A Stereographic Representation of Euler's Formula to Show Spirals and Topological Similarities to Micro-Structures in Ruminants and Forages

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A Stereographic Representation of Euler's Formula to Show Spirals and Topological Similarities to Micro–Structures in Ruminants and Forages

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This study suggests that the stereographic representation of Euler's formula and its conjugate complex in the combination with or without their phase shifts gives spirals showing topological similarities to micro-structures in ruminants and forages.

INTRODUCTION

We suggested that some mathematical treatments of Euler's formula gave a symbolic description of field–forage–ruminant relationships (Shimojo *et al.*, 2003a, b), and in addition, exponential functions with base e used for the growth analysis of ruminants and forages (Shimojo *et al.*, 2003c). These show an aspect of macro–structures in ruminant agriculture.

It is reported by Yoshida (2000) that a stereographic representation of Euler's formula gives spirals. If precise shapes are not considered, spirals suggested from Euler's formula might be expected to show topological similarities to helical structures in living things (Watson *et al.*, 1987; Calladine and Drew, 1992; Voet *et al.*, 1999, for example).

The present study was designed to investigate spirals suggested from Euler's formula and their topological similarities to helical structures in ruminants and forages.

EULER'S FORMULA AND ITS STEREOGRAPHIC DESCRIPTIONS

Euler's formula

Euler's formula is given by

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$$\exp(\boldsymbol{i}\beta)=\cos\beta+\boldsymbol{i}\sin\beta, \tag{1}$$

where $\boldsymbol{i}$=imaginary unit, β=real variable.

Stereographic description of Euler's formula gives a right-handed spiral

There are the following two sequences on the complex plane (Shimojo *et al.*, 2003a); a series of anti-clockwise $\pi/2$ rotations [$\times\boldsymbol{i}$],

$$(\cos\beta+\boldsymbol{i}\sin\beta)\xrightarrow{\times\boldsymbol{i}}(-\sin\beta+\boldsymbol{i}\cos\beta)\xrightarrow{\times\boldsymbol{i}}(-\cos\beta-\boldsymbol{i}\sin\beta)\xrightarrow{\times\boldsymbol{i}}(\sin\beta-\boldsymbol{i}\cos\beta), \tag{2}$$

and a series of clockwise $\pi/2$ rotations [$\times(-\boldsymbol{i})$],

$$(\sin\beta+\boldsymbol{i}\cos\beta)\xrightarrow{\times(-\boldsymbol{i})}(\cos\beta-\boldsymbol{i}\sin\beta)\xrightarrow{\times(-\boldsymbol{i})}(-\sin\beta-\boldsymbol{i}\cos\beta)\xrightarrow{\times(-\boldsymbol{i})}(-\cos\beta+\boldsymbol{i}\sin\beta). \tag{3}$$

In the present study, $\cos\beta+\boldsymbol{i}\sin\beta$ in (2), $\cos\beta-\boldsymbol{i}\sin\beta$ in (3) and their phase shifts will be described stereographically to show spirals.

Yoshida (2000) shows that the stereographic description of Euler's formula using coordinates (β, $\cos\beta$, $\boldsymbol{i}\sin\beta$) gives a right-handed spiral (Fig. 1a). Its planar projection is also shown in Fig. 1b. In the present study coordinate axes are eliminated in order to show the structure of spirals only, otherwise there will occur interferences of waves when spirals are combined.

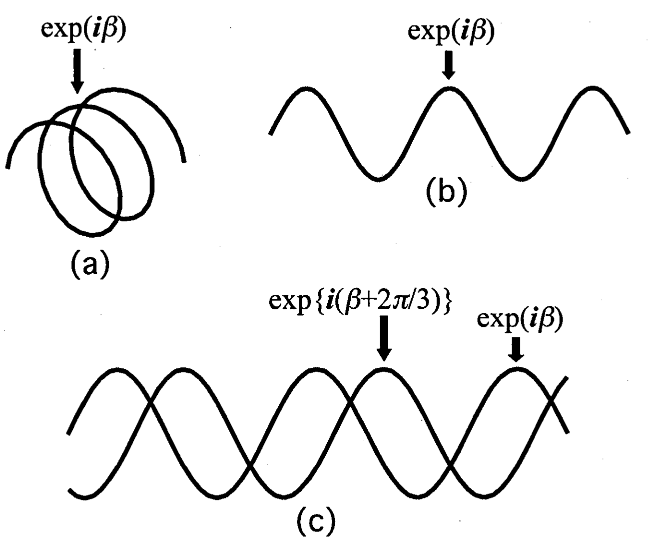

Fig. 1. A stereographic representation of Euler's formula [$\exp(\boldsymbol{i}\beta)=\cos\beta+\boldsymbol{i}\sin\beta$] using coordinates ($\beta$, $\cos\beta$, $\boldsymbol{i}\sin\beta$). (a) A right-handed spiral. (b) A planar projection of the right-handed spiral. (c) The planar projection of $\exp(\boldsymbol{i}\beta)$ in the combination with $\exp\{\boldsymbol{i}(\beta+2\pi/3)\}$. In each case the right-hand end of spirals is the starting point.

Phase shifts of Euler's formula

Let us take some phase shifts for $\exp(\boldsymbol{i}\beta)$, for example $2\pi/3$, $4\pi/3$ and π. Thus,

$$\exp\left(\boldsymbol{i}\frac{2\pi}{3}\right)\cdot\exp(\boldsymbol{i}\beta)=\exp\left\{\boldsymbol{i}\left(\beta+\frac{2\pi}{3}\right)\right\}, \tag{4}$$

$$\exp\left(\boldsymbol{i}\frac{4\pi}{3}\right)\cdot\exp(\boldsymbol{i}\beta)=\exp\left\{\boldsymbol{i}\left(\beta+\frac{4\pi}{3}\right)\right\}, \tag{5}$$

$$\exp(\boldsymbol{i}\pi)\cdot\exp(\boldsymbol{i}\beta)=\exp\{\boldsymbol{i}(\beta+\pi)\}, \tag{6}$$

It is shown in (4)$\sim$(6) that the form of Euler's formula is kept invariant when given any phase shift.

Combining Euler's formula and its phase shifts in the stereographic description

(A) Combination of (1) and (4)

Combining $\exp(\boldsymbol{i}\beta)$ and $\exp\{\boldsymbol{i}(\beta+2\pi/3)\}$ gives a pair of right-handed spirals (Fig. 1c).

(B) Combination of (1), (4) and (5)

The combination of $\exp(\boldsymbol{i}\beta)$, $\exp\{\boldsymbol{i}(\beta+2\pi/3)\}$ and $\exp\{\boldsymbol{i}(\beta+4\pi/3)\}$ gives right-handed three spirals (Fig. 2a).

(C) Combination of (1) and (6)

Combining $\exp(\boldsymbol{i}\beta)$ and $\exp\{\boldsymbol{i}(\beta+\pi)\}$ gives a pair of right-handed spirals (Fig. 2b).

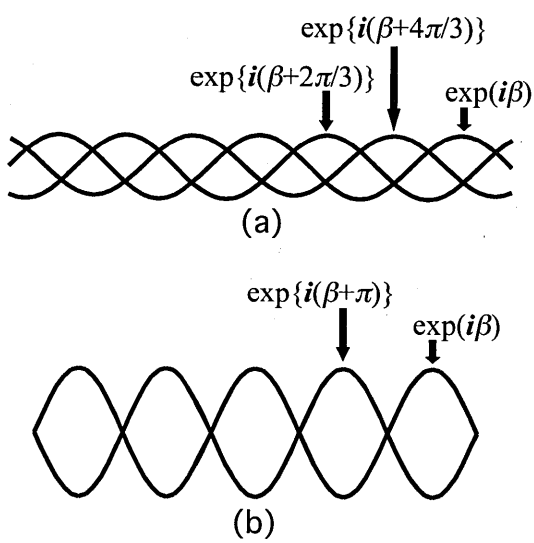

Fig. 2. The planar projection of the stereographic representation of Euler's formula [$\exp(\boldsymbol{i}\beta)$] in the combination with $\exp\{\boldsymbol{i}(\beta+2\pi/3)\}$ and $\exp\{\boldsymbol{i}(\beta+4\pi/3)\}$ in (a) and in the combination with $\exp\{\boldsymbol{i}(\beta+\pi)\}$ in (b). In each case the right-hand end of spirals is the starting point.

The conjugate complex to Euler's formula gives a left-handed spiral

As shown in (3), the conjugate complex to Euler's formula is given by

$$\exp(-\,\boldsymbol{i}\beta)=\cos\beta-\boldsymbol{i}\sin\beta. \tag{7}$$

The conjugate complex to Euler's formula gives a left-handed spiral when described stereographically using coordinates (β, $\cos\beta$, $-\,\boldsymbol{i}\sin\beta$), as shown in Fig. 3a. Its planar projection is also shown in Fig. 3b.

The $-\pi$ phase shift of $\exp(-\boldsymbol{i}\beta)$ is given by

$$\exp(-\boldsymbol{i}\pi)\cdot\exp(-\boldsymbol{i}\beta)=\exp\{-\boldsymbol{i}(\beta+\pi)\}. \tag{8}$$

It is shown in (8) that the form of conjugate complex to Euler's formula is kept invariant when given a phase shift of $-\pi$.

Combining (7) and (8), namely $\exp(-\boldsymbol{i}\beta)$ and $\exp\{-\boldsymbol{i}(\beta+\pi)\}$, gives a pair of left-handed spirals (Fig. 3c).

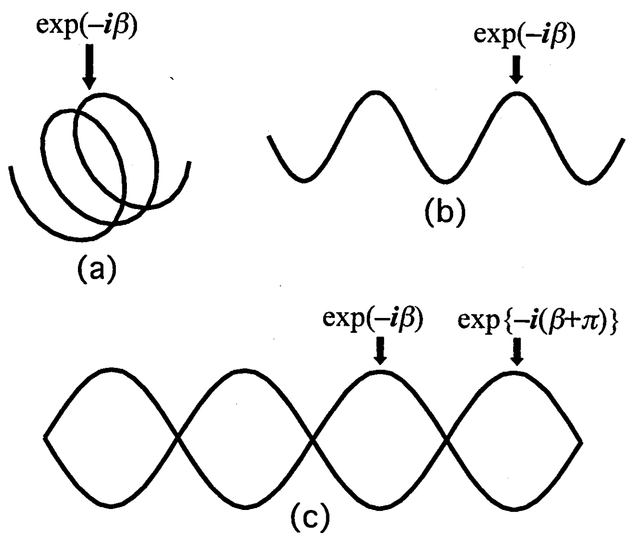

Fig. 3. A stereographic representation of the conjugate complex to Euler's formula [$\exp(-\boldsymbol{i}\beta)=\cos\beta-\boldsymbol{i}\sin\beta$] using coordinates ($\beta$, $\cos\beta$, $-\sin\beta$). (a) A left-handed spiral. (b) A planar projection of the left-handed spiral. (c) The planar projection of $\exp(-\boldsymbol{i}\beta)$ in the combination with $\exp\{-\boldsymbol{i}(\beta+\pi)\}$. In each case the right-hand end of spirals is the starting point.

Topological similarities to micro-structures in ruminants and forages

Sine curve and cosine curve are used for the analysis of helical structures in living things (Calladine and Drew, 1992). It is suggested from the present study that stereographic descriptions of Euler's formula and its conjugate complex in the combination with or without their phase shifts show topological similarities to the following helical structures: a right-handed helix [(1)], a left-handed helix [(7)], a right-handed double helix of two kinds [combining (1) and (4), combining (1) and (6)], a left–handed double helix [combining (7) and (8)], and a right–handed triple helix [combining (1), (4) and (5)]. A feature of the present approach is as follows. (a) Right–handed spirals and left–handed spirals are related to Euler's formula and its conjugate complex, respectively. (b) Double and triple spirals are given by Euler's formula or its conjugate complex in the combination with corresponding phase shifts. (c) The complementary property in the form of double spiral is related to keeping the form of Euler's formula or its conjugate complex invariant with respect to phase shifts. Therefore, the use of stereographic representation of Euler's formula and its conjugate complex in the combination with or without their phase shifts might give a rough image of helical structures in ruminants and forages.

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