Optimal Subsidy Policy and the Implementation Game

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Optimal Subsidy Policy and the Implementation Game[1]

Moriki Hosoe

1 Introduction

We consider about how subsidies should be given to each firm in a duopoly market. Providing subsidies for an infant-industry has often been discussed. For example, this problem may occur as a domestic problem or an international problem when the government wants to promote the development of a targeting industry from the point of view of the economic growth policy. Or it may be necessary to provide subsidies for the firms from the point of view of social welfare, even if each firm cannot survive under no subsidy.

Then what we would like to note at this point is a possibility of distortionary cost of financing the subsidies. To put a stress on this distortional characteristics of financing the subsidy payment has often been mentioned. For example, in Laffont=Tirole[8], they are saying that each dollar spent by the government costs the U.S.A. economy 1.3 dollars through distortional taxes, as a reasonable mean estimate. In fact, according to calculation by Browning[3] for U.S.economy, on the assumption that subsidies are financed by labour earnings, the cost takes a value between 1.10 and 4.03. Recently Neary[13] analyzes the cost asymmetries in international subsidy games, considering this shadow cost of public fund explictly. He points out the importance of the shadow cost of public fund on the determination of subsidy pattern.

In this paper we will investigate the optimal subsidy policy in a duopoly market. First attention is whether a more efficient firm should be provided more subsidy or not. Neary[13] shows that the winner (the more efficient firm) should take more subsidy in the context of a domestic monopoly in an international market. Second attention is about the effect of the optimal subsidy policy on the market structure itself. We need to know how this effect develops as the shadow price of public fund increases. Third attention

[1]When I was visiting the department of Business Economics and Public Policy in School of Business, Indiana University, this paper was written and presented at microeconomics workshop there. I would be very grateful to Professor Eric Rasmusen for helpful comments. I also would like to thank prof. Stephen Martin, Copenhagen University for his precious sugestion. All remaining errors are mine.

effect develops as the shadow price of public fund increases. Third attention is the implementation problem of the optimal subsidy policy. This means that we investigate the effect of the optimal subsidy policy on the established firms. If they have a right to reject the policy, it may not be implemented.

The remainder of the paper is organized as follows. In section 2 we present a monopoly model for the optimal subsidy policy as a reference. In section 3 a duopoly model is formulated and the properties of optimal subsidy plicy are showm. In section 4 the effect of the optimal policy and the value of the shadow price of public fund on the market structure is described. In section 5 the movement of social surplus under the optimal subsidy is given, depending on the cost structure. Section 6 shows how the incumbent firms are affected in terms of their profit. Section 7 shows the implementation problem of the optimal subsidy policy. Section 8 concludes the paper.

2 Monopoly market

Firstly we consider the problem of optimal subsidy policy in a monopoly market as a reference. For simplification of description, it is assumed that demands for the goods in the market are shown by a linear inverse demand function $p = a - x$, where x is the level of output in the market and the marginal cost of the firm is constant ($c > 0$). Here, let us consider about a subsidy policy of a government to this market. The subsidy we consider is assumed to be a form of a constant rate per output of the firm. Given the rate of subsidy s, the firm will produce so as to maximize the profit.

$$\max_x \quad \pi = \max_x \quad (a - x - (c - s))x \tag{1}$$

Here it is assumed that $a \geq c$. From this (1), the optimal output and the corresponding profit are given.

$$x = \frac{a - (c - s)}{2}, \qquad \pi = \frac{(a - (c - s))^2}{4} \tag{2}$$

Now we introduce the social surplus in order to get the optimal policy of subsidy. It should be noted that a social loss may occur when the government finances the subsidy. Therefore, we have to pay an attention to the cost of financing the subsidy or the opportunity cost. This is often called a shadow price of public fund (Laffont Tirole[8]) or distortionary factor. We represent this shadow price of public fund by a constant rate s per unit of the subsidy.

Taking the shadow price of public fund into consideration, the social surplus is

Social Surplus(SS)=Consumer Surplus + Producer Surplus - Social Cost

, where the term "Social Cost" denotes the cost of financing the subsidy. In the monopoly market, this is represented by

$$SS = \frac{3x^2}{2} - dsx \tag{3}$$

The government decides a rate of subsidy so as to maximize the social surplus. This is easily solved so that we obtain the optimal rate of subsidy.

$$s = \frac{(3-2d)(a-c)}{4d-3} \tag{4}$$

Therefore the optimal rate of subsidy depends on the value of the shadow price of public fund. That is,

$$s \geq (\leq) 0 \Leftrightarrow d \leq (\geq) \frac{3}{2} \tag{5}$$

Thus the government will provide a positive subsidy when the shadow price is low and, a negative subsidy (i.e., a tax) when the shadow price is high. From (5) it also follows that the subsidy is a decreasing function of the shadow price. Then we obtain the output and profit of the monopoly.

$$x = \frac{d(a-c)}{4d-3}, \pi = \frac{d(a-c)^2}{4d-3} \tag{6}$$

And the monopoly price is represented by

$$p = \frac{a+(c-s)}{4d-3} = \frac{(3a+c)d-3a}{4d-3} \tag{7}$$

For a comparison, we show the output, the price ,and the social surplus obtained by the profit-maximization of the monopoly when there is no subsidy.

$$p^o = \frac{a+c}{2}, x^0 = \frac{a-c}{2}, SS^0 = \frac{3(a-c)^2}{8} \tag{8}$$

Therefore the output (*i.e.*, the profit) when this optimal subsidy is positive is always higher than that under no subsidy. On the other hand these are lower when a tax is levied as optimal subsidy policy than under no subsidy. Anyway under government-intervention the output decreases and the price

increases with the shadow price of public fund d. The social surplus is represented by

$$SS = \frac{3x^2}{2} - dsx = \frac{(a-c)^2 d^2}{2(4d-3)} \tag{9}$$

From this we understand that the social surplus decreases in d in the region of positive subsidy, but it increases in d in the region of tax levied. This is represented as follows.

Proposition 1 *Under the optimal subsidy policy the profit of the firm is a decreasing in the shadow price of public fund d. If d is smaller (larger) than 3/2, the profit of the firm is larger (smaller) than that when there is no subsidy.*

Proposition 2 *The social surplus decreases in d, where* $1 < d \leq 3/2$, *and it increases in d , where* $3/2 < d$.

This implies that in high shadow price of public fund the government wants to scale down the industry by levying taxes and as a result the industry makes contribution to increase the social surplus by increasing the gevernment revenue.

3 Duopoly market and subsidy policy

We formulate a model of subsidies policy in a duopolistic industry. Suppose that in a market two firms 1 and 2 compete in Cournot-Nash setting, who produce a homogenous good, with marginal cost $c_i (i = 1, 2)$, respectively. If the government provides subsidies $s_i (i = 1, 2)$ per unit of production for each firm, the quantity-competition leads to the market equilibrium, where the following levels of price, production, and profit are given.

$$p = \frac{a - (c_1 - s_1) - (c_2 - s_2)}{3}$$

$$x_i = \frac{a - 2(c_i - s_i) + (c_j - s_j)}{3}$$

$$\pi_i = \frac{(a - 2(c_i - s_i) + (c_j - s_j))^2}{9} \qquad i \neq j = 1, 2$$

In this market, the social surplus is defined as follows.

$$SS = CS + \pi_1 + \pi_2 - ds_1 x_1 - ds_2 x_2$$

Then the consumer surplus becomes

$$CS = \frac{(2a - c_1 + s_1 - c_2 + s_2)^2}{18}$$

The government must decide the optimal level of subsidy so as to maximize the social surplus in the market equilibruim. That is,

$$\max_{s_1, s_2} \quad SS$$

$$subject \quad to \quad x_i \geq 0 \qquad (i = 1, 2)$$

By calculating the reaction function of the two firms, we have the optimal level of subsidy.

$$s_i = \frac{(8 - 15d + 6d^2)c_i + dc_j + 2(4 - 3d)a(d - 1)}{2(d - 1)(6d - 4)} \tag{10}$$

From this condition, we can get some properties of optimal subsidy. First, we have

Lemma 1

$$\frac{\partial s_i}{\partial c_j} \geq 0 \quad i \neq j = 1, 2$$

In other words, a firm is provided more subsidies if the marginal cost of the rival firm becomes higher.

we also have

Lemma 2

$$\frac{\partial s_i}{\partial c_i} \leq (\geq 0) \leftrightarrow d \leq (\geq) d^* \tag{11}$$

where d^* is $(15+\sqrt{33})/12$.[2] Therefore, when the shadow price of public fund is smaller(larger), increase of the marginal cost of a firm leads to decrease (increase) of the subsidies for the firm. We have to consider the reason why a firm is provided more subsidies when marginal cost becomes higher than in the past in the case that the social cost of public fund is high. To see this, the line $s_i = 0$ depending on the value d is depicted in Fig.1. We assume that the two marginal costs c_1 and c_2 do not exceed a. In the hatched

[2] d^* is one of two roots satisfying th equation $8 - 15d + 6d^2 = 0$, which takes a value in [5/3,7/4]

region the subsidy s_i is positive. The line $s_i = 0$ always passes a point (a, a). Therefore, we only need to see the movement of the point where the line hits on the vertical axis. In Fig.1(b), the movement of the intersection point m on the vertical axis is shown when d changes.

As d increases from 1 to $(4/3)^{1/2}$, this point goes down from the origin to point A. As d increases further from $(4/3)^{1/2}$ to d^*, the point goes down to negative infinity. In particular, this point passes again the origin at $d = 4/3$. Therefore, at $d = (4/3)^{1/2}$, the region of positive subsidy is the widest in cost space (c_j, c_i).

From this discussion, we can understand that there is a possibility that both of two firms are imposed taxes rather than positive subsidies if and only if d is larger than $4/3$.

In this situation, if d exceeds d^*, taxes are always levied to both firms and the tax is decreasing when the marginal cost on a firm increases. This implies that the positive effect of the decrease of marginal cost on the consumer surplus is less than the effect on the increase of public fund.

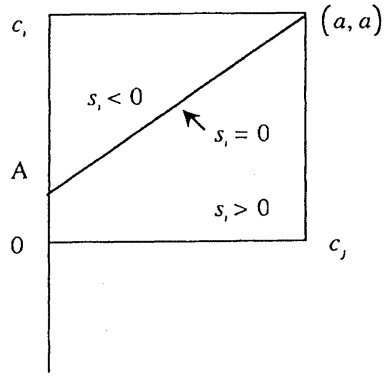


Fig. 1(a)

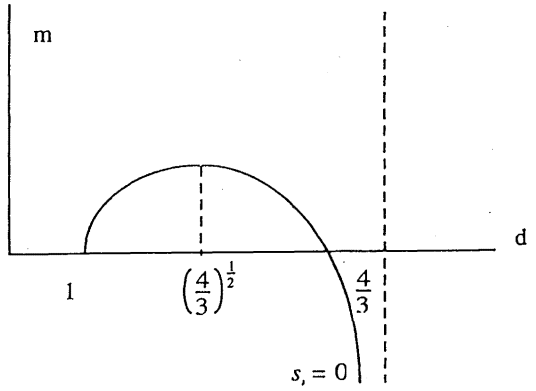


Fig. 1(b)

From(10), we also have the following properties about the movement of the optimal subsidy for the change of d.

Lemma 3 *If $c_i \leq c_j$, s_i is a decreasing function of d.*
If $c_i \geq c_j$, s_i is a single-peaked curve of d.

Finally we have

Lemma 4

$$s_i - s_j = (c_i - c_j)\frac{d-2}{2(d-1)}$$

4 Market structure and shadow price of public fund

Though we first assumed a duopoly market in the previous section, intruducing the subsidy policy may change the market itself. In particular, the duopoly market may change into a monopoly. This will depend on the cost gap between two firms and the level of the shadow price of the public fund.

When there is no-intervention of the government, it is well known about the relation between the market structure and the cost condition of two firms. That is, since the outputs of two firms in equilibrium under no-intervention of the government is represented

$$x_i^0 = \frac{a - 2c_i + c_j}{3}$$

, as the market structure is shown in Fig.2(a).

Let us consider the influence of the subsidy policy on the market structure. The equilibrium level of production of each firm under the optimal subsidy policy is written by

$$x_i = \frac{(-2(d-1)a - (2d-1)c_j + (4d-3)c_i)}{6(d-1)(4-6d)} \tag{12}$$

From this equation, we have the following properties.

$$\frac{\partial x_i}{\partial c_i} \leq 0, \frac{\partial x_i}{\partial c_j} \geq 0 \tag{13}$$

Furthermore,

$$x_i - x_j = \frac{(c_j - c_i)d}{6(2d-1)} \tag{14}$$

Fig. 2(a)

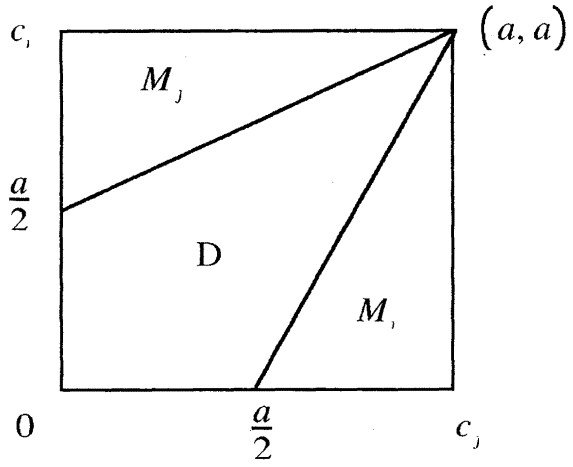
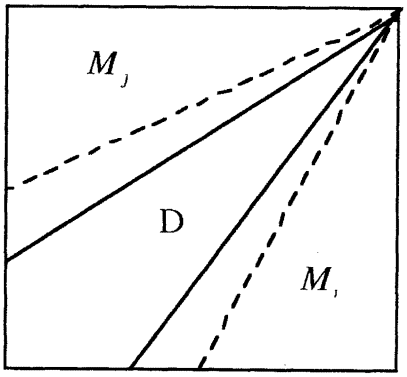


Fig. 2(b)



On the other hand, in the duopoly market without government intervention, we have

$$x_i^o - x_j^o = \frac{c_j - c_i}{3} \tag{15}$$

, where superscript o denotes the notation in the case of no-intervention of government. Therefore, in the equilibrium, the gap of the output level between two firms under the subsidy policy is smaller than that under no-intervention. And the gap of the output level decreases as d increases.

Next, consider the influence of the change of d on the level of production (therefore, profit). (12) is rewritten as

$$x_i = \frac{(-2(a - 2c_i + c_j)d + (2a - 3c_i + c_j))d}{2(d-1)(4d - 6d)} \tag{16}$$

In the case where $c_i \geq c_j$, the level of production of the firm i with a relatively low marginal cost shows a curve with a peak where the level of production increases as d increases to a point, and decreases as d increases from the point, finally approaching to a limit $(a - 2c_i - c_j)/6$. On the other hand, for a firm i with a relatively high marginal cost, this curve is an increasing function of d, approaching to the limit.

We examine about the total output (*i.e* the market price) in this market in both cases. Under the subsidy policy, from (16), the total output is

$$x_1 + x_2 = \frac{(2a - c_1 + c_2)d}{6d - 4} \tag{17}$$

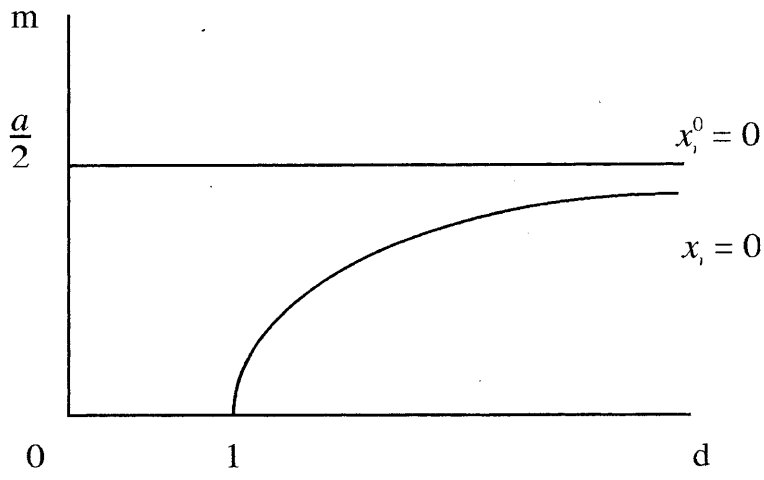
On the other hand, under no-intervention,

$$x_1^o + x_2^o = \frac{(2a - c_1 - c_2)d}{3} \tag{18}$$

is obtained. Therefore, the market price goes up to a limit as d increases and it also is lower (higher) than without intervention, when d is smaller (larger) than 4/3. Then, by comparing the zero-profit curve x_i=0 under the government-intervention and the zero-profit curve x_i^o=0 under no-intervention, we obtain the following proposition.

Proposition 3 *The region of costs of two firms where the duopoly market holds without any government intervention is wider than the corresponding region under the optimal subsidy policy. In particular, when the gap of marginal cost of two firms is relatively large, it is likely that a monopoly establishes, even though there exists a duopoly market before intervention.*

Fig. 3



(proof) In (c_j, c_i) space, we note that the line satisfing x_i=0 and the line satisfying x_i^0=0 pass through point(a, a). Therefore we only need to compare each intersection point on c_i axis of the two lines. Then we can easily show that the intersection of the line x_i is always larger than that of the line x_i^0, regardless of the level of the shadow price of public fund. In particular, the intersection point of the line x_i =0 on the c_i axis goes from the origin to the intersection point of the line x_i^0 as d goes to infinity.$(Q.E.D)$

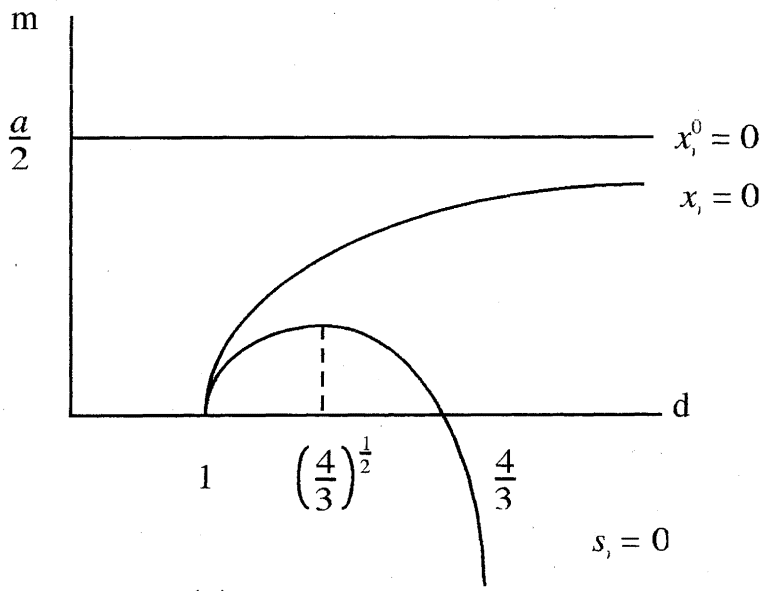
Corollary 1 *As the shadow price of public fund decreases, the region of a duopoly market under intervention shrinks and degenerates to the line* 45^o. *Therefore, if there is no distortion of financing the public fund, a duopoly market under the optimal subsidy policy holds if and only if they have the same marginal cost. In other words, a duopolistic market holds in two firms with different costs if and only if there is any distortion of financing the public fund.*

We will add the line $s_i = 0$ in (c_j, c_i) space. As shown before, this line has a positive (negative) tendency , passing through point (a, a), where d is smaller (larger) than d^*. It is pointed out that this line also has the largest value of the intersection point on the vertical axis at $(4/3)^{1/2}$. By comparing the two intersection points on the vertical axis of the line $x_i = 0$ and $s_i = 0$, we can see the movement of the profit of the two firms and the level of the optimal subsidy when the shadow price of public fund increases. The difference of two intersection points is

$$\frac{2(d-1)a}{4d-3} - \frac{2(4-3d)(d-1)a}{-6d^2+15d-18} = \frac{4a(d-1)^2(3d-2)}{(4d-3)(-6d^2+15d-18)}$$

. Therefore, when d is smaller than d^*, the line $x_i = 0$ is above the line $s_i = 0$. This movement is depicted in Fig.4.

Fig. 4



Proposition 4 case (1) *if the shadow price of public fund d is relatively small, there is a region where both two firms have positive subsidies and then*

the region expands until d becomes $(4/3)^{1/2}$. *In the region where a duopoly exists, only the inefficient firm is imposed taxes, if any. When the gap of marginal cost is large, a monopoly exists and a positive subsidy is always provided for it.*

case(2)*if the shadow price is larger than* $4/3$, *both firms are imposed taxes, when the gap of marginal costs of two firms is small. A monopolist is always provided subsidy when it exists.*

case(3) *if the shadow price becomes further large, there exists a duopoly only if they are imposed taxes. At the same time, when a monopoly market establishes, the monopoly may be imposed taxes if the gap of the marginal costs is not so large.*

case(4) *if the shadow price is larger than* d^*, *both duopoly market and monopoly market can exist under government-intervention, depending on the relative magnitude of marginal costs of two firms. In either market, taxes are always imposed. (See Fig.5)*

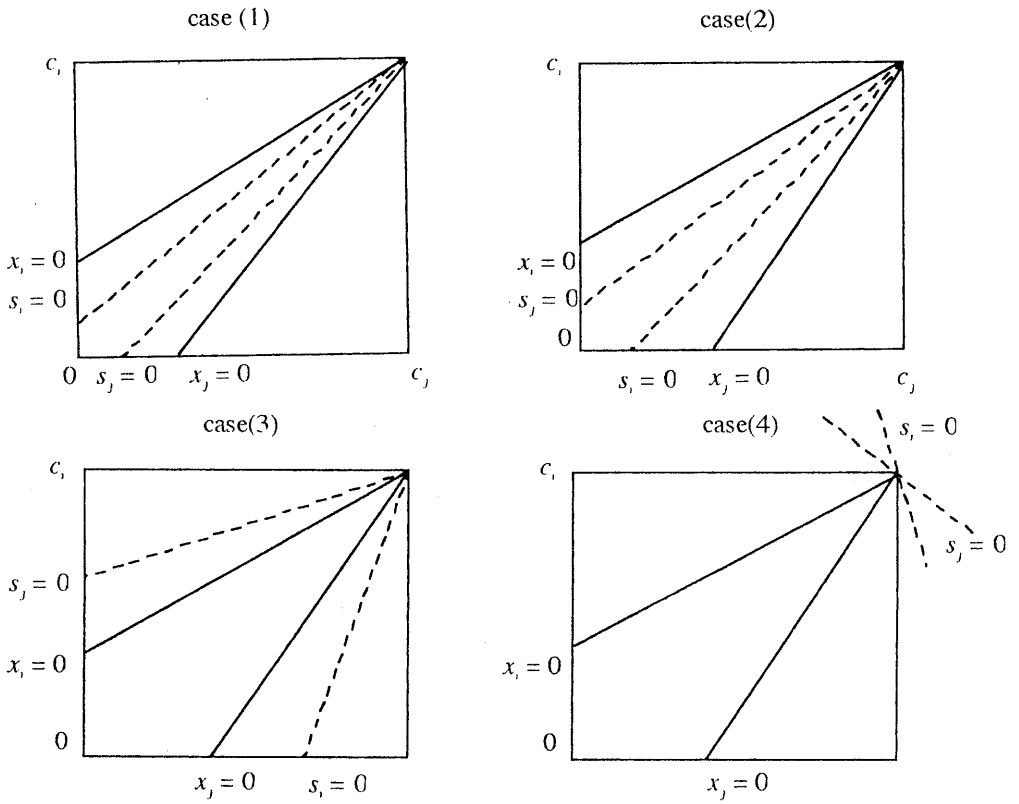


Fig. 5

We turn to investigate the feature of the monopoly policy as a part of

optimal policy. Let us suppose that firm 2 should be a monopoly under the optimal policy. As already stated, the condition is $x_1 \leq 0$. Firm 1 is detered to produce a positive level of output because the government levies high taxes on firm 1 and provides high subsidies for firm 2. However, the government revenue that incurs by levying taxes on firm 1 is not realized if there is no production of firm 1. Therefore, we have to consider about the optimal policy in monopoly region. If the government makes an judgement that firm 2 should be a monopolist, the problem to solve is

$$\max_{s_2} \quad \frac{3}{2}\pi_2(c_2 - s_2) - ds_2x_2(c_2 - s_2)$$

$$s.t. \quad \pi_1(c_1, c_2 - s_2) \leq 0$$

where $\pi_2(c_2 - s_2)$ is a monopoly profit of firm 2 when the marginal cost is $c_2 - s_2$ and $\pi_1(c_1, c_2 - s_2)$ is a duopoly profit of firm 1 when the marginal cost of firm 1 is c_1 and that of firm 2 is $c_2 - s_2$. This means that the government decides the subsidy rate for the monopolist so as to maximize the social surplus under the condition of non-entry of the other firm. Note that we can not simply apply the monopoly problem in section 2 to our problem and under this formulation the subsidy-tax revenue is actually realized. Using Kuhn-Tucker theorem, this problem is solved to be

$$If \quad (4d-3)(a-c_1) - d(a-c_2) \leq 0, \quad s_2 = \frac{(a-c_2)(3-2d)}{4d-3} \tag{19}$$

$$If \quad (4d-3)(a-c_1) - d(a-c_2) \geq 0, \quad s_2 = 2(a-c_1) - (a-c_2) \tag{20}$$

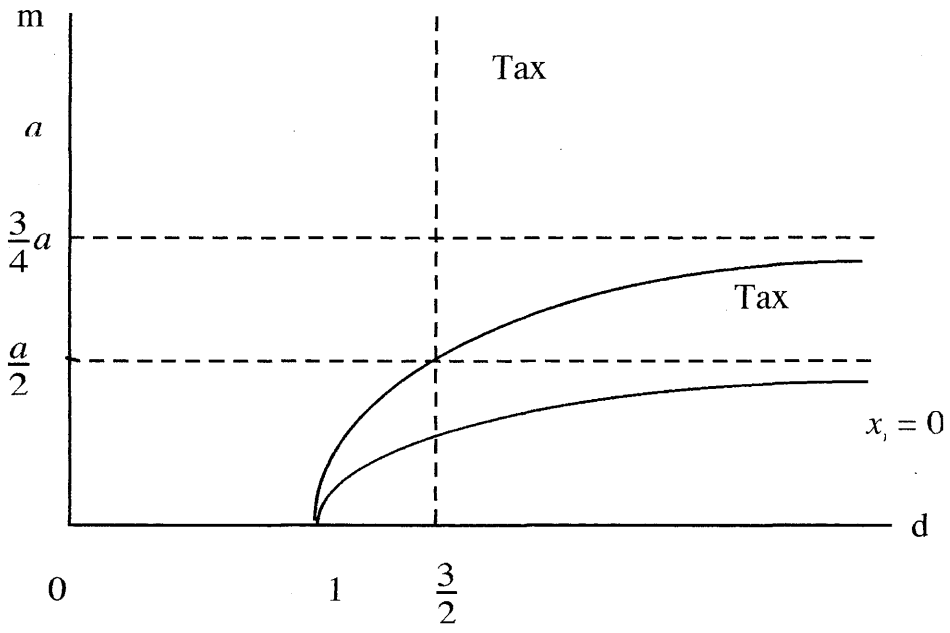
From these equation, the monopoly region is devided into two regions. The former (19) is refered to as an unconditional monopoly region, the analysis of which has already been done in section 2 as an optimal policy in a monopoly market. In this region, the monopolist profit and the corresponding social surplus are (6) and (9), respectively. On the other hand, the latter (20) is refered to as an entry-deterance monopoly region, since firm 2 is a monopoly under the threat of the entry of firm 1. In this region, the monopolist profit is obtained as follows.

$$\pi_m^e = (\frac{a - 2(c_2 - s_2) + c_1}{3})^2 = (a - c_1)^2 \tag{21}$$

$$SS_m^e = (\frac{3}{2} - 2d)(a - c_1)^2 + 2d(a - c_2)(a - c_1) \tag{22}$$

where the superscript "e" stands for entry-deterance and the subscript "m" stands for monopoly. The more efficient firm is provided an unconditional optimal subsidy and the less efficient monopoly is provided a subsidy necessary for the entry-deterance of firm 2 in the monopoly region. These two regions in space (d, m) is depicted in Fig.6. ED line is the boundary of the entry-deterance region. Note that the optimal subsidy is negative in the case of $d > 3/2$.

Fig. 6



5 Social surplus and optimal subsidy policy

In previos sections the market structure in the presence of optimal subsidy policy was shown. So we have to investigate the feature of the social surplus in the case. Let us note that the equilibrium social surplus in duopoly market without government-intervention is shown by

$$SS^o = \frac{1}{2}(\frac{2a - c_1 - c_2)}{3})^2 + (\frac{a - 2c_1 + c_2}{3})^2 + (\frac{a - 2c_2 + c_1}{3})^2$$

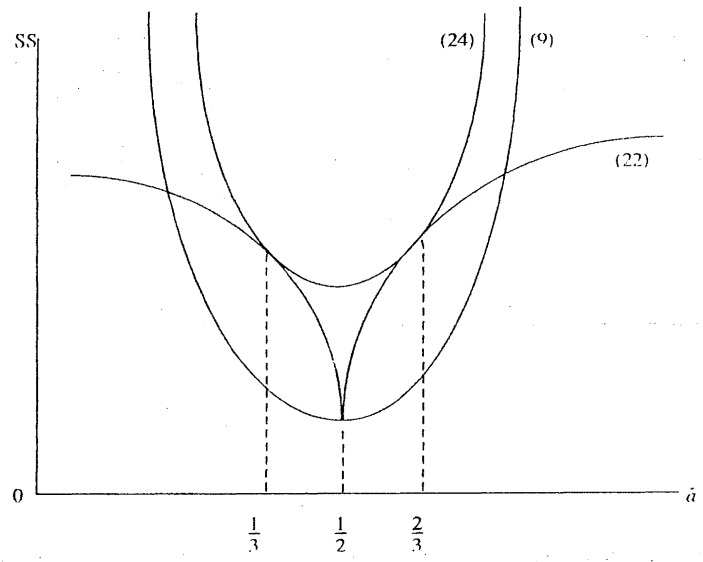
$$= \frac{11a_1^2 - 14a_1a_2 + 11a_2}{18} \tag{23}$$

where $a_i = a - c_i (i = 1, 2)$. To examine the effect of cost differentials of two firms on the social surplus, let us fix the sum of costs in the two firms. This is equivalent to normalizing $a_1 + a_2 = 1$. We denote $\bar{a} = a_1$. To have non-negative equilibrium level of output, $\bar{a}$ must be restricted to the region

$$\frac{1}{3} \leq \bar{a} \leq \frac{2}{3}$$

Then this social surplus is minimized at $\bar{a}$ =1/2 and is maximized at $\bar{a} = 1/3$, from the convexity of the social surplus function in $\bar{a}$. Since the sum of costs in the two firms is fixed, the consumer surplus is constant, whatever the value of $\bar{a}$ may be. Therefore, the value of the social surplus depends only on the total profits in the two firms. The large differentials of the costs leads to the increase of total profit in the duopoly market.

Fig. 7



We turn to seek the properties of the social surplus incured by the optimal subsidy policy, with the sum of two costs constant. Then, in the region where a duopoly market holds, the social surplus is reduced to

$$SS = \frac{d^2(-3 + 8\bar{a}^2 + (4 - 12\bar{a} + 12\bar{a}^2)d)}{8(d-1)(4d-3)} \tag{24}$$

In the region where a monopoly market holds, there are two cases, as shown before. The social surplus in the unconditional monopoly region is given by (10) and that in the entry-deterance region is given by (22).

In Fig.7, given d, the movement of the social surplus incured by the optimal subsidy policy is shown, corresponding to switching from duopoly market to monopoly market according to the magnitude of cost differentials.

6 Subsidy policy and the profit of a firm

It is interesting to show the feature of the region in the cost space where each firm obtains a benefit by this subsidy policy. If incumbent firms incur some loss by this intervention of government, they will resist the implementation of this policy.

(1)From duopoly to duopoly

Firstly let us consider the case where a duopoly market holds even after the intervention of government. For this discussion, we will take the defference of the two output levels x_i^o and x_i. This is written as

$$\frac{a(d-1)(8-6d)+(12d^2-31d+16)c_i+(-6d^2+17d-8)c_2}{2(d-1)(4-6d)} \tag{25}$$

Then the region satisfying $x_i^o - x_i = 0$ is easily shown to be linear and pass through point (a, a). Therefore, to see this movement of this line when d changes, it is sufficient to see the intersection point of this line on the vertical axis. Fig.8 shows the movement of this line. From this, we can explain the change of market structure corresponding to the change of d.

A firm with a relatively low marginal cost makes less profit than before intervention, in spite of having some positive subsidy. In this case, a firm that is levied a tax on lowers the profit in comparison with the profit before intervention.

On the other hand, a firm with a relatively low marginal cost increases the profit, in comparison with the profit when there is no-intervention, even though it is levies a tax on. It is obvious that a firm which is provided a subsidy for always increases the profit.

When d is larger than d^*, the two firms are always levied taxes on and always obtains less profit than when there is no-intervention of government.

(2)From duopoly to monopoly

In the above discussion, the profit of duopoly was compared in the case where a duopoly market existed in both after-intervention and before-intervention. There remains the study of the case where a monopoly market

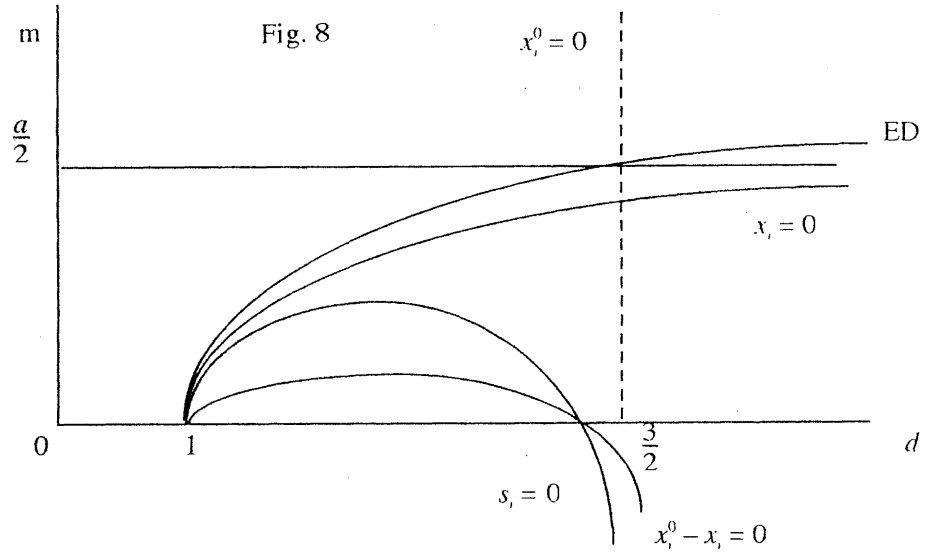
establishes after intervention, though a duopoly market exists before intervention of government. Though this possibility always exists whatever the value of d may be, the region where such a switiching occurs shrinks in cost space as d increases, as shown before. The camparison of the profit of the firm must be examined. There are two possible monopoly regions under the government intervention. Let start with an unconditional monopoly region. In this region, we seek the condition that the profit of the firm j as a monopolist when there is government intervention is equal to that of the firm j as a duopolist when there is no-intervention. This is given by

$$\frac{d(a-c_j)}{4d-3}=\frac{a-2c_j+c_i}{3}$$

The intersection point on c_i axis of the line satisfying this equation is

$$\frac{(3-d)a}{4d-3} \tag{26}$$

However this value is a decreasing function of d and it intersects at d=3/2 with the line $m=a/2$. Therefore, in this region, it is impossible that the profit of the firm j as a monopolist when there is the government intervention is equal to that of the firm j as a duopolist when there is no-intervention.



Let us go to the entry-deterance region. In the region the profit of the monopoly j is $(a-c_i)^2$. Comparing this with the before-intervention profit of the monopolist as a duopolist, the after-intervention profit is always larger

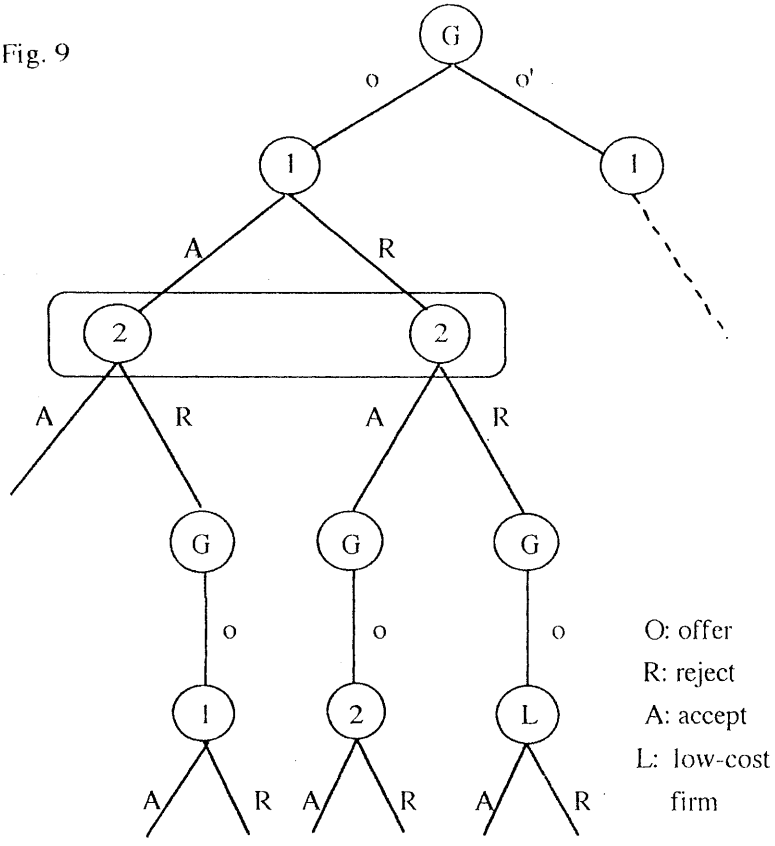
than the before-intervention profit, if m is smaller than $a/2$, which is corresponding to the region the monopoly is imposed taxes.

(3)From monopoly to monopoly

Finally let us consider a problem of whether a monopoly is better off by introduction of subsidy policy, when a monopoly market holds whenever there is government-intervention or not. This is a simple problem. As shown in section 2, the monopoly is better off (worse off) if and only if d is smaller (larger)than $3/2$. Therefore, it is irrelevant to the level of costs and demands. In Fig.8, the hatched region is one where monopolist j is better off after government-intervention.

7 Implementation of the optimal policy

Fig. 9



In the preceding section, the change of the profits of the firms under the optimal subsidy policy was investigated. We turn to consider the imple-

mentation problem of the optimal subsidy policy. It is important to inquire about how the optimal policy is implemented in the presence of incumbent firms in a duopoly market.

We assume that each firm has a right to reject the offer of government. In particular, it will take a strong opposition to the offer if the offer leads to a worse situation in comparison with the status quo, that is, the situation at the time of no-intervention. The strategies of firm are to accept, or , to reject the offer of subsidy rate. This is a kind of non-cooperative game. Then the problem is that the status quo of firm crucially depends on the strategy the rival firm takes. Note that, say, firm 2 may complain of the offer of positive subsidy. This is so if firm 1 has a relatively low cost. Likewise, firm 2 obtains some benifit in a region in spite that it may be offered a negative subsidy. This subsidy-provision game is developed as follows.

Note that we restrict the game to one where the accepting firm is offered a second menu if only one firm accepts the first menu and a more efficient firm is offered a second menu when both of firms reject the first offer. This protocol itself of negotiation may be a kind of threat against two firms.

7.1 Implementation of the optimal policy

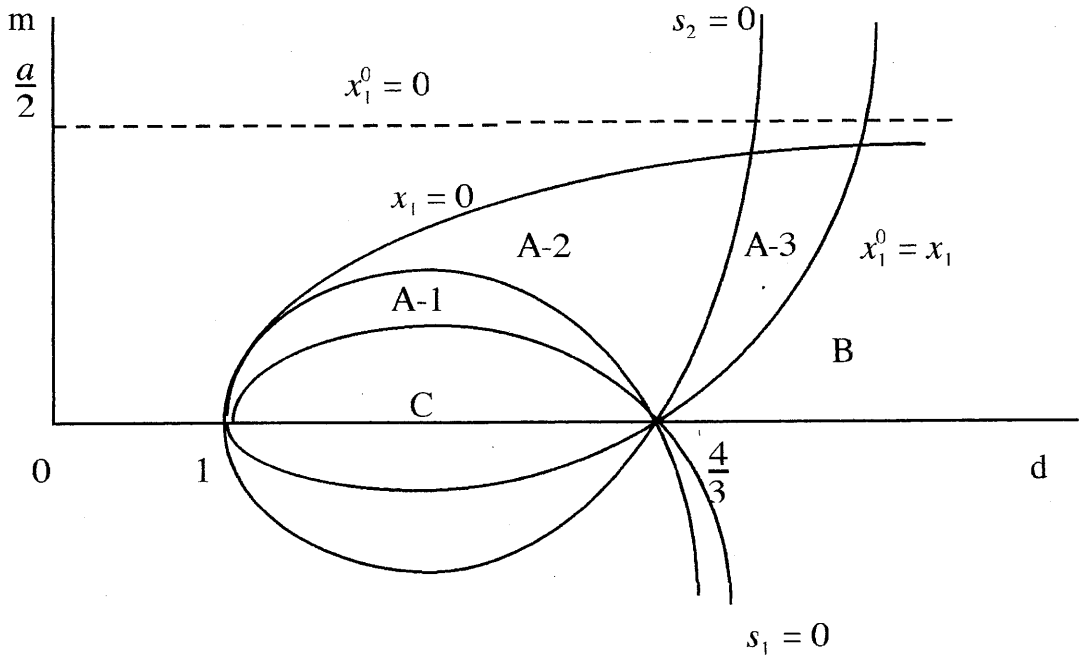
Without the loss of the generality, let us suppose that firm 1 is facing the problem of implementation. The region where firm 1 complains of this optimal subsidy policy is such a region that $x_1 \leq 0$ and $x_1 \geq 0$ are satisfied. This region is devided into two regions. One is the region where firm 1 complains of this optimal subsidy policy but firm 2 does not, which is refered as region (A). The other is the region where both of firm 1 and 2 complains of the government policy, which is refered to as region (B).

Let us start with the region (A)

(A-1) Firstly let us consider the case where both of two firms are offered a positive subsidy. In the case, firm 2 will not reject the offer when firm 1 rejects the offer, because firm 2 will get more benefit, when the rival firm rejects the offer and only firm 2 has the subsidy, compared with the situation of no-intervention incured by the rejection of both firms. Therefore, if firm 1 is offered a positive subsidy, it will not reject the offer, taking this strategy

of firm 2 into consideration, in spite of having a complaint.

Fig. 10



(A-2) Secondly let us consider the case where firm 2 is offered a positive subsidy and firm 1 is offered a negative subsidy. In the case firm 1 will reject the offer, whatever strategy the rival firm may take. On the other hand, firm 2 will accept the offer. Therefore in region (A-2), the optimal policy is not implemented if firm 1 is offered a negative subsidy. Then the government will make a further offer, facing the situation that the first best policy is rejected.

(A-3) In the region where both of two firms are offered a negative subsidy, it is obvious that to reject the offer is a dominant strategy for firm 1. Therefore, firm 2 will reject the offer reluctantly.

(1)Pre-commitment of second offer

In region (A-2), since firm 1 rejects the offer and firm 2 accepts the offer, the subsidy policy is revised as a result. So the government will offer a second menu as a threat to implement the first best policy. What is important is about the possibility of pre-commitment. If the government can make a pre-commitment about implementation of any policy, these firm may be

enforced to accept the first offer reluctantly. For example, the government could provide a high subsidy, for the firm that accepted the first offer, such that the deviating firm 1 has a lower profit compared with that obtained if it accepts the first offer.

(2) Impossibility of pre-commitment of second offer

However such a punishment may not be a credible one, because such a second offer may lower the social surplus. If the social surplus becomes lower than that realized when firm 1 rejected, the second offer may be incredible. In the case, the second offer is an empty threat, which leads to deviation of firm 1. A credible second offer must satisfy the following conditions when, say, firm 1 rejects the first offer.

$$SS^2 \geq SS^{1,*} \quad (credibility \quad condition) \tag{27}$$

$$\pi_2^2 \geq \pi_2^{1,*} \quad (acceptance \quad condition \quad of \quad firm\ 2) \tag{28}$$

$$\pi_1^2 \leq \pi_1^1 \quad (deviation - deterance \quad condition) \tag{29}$$

where the superscript $i(i = 1, 2)$ stands for the i-th offer and $\pi_2^{1,*}$ denotes the profit of firm 2 that would have had when it accepted and firm 1 rejected. If (s_1, s_2) is offered at the first stage and only firm 2 accepts, this value is

$$\pi_2^{1,*} = (\frac{a - 2(c_2 - s_2) + c_1}{3})^2.$$

$SS^{1,*}$ is the corresponding social surplus. Note that firm 1 rejects the subsidy (or tax). Then for a second offer to be a credible threat, the offer must increase the social surplus, compared with that incured under the deviation of firm 1, and must be such an offer that firm 2 accepts and firm 1 is damaged, compared with when it accepts the first offer. Therefore, the first offer is implemented since firm 1 does not reject it if there is a new offer satisfying (27)-(29). Then condition (28) is reduced to the condition $s_2^2 \geq s_2$. From the condition (29), we also obtain the condition $s_2^2 \geq s_2 - 2s_1$. Since, in the region in question, the optimal subsidy for firm 1 is negative, these three conditions are reduced to this inequality and (27). Then, it is shown that such an offer does not exist in this region. Intuitively saying, this means that there is no offer that provides more subsidies for firm 2 than in the first offer and incurs more social surplus than that in the deviation. Therefore, the first best policy is not implemented in the region $(A - 2)$.

In the region $(A-3)$ both of two firms reject the first best offer. Then the government can offer a second menu for the more efficient firm, satisfying

(27)-(29). Then it is easily shown that any offer with a positive subsidy does not satisfy (27).

On the other hand, since both of them complain of the offer in region (B) and a negative subsidy is offered, to reject the offer is also a dominant strategy for each firm. In this region (B) we can also show that the optimal policy is not implemented, by a similar discussion.

Finally let us consider about a region (C) in Fig.10 where the duopoly do not complain of the optimal policy. Since in this region the subsidy is positive, each firm will make a loss, compared with the situation under the optimal policy, if it takes an action strategically. Therefore, the optimal policy is implemented in this region. The above discussions are summarized.

Proposition 5 *In the case that each firm has a right to reject a subsidy policy, the optimal subsidy policy is implemented if the subsidy for two firms are positive. This is satisfied even if a firm still complains of the positive subsidy policy.*

7.2 Implementation of the optimal policy in the monopoly market

We turn next to consider the monopoly case. If the monopolist is provided a positive subsidy ($d \leq 3/2$), the loser has no way to do. The monopolist also does not have any reason to reject the optimal offer. If it rejects the offer to raise the subsidy more, the government can offer a new menu to the loser satisfying the credible conditions. Such a second menu is easily obtained. So the optimal policy is implementable when the subsidy is positive.

On the other hand, if the subsidy is negative, the monopolist may reject the offer. In the unconditional monopoly region, the monopolist can enjoy the market performance whithout intervention of the government. By being levied taxes on , the profit will obviously decrease. Therefore, the monopolist has a reason to reject the offer. In the entry-deterance region, as soon as the monopolist rejects the offer, the loser will enter the market. If entered, the profit of the former monopolist is

$$\left(\frac{2(a-c_2)-(a-c_1)}{3}\right)^2$$

If not entered, the monopolist has the profit $(a-c_1)^2$, which is obviously larger than the after-entry profit if and only if $(a-c_1)/(a-c_2)$ is larger than

1/2. This value is corresponding to $m = a/2$ on the c_1 axis. Therefore, the optimal policy is also likely to be rejected by the monopoly in the entry-deterance region with negative subsidy.

Can the government implement the offer in this situation ? The problem is if the government can offer a second offer to the loser as a credible threat againt the monopolist, or not. In this case a credible menu has to satisfy th following conditions.

$$SS^D(c_2, c_1 - s_1^2) \geq SS^M(c_2) \quad (credibility \quad condition) \tag{30}$$

$$\pi_2^M(c_2 - s_2) \geq \pi_2^D(c_2, c_1 - s_1^2) \quad (acceptance \quad condition \quad of \quad firm\ 2) \tag{31}$$

$$\pi_1^D(c_1 - s_1, c_2) \geq 0. \quad (deviation - deterance \quad condition). \tag{32}$$

where the superscript "D" and "M" stand for duopoly and monopoly, respectively. For example, SS^M is the social surplus in the case where the monopolist 2 is dominant without any subsidy. Likewise, $\pi_2^D(c_2, c_1 - s_1^2)$ is the profit of the former monopolist 2 when it rejects the first offer and the loser 1 accepts the new offer (s_1^2). (32) is the threat condition against the deviation of the monopolist 2. (31) is the condition that the loser accepts the new offer. The menu (s_2) is said to be weakly implemented when there is a second menu s_1^2 satisfying the above three conditions and at least the deviation-deterance condition condition with equality. The menu s_2 is said to be strictly implemented when there is a second menu satisfying these conditions and at least the deviation-deterance condition with strict inequality. Then, we obtain a proposition.

Proposition 6 *The first best policy in the monopoly market with positive subsidy is implemented. On the other hand, the first best policy in the monopoly market with negative subsidy is weakly implemented.*

(proof) (32) is equivalent to

$$s_1 \geq \frac{c_2 - 2c_1}{2} \tag{33}$$

From (31), we obtain

$$s_2 \geq -\frac{c_1 + c_2 + s_1}{3}$$

As for (30), we define a function $H(s_1) = SS^D(s_1) - SS^M$. Then it is shown that if $s_1 \geq \frac{c_2 - 2c_1}{2}$, $H(s_1) \leq 0$. Therefore, it is necessary that $s_1 \leq \frac{c_2 - 2c_1}{2}$ in order to satisfy the condition (30), with equality if $s_1 = \frac{c_2 - 2c_1}{2}$. Hence,

taking the inequality (33) into consideration, we have $s_1 = fracc_2 - 2c_12$. Then from (31), s_2 has to be larger than $-c_2/2$. The first best policy satisfies this condition either in unconditional monopoly region or in entry-deterance region. Therefore, the first best policy is weakly implemented. On the other hand, if the inequality (33) holds strictly, by the above properties about $H(s_1)$, the condition (30) is obviously not satisfied. This implies that the first best policy is not strictly implemented. (Q.E.D)

8 Concluding remarks

In summary, this paper has focused on the optimal subsidy policy in a duopoly market. As a result of investigation, It became clear that the market structure depends on the subsidy policy. By putting an attention on the shadow price of public fund, it is also shown that the optimal policy is strongly sensitive to the value of the shadow price.

In particular, we found that the increase of marginal cost of firm lead to lower subsidy and higher tax in a region of a low shadow price of public fund, and lower tax above a threshold value of the price. Though the possibility that a duopoly market establishes was smaller than that when there was no-intervention of government, we also found that the possibility became larger as the shadow price of public fund became large. In section 7 we considered about the incentive of firms about acceptance of the optimal policy. As a result, it turned out that the optimal subsidy policy in the duopoly market was implemented only when the subsidies was positive. A firm that is provided a negative subsidy may reject the offer to go back to the status quo. However, we would note that there is a possibility that a firm wants to reject the optimal policy even if the subsidy is positive. In this case, it is shown that the firm cannot reject the optimal policy.

Finally we pointed out that the optimal policy in the monopoly market with negative subsidy is weakly implemented.

We would like to finish this paper with pointing out the further fields to investigate along the line of this paper. First, it is very important to make the implementation mechanism explicit. By doing that, we will be able to obtain the second best subsidy policy.

Second, introducing an asymmetric information of cost structure seems to change the market structure and the optimal subsidy policy. Aurio=Laffont[1] and Dana=Spier[5] are trying to investigate this problem in the contex of regulation. In particular, Dana=Spier is very interesting, which considers

about designing a private industry through an auction. However the industry they dealt with is a new market but not an established one. In the case that incumbent firms exist, it will be necessary to take a different way of analysis.

Lastly, we will need to investigate a bureaucratic or political bias of the subsidy policy. This is a problem of collusion among the interest groups and regulatory institutions. To make the idea of the shadow price of public fund clearer, the further developments of research on this field are very important. In this respect, Spiller[14] is noteworthy.

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