Estimation of Connectivity among Units of Vector Auto-Regressive Models Based On the Edge Snapping and its Applications to the Network Formations / Disruptions Analysis

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1   Introduction

Until today, various methods have been proposed to synchronize nonlinear fluctuating dynamics by imposing appropriate inputs [1]-[4]. Recently, among many synchronization theories, the Edge Snapping is recognized as an efficient method focusing on the connectivity between units [5]-[8]. The Edge Snapping has been applied to the continuous chaotic dynamics and also to the discrete chaotic dynamics. In the Edge Snapping, connections (called as coupling gains are initially 0, and then the coupling gains are adjusted smoothly (to be connected, or disconnected) based on the relative relation between units [5]-[8]. But, the original method has several restrictions, and some extension is necessary [9]. Many conventional works related to synchronization postulate that objects to be synchronized should be nonlinear system and dynamics [1]-[8]. We also extend the network formation scheme to the network disruption as the counter part of the Edge Snapping [10]. But in real world, many systems are not represented by ideal nonlinear and chaotic dynamics, and in the sense a linear description is necessary. This paper deals with the estimation of connectivity among units of Vector Auto-Regressive (VAR) models [11]-[13] based on the Edge Snapping and its applications to the group formation analysis.

In our systems, units are represented by the linear VAR models [11]-[13], and the coupling gains are adjusted so that vectors between two units become closer (or strengthen the connectivity to unit having larger values) based on the Edge Snapping. As a result, coupling gains representing the connectivity become very close to 1 (however, gains fluctuate around 1). If the system is described by the chaotic dynamics, then the gains are rigidly 1 (due to the lead in phenomenon of chaos), However we treat here linear VAR models, and we see some fluctuation. But for real application, the basic characteristics is enough to see group formations.

At the beginning, we show the fundamentals of the Edge Snapping. The coupling gain σ_{ij} between unit i and unit j is described by the dynamics including the difference of characteristics

between two units. Namely, the coupling gains changes smoothly along the second order vibration system. In Case 1, we treat the system where the coupling gain approaches to 1 if vectors between two units become closer. But in Case 2, we define the system where the coupling gain approaches to 1 if the unit j has a relative larger value. We use the discretized system to approximate the dynamics of the coupling gain, then we can show qualitative behavior of coupling gains to figure out the convergence. As a result, we see the gain move at random if the coefficient is smaller than 0.5, but if the coefficient become larger than 0.5, then the gain rapidly approaches to 1.

As applications, we use the method to artificial data, and also discuss real application to group formation analysis. Our method treat smooth and natural corporation and group formation among units, and has several advantages over other works postulating a priori definition of corporation [14][15].

In the followings, in Section2 we show the basics and application of the Edge Snapping. In Section 3, we treat the representation of units by using VAR models. In Section 4 we show the applications.

2　Basics of Edge Snapping and Applications

2.1　Synchronization of chaos based on edge snapping and extension

For the sake of simple explanation of basics of the Edge Snapping, we show at first the synchronization of continuous chaotic system. There are N multiple units in the continuous time system, we suppose that the behavior (fluctuation) of the i th unit is described by the M dimensional vector $x_i(t) = x_{i1}(t), x_{i2}(t), ..., x_{iM}(t)$ as follows.

$$\dot{x}_i(t) = f(x_i(t)) + c_1 \sum_{j=1}^{N} \sigma_{ij}(t)(x_j(t) - x_i(t)) \tag{1}$$

where $\sigma_{ij}(t)$ is the coupling gain between unit i and unit j

The constant c_1 is used to normalize the range of variables depending on the cases of examples. The coupling gains behave along the following partial differential equations (pdf).

$$\ddot{\sigma}_{ij}(t) + d_D \dot{\sigma}_{ij} + \frac{\partial V(\sigma_{ij})}{\partial \sigma_{ij}} = g(x_i(t), x_j(t)) \tag{2}$$

The shape of the function $V(\sigma_{ij})$ is discussed later.

Here, the constant d_D is the damping factor to mitigate the fluctuation of the dynamics of the coupling gain. The constant c_2 is used to normalize the range of variables. The function $V(\sigma_{ij})$ takes 1 on points $\sigma_{ij} = 1$ and $\sigma_{ij} = 0$, and looks like a well-shaped potential function. The overview of the function $V(\sigma_{ij})$ is shown n Fig.1. We use the functional following the example in reference [5].

$$V(\sigma_{ij}) = b_V \sigma_{ij}^2 (\sigma_{ij} - 1)^2 \tag{3}$$

The purpose of the synchronization of vibration system is attaining the approximate coincidence of variables, and is represented as $\lim_{t \to \infty} [x_i(t) - x_j(t)] = 0$.

We use the discretized version of the continuous equations in equations (1)(2) [7][8][9]. At first, the transition of the variable $x_i(t)$ is described as follows.

$$x_i(t+1) = f(x_i(t)) + c_1 \sum_{j=1}^{N} \sigma_{ij}(t)(x_j(t) - x_i(t)) \tag{4}$$

where the constant c_1 is used to normalize the range of variable as in the continuous system. Then, we use the following third order discretized dynamics of equation to describe the vibration of coupling gain. The partial differentiation of the continuous time function $V(.)$ shown as $\partial V(\sigma_{ij})/\partial \sigma_{ij}$, and becomes a third order function of σ_{ij}.

$$(1 + d_D + B)\sigma_{ij}(t) - D - [2+d]\sigma_{ij}(t-1) + \sigma_{ij}(t-2) = 0 \tag{5}$$

$$D = c_2|x_j(t) - x_i(t)|, B(\sigma_{ij}) = \frac{\partial V(\sigma_{ij})}{\partial \sigma_{ij}} \tag{6}$$

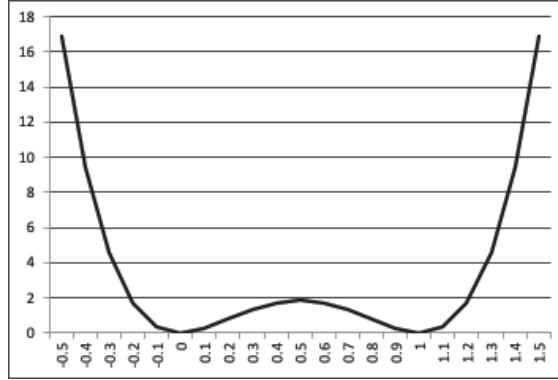$$B(\sigma_{ij}) = 2b_V \sigma_{ij}(\sigma_{ij} - 1)(2\sigma_{ij} - 1) \tag{7}$$

Figure 1: Overview of function $V(.)$

2.2 Definition of two cases

In this paper, we discuss following two cases by defining the function $g(.)$ in equation (2) as follows.
(1) Case 1: defining as the difference between two vectors

$$g(e_{ij}(t)) = c_2|x_j(t) - x_i(t)| \tag{8}$$

(2) Case 2: defining as the division of two vectors

$$g(e_{ij}(t)) = c_2|x_j(t)|/|x_i(t)| \tag{9}$$

In Case 1, the value of function $g(.)$ decreases if the value of variables in unit i come closer to the value of unit j. As a result, the coupling gain between two units approaches to 1, then we find the nearest neighbor unit in the whole sets.

In Case 2, if the value of unit i becomes large, then the value of the function $g(.)$ becomes small. Then, the coupling gain between two units approaches to 1. Then we find a dominant unit in the whole sets.

2.3　Qualitative performance of coupling gain

Now we examine the qualitative performance of the coupling gain $\sigma_{ij}(t)$ in equations (4)(5)(6). If we ignore the term including the second order differentiation with respect to the time (the term describes the gradual change of the gain $\sigma_{ij}(t)$), and even more we approximate the first order difference by the the difference between the time t and $t+1$ such as $\dot{\sigma}_{ij} = \Delta\sigma_{ij}$ (using $\Delta t = 1$). Then, we obtain following representation. Here we use the value of function $B(.)$ by using the value σ_{ij} at time t.

$$d_D \Delta\sigma_{ij}(t+1) = c_1[x_j(t) - x_i(t)]^2 - B(\sigma_{ij}(t)) \tag{10}$$

We see in case where $\sigma_{ij} > 0.5$, the function $B(.)$ is negative. Similarly, in case $\sigma_{ij} < 0.5$, the function $B(.)$ is positive. By considering these cases, we estimate the value $\sigma_{ij}(t+1)$ at time $t+1$.

(1) Tendency of convergence of coupling gains : over 0.5 then rapidly approaches to 1.

For the sake of simplicity, we focus on specific values of $\sigma_{ij}(t)$. In case $\sigma_{ij}(t) = 0$ and in case $\sigma_{ij}(t) = 0.5$, the right hand side of equation (10) is positive. The fact reduces $\sigma_{ij}(t+1) > 0$ for $\sigma_{ij}(t) = 0$, and $\sigma_{ij}(t+1) > 0.5$ for $\sigma_{ij}(t) = 0.5$. The coupling gain increases.

In case $\sigma_{ij}(t) = 0.75$, the right hand side of equation (10) is positive, and we see $\sigma_{ij}(t+1) > 0.75$, and the value still increases.

On the other hand in case $\sigma_{ij}(t) = 0.25$ the right hand side of equation (10) is ether positive or negative, and the fact reduces that $\sigma_{ij}(t+1)$ can take various values,

The analysis means that if $\sigma_{ij}(t) \geq 0.5$m the value $\sigma_{ij}(t+1) > 0.5$ will be realized. On the other hand if $\sigma_{ij}(t) < 0.5$, then $\sigma_{ij}(t+1) > 0.5$ can take various values. However, we must note that the additive term in equation(2) including second order differentiation of $\sigma_{ij}(t)$ with respect to the time deliver complicated effect to the behavior to coupling gains..

(2) if coupling gain approaches to 1 (or 0), gain remains around 1 (or 0)

It is possible to recall in Case 1, if the coupling gain $\sigma_{ij}(t)$ approaches to 1, then two vector variables x_i, x_j change along a similar pattern and become very close each other. Actually if the dynamics of vector variables x_i, x_j are described by the continuous time chaotic system, or the discrete time chaotic system, then after a lapse of time we have ,$x_i = x_j$ (due to the lead phenomenon of the chaotic synchronization).

However, in cases of general models about the unit coupling treated in the paper , we can not see the rigid coincidence between two vector variables. However, in case if $\sigma_{ij}(t)$ approaches to 1 very closely and the values of two vectors x_i, x_j are very close, then the value of $B(.)$ on the left hand side of equation (**) and the value of $x_j - x_i$ is very close to 0 Then, as a result, the value of $\Delta\sigma_{ij}(t)$ still remains around 0, The fact means the coupling gain still remains close to 1, and takes only small vibration around the value 1. In the same way, in case where the coupling gain

approaches to 0, the value of $B(.)$ becomes nearly equal to 0, then the possibility of case where the gain leaves from 0 will be very small.

2.4   Modeling of network disruptions

Most of conventional works discuss the formations of networks consisting of units by considering the connection of units and the synchronization scheme. However, several recent works describe the disconnection of networks through some kinds of external forces or internal sources to resolve the connections among units (called as the disruptions of networks) [11]. In the paper, for the analysis of network disruptions we utilize the counter part of the connectivity of units based on the Edge Snapping. We showed mostly the behavior of unit i and unit j where we focused on the difference between two vectors x_i and x_j which goes smaller (in Case 1) or the ratio of these vectors which goes smaller (in Case 2). Then, as the counter part of these schemes, we consider the cases where the vector variable x_i suddenly becomes large (in Case M), or suddenly changed to be sufficiently small (in Case M). However, we can copy with the problems by slightly changing our scheme of the Edge Snapping.

Now we start with the approximate representation of the behavior of coupling gain σ_{ij} in equation (10). We slightly rewrite the equation by using two vector variables as $\Delta x = |x_i(t) - x_j(t)|$ needed to get Δx.

$$d_D \Delta \sigma_{ij}(t+1) = c_1 \Delta x^2 - B(\sigma_{ij}(t)) \tag{11}$$

We consider the case where the gain σ_{ij} is already close to 1 but still fluctuating around 1. We would like to change (shift) the gain to 0. However, we can not attain the situation direct, because the right hand side of equation (99) is positive since $B(\sigma_{ij}) = 0$ for $\sigma_{ij} = 1$. We have

$$d_D \Delta \sigma_{ij}(t+1) = c_1 \Delta x^2 \tag{12}$$

Then, we need two steps to attain the disconnection of the coupling. At first, we shift the coupling gain larger than 1 ($\sigma_{ij}(t+1) = 1 + \Delta \sigma(t+1) > 1$, and then move $\sigma_{ij}(t+2)$ to 0.

(Step 1) move $\sigma_{ij}(t)$ by $\Delta \sigma(t+1)$

Since $B(1) = 0$ under $\sigma_{ij}(t) = 1$, then $d_D \Delta \sigma_{ij}(t+1) = c_1 \Delta x^2$, and $\sigma_{ij}(t+1) = \sigma_{ij}(t) + \Delta \sigma_{ij}(t+1) > 1$.

(Step 2) move $\sigma_{ij}(t+2)$ to 0

Suppose that the value of Δx is still remained unchanged at time $t+1$ and at time $t+2$, then we shift the value of $\sigma_{ij}(t+2)$ by setting following equation.

$$-d_D[1 + \Delta \sigma_{ij}(t+1)] = c_1 \Delta x^2 - B(1 + \sigma_{ij}(t+1)) \tag{13}$$

where we use the relation

$$\sigma_{ij}(t+1) = 1 + \Delta \sigma_{ij}(t+1) \tag{14}$$

and we need following equation.

$$\Delta \sigma_{ij}(t+2) = -[\sigma_{ij}(t+1) + \Delta \sigma_{ij}(t+1)] \tag{15}$$

By combining two equations (99) and (88), the optimal value of Δx is obtained.

We call the threshold value of the difference defined in equation (99) as Δx_{dis} in the followings. Actually, we meet the cases where we attain the disruption even if the value of $|x_i - x_j|$ is smaller than Δx_{dis} due to the second order fluctuation of the coupling gain (in equation (99) we omit the effect). Similarly we may find the cases where even if $|x_i - x_j|$ is sufficiently larger than Δx_{dis} we can not attain the disruption. However, the value of Δx_{dis} is still useful to examine the disruption.

In the same way, we have the relation of the disruption of networks for Case 2. We simply replace the value of $|x_i - x_j|$ on the right hand side of equation (99) by the ratio of two vector variables $|x_j|/|x_i|$ and we call the threshold value of as Rx_{dis} in the followings.

3　Representation of units by using VAR models

3.1　Background of introduction of VAR model

We quickly summarize the background of the introduction of VAR models in the socio-economic analysis [11]-[13]. The advantages of VAR model are 1) it is a simplified model reduced from nonlinear macro models, 2) easily we obtain the effect of external input (called shock analysis). Concerning about 1), the VAR model is developed to reduce complicated nonlinear macro model to simple forms and to obtain practical tool to analyze real world data. Therefore, the background of the VAR model comes from nonlinear macro economics. In ordinary macro models, we a priori need to separate explanatory (external) variables and explained variable (internal) variables, and therefore exist some restriction of the selection of variables. In VAR models, we do not need such separations among variable. Sims proposed the VAR model at first, and then we can analyze any size of economic models [11].

Concerning about 2), we can easily estimate the effect of sudden change of a certain variable. Assuming a certain variable raises a level (called as shock), then in the VAR framework, other variables are changed (response) from the current levels. In these events, we can estimate the amplitude of the response and also the length of lasting time of effect.

3.2　Basics of VAR models

In the following, we show the equations used for VAR models. Namely, the variables are represented by using simultaneous equation and also variables are characterized by using other variables including lagged components.. For the sake of simplicity, we assume that the lagged variables include only first order lag time $t - 1$ in equations. We have next representation [11]-[13].

$$X_i(t) = A^{(i)} X_i(t-1) + V_i(t) \tag{16}$$

$$X_i(t) = [x_{i1}(t), x_{i1}(t), x_{i2}(t), ..., x_{iM}(t)]^T \tag{17}$$

$$A^{(i)} = \begin{pmatrix} a_{11}^{(i)} & a_{12}^{(i)} & ... & a_{1M}^{(i)} \\ a_{21}^{(i)} & a_{22}^{(i)} & ... & a_{2M}^{(i)} \\ ... & & & \\ a_{M1}^{(i)} & a_{M2}^{(i)} & ... & a_{MM}^{(i)} \end{pmatrix} \tag{18}$$

$$V_i(t) = [v_{i1}(t), v_{i1}(t), v_{i2}(t), ..., v_{iM}(t)]^T \tag{19}$$

where terms $v_{i1}(t), v_{i1}(t), v_{i2}(t), ..., v_{iM}(t)]$ are white noises.

3.3   VAR models and shock analysis

As one of the most important analysis of VAR models, there exists a method call as the shock analysis [10]-[12]. The method can be explained by using the innovation theory, but for simplicity we explain it on the basis of engineering field. From the engineering point of view, the shock is explained as the impulse response when we impose the impulse in the term of random variable (white noises) in equation (11). We estimate the response (output) of every variables when we imposed a impulse on a certain white noise. Then, we summarize the response (amplitude) for each variables, and we see also how long the shock affects on each variable. For example, we can know the effect of the monetary policy of a country to other variable for example GDP (Gross Domestic Product).

4   Applications

4.1   Analysis by using artificial data

In the following, at first we show the ability of our paper by applying our method to the artificial data. Our procedure is organized as follows.

(1) coefficients of VAR models are taken from random numbers

In the simulation study, we generate random numbers and give them as the coefficients of VAR models, In this way, we avoid the cases where the coupling gain among several units are 1 or 0 from the beginning.

(2) we stop the generation of VAR coefficients if time series reveals divergence

Since the coefficients of VAR model are taken from random numbers, it will happen the time series generated from the VAR model diverges. In these cases, we stop further usage of the VAR model, and abandon these coefficients. Finally, we get sufficient number of sets of VAR coefficients for simulation study.

The parameters for units of VAR models are given as follows

$$N = 100, M = 5, d_D = 30, b_V = 30, c_1 = 0.5, c_2 = 0.2 \tag{20}$$

(1) Results in Case 1

Fig.2 shows a typical example where the coupling gain approaches to 1 or 0, ultimately. To distinguish two cases, we denote the case where the coupling gain approaches to 1 as "goto 1", and the case where the gain approaches to 0 as "goto 0" in this figure. In the simulation study, we see the coupling gain approaches very rapidly to 1 or to 0 after the beginning of coupling. The fact means if two variables instantly come closer, then the coupling gain raises to 1. On the other hand, if the coupling gain falls less than 0.5, then the coupling gain approaches to 0 rapidly.

In Fig,3 we also show the change of two corresponding time series x_i, x_j along the time. We only show the case the coupling gain approaches to 1 (case "go to 1") , and we show only the first
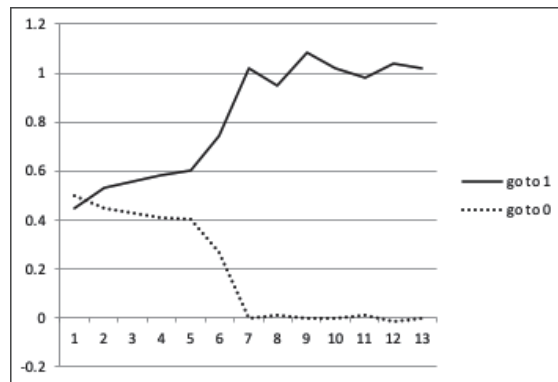
Figure 2: Change of coupling coefficients along time in Case 1 (upper:when the coefficient approaches to 1 (go to 1), lower:when the coefficient approaches to 0) (goto 0)
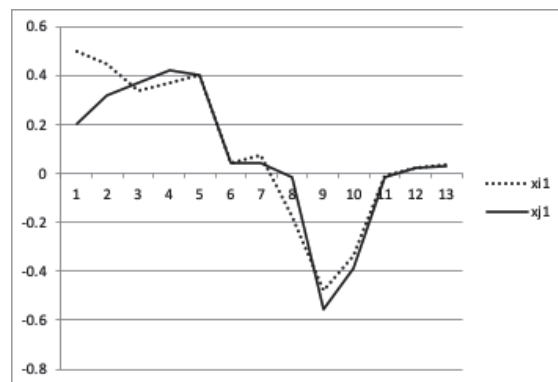

Figure 3: Change of vector variables x_i, x_j along time (only first elements)

element of vectors, since the other elements have the same performances. As shown Fig.3, two variables come close each other after the lapse of time. We omit the cases where the coupling gain approaches to 0 ("go to 0"). We see from this figure that two vector variables come closer along the time very rapidly (other elements except the first element show the same performance, but omitted here).
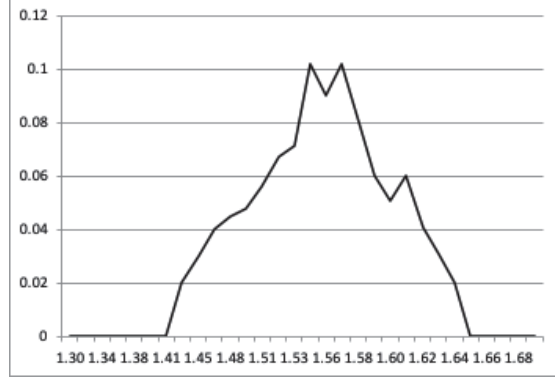

Figure 4: Distribution of $p(\log k)$ in Case 1

Further more, we estimate the distribution of connected number k for units (by taking $(\log k)$ among units by using the probability distribution of denoted as $p(\log k)$. The result is shown Fig.4. As is seen form the figure, the probability $p(\log k)$ is concentrated around the center of $\log k$. The fact means the number of connectivity among units is not enough small or enough large, and the behaviors of units are not distinguished.

(2) Results in Case 2

Similarly, in Fig.5 we show a typical example in Case 2 where the coupling gain approaches to 1 or 0. As in Case 1, if there exits a variable instantaneously having relatively large value, then the coupling gain between another unit that wish to connect to underlying unit and the coupling gain is already greater than 0,5 raises rapidly to 1. On the other hand, if the underlying coupling gain remains still under 0.5, then the coupling gain approaches rapidly to 0.

In Fig.6, we show the change of two corresponding time series x_i, x_j along the time (we only show the case the coupling gain approaches to 1 case "go to 1") where one of the time series has at the beginning relatively large value. We show only first elements of vectors, since the other elements have the same performances.

Now, we can find very interesting feature in Case 2 from the properties of coupling gains among units . Namely, by using the connection where the coupling gains are close to 1, we depict the relation between the number of connected units (denoted as k) meaning the number of connections between a dominant unit having large value, and the histogram $p(\log k)$ about k. Then, the relation between $\log k$ (horizontal line) and $p(k)$ (vertical line), we get linear relation, which is typically found in the scale-free networks. Fig.7 show an example of the relation $p(\log k)$ and $\log k$, and we find the linear relation between them showing the network is scale-free. The fact means that units
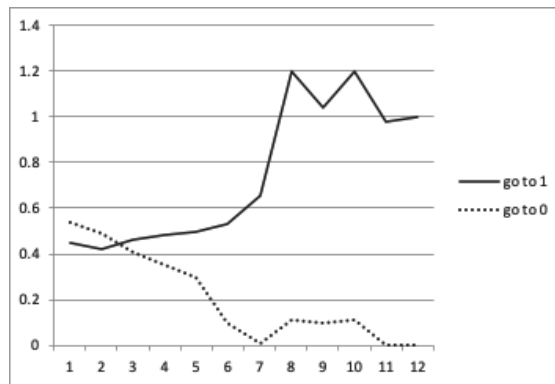
Figure 5: Change of coupling coefficients along time in Case 1 (upper:when the coefficient approaches to 1 (go to 1), lower:when the coefficient approaches to 0) (goto 0)
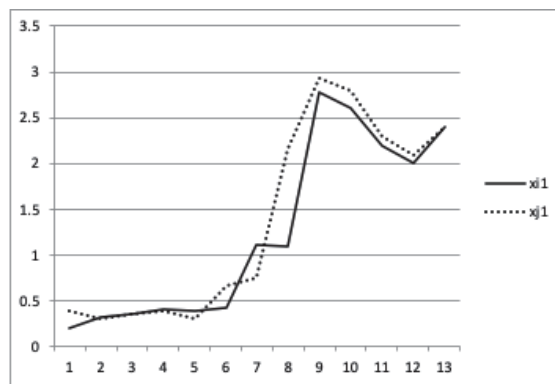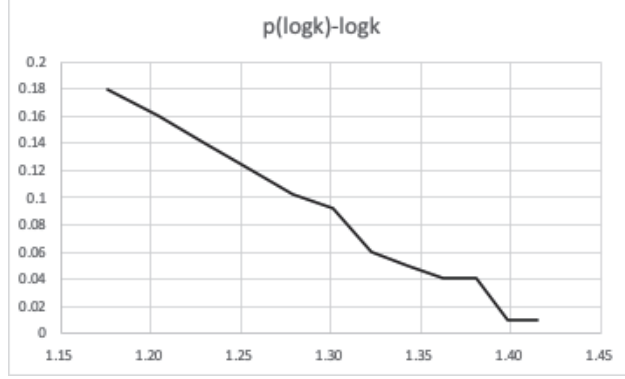

Figure 6: Change of vector variables x_i, x_j along time (only first elements)

Figure 7: Distribution of $p(\log k)$ in Case 2

having relatively large values can have many connections (links), but the number of these cases are small (rare cases). On the other hand, almost all units have only small sized links or middle-sized links, and can not get sufficient connections.

4.2   Simulation study of network disruption

Now we show the simulation study of the network disruption due to the sudden change of vector variables of units. We assume in Case 1 that units i and unit j realized the coupling state where the gain $\sigma_{ij} = 1$. Then, we impose a sudden change on the vector variable x_i such as the indicator Δx_{dis} is equal to the value as in equation (99). We slightly changed the parameter from $b_V = 30$ to $b_V = 8$ so that the coupling coefficient can be easily moved from 1 to 0.
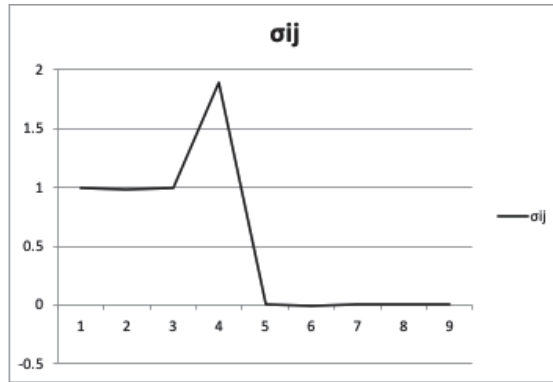

Figure 8: Change of coupling coefficient σ_{ij} fro 1 to 0

Table 1.Meaning of economic indicator for each country

name of variable	meaning	unit of data increase rate
GDP	increase rate of GDP from previous term	%
CPI	increase rate of consumer price index	%
Rate	short term interest rate	%
Employment	unemployment rate in whole workers	%
Monetary base	money supply in monetary policy	recalculated by dollar

Fig.8 shows the change of the coupling gain before and after the change of x_i, As is seen from Fig.8, the coupling gain become quickly around 0 that means two units are almost disconnected. The situation continues still after the sudden change of vector variable x_i. Even though we can show the same result of sudden change of vector variable x_i and the indicator Rx_{dis}, but the performance is almost the same and we skip them.

4.3　Analysis of economic data based on Case 1

As an application of Case 1 to real world data, we show an example of the analysis of economic data [16]. Namely, at first we formulize the VRA models for each dominant country in world economics by using typical economic indicators and obtain VAR models for each unit (country). Then, we apply the method of our paper to find the relation (connectivity) among the countries. We see tight connectivity among countries. We examine whether there exists large coupling gains among units (countries) by using the Edge Snapping. We focus on the economic indicators of following 18 countries :Japan, USA, Korea, France, Germany, England, Canada, Australia, Sweden, Denmark, Norway, Poland, Russia, China, Turkey, South Africa, Mexico, Brazil.

To characterize the economy of each country, we use typical economic indicators shown in Table 1 in each year form 1994 to 2006. Then, we obtain VAR models to get VAR coefficients for each country representing the economic behavior of each country (well known software package TSP is available [17]). Then, we apply the Edge Snapping to these data to get the connectivity among countries. The same procedure for artificial data in Case 1 is applied.

Here, we show only the result of grouping for each country where we make a group by recognizing the coupling gains are very close to 1. We obtain the result as follows. The numbers in the parenthesis denote the number of other countries which are assumed to belong the same group.

Japan(3), USA(4), Korea(2), France(4), Germany(5), England(3), Canada(4), Australia(4), Sweden(1), Denmark(3), Norway(4), Poland(0), Russia(0), China(0), Turkey(0), South Africa(0), Mexico(0), Brazil(0)

As is seen from the result, the number of connection k is distributed mostly in 3 ,4, 5. The fact means the number of connectivity among units are not distinguished.

Now we consider the shock analysis focusing only on the USA-Japan relation. More precisely, we try two kinds of shock analysis by assuming without the connection between two units (the USA and Japan). and with the underlying connection. The aim of the analysis is to see the induction of larger response by the shock through the USA economy. However, the following discussion is only a simulation study based on a assumption, and is not the real event.

At first, we explain the case where the connection between two units (the USA and Japan)

is assumed. We calculate the responses of the VAR variables of Japan by connecting the VAR variables of the USA. For example, we impose a impluse (shock) to the variable 'Monetary base" of the USA, and calculate every responses of VAR variables (including Monetary base). But, at the same time, we evaluate also the response of VAR variables of Japan, while the responses of the USA side are transmitted to these of Japan. Generally, these response are increased by such kinds of induction. Then, we can compare the shock analysis in two cases (assuming without connection and with connection).
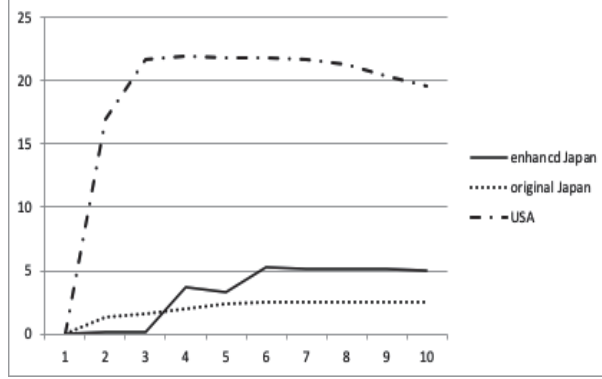

Figure 9: Shock analysis without connectivity and considering connectivity

Fig.9 shows the diagram of these responses. In this figure the symbol 'USA' means the response of 'GDP' induced by the shock from 'Monetary base' of the USA. The symbol 'original Japan', and 'enhanced Japan' mean responses of 'GDP' of Japan, in two cases (assuming without connection and with connection). In this figure, the vertical line means the rate (%) of the GDP response in the whole response and the horizontal line means the time (years) of the continuation of responses. As is seen, the line 'enhanced Japan' is larger than the line 'original Japan'.

4.4 Analysis of formation of scale-free networks based on Case 2

In the following, we discuss the analysis of real world data as examples based on Case 2. We focus on the scale-free networks in real world, and find they are generated through similar procedures of collaboration among units as in Case 2 based on the Edge Snapping. However, we only show the result of formation of scale-free networks as a result, and do not trace the process of group formation (it is very difficult to apply the Edge Snapping to real data). Simply say, we show only the ultimate results of formation of scale-free networks, but it is easily imagine that the growth of scale-free networks in following examples are generated through dominant units governing the coupling gains and the group formation.

(Example 1) Patent application of hybrid car development

About the application of patent for the development of hybrid cars, there is a set of data applied from dominant car makers in Japan [18]. From 1996 to 2013, there are lists of groups

of engineers including names of members joined to the development and names of applications. Then, we can reorganize the lists as the collaboration among the engineers in the underlying firm. Namely, we focus on a certain engineer and count the number of his name (denoted as k) in all lists, and then we regard k as the links of collaboration. If the number k is large, we find the engineer is appreciated in the firm. After surveying the data all through the lists for every patents application and firms, we finally summarize the relation between $\log k$ and the probability $p(\log k)$. Then, we find the relation $p(\log k)$ is depicted as a linear line, and the network of engineers reveals to be scale-free. The fact means, around better and more dominant engineer, the collaboration networks are formalized. This means dominant engineers can have many connections (links), but the number of these cases are small (rare cases). Almost all engineers have only small sized links or middle-sized links, and perform applications with small group.

(Example 2) Collaboration in the life science field among research institutes

We can find an empirical study analyzing the collaboration in the life science field among research institutes within the USA, Europe and Japan [19]. In the paper, the number of links between research institutes are represented by the links, and we can know the number of links k (we use $log k$) for each institute and its distribution denoted as $p(\log k)$. Then the probability $p(\log k)$ is depicted as a typical linear line observed in scale-free networks. The fact means that each institutes give high priority to the profit and better result, and tend to corporate with better and more dominant institutes.

Fig.10 shows the diagram $p(\log k)$ for the example estimated from the data in reference [18]. As in example 1, The fact means dominant institutes can have many connections, but the number of these cases are small (rare cases). Almost all institutes have only small sized links or middle-sized links, and the collaboration for the development is small.
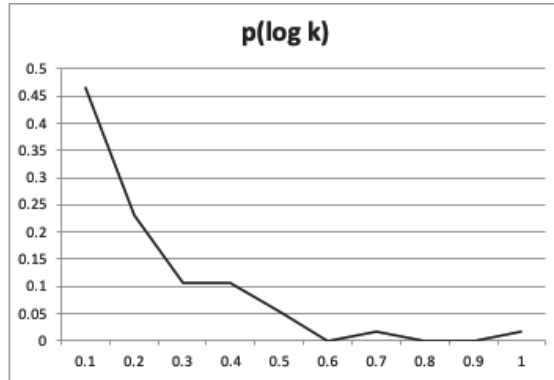

Figure 10: Distribution of $p(\log k)$ in Example 2

(Example 3) Behavior of members in SNS

To know the behavior of members of communities on the SNS (Social Network Service), we would like to summarize the number of links among communities [20]. At first, we collect the statistics for the number of common topics among communities (denoted as w, and w ranges from

3 to 672). Now we change the aspect to the data in such a way that we get the frequency for w (denoted as k). Then k means the number of links among communities. We have k as the number of links among communities (units) and the probability $p(\log k)$, which is depicted as a linear line as found in many scale-free networks. As in examples 1 and 2, this means there exist active communities having many links, but these active communities are rare cases. On the other hand, almost all communities share topics with relatively smaller number of communities.

4.5   Examples of network disruptions

Different from network formations, we ca easily find the examples of disruptions of network in real world. The reasons of the disruption are usually found in either the external sources or the internal sources. In any way, sudden changes of vector variables on a unit of connections induces the disruption. For example, in the disruption of big firm, a kind of large loss of profit induces the bankruptcy. In the same way, the conflict inside the firm can also induces the disruption. Similarly, in human relationships we can find the disruptions due to the external or internal reasons. Then, we skip the details.

5   Conclusion

This paper showed the estimation of connectivity among units of VAR models based on the Edge Snapping and its applications to the group formation analysis. The coupling gains were adjusted so as to minimize the difference between two characteristics of units, or strengthen the coupling with unit with large value As applications, we show simulation studies using the artificial data , and also real world applications of group formation.

For future works, we must apply the method to another and more complicated units and models rather than VAR models.

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