

## Note on categorical quantum fields in curved spacetimes

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# Note on categorical quantum fields in curved spacetimes

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## Abstract

The quantum fields in curved spacetimes are discussed in the language of category.

The quest for a quantum gravity is the hardest problem in theoretical physics. Reflecting over the long history of unsuccessful attempts I suspect that such a quest is a wrong attempt.

I think that a spacetime is a mathematical space to represent a causal structure of events. We need an observer to define the events and each observer uses her own spacetime. The metric of the mathematical space is determined by the rays of light. In the absence of gravity each ray is a straight line. In the presence of gravity rays are curved. An observer determine a curved spacetime by the observation of curved rays. Thus, we can obtain spacetimes observationally without any knowledge of the theories of gravity. A spacetime is classical and emergent [1].

In other words the theory of gravity is a kind of hydrodynamic description and not the target of quantization.

A spacetime determined by above-mentioned classical observation is a precondition for a quantum measurement. An observer equips her apparatus in her own spacetime. The apparatus is a classical object. Events are recorded by the classical apparatus.

The consequence of a quantum measurement is described by a map  $\omega$  from observables to data. The output of the map is the expectation value of observables in the situation prepared by the observer. A micro quantum system is indefinite before observations. We can put questions to a quantum system and get answers with the help of a macro classical system, an apparatus.

Through an interaction between a micro quantum system and a macro classical system we can establish a dual relation between systems so that we can guess the quantum system by the knowledge appeared in our classical system [2].

In the case of flat spacetime the description of the local observations of quantum systems is established in the categorical framework [3]. Thus, the remaining task is to develop the categorical quantum theory in curved spacetimes. This task is almost completed [4] and in the following I will roughly sketch the essence of it omitting

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the discussions, e.g. Einstein causality, time-slice axiom and so on, necessary for implementing the theory.

An observer use a specific spacetime  $M$ . Another observer use a different specific spacetime  $N$ . In order to obtain a coherent description we have to consider all the spacetime manifolds at once. Thus, we are led to the category of spacetime manifolds. The local structure of this category is written as

$$M \xrightarrow{\psi} N.$$

The objects,  $M$  and  $N$ , are globally hyperbolic spacetimes. The morphism  $\psi$  is diffeomorphic.

By a functor  $\mathcal{D}$  this local structure is mapped to

$$\mathcal{D}(M) \xrightarrow{\mathcal{D}(\psi)} \mathcal{D}(N).$$

The objects of this category,  $\mathcal{D}(M)$  and  $\mathcal{D}(N)$ , are smeared test functions on spacetimes.

By another functor  $\mathcal{A}$

$$\mathcal{A}(M) \xrightarrow{\mathcal{A}(\psi)} \mathcal{A}(N).$$

The objects of this category,  $\mathcal{A}(M)$  and  $\mathcal{A}(N)$ , are observable algebras on spacetimes. Each object  $\mathcal{A}(M)$  is constructed from local data  $\omega$  which can be determined without the knowledge of outside of the spacetime region of the local observation.

A natural transformation between functors  $\mathcal{D}$  and  $\mathcal{A}$  is a locally covariant quantum field  $\Phi$ :

$$\mathcal{D}(M) \xrightarrow{\Phi_M} \mathcal{A}(M).$$

The field  $\Phi_M$  depends on the spacetime  $M$  and the set of these fields describes all the spacetimes in a coherent manner. Here we do not assume any symmetries of spacetimes.

An observer uses her own spacetime  $M$ . The results of her observations are different from those of the other observers using different spacetimes. This observer dependence is described coherently by the spacetime-dependent field  $\Phi_M$ . A typical observer dependence is seen in the Hawking-Unruh effect and such an effect is naturally described by the present scheme.

## References

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- [4] Rejzner, arXiv:1603.06993v1 for a review.