CHARACTERIZATION OF AGREEABLE PLANS IN AN ECONOMY WITH HETEROGENEOUS CAPITAL GOODS: THE CONTINUOUS-TIME CASE

大住, 圭介

https://doi.org/10.15017/4493041

出版情報：經濟學研究. 57 (5/6), pp.143-152, 1992-09-10. 九州大学経済学会

Characterization of Agreeable Plans in an Economy with Heterogeneous Capital Goods: The Continuous-Time Case

Keisuke Ohsumi

1. INTRODUCTION

The purpose of this paper is to provide necessary conditions for a plan to be agreeable in the context of a continuous-time dynamic economic model with many capital goods. Since the pioneering paper by Hammond and Mirrlees [1973], fairly detailed analyses of agreeable plans have been conducted in the familiar one-good model and in a class of discrete-time models with many capital goods (see Hammond and Mirrlees [1973], Heal [1973], Hammond [1975], Hammond and Kennan [1979], and Ohsumi [1982]). However, in a class of continuous-time dynamic economic models with more than one capital goods, little is known about the properties of agreeable plans. For the results concerning agreeable plans in the familiar one-good continuous-time model, see Hammond and Mirrlees [1973].

In this paper we will prove the two important propositions concerning agreeable plans. First we will prove that, if an agreeable plan is interior, then it is a relatively maximal accumulation path — secondly that, if an agreeable plan is interior, then it is a locally optimal accumulation path. Although these results are very important in the study of agreeable plans in the context of the continuous—time model with many capital goods, these results have not been published to date.

2. THE MODEL

We suppose that there is a finite number, n, of different kinds of capital goods, which are arbitrarily numbered from 1 to n. We also suppose that the population grows at an exogenous nonnegative rate (m) from the given initial magnitude $\overline{P}_0 > 0$. Let P(t) denote the magnitude of the population at time t. Then, $P(t) = \overline{P}_0 e^{mt}$.

Let

$k_i(t)$ = per capita quantity of the ith capital good at time t,

$k(t) = [k_1(t), \ldots, k_n(t)]' =$ n−vector of capital good quantities in per capita terms at time t,

$i_i(t) =$ i th investment − good output in per capita terms at time t,

$i(t) = [i_1(t), \ldots, i_n(t)]' =$ n−vector of investment−goods output in per capita terms at time t,

$z(t) = i(t) - mk(t)$,

$X(t) =$ set of technically possible pairs $(k(t), z(t))$ at time t.

Also, for each time t, let $u(k, z, t)$ be the maximum instantaneous social utility realizable at time t, given k and z at time t. Then the instantaneous social utility function at time t is defined by

$$u(k, z, t) = u([k_1, k_2, \ldots, k_n]', [z_1, z_2, \ldots, z_n]', t) : X(t) \to R.$$

Now, we make the following assumptions which are very standard in the theory of optimal economic growth.

(A.1) $X(t) = X \subset R^n_+ \times R^n$, for all $t \in R_+$.

(A.2) The set X is a closed set.

(A.3) There is a continuous, piecewise continuously differentiable function $k(t) : [0, \infty) \to R^n_+$ such that $(k(t), \dot{k}(t)) \in X$, for almost every $t \in R_+$.

(A.4)
$$\begin{aligned} u(k, z, t) &= e^{-\delta t} P(t) v[k, z] \\ &= e^{-(\delta - m)t} \bar{P}_0 v[k, z] \\ &= e^{-\rho t} \bar{P}_0 v[k, z] \end{aligned}$$

where $\delta \in R$.

(A.5) $v[k, z] : X \to R$ is continuous and twice continuously differentiable on the interior of X.

Here, $v[k, z] : X \to R$ is the utility function of the representative person, δ is a planning rate of discount and ρ is a planning rate of effective discount.

Now, given the dynamic model described above, we introduce some concepts.

Definition 2.1: A continuous, piecewise continuously differentiable function $k(t) : [t', t''] \to R^n_+$ is said to be a feasible accumulation path from k' at time t' to k'' at time t'' if

$$k(t') = k', \ k(t'') = k''$$

and, for almost every $t \in [t', t'']$,

$$(k(t), (\dot{k}(t)) \in X, \tag{2.1}$$

where $0 \leqq t' < t'' \leqq +\infty$.

We will denote by $F(t', k', t'', k'')$ the set of all feasible accumulation paths from k' at time t' to k'' at time t''. In the case where k'' is not specified, we will denote this set by $F(t', k', t'')$. Also, in order to simplify the notation, we will write

$$k(t) : [t', t''] \to R^n_+ \equiv (k(t))_{t=t'}^{t''}$$

Definition 2.2 : A feasible infinite accumulation path $(k(t))_{t=0}^{\infty}$ from $\bar{k}(0)$ is said to be interior if

$$(k(t), \dot{k}(t)) \in \text{ interior } X, \tag{2.2}$$

for every $t \in [0, \infty)$ at which $\dot{k}(t)$ exists. Here, we will write $F_i(0, \bar{k}(0), \infty)$ for the set of all interior accumulation paths from $\bar{k}(0)$.

Definition 2.3 : A T-period ν-neighborhood of an interior accumulation path $(\tilde{k}(t))_{t=0}^{\infty} \in F(0, \bar{k}(0), \infty)$ is the collection of all feasible accumulation paths $(k(t))_{t=0}^{T} \in F(0, \bar{k}(0), T, \tilde{k}(T))$ such that

$$\|\tilde{k}(t) - k(t)\| \leqq \nu \text{ for every } t \in [0, T] \tag{2.3}$$

and

$$\|\dot{\tilde{k}}(t) - \dot{k}(t)\| \leqq \nu \tag{2.4}$$

for every $t \in [0, T]$ at which $\dot{\tilde{k}}(t)$ and $\dot{k}(t)$ exist. Here, we will write $N((\tilde{k}(t))_{t=0}^{\infty}, T, \nu)$ for a T-period ν-neighborhood of $(\tilde{k}(t))_{t=0}^{\infty}$.

Definition 2.4 : An interior accumulation path $(\tilde{k}(t))_{t=0}^{\infty} \in F(0, \bar{k}(0), \infty)$ is said to be relatively maximal if

$$\forall T \in R_+,\ \exists \nu > 0,\ \forall (k(t))_{t=0}^{T} \in N((\tilde{k}(t))_{t=0}^{\infty}, T, \nu);$$

$$\int_0^T e^{-\rho t}\bar{P}_0\ v[\tilde{k}(t), \dot{\tilde{k}}(t)]dt \geqq \int_0^T e^{-\rho t}\bar{P}_0\ v[k(t), \dot{k}(t)]dt. \tag{2.5}$$

Here, we will write $F^{rm}(0, \bar{k}(0), \infty)$ for the set of all relatively maximal paths from $\bar{k}(0)$.

Definition 2.5 : An infinite accumulation path $(\tilde{k}(t))_{t=0}^{\infty} \in F(0, \bar{k}(0), \infty)$ is said to be a locally optimal accumulation path from $\bar{k}(0)$ if $\tilde{k}(t) : [0, \infty) \to R^n_+$ is twice differentiable and it satisfies the following differential equation of the second order

$$v_{zz}(\tilde{k}(t), \dot{\tilde{k}}(t))\ddot{\tilde{k}}(t) + v_{zk}(\tilde{k}(t), \dot{\tilde{k}}(t))\dot{\tilde{k}}(t) - \{\rho v_z(\tilde{k}(t), \dot{\tilde{k}}(t)) + v_k(\tilde{k}(t), \dot{\tilde{k}}(t))\} = 0$$

$$(\text{for every } t \in [0, \infty)). \tag{2.6}$$

Here, we will write $F^{\ell o}(0, \bar{k}(0), \infty)$ for the set of all locally optimal accumulation paths from $\bar{k}(0)$.

Also, given an initial capital stock $\bar{k}(0)$, we give a definition of agreeability-choice criterion with which we shall be concerned in this paper.

Definition 2.6 : A feasible infinite accumulation path $(\tilde{k}(t))_{t=0}^{\infty} \in F(0, \bar{k}(0), \infty)$ is said to be :

(1) an agreeable plan in the sense of Hammond and Mirrlees if

$$\forall T_0 > 0,\ \forall \varepsilon > 0,\ \exists T_1 > T_0,\ \forall T > T_1,\ \forall (k(t))_{t=0}^{T} \in F(0, \bar{k}(0), T),$$

$$\exists (k'(t))_{t=0}^{T} \in \{(k'(t))_{t=0}^{T} \mid k'(t) = \tilde{k}(t),\ 0 \leqq t \leqq T_0,\ (k'(t))_{t=0}^{T} \in F(0, \bar{k}(0), T)\};$$

$$\int_0^T e^{-\rho t}\bar{P}_0\ v[k'(t), \dot{k}'(t)]\,d\,t \geqq \int_0^T e^{-\rho t}\bar{P}_0\ v[k(t), \dot{k}(t)]\,d\,t - \varepsilon \tag{2.7}$$

(2) an agreeable plan in the sense of Hammond if, for $\forall T_0 > 0$,

$$T \to +\infty (T > T_0) \Rightarrow \bar{W}^T - W^T((\tilde{k}(t))_{t=0}^{\infty} | T_0) \to 0, \tag{2.8}$$

where $\bar{W}^T = \sup \int_0^T e^{-\rho t}\bar{P}_0\ v[k(t), \dot{k}(t)]\ dt$ over all $(k(t))_{t=0}^T \in F(0, \bar{k}(0), T)$, $W^T((k(t))_{t=0}^{\infty} | T_0) = \sup$ $\int_0^T e^{-\rho t}\bar{P}_0\ v[k'(t), \dot{k}'(t)]\ dt$ over all $(k'(t))_{t=0}^T \in \{(k'(t))_{t=0}^T | k'(t) = k(t), 0 \leqq t \leqq T_0, (k'(t))_{t=0}^T \in F(0, \bar{k}(0), T)\}$ $(T > T_0)$.

Although it seems at first glance that these two definitions of agreeability are considerably different, these definitions are in fact equivalent for the model described above (see Ohsumi 〔1986〕). Here, we will write $F_{ag}(0, \bar{k}(0), \infty)$ for the set of all agreeable plans from $\bar{k}(0)$.

3. NECESSARY CONDITIONS FOR AGREEABILITY

In this section, given the continuous-time dynamic economic model described above, we prove the important proposition concerning agreeable plans. We first prove that, if an agreeable plan is interior, then it is a relatively maximal path. Then, we prove that, if an agreeable plan is interior, it is a locally optimal accumulation path. These conclusions are very important in the study of agreeable plans for a class of widely-used continuous-time models with many capital goods because they allow us to make use of some results in the extensive theoretical literature on optimal growth (see Levhari and Liviatan 〔1972〕, Magill 〔1977〕, 〔1979〕, and Magill and Scheinkman 〔1979〕).

Now, in order to proceed, the following assumptions will be made:

(A.6) For any T'', $T'(T'' > T' \geqq 0)$ and any $k \geq \mathbf{0}$,

$\sup \int_{T'}^{T''} e^{-\rho t}\bar{P}_0\ v[k(t), \dot{k}(t)]\ dt$ over $(k(t))_{t=T'}^{T''} \in F(T', k, T'')$ is finite.

Assumption (A.6) does not seem to be stringent.

Now, we can prove the following result.

Proposition 3.1: Under assumptions (A.1)-(A.6) and given an initial capital stock $\bar{k}(0) \geq \mathbf{0}$, if an agreeable plan is interior, then it is a relatively maximal accumulation path, that is,

$$F_i(0, \bar{k}(0), \infty) \cap F_{ag}(0, \bar{k}(0), \infty) \subset F^{rm}(0, \bar{k}(0), \infty). \tag{3.1}$$

Proof. Suppose that

$\{F_{ag}(0, \bar{k}(0), \infty) \cap F_i(0, \bar{k}(0), \infty)\} \cap \{F(0, \bar{k}(0), \infty) - F^{rm}(0, \bar{k}(0), \infty)\}$ is not empty.

Then, there exists $(\bar{k}(t))_{t=0}^{\infty} \in F(0, \bar{k}(0), \infty)$ such that

$$(\bar{k}(t))_{t=0}^{\infty} \in \{F_{ag}(0, \bar{k}(0), \infty) \cap F_i(0, \bar{k}(0), \infty)\} \cap \{F(0, \bar{k}(0), \infty) - F^{rm}(0, \bar{k}(0), \infty)\}.$$

Since $(\bar{k}(t))_{t=0}^{\infty} \in F(0, \bar{k}(0), \infty) - F^{rm}(0, \bar{k}(0), \infty)$, there exists $T_0 > 0$ such that

$\forall \nu > 0,\ \exists (k(t))_{t=0}^{T_0} \in N((\bar{k}(t))_{t=0}^{\infty}, T_0, \nu)$;

$$\int_0^{T_0} e^{-\rho t}\bar{P}_0\ v[\bar{k}(t), \dot{\bar{k}}(t)]\,dt < \int_0^{T_0} e^{-\rho t}\bar{P}_0\ v[k(t), \dot{k}(t)]\,dt.$$

Thus, given $T=T_0>0$ and $\nu=\nu'>0$, there exists $(k'(t))_{t=0}^{T_0}\in N((\bar{k}(t))_{t=0}^{\infty}, T_0, \nu')$ such that there is $\varepsilon^*>0$ such that

$$\int_0^{T_0} e^{-\rho t}\bar{P}_0\ v[\bar{k}(t), \dot{\bar{k}}(t)]\,dt < \int_0^{T_0} e^{-\rho t}\bar{P}_0\ v[k'(t), \dot{k}'(t)]\,dt-\varepsilon^*.$$

Hence, by (A.6), for any $T>T_0$,

$$\begin{aligned}\overline{W}^T &\geqq \int_0^{T_0} e^{-\rho t}\bar{P}_0\ v[k'(t), \dot{k}'(t)]\,dt.\\ &\quad +\sup \int_{T_0}^{T} e^{-\rho t}\bar{P}_0\ v[k''(t), \dot{k}''(t)]\,dt \text{ over all } (k''(t))_{t=T_0}^{T}\in F(T_0, \bar{k}(T_0),T)\\ &> \int_0^{T_0} e^{-\rho t}\bar{P}_0\ v[\bar{k}(t), \dot{\bar{k}}(t)]\,dt+\varepsilon^*\\ &\quad +\sup \int_{T_0}^{T} e^{-\rho t}\bar{P}_0\ v[k''(t), \dot{k}''(t)]\,dt \text{ over all } (k''(t))_{t=T_0}^{T}\in F(T_0, \bar{k}(T_0),T)\\ &= W^T((\bar{k}(t))_{t=0}^{\infty}\,|\,T_0)+\varepsilon^*.\end{aligned}$$

Therefore, there exists $T_0>0$ such that, for any $T>T_0$,

$$\overline{W}^T-W^T((\bar{k}(t))_{t=0}^{\infty}\,|\,T_0)>\varepsilon^*>0.$$

This is a contradiction. Thus,

$$\{F_{ag}(0, \bar{k}(0), \infty)\cap F_i(0, \bar{k}(0), \infty)\}\cap\{F(0, \bar{k}(0), \infty)-F^{rm}(0, \bar{k}(0), \infty)\} = \phi.$$

Therefore, $F_{ag}(0, \bar{k}(0), \infty)\cap F_i(0, \bar{k}(0), \infty)\subset F^{rm}(0, \bar{k}(0), \infty)$.

Q.E.D.

We can prove the following important fact concerning agreeable plans:

Proposition 3.2: Under assumptions (A.1)−(A.6) and given an initial capital stock $\bar{k}(0)\geq\mathbf{0}$, if an agreeable plan $(\hat{k}(t))_{t=0}^{\infty}\in F(0, \bar{k}(0), \infty)$ is interior, then,

(1) in any part of $[0, \infty)$ in which $\dot{\hat{k}}(t)$ is continuous,

$$d/dt\{e^{-\rho t}v_{z_i}[\hat{k}(t), \dot{\hat{k}}(t)]\}-e^{-\rho t}v_{k_i}[\hat{k}(t), \dot{\hat{k}}(t)] = 0 \quad (i=1, 2, \ldots, n); \qquad (3.2)$$

(2) at any corner point (a point of discontinuity of $\dot{\hat{k}}(t)$), t_0,

$$v_{z_i}[\hat{k}(t_0), \dot{\hat{k}}(t_0-0)] = v_{z_i}[\hat{k}(t_0), \dot{\hat{k}}(t_0+0)].$$

Proof. Suppose that $(\hat{k}(t))_{t=0}^{\infty}\in F_i(0, \bar{k}(0), \infty)\cap F_{ag}(0, \bar{k}(0), \infty)$. Then by Proposition 3.1, for all $t\in[0, \infty)$, there exists $T'>t$ such that

$$\exists\,\nu>0,\ \forall(k(t))_{t=0}^{T'}\in N((\hat{k}(t))_{t=0}^{\infty}, T', \nu):$$

$$\int_0^{T'} e^{-\rho t}\bar{P}_0\ v[\hat{k}(t), \dot{\hat{k}}(t)]\,dt \geqq \int_0^{T'} e^{-\rho t}\bar{P}_0\ v[k(t), \dot{k}(t)]\,dt.$$

Now, we choose some vector function $h(t)=[h_1(t), \ldots, h_n(t)]$ that satisfies the following conditions:

(1) $h(t):[0, T']\rightarrow R^n$ is a continuous, piecewise continuously differentiable function;

(2) $h(0)=h(T')=0$;

(3) there exists $t \in [0, T']$ such that $h(t) \neq \mathbf{0}$.

Given $h(t): [0, T'] \to R^n$, there exists a sufficiently small $\varepsilon > 0$ such that, for $\delta \in (-\varepsilon, \varepsilon)$, the function $k(t): [0, T'] \to R^n$ defined by

$$k(t) = \hat{k}(t) + \delta h(t) \qquad (t \in [0, T'])$$

lies in $N(\langle \hat{k}(t) \rangle_{t=0}^{\infty}, T', \nu)$. For $\delta \in (-\varepsilon, \varepsilon)$, we now define the function as follows:

$$\phi(\delta) = \int_0^{T'} e^{-\rho t}\ v[\hat{k}(t) + \delta h(k),\ \dot{\hat{k}}(t) + \delta \dot{h}(t)]\ dt.$$

Also, for $\Delta\delta \neq 0$,

$$\frac{\phi(\delta + \Delta\delta) - \phi(\delta)}{\Delta\delta}$$

$$= \frac{1}{\Delta\delta}[\int_0^{T'} e^{-\rho t}\{v[\hat{k}(t) + (\delta + \Delta\delta)h(t),\ \dot{\hat{k}}(t) + (\delta + \Delta\delta)\dot{h}(t)]$$

$$- v[\hat{k}(t) + \delta h(t),\ \dot{\hat{k}}(t) + \delta \dot{h}(t)]\}\ dt]$$

$$= \frac{1}{\Delta\delta}\int_0^{T'} e^{-\rho t}[\{v_k[\hat{k}(t) + \delta h(t),\ \dot{\hat{k}}(t) + \delta \dot{h}(t)]\}^T(\Delta\delta h(t)) + \{v_z[\hat{k}(t) + \delta h(t),\ \dot{\hat{k}}(t) + \delta \dot{h}(t)]^T(\Delta\delta \dot{h}(t))$$

$$+ \frac{1}{2}(\Delta\delta\, h(t))^T\ v_{kk}[\xi,\ \eta](\Delta\delta h(t)) + \frac{1}{2}(\Delta\delta h(t))^T\ v_{kz}[\xi,\ \eta](\Delta\delta \dot{h}(t))$$

$$+ \frac{1}{2}(\Delta\delta \dot{h}(t))^T\ v_{zk}[\xi,\ \eta](\Delta\delta h(t)) + \frac{1}{2}(\Delta\delta \dot{h}(t))^T\ v_{zz}[\xi,\ \eta](\Delta\delta \dot{h}(t))]dt$$

where $\xi = \hat{k}(t) + \delta h(t) + \theta\Delta\delta h(t)$, $\eta = \dot{\hat{k}}(t) + \delta \dot{h}(t) + \theta\Delta\delta \dot{h}(t)$, $0 < \theta < 1$.

Thus,

$$\Delta\delta \to 0 \Rightarrow$$

$$\frac{\phi(\delta + \Delta\delta) - \phi(\delta)}{\Delta\delta} \to \phi'(\delta)$$

$$= \int_0^{T'} e^{-\rho t}\{v_k[\hat{k}(t) + \delta h(t),\ \dot{\hat{k}}(t) + \delta \dot{h}(t)] \cdot h(t) + v_z[\hat{k}(t) + \delta h(t),\ \dot{\hat{k}}(t) + \delta \dot{h}(t)] \cdot \dot{h}(t)\}\ dt.$$

Furthermore, for any $\delta \in (-\varepsilon, \varepsilon)$,

$$\phi(0) \geqq \phi(\delta).$$

Therefore we obtain

$$\phi'(0) = 0.$$

Hence,

$$\int_0^{T'} e^{-\rho t}\Big[\sum_{i=1}^{n}\{v_{k_i}[\hat{k}(t),\ \dot{\hat{k}}(t)]\ h_i(t) + v_{z_i}[\hat{k}(t),\ \dot{\hat{k}}(t)]\ \dot{h}_i(t)\}\Big]\ dt = 0.$$

Now, since

$$\frac{d}{dt}\int_0^{t} e^{-\rho\tau}\ v_{k_i}[\hat{k}(\tau),\ \dot{\hat{k}}(\tau)]\ d\tau = e^{-\rho t}\ v_{k_i}[\hat{k}(t),\ \dot{\hat{k}}(t)],$$

with integration by part,

$$\int_0^{T'} e^{-\rho t}\ v_{K_i}[\hat{k}(t), \dot{\hat{k}}(t)]\ h_i(t)\ dt$$

$$= [h_i(t) \int_0^t e^{-\rho\tau}\ v_{K_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau]_0^{T'} - \int_0^{T'} \dot{h}_i(t)\left\{\int_0^t e^{-\rho\tau}\ v_{K_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau\right\}dt.$$

Here, $h(0)=h(T')=0$. Thus,

$$\sum_{i=1}^{n}\left\{\int_0^{T'} e^{-\rho t}\ v_{K_i}[\hat{k}(t), \dot{\hat{k}}(t)]\ h_i(t)\ dt + \int_0^{T'} e^{-\rho t}\ v_{Z_i}[\hat{k}(t), \dot{\hat{k}}(t)]\ \dot{h}_i(t)\ dt\right\}$$

$$= \sum_{i=1}^{n}\left[\int_0^{T'} e^{-\rho t}\ v_{Z_i}[\hat{k}(t), \dot{\hat{k}}(t)]\dot{h}_i(t)\ dt - \int_0^{T'} \dot{h}_i(t)\left\{\int_0^t e^{-\rho\tau}\ v_{K_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau\right\}\ dt\right].$$

$$= \int_0^{T'}\left[\sum_{i=1}^{n}\{e^{-\rho t}\ v_{Z_i}[\hat{k}(t), \dot{\hat{k}}(t)] - \int_0^t e^{-\rho\tau}\ v_{K_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau\}\ \dot{h}_i(t)\right]\ dt.$$

$$=0.$$

Now, we define the function $L_i(t): [0, T']\to R$ $(i=1, 2, \dots, n)$ as follows:

$$L_i(t) = e^{-\rho t}\ v_{Z_i}[\hat{k}(t), \dot{\hat{k}}(t)]\ - \int_0^t e^{-\rho\tau}\ v_{K_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau.$$

Also, we choose the constant $C_i (i=1, 2, \dots, n)$ such that

$$C_i = \frac{1}{T'}\int_0^{T'} L_i(t)\ dt.$$

and we define the function $\hat{h}_i(t): [0, T']\to R$ $(i=1, 2, \dots, n)$ as follows::

$$\hat{h}_i(t) = \int_0^t \{L_i(s)-C_i\}\ ds.$$

The function $\hat{h}_i(t): [0, T']\to R$ $(i=1, 2, \dots, n)$ is continuous and piecewise continuously differentiable. Furthermore, for any $i \in \{1, 2, \dots, n\}$,

$$\hat{h}_i(0) = 0,$$

$$\hat{h}_i(T') = \int_0^{T'} \{L_i(s)-C_i\}\ ds = \int_0^{T'} L_i(s)\ ds - C_iT' = 0.$$

Therefore, $\hat{h}_i(t): [0, T']\to R(i=1, 2, \dots, n)$ satisfies the above conditions (1), (2), (3). Since the above equation still holds for $\hat{h}_i(t): [0, T']\to R$ $(i=1, 2, \dots, n)$, we obtain

$$\int_0^{T'} \sum_{i=1}^{n} \{L_i(t)-C_i\}\ dt = 0.$$

Also, since, for any $i \in \{1, 2, \dots, n\}$,

$$\int_0^{T'} \{L_i(t)-C_i\}\ dt = 0,$$

we obtain

$$\int_0^{T'} \sum_{i=1}^{n} \{L_i(t)-C_i\}^2\ dt$$

$$= \int_0^{T'} \sum_{i=1}^{n} L_i(t)\ \{L_i(t)-C_i\}\ dt - \sum_{i=1}^{n} \{C_i \int_0^{T'} (L_i(t)-C_i)\ dt\}$$

$$= 0.$$

Hence, for any $t \in [0, T']$, $L_i(t)=C_i$ $(i=1, 2, \dots, n)$. Thus, for any $t \in [0, T']$

$$e^{-\rho t}\ v_{z_i}[\hat{k}(t), \dot{\hat{k}}(t)] - \int_0^t e^{-\rho\tau}\ v_{k_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau = C_i, \tag{3.4}$$

where C_i (i=1, 2, ..., n) is some constant.

Now, we consider any bounded interval $(t', t'') \subset [0,\infty)$ in which $\dot{\hat{k}}(t)$ is continuous and choose $T' > t''$. Since, on (t', t''), $v_{k_i}\ [\hat{k}(t), \dot{\hat{k}}(t)]$ is continuous, $\int_0^t e^{-\rho\tau}\ v_{k_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau$ is differentiable on (t', t''). Thus, $e^{-\rho t}\ v_{z_i}[\hat{k}(t), \dot{\hat{k}}(t)]$ is differentiable on (t', t'').

Therefore, differentiating with respect to t gives the equation

$$\frac{d}{dt}\{e^{-\rho t}\ v_{z_i}[\hat{k}(t), \dot{\hat{k}}(t)]\} - e^{-\rho t}\ v_{k_i}[\hat{k}(t), \dot{\hat{k}}(t)] = 0. \qquad (t \in (t', t''))$$

Also, let t_0 be any corner point. Then, we choose $T' > t_0$. Since, on $[0, T']$,

$$\int_0^t e^{-\rho\tau}\ v_{k_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau + C_i$$

is continuous, $e^{-\rho t}\ v_{z_i}[\hat{k}(t), \dot{\hat{k}}(t)]$ is continuous. Therefore, at t_0,

$$e^{-\rho t_o}\ v_{z_i}[\hat{k}(t_o), \dot{\hat{k}}(t_o-0)] = e^{-\rho t_o}\ v_{z_i}[\hat{k}(t_o), \dot{\hat{k}}(t_o+0)]\ .$$

Q.E.D.

Now, in order to proceed, the following assumptions will be made.

(A.7)　(1)　$\overset{\circ}{X}$ is convex.

(2)　For all $(k, z) \in \overset{\circ}{X}$,

$\det v_{zz}\ [k, z] \neq 0$.

(3)　For any $t_0 \in [0, \infty]$ and any $(k(t))_{t=0}^{\infty} \in F_i(0, \bar{k}(0), \infty)$,

$$\lim_{\substack{t \to t_o \\ t < t_o}} (k(t), \dot{k}(t)) = (k(t_o), \dot{k}(t_o-0)) \in \overset{\circ}{X},$$

$$\lim_{\substack{t \to t_o \\ t > t_o}} (k(t), \dot{k}(t)) = (k(t_o), \dot{k}(t_o+0)) \in \overset{\circ}{X}.$$

Assumptions (A.7) (1), (2) are standard assumptions.

Thus, we have the following proposition.

Proposition 3.3 : Under assumptions (A.1) − (A.7), and given an initial capital stock $\bar{k}(0) \geq \mathbf{0}$, if an agreeable plan is interior, then it is a locally optimal accumulation path from $\bar{k}(0)$, that is,

$$F_i(0, \bar{k}(0), \infty) \cap F_{ag}(0, \bar{k}(0), \infty) \subset F^{lo}(0, \bar{k}(0), \infty).$$

Proof. Let $(\hat{k}(t))_{t=0}^{\infty}$ be an interior agreeable plan from $\bar{k}(0)$. Now suppose that there exists a corner point t′ of $\langle \hat{k}(t) \rangle_{t=0}^{\infty}$. Then,

$$\dot{\hat{k}}(t'-0) \neq \dot{\hat{k}}(t'+0).$$

By Proposition 3.2, however,

$$v_z\ [\hat{k}(t'), \dot{\hat{k}}(t'-0)] = v_z\ [\hat{k}(t'), \dot{\hat{k}}(t'+0)].$$

Thus, by the theorem of Rolle, we obtain

$$v_{zz}\,[\hat{k}(t'),\ \dot{\hat{k}}(t'-0)+\theta\{\dot{\hat{k}}(t'+0)-\dot{\hat{k}}(t'-0)\}]\cdot(\dot{\hat{k}}(t'+0)-\dot{\hat{k}}(t'-0))=\mathbf{0}.$$

Here, $(\hat{k}(t'),\ \dot{\hat{k}}(t'-0)+\theta\{\dot{\hat{k}}(t'+0)-\dot{\hat{k}}(t'-0)\})\in\mathring{X}$, $\theta\in(0, 1)$.

This is a contradiction of (A.7−2). Thus, there exist no corner points of $(\hat{k}(t))_{t=0}^{\infty}$.

Next, for any $t\in[0, \infty)$, we choose $T'>t$. We define the function $F_i[t, z_1, z_2, \ldots, z_n]: [0, T']\times R^n\to R$ $(i=1, 2, \ldots, n)$ as follows:

$$F_i[t, z_1, z_2, \ldots, z_n]$$
$$=e^{-\rho t}\ v_{z_i}[\hat{k}(t), z_1, z_2, \ldots, z_n]-\int_0^t e^{-\rho\tau}\ v_{k_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau-C_i,$$

where C_i $(i=1, 2, \ldots, n)$ are the constants associated with the above equations (3.4). Since there exist no corner points of $(\hat{k}(t))_{t=0}^{\infty}$, the function $\int_0^t e^{-\rho\tau}\ v_{k_i}[\hat{k}(\tau), \dot{\hat{k}}(\tau)]\ d\tau: [0, T']\to R$ is continuously differentiable. Thus, $F_i[t, z_1, z_2, \ldots, z_n]: [0, T']\times R^n\to R$ $(i=1, 2, \ldots, n)$ is continuously differentiable.

Furthermore, for any $t'\in[0, T']$, the following conditions hold:

(1) $F_i[t', \dot{\hat{k}}(t')]=0\qquad(i=1, 2, \ldots, n)$;

(2) $$\det\begin{bmatrix}\dfrac{\partial F_1[t', \dot{\hat{k}}(t')]}{\partial z_1} & \cdots\cdots & \dfrac{\partial F_1[t', \dot{\hat{k}}(t')]}{\partial z_n}\\ \vdots & & \vdots\\ \dfrac{\partial F_n[t', \dot{\hat{k}}(t')]}{\partial z_1} & \cdots\cdots & \dfrac{\partial F_n[t', \dot{\hat{k}}(t')]}{\partial z_n}\end{bmatrix}$$
$$=\det e^{-\rho t'}\ v_{zz}\ [\hat{k}(t'), \dot{\hat{k}}(t')]\ \neq 0.$$

Therefore, according to a theorem on implicit functions, there exists a neighborhood $I\times\Omega$ of $(t', \dot{\hat{k}}(t'))$ such that there exists a unique function $z(t): I\to\Omega$, satisfying the condition $z(t')=\dot{\hat{k}}(t')$. Also, $z(t): I\to\Omega$ is a continuously differentiable function. Thus $z(t)=\dot{\hat{k}}(t)$ in some neighborhood of t'. Hence, $\ddot{\hat{k}}_i(t)$ values exist and are continuous in some neighborhood of t'. By repeating this process for any $t\in[0, \infty)$, we can show that $\hat{k}(t): [0, \infty)\to R_+^n$ is twice continuously differentiable.

Therefore, by carrying out the differentiation of equations (3.2)

$$v_{zz}[\hat{k}(t), \dot{\hat{k}}(t)]\ddot{\hat{k}}(t)+v_{zk}[\hat{k}(t), \dot{\hat{k}}(t)]\dot{\hat{k}}(t)$$
$$-\{\rho\ v_z[\hat{k}(t), \dot{\hat{k}}(t)]+v_k[\hat{k}(t), \dot{\hat{k}}(t)]\}=\mathbf{0}.$$

Therefore, $(\hat{k}(t))_{t=0}^{\infty}$ is a locally optimal accumulation path from $\bar{k}(0)$.

Q.E.D.

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