# Introducing Complex Numbers into Basic Growth FUnctions (2) : Applying Complex Representation of '(-1) +1' to Definite Integral of Exponential Function with Base e Expanded into Infinite Series

Shimojo, Masataka
Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Faculty of Agriculture, Kyushu University

Ikeda, Kentarou
Research Fellow, Faculty of Agriculture, Kyushu University

Ishiwaka, Reiko
Research Fellow, Faculty of Agriculture, Kyushu University

Sato, Hiroyuki
Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Graduate School of Bioresource and Bioenvironmental Sciences, Kyushu University

他

https://doi.org/10.5109/4594

出版情報：九州大学大学院農学研究院紀要. 49 (2), pp.337-341, 2004-10-01. Faculty of Agriculture, Kyushu University
バージョン：
権利関係：

J. Fac. Agr., Kyushu Univ., **49** (2), 337–341 (2004)

# Introducing Complex Numbers into Basic Growth Functions – (II) Applying Complex Representation of '(−1)+1' to Definite Integral of Exponential Function with Base *e* Expanded into Infinite Series –

**Masataka SHIMOJO*, Kentaro IKEDA[1], Reiko ISHIWAKA[1], Hiroyuki SATO[2], Yoki ASANO[3], Manabu TOBISA[4], Yutaka NAKANO[5], Noriko OHBA[6], Minako EGUCHI[7] and Yasuhisa MASUDA**

Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Faculty of Agriculture, Kyushu University, Fukuoka 812–8581, Japan
(*Received June 30, 2004 and accepted July 13, 2004*)

The present study was conducted to investigate the application of complex representation of '(−1)+1' to the definite integral of exp($t$) expanded into infinite series. The results obtained were as follows. There were two sets of '(−1)+1' appearing in the calculation of definite integral of exp($t$). The complex representation of '(−1)+1' left a kind of complex number by the hypothetic breakdown of multiplication form connecting complex numbers constructing '(−1)+1', but the complex number which came form the first '(−1)+1' was offset by that coming from the second '(−1)+1'. This hypothetic phenomenon might be related to the fluctuation between '0' and '(−1)+1' occurring whenever the increase in weight was calculated. This application did not affect the calculation of weight increase, suggesting that the definite integral of exp($t$) was attended hypothetically by pair appearances and disappearances of complex numbers with their opposites.

## INTRODUCTION

In the preceding report (Shimojo *et al.*, 2004b) in this issue, the differentiation of exp($t$) expanded into infinite series was attended hypothetically by pair disappearances of complex numbers with their opposites occurring after the hypothetic breakdown of multiplication form connecting them to construct '1'. Shimojo *et al.* (2004a) reported that the complex representation of '0=(−1)+1' led to incomplete pair disappearances of complex numbers by the hypothetic breakdown of multiplication form connecting them constructing '−1', leaving a kind of complex number. It was also shown by Shimojo *et al.*

[1] Research Fellow, Faculty of Agriculture, Kyushu University
[2] Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Graduate School of Bioresource and Bioenvironmental Sciences, Kyushu University
[3] Employed Research Scientist, Miyazaki University, Miyazaki 889–2192, Japan
[4] Faculty of Agriculture, Miyazaki University, Miyazaki 889–2192, Japan
[5] University Farm, Faculty of Agriculture, Kyushu University
[6] Research Student, School of Agriculture, Kyushu University
[7] Technical Official, School of Agriculture, Kyushu University
* Corresponding author (E-mail: mshimojo@agr.kyushu-u.ac.jp)

(2002) that there was an appearance of '(-1)+1' from '0' after a disappearance of '1' in the calculation of definite integral of $\exp(t)$ expanded into infinite series.

This second study was designed to investigate applying the complex representation of '(-1)+1' to the definite integral of $\exp(t)$ expanded into infinite series.

## COMPLEX REPRESENTATION OF '(-1)+1' IN DEFINITE INTEGRAL OF EXP($t$)

As shown in the preceding report (Shimojo *et al.*, 2004b), the following simple equation will be used instead of $W = W_0 \cdot \exp((\mathrm{RGR}) \cdot t)$

$$S = \exp(t) = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots. \tag{1}$$

### Definite integral of exp($t$) with respect to $t$

The definite integral of $\exp(t)$ for calculating the increase in weight is given by

$$\begin{aligned}\int_{t_1}^{t_2} \exp(t)dt &= \int_{t_1}^{t_2} \left(1 + \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right)dt \\ &= \left[\frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right]_{t_1}^{t_2} \\ &= \left[\left(1 + \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right) - 1\right]_{t_1}^{t_2} \\ &= \left[\exp(t) - 1\right]_{t_1}^{t_2} \\ &= \exp(t_2) - \exp(t_1).\end{aligned} \tag{2}$$

Another description is given by

$$\begin{aligned}\int_{t_1}^{t_2} \exp(t)dt &= \int_{t_1}^{t_2} \left(1 + \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right)dt \\ &= \left[\frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right]_{t_1}^{t_2} \\ &= \left[\left(1 + \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right) + (-1)\right]_{t_1}^{t_2} \\ &= \left[\left(\frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right) + \{(-1) + 1\}\right]_{t_1}^{t_2}\end{aligned}$$

$$= \left[\frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right]_{t_1}^{t_2} + \{(-1)+1\} - \{(-1)+1\}, \tag{3}$$

$$= \left[\frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots\right]_{t_1}^{t_2} + \{(-1)+1\} + \{1+(-1)\}, \tag{4}$$

where equations (3) and (4) are equal to equation (2), $\exp(t_2) - \exp(t_1)$.
There are not differences between equations (3) and (4) except for the second braces with plus or minus sign. However, there will occur a difference when '(−1)+1' is subjected to the description using complex numbers and the hypothetic breakdown of multiplication form connecting them (Shimojo *et al.*, 2004a). This will be taken up in the next section.

**Applying complex representation of '(−1)+1' to equations (3) and (4)**

An example of the complex representation of '(−1)+1' is given by the following description when expressed using $\exp(\boldsymbol{i}\theta)$,

$$\begin{aligned}(-1)+1=&\big[\exp(\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))\\&\quad\cdot\{-(-\exp(\boldsymbol{i}\theta))\}\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta)\big]\\&+\{\exp(\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))\\&\quad\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta)\},\end{aligned}\tag{5}$$

where one of the eight components constructing '1' is given a minus sign in order to obtain the complex representation of '−1'.

The hypothetic breakdown of multiplication form in equation (5) leaves a sort of complex number as follows,

$$\begin{aligned}&\big[\exp(\boldsymbol{i}\theta)+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+(-\exp(-\boldsymbol{i}\theta))\\&\quad+\{-(-\exp(\boldsymbol{i}\theta))\}+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)\big]\\&+\{\exp(\boldsymbol{i}\theta)+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+(-\exp(-\boldsymbol{i}\theta))\\&\quad+(-\exp(\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)\}\\&=2\exp(\boldsymbol{i}\theta).\end{aligned}\tag{6}$$

If inserting equation (5) into equation (3) is followed by the hypothetic breakdown of multiplication form shown in equation (6), then

$$\begin{aligned}\int_{t_1}^{t_2}\exp(t)dt&=\left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2}+\{(-1)+1\}-\{(-1)+1\}\\&=\left[\frac{t}{1!}+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots\right]_{t_1}^{t_2}+2\exp(\boldsymbol{i}\theta)-2\exp(\boldsymbol{i}\theta)\\&=\exp(t_2)-\exp(t_1).\end{aligned}\tag{7}$$

This is why the two sets of '(−1)+1' are required in order to vanish $2\exp(\boldsymbol{i}\theta)$ that is left by the hypothetic breakdown of multiplication form. In other words, this hypothetic vanishing effect might be related to the fluctuation between '0' and '(−1)+1' that occurs whenever the weight increase is calculated.

However, an inequality is given when applied to equation (4). Thus,

$$\int_{t_1}^{t_2} \exp(t)dt = \left[ \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right]_{t_1}^{t_2} + \{(-1)+1\} + \{1+(-1)\}$$

$$\neq \left[ \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right]_{t_1}^{t_2} + 2\exp(\boldsymbol{i}\theta) + 2\exp(\boldsymbol{i}\theta). \qquad (8)$$

This inequality will be turned into the equality when the hypothetic breakdown of multiplication form is different between the first '(−1)+1' and the second '1+(−1)' in order to obtain the offset effect. Thus, the hypothetic breakdown of multiplication form in the second '1+(−1)' is given by

$$\begin{aligned}&[\exp(\boldsymbol{i}\theta) + \boldsymbol{i}\exp(-\boldsymbol{i}\theta) + \boldsymbol{i}\exp(\boldsymbol{i}\theta) + (-\exp(-\boldsymbol{i}\theta))\\ &\quad + (-\exp(\boldsymbol{i}\theta)) + (-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)) + (-\boldsymbol{i}\exp(\boldsymbol{i}\theta)) + \exp(-\boldsymbol{i}\theta)]\\ &+ \{(-\exp(\boldsymbol{i}\theta)) + \boldsymbol{i}\exp(-\boldsymbol{i}\theta)) + \boldsymbol{i}\exp(\boldsymbol{i}\theta)) + (-\exp(-\boldsymbol{i}\theta))\\ &\quad + (-\exp(\boldsymbol{i}\theta)) + (-\boldsymbol{i}\exp(-\boldsymbol{i}\theta)) + (-\boldsymbol{i}\exp(\boldsymbol{i}\theta)) + \exp(-\boldsymbol{i}\theta)\}\\ &= -2\exp(\boldsymbol{i}\theta).\end{aligned} \qquad (9)$$

Inserting equations (6) and (9) into equation (4) leads to

$$\begin{aligned}\int_{t_1}^{t_2} \exp(t)dt &= \left[ \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right]_{t_1}^{t_2} + \{(-1)+1\} + \{1+(-1)\}\\ &= \left[ \frac{t}{1!} + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots \right]_{t_1}^{t_2} + 2\exp(\boldsymbol{i}\theta) - 2\exp(\boldsymbol{i}\theta)\\ &= \exp(t_2) - \exp(t_1).\end{aligned} \qquad (10)$$

It is suggested from equation (10) that the offset effect in the way of hypothetic breakdown of multiplication form is transmitted instantaneously from the first '(−1)+1' to the second '1+(−1)'.

## Conclusions and suggestions from the present study

The calculation of weight increase is not affected by applying the complex representation of '(−1)+1' to the definite integral of $\exp(t)$ expanded into infinite series. This is similar to the case of calculating growth rate under applying the complex representation of '1' to the differentiation of $\exp(t)$ expanded into infinite series (Shimojo *et al.*, 2004b). Broadly speaking at the risk of making mistakes, the basic growth analysis might be attended hypothetically by pair appearances and disappearances of complex numbers with their opposites. This is why we have tried to apply the complex representation to some aspects of ruminant agriculture (Shimojo *et al.*, 2003a, b, c, d, e, f).

## REFERENCES

Shimojo, M., Y. Asano, K. Ikeda, R. Ishiwaka, T. Shao, N. Ohba, H. Sato, Y. Matsufuji, M. Tobisa, Y. Yano and Y. Masuda 2002 Basic growth analysis and symmetric properties of exponential function with base *e*. *J. Fac. Agr., Kyushu Univ.*, **47**: 55–60

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003a A symbolic representation of field–forage–ruminant relationships using polar form on the complex plane. *J. Fac. Agr., Kyushu Univ.*, **47**: 359–366

Shimojo, M., Y. Asano, K. Ikeda, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003b Complex representation of field–forage–ruminant relationships using symmetric properties of Euler's formula. *J. Fac. Agr., Kyushu Univ.*, **47**: 367–372

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003c Exponential functions with base *e* in growth analysis and deriving them from rotations of axes of time described using Euler's formula. *J. Fac. Agr., Kyushu Univ.*, **48**: 65–69

Shimojo, M., Y. Asano, K. Ikeda, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003d A stereographic representation of Euler's formula to show spirals and topological similarities to micro–structures in ruminants and forages. *J. Fac. Agr., Kyushu Univ.*, **48**: 71–75

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2003e Philosophical properties of Euler's formula in its application to symbolic representation of some aspects of ruminant agriculture. *J. Fac. Agr., Kyushu Univ.*, **48**: 77–83

Shimojo, M., Y. Asano, K. Ikeda, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano and Y. Masuda 2003f Simple descriptions of some micro– and macro–structures in ruminant agriculture. Abstracts of Mie Bioforum 2003 on biotechnology of lignocellulose degradation and biomass utilization., Mie, Japan, pp. 83

Shimojo, M., K. Ikeda, Y. Asano, R. Ishiwaka, T. Shao, H. Sato, M. Tobisa, Y. Nakano, N. Ohba, Y. Yano and Y. Masuda 2004a Hypothetic incomplete pair disappearances of complex numbers forming '0=(−1)+1' in the application to complex representation of some aspects of ruminant agriculture. *J. Fac. Agr., Kyushu Univ.*, **49**: 61–67

Shimojo, M., K. Ikeda, R. Ishiwaka, H. Sato, Y. Asano, M. Tobisa, Y. Nakano, N. Ohba, M. Eguchi and Y. Masuda 2004b Introducing complex numbers into basic growth functions – (I) Applying complex representation of '1' to differentiation of exponential function with base *e* expanded into infinite series –. *J. Fac. Agr., Kyushu Univ.*, **49**: 331–335