(-\boldsymbol{i}\theta).\end{aligned} \tag{15}$$

The hypothetic breakdown of multiplication form in (15) gives

$$\begin{aligned}&(-\exp(\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+\{-(-\exp(-\boldsymbol{i}\theta))\}\\ &\quad+\{-(-\exp(\boldsymbol{i}\theta))\}+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+\{-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\}+\exp(-\boldsymbol{i}\theta)\\ &=-2\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+2\boldsymbol{i}\exp(\boldsymbol{i}\theta)+2\exp(-\boldsymbol{i}\theta).\end{aligned} \tag{16}$$

Equation (16) shows that selecting a pair does not make changes due to the exchange between plus and minus signs. This is, therefore, the same case as that of the group (i) in giving a minus sign to each of the three complex numbers out of eight.

(ii) The group including two pairs and a complex number from the other four complex numbers

Two pairs are obtained from the combination of four taken two at a time. A complex number is obtained from the combination of four taken one at a time. Therefore, the total number of cases in this group is given by

$$({}_4C_2)\cdot({}_4C_1)=24. \tag{17}$$

An example of this case is given by

$$\begin{aligned}-1 &= (-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot(-\exp(-\boldsymbol{i}\theta))\\ &\quad\cdot\{-(-\exp(\boldsymbol{i}\theta))\}\cdot\{-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\}\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta).\end{aligned} \tag{18}$$

The hypothetic breakdown of multiplication form in (18) gives

$$(-\exp(\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+(-\exp(-\boldsymbol{i}\theta))$$
$$+\{-(-\exp(\boldsymbol{i}\theta))\}+\{-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\}+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)$$
$$=-2\boldsymbol{i}\exp(\boldsymbol{i}\theta). \tag{19}$$

Equation (19) shows that selecting two pairs does not make changes due to the exchange between plus and minus signs. This is, therefore, the same case as that of giving a minus sign to one of the eight complex numbers.

Giving a minus sign to each of the seven complex numbers out of eight

Giving a minus sign to each of the seven complex numbers changes '1' into '−1'. Thus, the combination of eight complex numbers taken seven at a time is given by

$${}_8C_7=\frac{{}_8P_7}{7!}=\frac{8!}{(7!)\cdot(1!)}=8. \tag{20}$$

Each of the 8 cases includes three pairs and a complex number from the other two complex numbers. Three pairs are obtained from the combination of four taken three at a time. A complex number is obtained from the combination of two taken one at a time. Therefore, the total number of cases in this group is given by

$$({}_4C_3)\cdot({}_2C_1)=8. \tag{21}$$

An example of this case is given by

$$-1=(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\{-(-\exp(-\boldsymbol{i}\theta))\}$$
$$\cdot\{-(-\exp(\boldsymbol{i}\theta))\}\cdot\{-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\}\cdot\{-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\}\cdot\exp(-\boldsymbol{i}\theta). \tag{22}$$

The hypothetic breakdown of multiplication form in (22) gives

$$(-\exp(\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\{-(-\exp(-\boldsymbol{i}\theta))\}$$
$$+\{-(-\exp(\boldsymbol{i}\theta))\}+\{-(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\}+\{-(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\}+\exp(-\boldsymbol{i}\theta)$$
$$=2\exp(-\boldsymbol{i}\theta). \tag{23}$$

Equation (23) shows that selecting three pairs does not make changes due to the exchange between plus and minus signs. This is, therefore, the same case as that of giving a minus sign to one of the eight complex numbers.

Conclusions and suggestions from the present study

Giving a minus sign to each of the odd–numbered pieces (1, 3, 5, 7) of complex number out of eight constructing '1' leads to '−1'. The hypothetic breakdown of multiplication form connecting complex numbers constructing '−1' leaves one or three sorts of complex numbers. The symmetry breakdown, which is given by the breakdown of multiplication form, is larger when a minus sign is given to each of the three and five pieces of complex number than when given to one piece and each of the seven pieces out of eight. The larger breakdown of symmetry leaves more pieces of complex number. However, what causes the symmetry to break differently is not known.

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