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HANADA, Toshiya
International Centre for Space Weather Science and Education, Kyushu University

HINAGAWA, Hideaki
Department of Aeronautics and Astronautics, Kyushu University

Chen, Hongru
Department of Aeronautics and Astronautics, Faculty of Engineering, Kyushu University

Hamada, Hiroaki
Department of Aeronautics and Astronautics, Kyushu University

他

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Attitude Motion Under Full Orbit Perturbations

By Toshiya HANADA,[1)] Hideaki HINAGAWA,[2)] Hongru CHEN,[2)] Hiroaki HAMADA[2)] and Shingo IKEMURA[2)]

[1)]*International Centre for Space Weather Science and Education, Kyushu University, Fukuoka, Japan*
[2)]*Department of Aeronautics and Astronautics, Kyushu University, Fukuoka, Japan*

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This paper introduces an effort to precisely describe attitude motion under full orbit perturbations. To define the attitude of an Earth-orbiting spacecraft, this study introduces a reference frame being affected by orbit perturbations. Unlike the commonly adopted reference frame, the reference frame introduced here is fully perturbed, so that it rotates about not only the pitch axis, but also the yaw axis. To incorporate the mutual coupling effect between attitude motion and orbit motion, the method introduced considers the spacecraft as consisting of multiple facets and carefully models orbit perturbation-induced torque that varies the attitude. This paper focuses on the attitude dynamics of a small satellite with relatively low moments of inertia under full orbit perturbations, and provides some interesting results and the outcome from the method introduced.

Key Words: Orbit Perturbations, Attitude Motion, Astrodynamics

Nomenclature

a	:	semi-major axis, km
A	:	area of a facet, m^2
C	:	rotation matrix about an axis specified by the superscript with an angle given by the subscript,
e	:	eccentricity
f	:	true anomaly, radian
F	:	perturbing acceleration, km/day^2
h	:	angular momentum, km^2/day
i	:	inclination, radian
I	:	moment of inertia about an axis specified by the subscript, kg m^2
J	:	zonal gravitational coefficient
$\boldsymbol{\ell}$	:	position vector of a facet
M	:	mean anomaly, radian
n	:	mean motion, rev/day
$\boldsymbol{n}$	:	a unit vector inward normal to a facet
r	:	radius, km
t	:	time, day
u	:	$\omega + f$, radian
θ	:	pitch angle, radian
ϕ	:	roll angle, radian
ψ	:	yaw angle, radian
Ω	:	right ascension of the ascending node, radian
ω	:	argument of perigee, radian
$\boldsymbol{\omega}$	:	angular velocity vector, radian/day

1. Introduction

The attitude motion of an object is mutually associated with the orbit motion of the object. The attitude motion results in changing the area-to-mass ratio, thereby affecting the orbit motion. The orbit motion results in environmental change that affects the attitude motion. Furthermore, the area-to-mass ratio in the calculation of atmospheric drag is completely different from that in the calculation of solar radiation pressure effect. The former ratio uses the area normal to the object's velocity vector relative to the rotating atmosphere, whereas the latter ratio uses the area normal to the Sun direction. Neigher are time-averaged cross-sectional areas, which is usually assumed to propagate the orbit, so both should be evaluated independently based on attitude motion.

This paper introduces a method to precisely describe attitude motion under full orbit perturbations. This method can be applied to understand the dynamics of light curve, the change in light intensity or visible magnitude of space objects in optical measurements.[1,2)] It can also be applied to conduct a feasibility study of using small satellites to demonstrate wireless power transmission in space.[3)] In addition, this method will contribute to orbital anomaly, reentry prediction, and collision avoidance.

To define the attitude of an Earth-orbiting spacecraft, this method introduces a reference frame that is affected by orbit perturbations. This reference frame rotates about the axis normal to the orbital plane or the angular momentum vector of the orbit as the commonly adopted reference frame does. The reference frame also rotates about the position vector due to the perturbing force normal to the orbital plane. However, the latter rotation represents the precession of the angular momentum vector of the orbit. Gim and Alfriend[4)] introduced a similar perturbed reference frame to describe the relative motion of satellites flying in formation, considering only the Earth's oblateness. This paper, however, includes orbit perturbations such as non-spherical parts of the Earth's gravitational attraction, atmospheric drag, gravitational attractions due to the Sun and the Moon, and solar radiation pressure to describe the attitude motion.

Since some orbit perturbations are also affected by the attitude of the spacecraft, the perturbing forces should be evaluated by taking into account the attitude of the spacecraft. Therefore, this method also introduces the mutual coupling effect between attitude motion and orbit motion. To incorporate this mutual coupling effect, orbit-dependent torque that varies the attitude should be carefully modeled. Therefore, this method considers a spacecraft as consisting of multiple facets. The resultant perturbing forces and external torque are evaluated after calculating the perturbing forces and external torque acting on each facet.

A preliminary study has revealed that the reference frame introduced in this paper to define the attitude of a spacecraft strongly affects small satellites with relatively low moments of inertia. Therefore, this paper focuses on such a small satellite, and provides some interesting results and outcomes from attitude motion under full-orbit perturbations.

2. Orbit Propagators

There seem to be two different types of orbit propagators. One integrates the rates of change of the classical orbital elements in the Gaussian form of the variation of parameter equations,[5,6] whereas the other is based on Cowell's formulation.[7] As will be described later in this paper, the rates of change of the classical orbital elements in the Gaussian form of the variation of parameter equations are convenient for introducing a perturbed reference frame. Therefore, the first one has been adopted for this study.

For finding the rates of change of the classical orbital elements, this method uses the perturbing forces with specific components resolved in the RSW spacecraft coordinate system. The R axis always points from the Earth's center along the radius vector toward the spacecraft as it moves through the orbit. The S axis points in the direction of (but not necessarily parallel to) the velocity vector and is perpendicular to the radius vector – an important requirement. The W axis is normal to the orbital plane. The Gaussian form of the variation of parameter equations is advantageous for non-conservative forces because it is expressed directly from the perturbing acceleration. It also works well for conservative forces because the forces are simple gradients of the potential functions.

In addition to the spherically symmetric gravitational force of the Earth, a number of perturbing accelerations affect the orbit of an Earth-orbiting spacecraft. The forces that need to be considered for this method are: 1) non-spherical part of the Earth's gravitational attraction, 2) atmospheric drag, 3) gravitational attractions due to the Sun and the Moon (approximated as point masses), and 4) solar radiation pressure. As will be described later, the perturbing accelerations due to the atmospheric drag and solar radiation pressure should be evaluated by taking into account the attitude of the spacecraft.

The EGM96 Earth gravity model is used for calculating the non-spherical part of the Earth's gravitational attraction. A simple exponential atmosphere model is used for drag calculation. Solar radiation pressure is modeled by assuming Lambertian diffusion.

Finally, the rates of change of the classical orbital elements in the Gaussian form of the variation of parameters equations may be given by

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left[e\sin f \cdot F_R + (1 + e\cos f)\cdot F_S\right]$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\left[\sin f \cdot F_R + \left(\cos f + \frac{e+\cos f}{1+e\cos f}\right)\cdot F_S\right]$$

$$\frac{di}{dt} = \frac{r\cos u}{na^2\sqrt{1-e^2}}F_W$$

$$\frac{d\Omega}{dt} = \frac{r\sin u}{na^2\sqrt{1-e^2}\sin i}F_W$$

$$\frac{d\omega}{dt} = -\frac{\sqrt{1-e^2}}{nae}\left[\cos f \cdot F_R - \left(\sin f + \frac{\sin f}{1+e\cos f}\right)\cdot F_S\right] - \frac{d\Omega}{dt}\cos i$$

$$\frac{df}{dt} = \frac{h}{r^2} - \frac{d\omega}{dt} - \frac{d\Omega}{dt}\cos i$$

3. Perturbed Reference Frame

As mentioned in the previous section, the RSW spacecraft coordinate system is used in orbit mechanics and moves with a spacecraft. In the case of a circular orbit, the RSW system rotates at a constant rate of n about the W axis. In the case of an elliptic orbit, however, the RSW system rotates at a rate of df/dt about the W axis. Additionaly, if orbit perturbations are fully taken into account, then the RSW system rotates at

$$[C_u^3][C_i^1]\begin{Bmatrix}0\\0\\\frac{d\Omega}{dt}\end{Bmatrix} + [C_u^3]\begin{Bmatrix}\frac{di}{dt}\\0\\0\end{Bmatrix} + \begin{Bmatrix}0\\0\\\frac{du}{dt}\end{Bmatrix}$$

$$= \begin{Bmatrix}\frac{d\Omega}{dt}\sin u\sin i + \frac{di}{dt}\cos u\\ \frac{d\Omega}{dt}\cos u\sin i - \frac{di}{dt}\sin u\\ \frac{d\Omega}{dt}\cos i + \frac{du}{dt}\end{Bmatrix}$$

Substituting the rates of change of the classical orbital elements in the Gaussian form of variation of parameter equations given in the previous section, then the above rotation rates can be reduced to

$$\begin{Bmatrix}\frac{rF_W}{h} & 0 & \frac{h}{r^2}\end{Bmatrix}^T$$

The RSW system under full-orbit perturbations rotates about not only the W axis, but also the R axis. The rotation of the R axis represents the nodal regression mainly due to the Earth's oblateness (or J_2 perturbation). Gim and Alfriend[4] introduced a similar perturbed coordinate system to describe the relative motion of satellites flying in formation, considering only J_2 perturbation not to be fully perturbed. The rotation of the W axis is also subject to orbit perturbations since the angular momentum is not constant in perturbed orbit motion. It may be noted that the rate of change of the

Table 1. Areas, position vectors, normal vectors, and surface properties (absorption, specular reflection, diffusion) of facets.

	A	$\boldsymbol{\ell}$			$\boldsymbol{n}$			Surface properties		
1	0.250000	0.250	0.000	0.000	–1.000	0.000	0.000	0.700	0.200	0.100
2	0.250000	0.000	0.250	0.000	0.000	–1.000	0.000	0.700	0.200	0.100
3	0.250000	0.000	0.000	0.250	0.000	0.000	–1.000	0.700	0.200	0.100
4	0.250000	0.000	0.000	–0.250	0.000	0.000	1.000	0.700	0.200	0.100
5	0.250000	0.000	–0.250	0.000	0.000	1.000	0.000	0.700	0.200	0.100
6	0.250000	–0.250	0.000	0.000	1.000	0.000	0.000	0.700	0.200	0.100

angular momentum is given by rF_S.

A roll-pitch-yaw spacecraft coordinate system is used in attitude dynamics and instrument pointing to provide an "aircraft-like" reference system for spacecraft. The roll-pitch-yaw system is very similar to the RSW system and is actually only one (standard) rotation from that system. The roll axis is the same as the S axis of the RSW system. The yaw axis is opposite to the position vector, points toward the center of the Earth, and also lies in the orbital plane. The pitch axis is opposite to the angular momentum vector. Therefore, the roll-pitch-yaw system rotates at

$$\left\{0 \quad -\frac{h}{r^2} \quad -\frac{rF_W}{h}\right\}^T$$

This method adopts the aforementioned roll-pitch-yaw spacecraft coordinate system as a reference frame to describe attitude motion under full-orbit perturbations. Unlike the commonly adopted reference frame, the aforementioned reference frame is fully perturbed, so that it also rotates about the yaw axis.

4. Attitude Motion Simulators

Two different attitude motion simulators have been developed for this method. One uses the classical Euler equations of motion to compute the angular velocity derivatives, whereas the other introduces quaternions to avoid encountering the singularities observed for the sequence of roll, pitch, and yaw rotations when the pitch angle θ approaches 90 degrees. In both simulators, the angular velocity vector in the roll-pitch-yaw spacecraft coordinate system has to be related to the rate of change of the roll, pitch, and yaw angles. Thus, this paper provides a numerical expression of the angular velocity vector in the roll-pitch-yaw system, instead of describing details of each attitude motion simulator.

The angular velocity vector in the roll-pitch-yaw system can be related to the rate of change of the roll, pitch, and yaw angles as follows:

$$\boldsymbol{\omega} = [C_\phi^1][C_\theta^2][C_\psi^3]\begin{Bmatrix} 0 \\ -\dfrac{h}{r^2} \\ -\dfrac{rF_W}{h} \end{Bmatrix} + [C_\phi^1][C_\theta^2]\begin{Bmatrix} 0 \\ 0 \\ \dfrac{d\psi}{dt} \end{Bmatrix} + [C_\phi^1]\begin{Bmatrix} 0 \\ \dfrac{d\theta}{dt} \\ 0 \end{Bmatrix} + \begin{Bmatrix} \dfrac{d\phi}{dt} \\ 0 \\ 0 \end{Bmatrix}$$

On the other hand, the roll-pitch-yaw system can be related to the RSW spacecraft coordinate system using the following transformation matrix:

$$[Q] \equiv [C_\phi^1][C_\theta^2][C_\psi^3]\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

Thus, the angular velocity vector in the roll-pitch-yaw system can be rewritten in

$$\boldsymbol{\omega} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix}\begin{Bmatrix} \dfrac{d\phi}{dt} \\ \dfrac{d\theta}{dt} \\ \dfrac{d\psi}{dt} \end{Bmatrix} + \frac{rF_W}{h}\begin{Bmatrix} Q_{11} \\ Q_{21} \\ Q_{31} \end{Bmatrix} + \frac{h}{r^2}\begin{Bmatrix} Q_{13} \\ Q_{23} \\ Q_{33} \end{Bmatrix}$$

The second term is newly introduced in this method to describe attitude motion under full-orbit perturbations.

In addition to gravitational torque, various types of external torque affect the attitude of an Earth-orbiting spacecraft under orbit perturbations. The types of torque that need to be considered for this method are: 1) aerodynamic torque, 2) solar radiation torque, and 3) magnetic torque. The torque considered herein should be evaluated by taking into account the attitude of the spacecraft. In addition, perturbing forces generating the aerodynamic or solar radiation torque also affect the orbit of the spacecraft. To take into account the mutual coupling effect between attitude motion and orbit motion, this method considers a spacecraft as consisting of multiple facets. After calculating the perturbing forces and external torque acting on each facet, the resultant perturbing forces and external torque are evaluated. It should be noted that the 11th generation of the IGRF model is used for magnetic torque calculation.

5. Results and Discussions

To demonstrate attitude motion under full-orbit perturbations, this method chooses a cubic satellite with a mass of 50 kg. This cubic satellite consists of six facets with the areas, position vectors, normal vectors, and surface properties (absorption, specular reflection, and diffusion) summarized in Table 1. It is assumed that no control torque is active. It is also assumed that the center of mass is slightly shifted from the geometric center of the cubic satellite (0.001, 0.001, 0.001 meters).. In addition, the remnant magnetism is assumed to be 0.0, 0.0, 0.3 Am^2.

Figures. 1, 2, 3, and 4 compare attitude motion between the unperturbed and perturbed reference frames. An important difference between the four figures is that torque is taken into account. Figure 1 takes into account only gravitational torque. In addition to the gravitational torque, Fig. 2 also

Table 2. Moments of inertia about the roll, pitch, and yaw axes. Initial values of the three Euler angles, and the rates of change of the three Euler angles.

I_{roll}	1.69 [kg m^2]
I_{pitch}	1.71 [kg m^2]
I_{yaw}	1.56 [kg m^2]
ϕ	0.0 [deg]
θ	1.0 [deg]
ψ	1.0 [deg]
$d\phi/dt$	0.0 [deg/s]
$d\theta/dt$	0.0 [deg/s]
$d\psi/dt$	0.0 [deg/s]

Table 3. Initial values of classical orbital elements.

Epoch	0000 UT on 1 January 2012
i	97.0347 [deg]
Ω	0.0000 [deg]
e	0.0000000
ω	0.0000 [deg]
M	0.0000 [deg]
n	15.55741064 [rev/day]

considers solar radiation torque, while Fig. 3 includes aerodynamic torque as well. Finally, Fig. 4 takes into account all four external torques that need to be considered for this method. Moments of inertia and initial values of the three Euler angles and the rates of the three Euler angles are summarized in Table 2, whereas Table 3 summarizes initial values of orbital parameters.

Figure 1 clearly demonstrates the effect of the perturbed reference frame introduced in this method on the attitude motion. The rotation of the roll-pitch-yaw spacecraft coordinate system about the yaw axis (the R axis of the RSW spacecraft coordinate system) affects not only the yaw motion, but also the roll motion because of yaw and roll coupling. However, this rotation does not contribute to change the angular momentum vector, and therefore does not affect the pitch motion. Higher moments of inertia indicate that more force has to be applied in order to cause a rotation, whereas lower moments of inertia mean that only less force is necessary. It may be noted, therefore, that this rotation cannot be effective in attitude motion of a spacecraft with relatively high moments of inertia.

The effect of the perturbed reference frame can still be observed in Fig. 2, which considers solar radiation torque in addition to gravitational torque. As demonstrated in Fig. 3, aerodynamic torque largely affects the attitude motion, so that the difference between the unperturbed and perturbed reference frames becomes relatively too small.

It is well known, as demonstrated in Fig. 4, that small satellites are strongly affected by magnetic torque. Unlike Figs. 1, 2, and 3, however, a significant difference between the unperturbed and perturbed reference frames can be observed in Fig. 4. The rotation of the reference frame about the yaw axis may cause this significant difference in the case where the attitude motion is sensitive to external torque as in Fig. 4 (i.e., small satellites with relatively low moments of inertia under magnetic torque). Deviations resulting from the difference in modeling cause further changes in orbit perturbations and therefore external torque to be enhanced. Therefore, it is quite important to be aware that the attitude motion can change dramatically when precisely described.

6. Conclusions

This paper introduced a reference frame being affected by orbit perturbations to define the attitude of an Earth-orbiting spacecraft. The perturbed reference frame also rotates about the yaw axis due to nodal regression in addition to rotating about the pitch axis. This paper demonstrated the attitude motion of a small satellite with relatively low moments of inertia under different external torques to investigate the effect of the perturbed reference frame. A short-periodic difference in the roll and yaw angles between the unperturbed and perturbed reference frames was observed. However, the aerodynamic torque largely affects the attitude motion in comparison to the gravitational and solar radiation torque. Accordingly, the difference between the unperturbed and perturbed reference frames becomes relatively too small. However, the rotation of the reference frame about the yaw axis can cause a significant difference when the attitude motion is sensitive to external torque, as in the case of small satellites with relatively low moments of inertia under magnetic torque. Deviations resulting from a difference in modeling cause further changes in orbit perturbations, and therefore external torque to be enhanced. It is concluded, therefore, that a precise description of attitude motion mutually associated with orbit perturbations is essential to better understand attitude motion under full-orbit perturbations.

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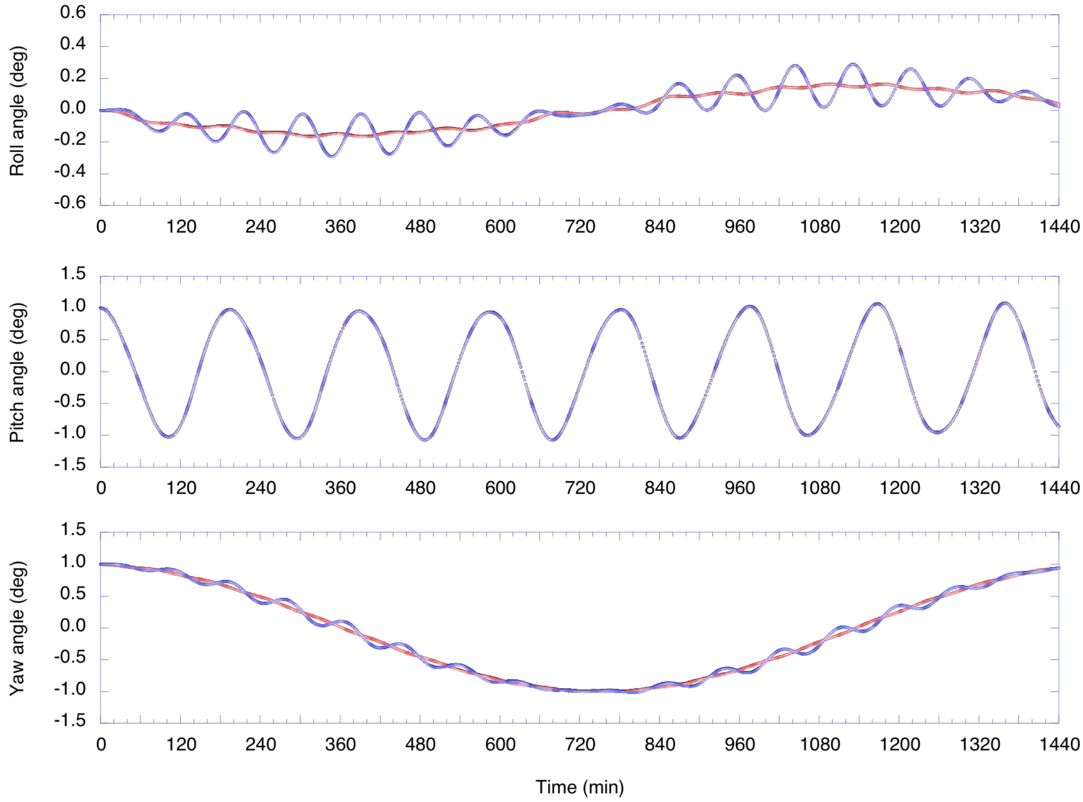

Fig. 1. Comparison of attitude motion under gravitational torque: (red) unperturbed, and (blue) perturbed reference frames.

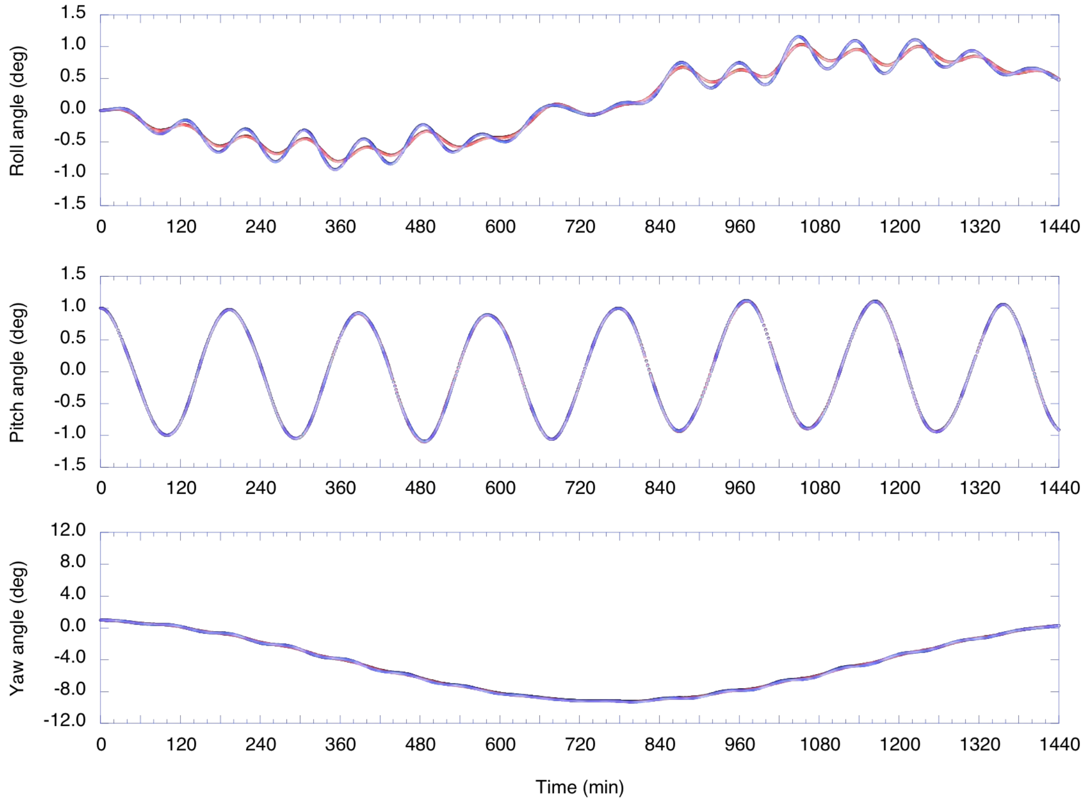

Fig. 2. Comparison of attitude motion under gravitational and solar radiation torques: (red) unperturbed, and (blue) perturbed reference frames.

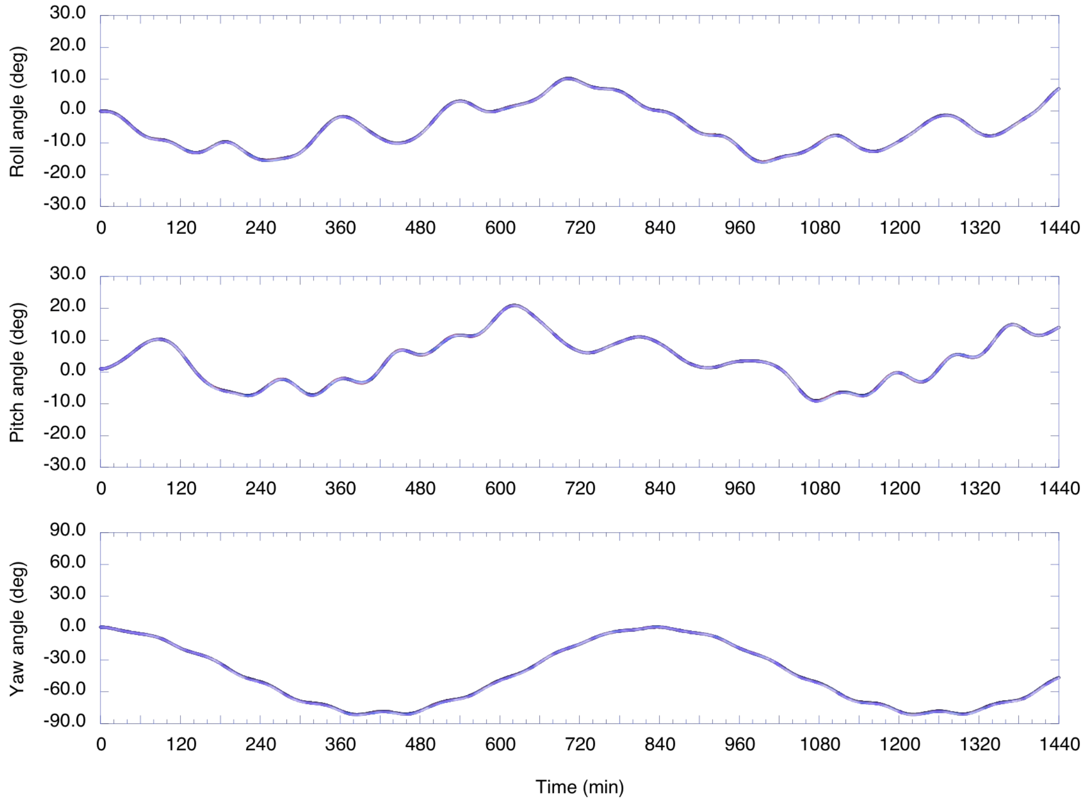

Fig. 3. Comparison of attitude motion under gravitational, solar radiation and aerodynamic torques: (red) unperturbed, and (blue) perturbed reference frames.

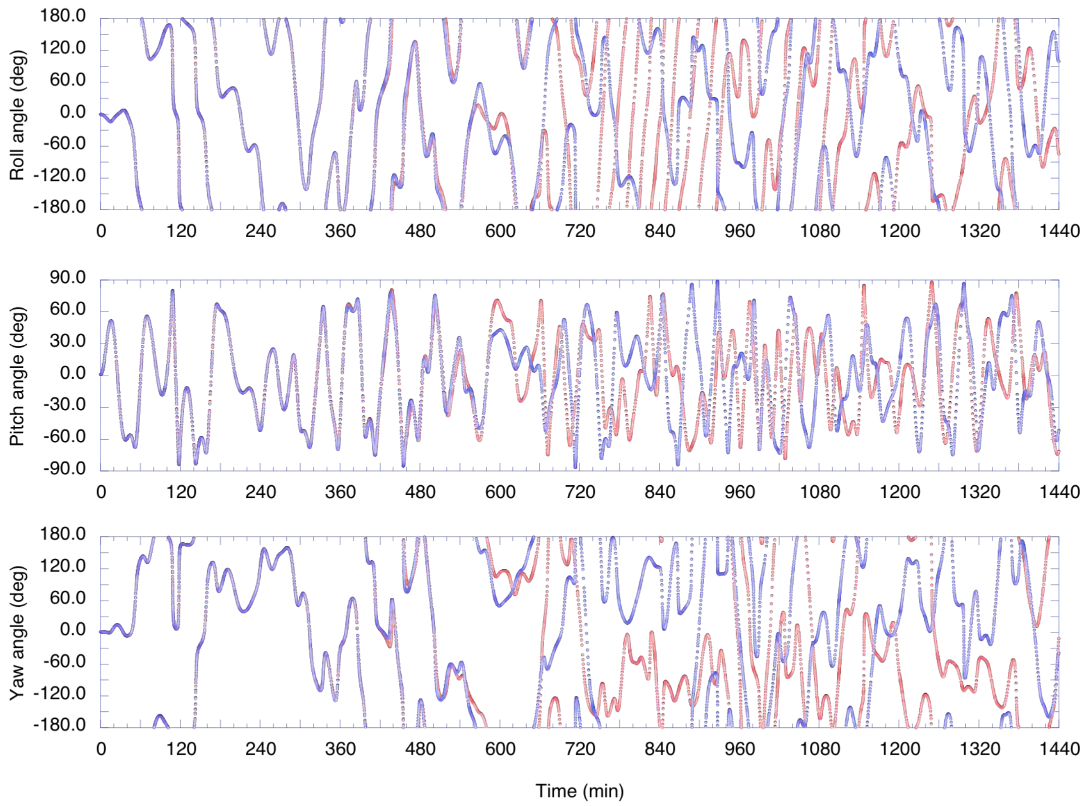

Fig. 4. Comparison of attitude motion under gravitational, solar radiation, aerodynamic, and magnetic torques: (red) unperturbed, and (blue) perturbed reference frames.