Introducingg Complex Numbers into Basic Growth Functions (1) : Applying Complex Representation of '1' to Differentiation of Exponential Function with Base e Expanded into Infinite Series

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Introducing Complex Numbers into Basic Growth Functions – (I) Applying Complex Representation of '1' to Differentiation of Exponential Function with Base *e* Expanded into Infinite Series –

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The basic growth analysis using exponential function with base *e* is a primary factor in the management of forages and ruminants. The present study was conducted to investigate the application of complex representation of '1' and '0' to the differentiation of $\exp(t)$ expanded into infinite series. The results obtained were as follows. The differentiation of '1', resulting in '0', was replaced by pair disappearances of complex numbers with their opposites occurring after the hypothetic breakdown of multiplication form connecting them to construct '1'. The differentiation of $(t/1!)$ with respect to t, resulting in '1', was replaced by the product of eight complex numbers constructing '1'. These results suggested that there were hypothetic pair appearances and disappearances of complex numbers according to the fluctuation between '1' and '0' occurring whenever the rate of growth was calculated.

INTRODUCTION

We applied complex numbers to the description of some aspects of ruminant agriculture; (1) relationships between leaf inclination and light extinction coefficient in a forage canopy (Shimojo, 1998), (2) changes in digestibility and indigestibility with growth of forages (Shimojo *et al.*, 1998a, b). In our recent reports using Euler's formula [$\exp(\boldsymbol{i}\theta) = \cos\theta + \boldsymbol{i}\sin\theta$], we suggested that (3) a series of $\pi/2$ phase shifts of Euler's formula gave a symbolic representation to the cycling of matter in field–forage–ruminant relationships (Shimojo *et al.*, 2003a, b), (4) $\pm\pi/2$ rotations of axes of time described using Euler's formula gave exponential functions with base e used for basic growth

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analysis (Shimojo *et al.*, 2003c, f) and for degradation analysis of forages in the rumen (Shimojo *et al.*, 2003f), (5) a stereographic representation of Euler's formula gave spiral structures showing topological resemblance to micro-structures in microbes, forages and ruminants (Shimojo *et al.*, 2003d, f). We also suggested where we got complex numbers; (6) the complex representation of '1' using four pairs of a complex number and its opposite (Shimojo *et al.*, 2003e, 2004), (7) the complex representation of '$0=(-1)+1$' where '-1' was obtained by giving a minus sign to one of the eight complex numbers constructing '1' (Shimojo *et al.*, 2004). Before then, (8) Shimojo *et al.* (2002) had investigated with exponential function with base e expanded into infinite series that its differentiation had been attended by a disappearance of '1' followed by a reappearance of '1' and the definite integral was attended by a disappearance of '1' followed by an appearance of '$1+(-1)$' from '0'. These three [(6), (7), (8)] suggest hypothetic relationships of complex numbers to exponential function with base e used for basic growth function. We would like to take up this subject in four reports in this issue.

This first study was designed to investigate applying the complex representation of '1' to the differentiation of exponential function with base e expanded into infinite series.

COMPLEX REPRESENTATION OF '1' IN DIFFERENTIATION OF EXP(t)

The exponential function with base e that is used for basic growth analysis (Brody, 1945; Watson, 1952; Radford, 1967; Hunt, 1990) is given by

$$W=W_0\cdot\exp((\mathrm{RGR})\cdot t), \tag{1}$$

where W=weight, RGR=relative growth rate, t=time, W_0= the weight at $t=0$.
In the present study we will use $\exp(t)$ instead of equation (1) for the simplification.
Thus,

$$S=\exp(t). \tag{2}$$

Expanding exp(t) into infinite series

Expanding $\exp(t)$ into infinite series is given by

$$\begin{aligned}\exp(t)&=\sum_{n=0}^{\infty}\frac{t^n}{n!}\\&=1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots\cdots+\frac{t^n}{n!}+\cdots\cdots.\end{aligned} \tag{3}$$

Differentiation of exp(t) with respect to t

The differentiation of $\exp(t)$ for calculating the rate of growth is given by

$$\begin{aligned}\frac{d}{dt}(\exp(t))&=\frac{d}{dt}\left(1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots\cdots+\frac{t^n}{n!}+\cdots\cdots\right)\\&=0+\left(1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots\cdots+\frac{t^n}{n!}+\cdots\cdots\right).\end{aligned} \tag{4}$$

It is suggested from equations (3) and (4) that there is a hypothetic fluctuation between '1' and '0' whenever the growth rate is calculated.

Applying complex representation of '1' and '0' to equation (4)

The complex representation of '1' (Shimojo *et al.*, 2003e, 2004) is given by

$$1=(\cos\theta+\boldsymbol{i}\sin\theta)\cdot(\sin\theta+\boldsymbol{i}\cos\theta)\cdot(-\sin\theta+\boldsymbol{i}\cos\theta)\cdot(-\cos\theta+\boldsymbol{i}\sin\theta)$$
$$\cdot(-\cos\theta-\boldsymbol{i}\sin\theta)\cdot(-\sin\theta-\boldsymbol{i}\cos\theta)\cdot(\sin\theta-\boldsymbol{i}\cos\theta)\cdot(\cos\theta-\boldsymbol{i}\sin\theta). \tag{5}$$

The hypothetic breakdown of multiplication form in (5) gives pair appearances and disappearances of complex numbers, resulting in '0' (Shimojo *et al.*, 2003e, 2004). Thus,

$$(\cos\theta+\boldsymbol{i}\sin\theta)+(\sin\theta+\boldsymbol{i}\cos\theta)+(-\sin\theta+\boldsymbol{i}\cos\theta)+(-\cos\theta+\boldsymbol{i}\sin\theta)$$
$$+(-\cos\theta-\boldsymbol{i}\sin\theta)+(-\sin\theta-\boldsymbol{i}\cos\theta)+(\sin\theta-\boldsymbol{i}\cos\theta)+(\cos\theta-\boldsymbol{i}\sin\theta)=0. \tag{6}$$

Rewriting equations (5) and (6) using exponential functions with base e leads to

$$1=\exp(\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))$$
$$\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta), \tag{7}$$

$$\exp(\boldsymbol{i}\theta)+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+(-\exp(-\boldsymbol{i}\theta))$$
$$+(-\exp(\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)=0. \tag{8}$$

Inserting equations (7) and (8) into equation (4) gives

$$\begin{aligned}\frac{d}{dt}(\exp(t))=0+\left(1+\frac{t}{1!}+\frac{t^2}{2!}+\cdots\cdots+\frac{t^n}{n!}+\cdots\cdots\right)\\
=\{\exp(\boldsymbol{i}\theta)+\boldsymbol{i}\exp(-\boldsymbol{i}\theta)+\boldsymbol{i}\exp(\boldsymbol{i}\theta)+(-\exp(-\boldsymbol{i}\theta))\\
+(-\exp(\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))+\exp(-\boldsymbol{i}\theta)\}\\
+\exp(\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta)\cdot(-\exp(-\boldsymbol{i}\theta))\\
\cdot(-\exp(\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta))\cdot\exp(-\boldsymbol{i}\theta)\\
+\left(\frac{t}{1!}+\frac{t^2}{2!}+\cdots\cdots+\frac{t^n}{n!}+\cdots\cdots\right).\end{aligned} \tag{9}$$

It is suggested from equation (9) that (i) the differentiation of '1', resulting in '0', is replaced by pair disappearances of complex numbers with their opposites occurring after the hypothetic breakdown of multiplication form connecting them to construct '1', (ii) the differentiation of (t/1!) with respect to t, resulting in '1', is replaced by the product of eight complex numbers constructing '1'. In other words, these hypothetic phenomena might be related to the fluctuation between '1' and '0' that occurs whenever the rate of growth is calculated. In addition, many complex numbers will show pair appearances and disappearances if the following equation is used.

$$\frac{d}{dt}(\exp(t))=\sum_{k=1}^{n}\{\exp(\boldsymbol{i}\theta_k)+\boldsymbol{i}\exp(-\boldsymbol{i}\theta_k)+\boldsymbol{i}\exp(\boldsymbol{i}\theta_k)+(-\exp(-\boldsymbol{i}\theta_k))$$
$$+(-\exp(\boldsymbol{i}\theta_k))+(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta_k))+(-\boldsymbol{i}\exp(\boldsymbol{i}\theta_k))+\exp(-\boldsymbol{i}\theta_k)\}$$
$$+\prod_{k=1}^{n}\{\exp(\boldsymbol{i}\theta_k)\cdot\boldsymbol{i}\exp(-\boldsymbol{i}\theta_k)\cdot\boldsymbol{i}\exp(\boldsymbol{i}\theta_k)\cdot(-\exp(-\boldsymbol{i}\theta_k))$$
$$\cdot(-\exp(\boldsymbol{i}\theta_k))\cdot(-\boldsymbol{i}\exp(-\boldsymbol{i}\theta_k))\cdot(-\boldsymbol{i}\exp(\boldsymbol{i}\theta_k))\cdot\exp(-\boldsymbol{i}\theta_k)\}$$
$$+\left(\frac{t}{1!}+\frac{t^2}{2!}+\cdots\cdots+\frac{t^n}{n!}+\cdots\cdots\right), \tag{10}$$

where $0<\theta_1<\cdots<\theta_k<\cdots\theta_n<\pi/2$.

Conclusions and suggestions from the present study

The basic growth analysis using exponential function with base e is a primary factor in the management of forages and ruminants in ruminant agriculture. Applying the complex representation of '1' to the differentiation of $\exp(t)$ expanded into infinite series does not affect the calculation of growth rate, suggesting that pair appearances and disappearances of complex numbers are something like air prevailing everywhere. However, descriptions using complex numbers give rough images of micro–, medium– and macro–structures of ruminant agriculture (Shimojo *et al.*, 2003a, b, c, d, e, f, 2004). This might be associated with the nature of Euler's formula that is considered of importance to those who are engaged in sciences (Yoshida, 2000).

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