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The Foundations of Statistical Activity Cost Theory with Applications to Some Old Accounting Measurement Issues : A Simulation Approach

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1. Introduction

This article outlines an axiomatic theory of accounting measurement and provides some examples of its application to shed new light on some old accounting measurement problems. Those examples relate to the appropriate statistical choice of depreciation method ; a statistical model of standard cost 'variances' ; the statistical nature of the concept of accounting goodwill ; and the issue of whether transaction costs have to have *realised* transactions in order for us to be able to make sensible statements about them.

The theory is referred to as Statistical Activity Cost Theory (SACT) and has many ramifications for financial accounting, management accounting, auditing and, ultimately, finance and economic theory. The purpose of this article is to draw the attention of this relatively unknown, new theory to a wider audience in the hope that others may be encouraged to apply their skills to investigate some of the vast and potentially useful applications of SACT in both practical and theoretical areas. The details of this theory have been published to date in British and Australian journals. A recent discussion of its nature and potential can be found in Gibbins and Willett (1997). This article illustrates applications of the theory and develops some earlier work through simulation experiments.

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2. Foundations of SACT

The crucial ideas and assumptions underlying SACT other than those which are physically obvious are shown in Table 1. The accounting space is a set of concepts forming the basis of accounting measurements. The two fundamental concepts are the *transaction cost* defined as the debt resulting from an exchange of resources between two accounting entities in some time interval and the *production relation* over time between elements of the resource set. The axioms define the conditions under which costs are additive and what we mean by 'continuous control' over a resource set and by 'jointness' in production. With these axioms, our normal measurement systems for physical quantities and time and our knowledge of established legal relationships we can use strings of production relations over time to match the economic processes of firms into sets of activities each having a start time (usually the date of a purchase invoice), a finish time (usually the date of a sales invoice), a duration (the difference between finish and start times) and a set of invoiced costs (of the inputs and outputs). On the basis of this structure it is possible then to prove that the basic financial statement identities (such as the capital maintenance assumption) are true (Willett [1987, 1988, 1991a]) and furthermore that it is possible to construct probabilistic models of a wide variety of accounting numbers including earnings (Willett [1991b], Lane and Willett [1997]), standard costing variances (Booth and Willett [1996]) and the market value of assets (Hodgson et al. [1993]).

Table 1.

An accounting space is a system of random variables which takes for granted the properties of physical and time measurement, the obvious properties of debt relationships and physical attributes of production processes. In addition the following three axioms have to be satisfied:

Axiom 1	*Additivity of costs*: The debt value characterising numerical costs are additive when exactly one of their components are entirely distinct and the others are identical.
Axiom 2	*Continuity of production relations*: If an accounting entity holds a resource at two points in time without a change of ownership then the entity holds that resource continually over the interval.
Axiom 3	*Separability of production relations*: If a smaller production relation is contained in a larger production relation then the resources remaining also form a production relation.

Definitions: Costs consist of an ordering of four 'component' sets: two accounting entities between which a debt has arisen, a resource set and a time interval. Production relations are relations between resource sets over just noticeable time differences.

Costs are 'matched' in activities if there is some sequence of production relations linking them together.

As may be appreciated, SACT follows the theories of Littleton [1953], using the theoretical approach pioneered by Mattessich [1964] and Ijiri [1965] and interpreting accounting numbers in a manner similar to that suggested by Brief and Owen [1970]. The result is the 'atomic' framework visualised in Figure 1. The continuous arrowed lines indicate that certain individual production relations have been linked together over time into an activity and thus show the period during which the activity is 'live'. An idealised example explaining the process of separability in more detail and showing how production relations relating to fixed assets may be specifically interpreted can be found in the Appendix[1]. Following Littleton *assets* are defined as unfinished activities. Thus activity Ai in Figure 1 is an asset at time twhile Aj is not an asset at t (it is an 'equity'). This is one way of stating that an asset has 'service potential' and is preferred here as it relates the definition to a currently observable condition.

The fact that with these axioms and definitions it is possible to derive all the usual financial statement arithmetic demonstrates that the SACT model is consistent with conventional accounting measurement practice. What is of more interest for the practical implications of the theory, however, is what results if a statistical interpretation is imposed on the basic structural variables, i.e. if invoice costs, start times and durations are all assumed to be random variables generated by chance processes. This leads to modelling techniques which produce new (and sometimes not

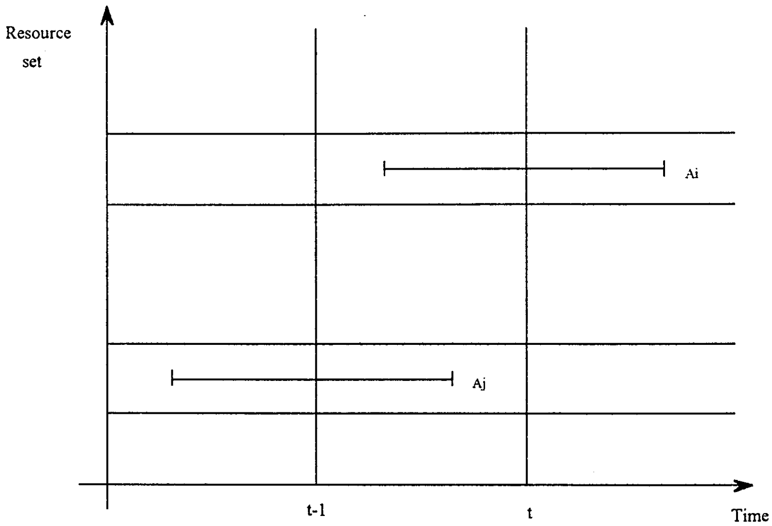

Figure 1 Activity model of the accounting process

[1] This example is of particular importance since it illustrates conditions under which certain types of holding activity (i.e. in this case a fixed asset) can be sectioned off and costed separately from other activities.

at all obvious) insights about such things as the probability distributions of accounting numbers and the possible ways more information could be extracted from accounting systems about the statistical characteristics of accounting numbers. These matters are discussed in the context of some specific illustrations in the next three sections.

3. Earnings calculations as statistics

The framework envisaged in Figure 1 and the definition of activities as 'matched' production relations suggests that *any* accounting number which is based on underlying invoice and physical production data can be modelled as an *estimate of the expected value of a function of a random sum of random numbers*. Activity costs are sums of input and output costs and it is not known with certainty beforehand exactly how many costs will be incurred in particular realisations of the process and it is not know either what will be the value of the costs incurred. Nevertheless, probability theory allows us to quantify the extent of our uncertainty about accounting numbers if a formula can be written down which effectively describes them. SACT allows us to do this. In doing so it also permits us to understand exactly in what sense observed activity costs, cash flows, funds flow and earnings calculations (which are usually conceived as being deterministic, direct measures) are, in fact, sample realisations of a potentially infinite population of outcomes and which, therefore, possess sampling distributions. It is this feature of SACT which enables us to express our feelings of uncertainty about accounting numbers such as earnings and also to engineer better accounting statistics for a chosen purpose-something which could not be done, otherwise.

Earnings provides us with an example of how the modelling process works. Undepreciated earnings in period t, ($K(t)$) may be described in SACT terminology as:

$$K(t)=\sum_i C_i \quad i=1 \ to \ N \tag{1}$$

where C_i is the accumulated ('accrued') cost of the ith activity and N is the random number of activities finishing in the accounting period ending at t.

The contribution margin (i.e. revenues less expenses before overhead costs) is that portion of $K(t)$ due to costs incurred in activities with short term durations (not greater than one accounting period) and in which the physical volume of outputs is a function of the physical volume of inputs. There are quite strong analytic reasons for believing the contribution margin as so defined will be approximately normally distributed under quite general conditions (Willett [1991b]). To gain an intuitive understanding of the insights provided by this theory, it is probably best to consider a simple simulation experiment which generates a sample of possible activities using a SACT model. It will help to interpret this experiment in terms of Figure 1.

Suppose a firm consists of just 100 activities over its entire lifetime and that the start times, durations, expenses and revenues are uniformly distributed. This is a kind of minimum information scenario. There are two classes of activities. Eighty short term activities have start times distributed across the firm's lifetime, durations distributed over the range one half to to one accounting period, expenses over the range 0.2 to 0.4, and revenues over 1.0 to 1.2. In addition twenty long term activities have start times distributed across the lifetime, durations distributed over 1 to 5 periods, expenses over 4 to 5 and revenues over zero to 3. The second category may intuitively be thought of as fixed assets with negative revenues. Consider a continuous subset of 5 periods within the lifetime chosen such that there is no constraint on the start times or durations of activities. When 10000 undepreciated earnings numbers are generated by 2000 repetitions of a Monte Carlo simulation of the activities over the firm's lifetime, an empirical distribution like that shown in Figure 2 is produced. As expected the shape of the distribution is normal looking despite the uniform patterns of the input distributions. The undepreciated earnings calculation, as modelled by expression (1) possesses characteristics relative to its input data which are similar to those of the arithmetic mean relative to its input data. The motivation to its use may be likewise analysed, i.e. we can use the undepreciated earnings as a statistic through which to make

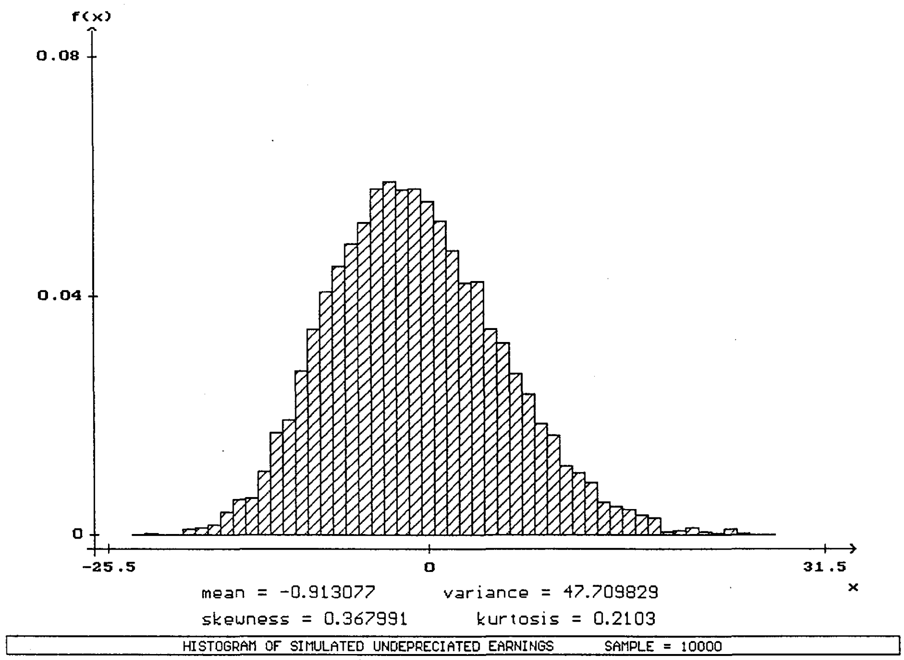

Figure 2

inferences about the nature of the underlying stochastic processes which produce the raw activity cost data. The question therefore arises: Is it possible to improve the inferential properties of the undepreciated earnings statistic?

Similar distributions can be generated for any accounting number as long as it is possible to describe it in a form similar to expression (1). This can done, for example, with the depreciated earnings number ($Y(t)$) by adjusting the undepreciated earnings number in (1) by a depreciation adjustment ($D(t)$).

$$Y(t)=K(t)+D(t) \tag{2}$$

In the case of straight line depreciation, $D(t)$ is a linear function of elapsed time the value of which depends upon the number of activities unfinished at the end of each accounting period.

Using the same simulation approach as with undepreciated earnings we could assume that the accountant makes reasonably educated guesses at estimated residual values and useful lives of fixed assets (in this case by accurately estimating their expected values) and then see what effect such additional information has on the sampling distribution of the resulting calculation. Typically it has the effect shown in Figure 3, i.e. the distribution of the depreciated version exhibits less variability than its undepreciated counterpart. In our simulation example, this can be seen from the appearance of the histogram, given the identical scale of Figures 2 and 3. The variance of depreciated earnings is approximately half that of undepreciated earnings. Therefore if one wanted to estimate the *long run* average earnings of this firm the depreciated number would produce a lower variance estimate than the undepreciated number[2]. Practicing accountants would probably say that this is what they would have expected, i.e. depreciation smoothes the earnings number about its long term average. Nevertheless academics have had considerable difficulty in the past in explaining in what sense depreciation could be a meaningful, useful adjustment and so this is one reason why the theoretical approach described here may be a significant development.

Mathematical analysis demonstrates that this idea can be generalised. Lane and Willett [1997] for example showed that the components of the earnings number in (2) can be modelled using activity cost variables in the following form.

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[2] Apart from motivating the choice of some clearly defined parameter there is no suggestion that mean per period eamings *should* be the parameter to be estimated. Clearly it would be possible by the method described to examine the statistical properties of earnings calculations relative to any parameter.

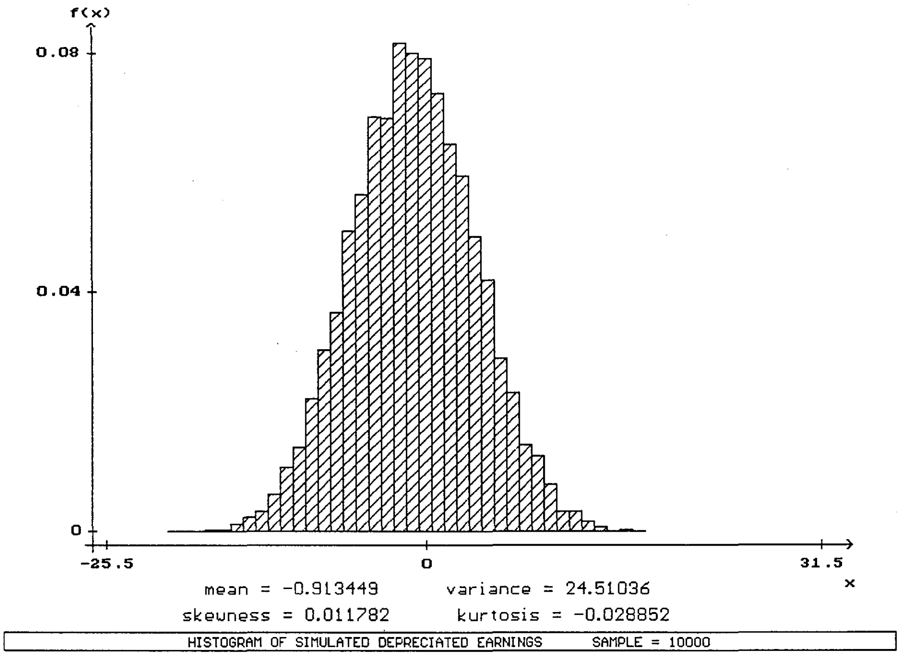


Figure 3

$$D(t)=\sum_{i=1}^{i=\delta_k}\alpha_k(i)M_k(i,t) \tag{3}$$

and,

$$K(t)=\sum_{j=1}^{j=M_k(t)}C_k(j) \tag{4}$$

where: $\alpha_k(i)$ and δ_k are k class specific decision variables standing for the formula used to calculate the depreciation charge up to t on the basis of elapsed activity time, i, and the anticipated duration time of the *kth* activity respectively; $M_k(i,t)$ is the random number of activities of the *kth* class starting i periods before t and still unfinished at t; $C_k(j)$ is the random cost in the *jth* finished activity of the *kth* class at t; and $M_k(t)$ is the random number of finished activities of the *kth* class at t.

When start times are governed by a Poisson process and duration times are exponential with parameter θ the minimum variance rate of 'straight line' depreciation, $\alpha^\wedge(\delta)$, is given by:

$$\alpha^\wedge(\delta)=\mu_c(1-e^{-\delta\theta})(1-e^{-\theta})/2(1-(\delta+1)e^{-\delta\theta}+\delta e^{-\delta(\delta+1)\theta}) \tag{5}$$

and the minimum variance for $Y(t)$ (which is less than the variance of $K(t)$) is obtained for any δ by numerical analysis. This turns out to be different from the usual rule for calculating straight line depreciation and, in fact, further analysis demonstrates that a special form of reducing balance depreciation produces an overall, minimum variance 'optimal' result in this case.

The observable and measurable nature of the variables used in this modelling process suggest that this optimisation technique could be applied to examine the behaviour of actual processes on the shop floor. Such investigations would basically require amendments to existing accounting software to handle the coding and analysis of additional data on the variability of invoice costs and production relations. It is also apparent that the model described in (3) could be extended to include depreciation variables other than elapsed time. One could, for example replace the scalar $\alpha(i)$ with a vector valued variable $\alpha(i,u)$ in which u is a physical usage measure and use this new information to recompute the optimal value. This modification would allow the investigation of some very old questions regarding the role of depreciation as a measure of the 'wearing out of assets'.

4. Standard costing variances

The foregoing examples, based upon analysing the statistical properties of accounting income, are both usually associated with the study of *financial accounting*. An illustration of the usefulness of this theory in analysing *management accounting* problems is the following. Managements adopting standard costing systems use reports similar to those shown in Table 2 given the data described. The variances calculated are used for a variety of purposes which have consequences for economic performance and organisational harmony. For instance, variances

Table 2

Inputs	Budget		Actual	
	Units	$	Units	$
	50	25.0	42	25.2

Budgeted data is based upon 100 units of output. 90 units of output were actually produced.

Reconciliation of budgeted to actual costs	
	$
Master budget costs (50× $0.5)	25.0
Volume variance [(50−45) × $0.5]	(2.5)
Flexible budget costs (45× $0.5)	22.5
Efficiency variance [(45−42) × $0.5]	(1.5)
Price variance [42× $ (0.6−0.5)]	4.2
Actual Costs	25.2

Source: Booth and willett (1996)

may identify inefficiencies in production or lack of endeavour on the part of those responsible for purchasing materials. Assessment of management performance is clearly of considerable interest to many participants in the communal production effort. The question however arises: Are the variances disclosed in such reports statistically significant and are they correlated? If the variances are statistically insignificant then managers blamed for unfavourable variances may feel justifiably aggrieved and if variances are correlated those congratulated for some favourable variances may be receiving undeserved praise.

SACT gives us a basis for assessing this matter (Booth and Willett [1996]). Henceforth, to distinguish them from statistical variances, we refer to standard cost 'variances' as 'deviations'. It is possible to define, in very general terms, what is meant by standard costing deviations, assess their statistical significance and show that the normal versions of these calculations will only be uncorrelated under particular circumstances. The total 'master budget deviation', Δ_T, is defined using the notions of conditional expectation, as the sum of a 'volume deviation', Δ_V, an 'efficiency deviation', Δ_E, and a 'price deviation', Δ_P, as follows:

Δ_T	=	Δ_P	+	Δ_E	+	Δ_V	=	C	−	$E(C)$	(6)
Total Deviation		Price Deviation		Efficiency Deviation		Volume Deviation		Actual Cost		Expected Cost	

where:

$$\Delta V=(E[C|Q]-E[C])$$
$$\Delta E=(E[C|Q,I]-E[C|Q]) \tag{7}$$
$$\Delta P=(C-E[C|Q,I])$$

Q and I are the output and input levels respectively.

Analysis then shows that when the variables for the input prices are identically distributed with a mean and variance independent of the output level, the number of invoices N and the invoice quantities I_i, $i=1,\ldots, N$ of the N purchases of input material, the following familiar textbook formulae for each of the deviations can be derived:

$$\Delta V=(Q-\lambda)\alpha\mu$$
$$\Delta E=(I-Q\alpha)\mu \tag{8}$$
$$\Delta P=\Sigma_i I_i(P_i-\mu)$$

Here, λ is the average output level, α is the physical input-output coefficient and μ is the average price per input unit.

Furthermore the appropriate conditional variances of the deviations can be computed from the following expressions:

$$\sigma^2{}_V=\chi\alpha^2\mu^2$$

$$\sigma^2{}_{E/Q}=Q\beta\mu^2 \tag{9}$$
$$\sigma^2{}_{P/Q,I}=\sigma^2E(\Sigma_i I_i^2|Q,I)$$

where, $\varkappa$ is the variance of the output variable, β is the common variance of the input-output coefficient and σ^2 is the variance of the input price per unit.

Under the circumstances described the formulae in (8) are uncorrelated. However if the basic variables are not independent this is no longer the case. For example consider the following simulation experiment in which knowledge of the processes generating output, input per unit output and input prices is assumed.

Output is presumed to have lower and upper bounds but otherwise there is minimal information, so we set

$Q\sim$ *discrete uniform* [2,6].

Input per unit output is presumed to have a lower bound but no upper bound and we set

$\alpha_j\sim$ *geometric* $(0.5)+2, j=1,\ldots,Q$.

Input is asumed to be invoiced once per accounting period and as the α_j are independent,

$I=\Sigma_{j=1}{}^{j=Q}\ \alpha_j\sim$ *negative binomial* $(Q,0.5)+2Q$.

It is also assumed that the input price depends upon the invoice quantity so that if $I\geq 10$, a 10% discount applies to the normal price. We set the normal price as $P\sim N(10,1)$.

With these assumptions,

$$\lambda=E(Q)=4$$
$$\alpha=E(\alpha_i)=3$$
$$\mu=E(P)=[\{\Sigma_{k=2}{}^{k=6}Prob(Q=k)Prob(I<10)|Q=k)\}E(P|I<10)]+$$
$$[\{\Sigma_{k=2}{}^{k=6}Prob(Q=k)Prob(I>=10|Q=k)\}E(P|I>=10)]$$
$$=9.35625.$$

Simulations were executed by sampling 20,000 sets of deviations. Although the numerical values fluctuated to a small extent, the pattern of the results was clear and the following results

Table 3

DEVIATION	MEAN	STD ERROR
ΔV	−0.0758	0.2800
ΔE	0.0122	0.1865
ΔP	−1.9592	0.0899
CORRELATIONS	STATISTIC	STD ERROR
ΔV, ΔE	0.0063	0.0071
ΔV, ΔP	−0.2914	0.0068
ΔE, ΔP	−0.1548	0.0070

set is representative (see Table 3). The volume-price and efficiency-price correlations are highly significant and negative as the above development would lead us to expect. The negative correlations reflect the likelihood of positive volume or efficiency deviations being associated with negative price deviations. This is due to relatively high output levels or input per unit output levels being associated with high input requirements and thus low input prices through the discounting condition. Note also however that the mean price deviation is significantey negative. This is due to the expectation of the price deviation being the covariance of I and P (in this case) and in general, the sum of the covariances of the I_i and P_i. In this case, it would be expected that this covariance would be negative.

Inasmuch that in practice, standard cost deviations are calculated as above and basic variables are not independent, it is not valid to assess each deviation in isolation. Furthermore, in such circumstances the expectations of deviations (i.e. the typical standard cost 'variances') will not necessarily be zero. For example in the above situation, a price deviation of approximately -1.9 to -2.0 would be a typical value rather than zero. A zero deviation in such circumstances might actually require investigation, a point which would not normally be appreciated in practice.

The simulation experiment indicates the potential of this theory for putting the analysis of standard cost deviations on a more rigorous basis. Some of these results may have already have been suspected (i.e. that correlations between the deviations exist). However by rigorous analysis they can also be quantified.

5. Market values and accounting for goodwill

Goodwill accounting is another example of how the statistical activity cost approach can provide new insight into understanding old questions. One such question relates to the aggregation problem analysed by McKeown [1972] and Vickrey [1975]. This matter, which concerns the efficacy of adding the market values of individual assets to obtain a valid representation of the market value of the assets in aggregate[3], is evidently impossible to explain in a deterministic context. The essence of the answer to this problem lies in a statistical approach to the definition of synergy and market values.

Synergy costs are defined in SACT as the costs of combining resources in a particular manner. For instance, in the case of the two element resource set $A_P=\{a_1,a_2\}$ there are two possible ways of combining the two 'proper' resource elements in transactions, i.e. transferring

[3] That is, if $v(A)$ is the market value of asset A and $v(B)$ is the market value of assest B, $v(A)+v(B)$ does not necessarily equal $v(A \cup B)$, the value of both A and B taken together.

them separately or transferring them together. These possibilities give rise to two 'improper' synergy elements denoted in the former case by the notation $<a_1;a_2>$ and in the latter by $<a_1,a_2>$. The extended set consisting of all proper and improper elements is denoted as A. The synergy element $<a_1;a_2>$ is termed null and its numerical cost denoted $c(<a_1;a_2>)$ is set equal to zero. A subset of A consisting of the proper resource elements and the non-null synergy elements in a realised transaction is denoted A_r and in the case of $A_r=\{a_1,a_2,<a_1,a_2>\}$ the numerical cost of $<a_1,a_2>$ is defined as:

$$c(<a_1,a_2>)=c(A_r)-c(a_1)-c(a_2) \tag{10}$$

For example if $c(A_r)=\$5$, $c(a_1)=\$1$ and $c(a_2)=\$2$ then $c(<a_1,a_2>)=\$2$.

Similarly, for the three element proper resource set, the non-null synergy elements consists of:

$$<a_1,a_2,a_3>,<a_1,a_2>,<a_1,a_3>,<a_2,a_3>. \tag{11}$$

Considering in general the n element proper resource set $A_p=\{a_1,a_2,...,a_n\}$, the cost of each non-null synergy element is defined as:

$$c(<a_i,...,a_m>)=c(A_r)-c(a_i)-...-c(a_m),\ 1\leq i<m\leq n. \tag{12}$$

The subset of A resulting from a set of transactions, realised in a small time interval, which partitions A_p is defined as

$$A_R=\{a_1,a_2...a_n,<a_1:a_2:...:a_n>\} \tag{13}$$

where the last term represents the set of synergy elements created by the relevant partition. The interpretation is that the transfer of the n resource elements is accompanied by a set of synergy elements created by the particular combination in which the resource elements are transferred. The total cost of such transactions is therefore defined as

$$c(A_R)=c(a_1)+c(a_2)+...+c(a_n)+c(<a_1:a_2:...:a_n>) \tag{14}$$

The last term represents the cost of the set of synergy elements.

These realised measures allow us to construct a workable notion of market values of resources. From a market value point of view each element and combination of elements in a resource set has the potential to generate random purchase or sale prices which in turn generate a random value for each synergy element. Consequently by assessing the probability of each possible set of transactions in a bundle of assets at a point in time (or more realistically in some appropriately small interval) it is possible to *define* the following measure of the market value of A, $v(A)$ as follows:

$$v(A)=E[c(a_1)]+E[c(a_2)]+...+E[c(a_n)]+E\{E[c(<a_1:a_2:...:a_n>)]\}. \tag{15}$$

The final term in (15) denotes the expected cost of each synergy element weighted by its probability of occurence (the number of synergy elements being determined by Sterling's formula for the number of ways a set can be partitioned (Willett [1987])). If we call this term, which is the market value of the synergy elements, the market value of goodwill, or *goodwill* for short, then it can be seen that the market value of A is the sum of its individual elements plus goodwill, a representation which accommodates the interpretation of the term 'goodwill' in practice.

As before, a simulation experiment can be used to illustrate the statistical characteristics of this notion of goodwill. The basic shape of the distribution of synergy costs can be seen from the two resource element case. With perfect and complete markets synergy costs will be zero with probability one. However, generally speaking, synergy costs will be realised in imperfect and incomplete markets even when there is no goodwill (i.e. the expected value of the synergy cost is zero). This idea is illustrated in the following simulation experiment. Let the costs of the individual resource elements a_1 and a_2 each be uniformly distributed on [0,1). The cost of $A_r = \{a_1,a_2,<a_1,a_2>\}$ then has a triangular distribution on [0,2,1], this distribution being that of the sum of the two $U[0,1)$ random variables. Thus the distribution of the cost of A_r when there is no goodwill is equivalent to the the distribution of the sum of the costs of resource elements a_1 and a_2 in separate transactions. The simulation experiment involved sampling costs for a_1, a_2, and A_r and subtracting the costs of a_1 and a_2 from A_r to compute $c(<a_1,a_2>)$. Output from this experiment is displayed in Figure 4. As may be observed, the fact that there is no goodwill is indicated by the approximately zero value of the mean. Most individual synergy costs however typically have non zero values due to chance fluctuations, this indicated by the shape and variance

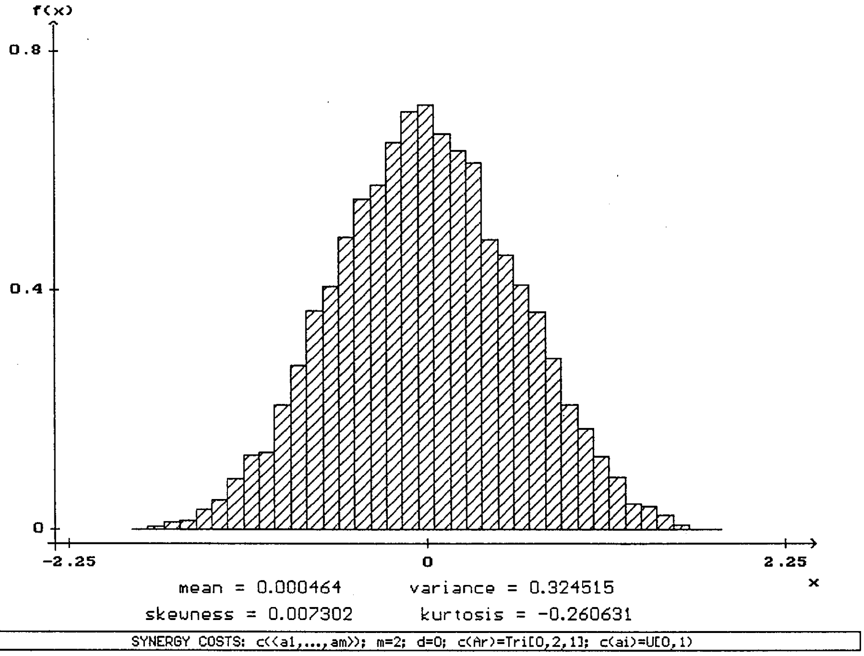


Figure 4

of the distribution. This illustrates how it is possible to report on the basis one transaction (i.e. by calculation (12)) the existence of 'historic cost' goodwill when in fact it does not represent any systematic economic underlying condition but only a chance fluctuation in values.

To see what happens when goodwill does exist, consider the following development of the simulation experiment. Here the sampled $c(A_r)$ is multiplied by 0.5 to represent a 50% discount when a transaction involves the two resource elements in combination. The output for this experiment is displayed in Figure 5. The shape of the distribution still looks normal but the existence of a systematic influence on the cost of the synergy element creates goodwill by shifting the location of the distribution, as indicated by the non-zero value of the mean. As in the first phase of the simulation experiment, a non-zero synergy cost will very frequently be observed. Consequently it is possible that a single observation of a positive synergy cost could be used to conclude that positive historic cost goodwill exists when in fact the systematic factors determining goodwill according to our definition produce a negative value. Generally, due to the large overlap in possible synergy costs it would be difficult to distinguish the situation represented in Figure 4 from that in Figure 5 on the basis of a single transaction. Consequently goodwill, as measured by historic cost practice typically may not accurately represent any systematic factors underlying the firm's economic values.

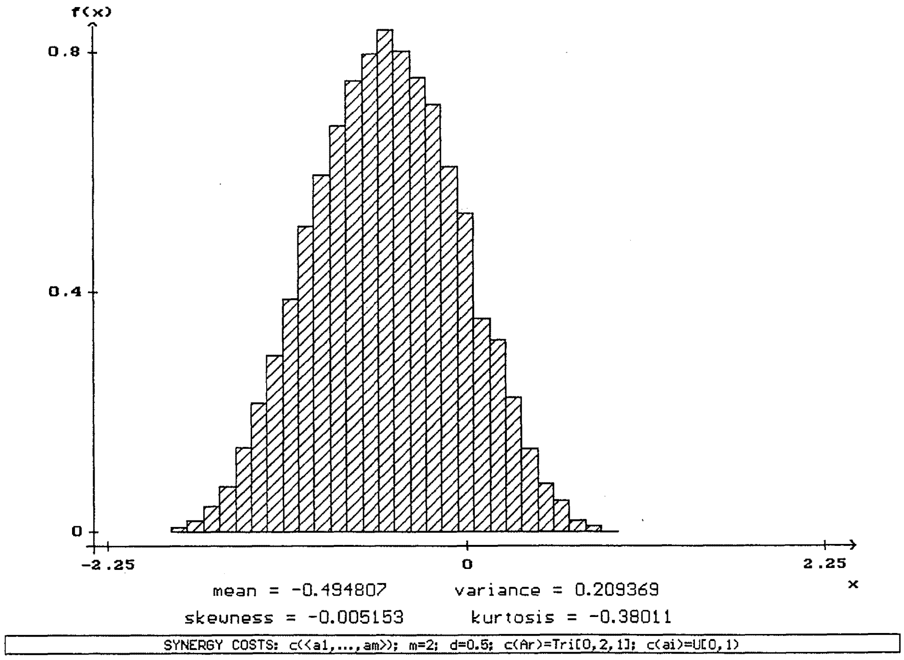


Figure 5

In principle, it should be possible with sufficient information to test for the existence of market value goodwill in practice. The statement that goodwill exists with probability $1-\alpha$ with respect to an asset set is equivalent to the statement that there is a significant difference at the *α per cent* level between the mean of a sample of aggregated individual asset costs and the mean of a sample of costs of aggregated assets. However given the limited amount of data which will typically be available, particularly with respect to the latter values, judgments about the existence of goodwill will probably be best guided in the forseeable future by the assessment of more qualitative factors such as the presence of discounting arrangements, the benefits of particular combinations of assets and other market 'imperfections'. Simulations of the type used above might then provide a method by which such information could be used to substantiate judgments concerning the existence of a material goodwill effect.

Goodwill is, in our sense, the value of the average synergy cost. The original motivation for this approach was to allow us to tag goodwill on the end of a list of asset values so that the total asset value is a meaningful sum which avoids the market value aggregation problem. However having defined the concept it also allows us to ask such questions as: What are the statistical properties of goodwill? What is the variance of the distribution of synergy costs and is the value of goodwill significantly different from zero? Are the traditional estimates of super-profits reliable indicators of the existence of goodwill? The answers to these and similar questions have implications for the debate on whether goodwill has information content and whether it can be reliably measured and, indeed, if it should be disclosed in financial statements at all.

6. The relationship of SACT measurements to traditional economic analysis

A criticism sometimes levelled at the transactions type of accounting theory on which SACT is based and which might bother some readers of the above analysis, is that it appears to lack economic motivation. The source of concern here seems to be that accounting numbers should inform us directly about the economic value of things. Traditionally the theory of economic value has been closely linked to the idea of utility measurement or some closely allied concept such as revealed preference. Unfortunately the most cursory examination of the accounting measurement procedures used in practice, at least in their pure historic cost form, reveals that it has very little to do with a relatively comprehensive and direct measurement of economic value, usually rationalised as the discounted value of future cash flows. The viewpoint expressed in SACT is that accounting measurement is better understood as providing only incomplete and indirect information about the manner in which the firm's use of it's technology, consumer and supplier 'utility' functions combine to affect the firm's debt levels. The relationship between the traditional 'economic value' approach to understanding accounting measurement and the interpretation proposed under the form of 'transactions' approach advocated via SACT is visualised in Figure 6. Under the traditional approach, accounting is seen as a relatively neutral and impassive vehicle through which economic values which already exist are reported to accounts users. Under SACT, accounting is seen as a process through which the concepts of economic value are operationalised in an attempt to reflect some of the attributes of consumer choice, producer decision making and the state of technology. The following simple example helps to explain the way in which the measurable elements of SACT are related to these latter, fundamental aspects of the economic environment. For this purpose we define the concepts of the supplier's marketing strategy, reflecting supply considerations, and a consumers's purchasing strategy, reflecting demand considerations. The example illustrates the way transaction costs are generated by the underlying factors of demand and supply and it also shows that the application of SACT is not restricted to situations where transactions actually take place. It also helps to make clear, if it was not already obvious, that a transaction cost in this theory is identical to the economic concept of a realised, traded price.

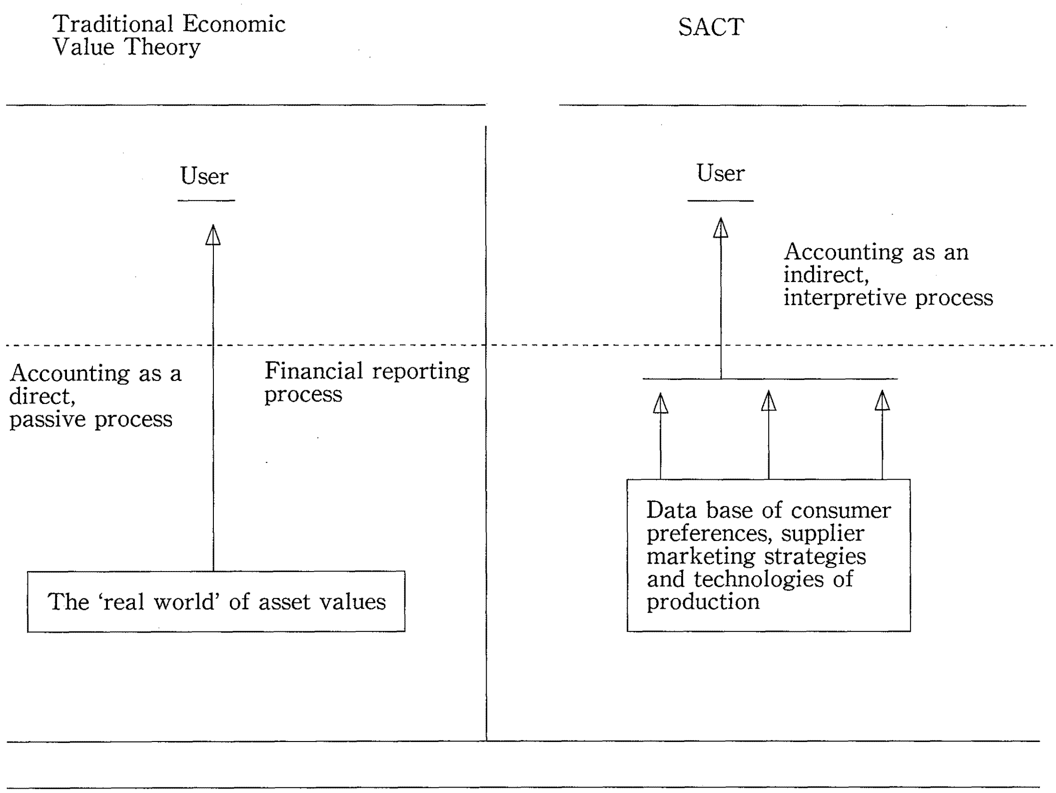


Figure 6

Let the probability distribution of the price charged by a supplier of a good (i.e. the 'offered' cost) depending on a variety of uncertain states of the world be:

$$P(C=0)=0; \quad P(C=1)=0.15; \quad P(C=2)=0.50; P(C=3)=0.35 \tag{16}$$

This is the *suppliers marketing strategy*.

Then let the conditional probability distribution of a buyer accepting ($A=1$) a supplier's offer be:

$$P(A=1|C=0)=1; \quad P(A=1|C=1)=0.75; \quad P(A=1|C=2)=0.25;$$
$$P(A=1|C=3)=0.10 \tag{17}$$

where it is assumed that either $A=1$ or $A=0$ (the buyer not accepting a supplier's offer) can occur but not both at the same time.

These conditional probabilities constitute the *consumers purchasing strategy*, the analogy of the traditional demand curve. The histogram is shown in Figure 7.

Then the distribution of transaction costs would be given by:

$$P(C=x|A=1)=P(A=1|C=x)P(C=x)/P(A=1) \tag{18}$$

For example,

$$P(C=1|A=1)=P(A=1|C=1)P(C=1)/P(A=1)=0.75\times0.15/0.2725=0.4128 \tag{19}$$

Similarly:

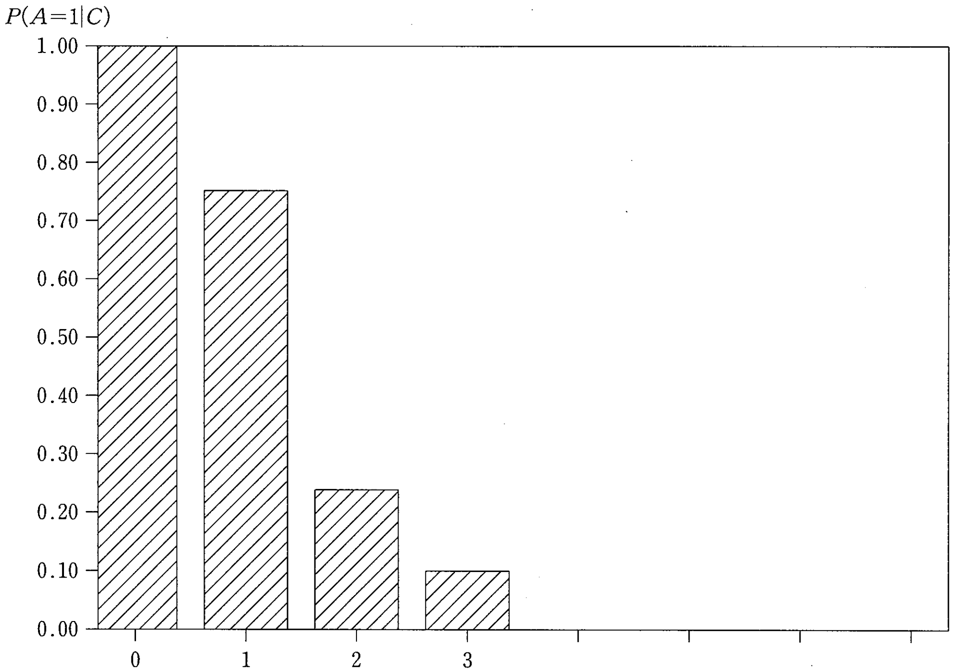


Figure 7 Histogram of consumer's purchasing strategy

$$P(C=2|A=1)=0.4587 \tag{20}$$

and,

$$P(C=3|A=1)=0.1284 \tag{21}$$

The frequency function of transaction costs in this case is therefore as indicated by the histogram shown in Figure 8

This example is highly simplified but could clearly be generalised to an infinite variety of conceivable shapes of consumers' buying and suppliers' selling strategies. Equally clearly we could survey or interview consumers and producers to discover facts about their buying and selling strategies and use this information to reason about the form of the accounting statistics this would produce. The example therefore not only relates the statistics commonly produced by accounting systems back to their underlying economic causes but, in a more explicit sense than usual, it demonstrates the possibility of investigating whether accounting numbers provide 'useful' information for economic decision making in particular circumstances. Furthermore it is evident that we do not need either 'perfect' or 'complete' markets to justify valuing resources or interpreting accounting data. SACT gives us, therefore, an improved framework in which to fit some different bits of the accounting jigsaw together.

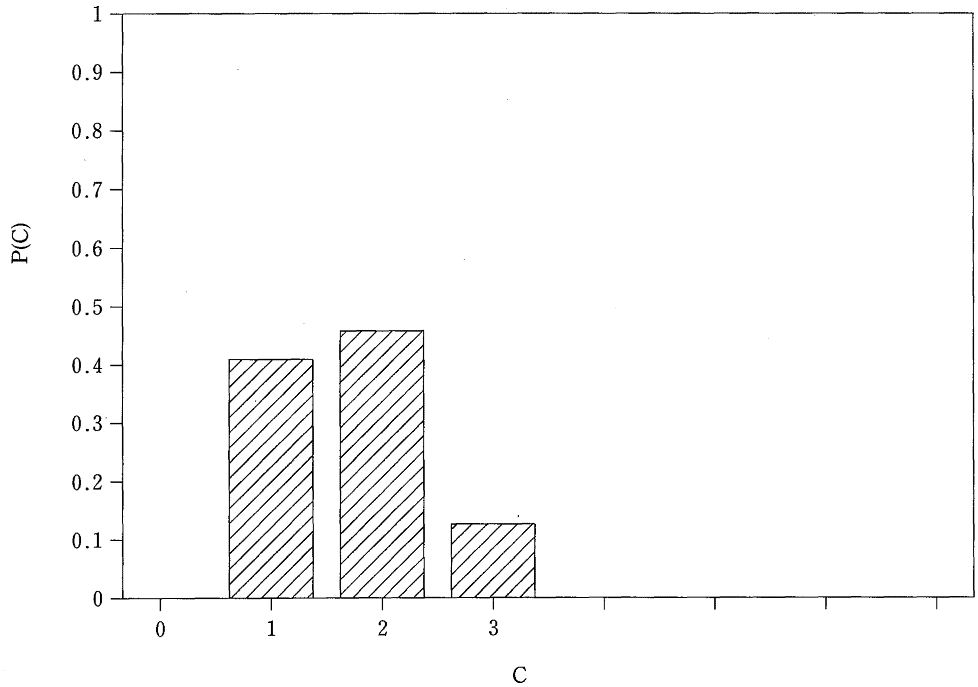


Figure 8 Histogram of theoretical transaction costs

7. Conclusions

Much of the developmental work on the theory outlined in this paper is in its early stages. The examples given in sections 2 to 6 provide a few instances where a SACT approach to research is already providing new ways of understanding accounting problems and in some instances giving us new results. The potential for the application of the SACT framework seems to be enormous as can be appreciated by perusing a book such as Feller's '*An Introduction to Probability Theory and its Applications*' (Feller, 1968; 1971). This is a reference source for the application of stochastic modelling in many disciplines. Perhaps not suprisingly there are no references to accounting in Feller's classic text but it is not hard to find problems (e.g. Satterthwaite's insurance claim problem or Einstein's dam problem) for which the analytic results can be reinterpreted in an accounting context. There is consequently a veritable mine of results waiting to be excavated and applied to unanswered accounting problems.

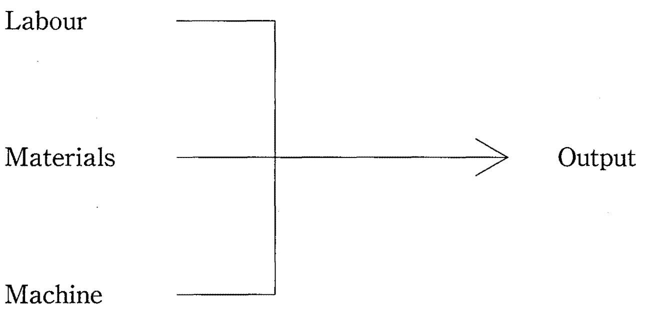
At the present time several academics from the UK, US, Canada, Australia and New Zealand are collaborating to extend analytic work on SACT and in some cases to apply it in practice. Apart from the applications described here, the theory has the potential to develop different approaches in business failure prediction, modelling the time series of earnings and cash flow, ratio analysis and segmental reporting. Investigations of the statistical patterns of the contribution margin from manufacturing processes have been being carried out. In auditing, research is taking place to use the SACT modelling procedures to examine the characteristics of analytic review procedures and a survey is currently under way to attempt to apply the concepts of the theory to improve the techniques for quantifying the inherent risk component of the audit risk model. The SACT paradigm has the potential to restructure and reinvigorate an area of accounting research, that concerned with attempting to develop a better understanding of what accounting numbers tell us about the real world, which has in recent years fallen from favour and been largely ignored.

Appendix

Production relations, separability and activity cost statistics

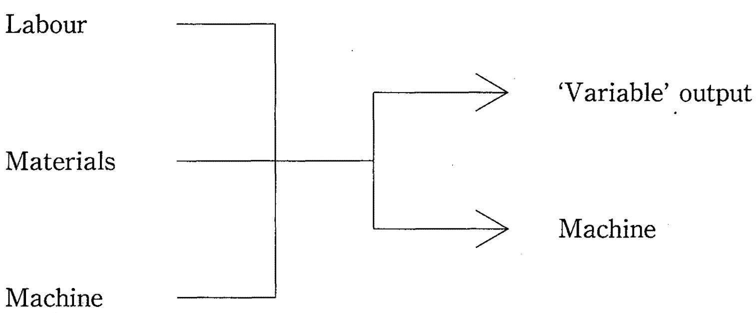
Traditionally both in the economics and accounting literatures, accounting for fixed assets has been interpreted as an attempt to measure the fall in value of fixed assets due to the 'contribution' they make to the value of output. Thomas [1969] showed that this view is untenable. SACT approaches the problem of accounting for fixed assets differently, interpreting the process as an attempt to average the expected cost of the activity of holding and using a fixed asset with respect to time - a statistic which can then be used for a variety of purposes. There are two key elements in the axiomatic foundations of the theory, which allow it to circumvent Thomas's allocation problem.

Firstly, the axiom of separability justifies the separate costing of fixed assets and other overhead activities by basing the criteria for separation on physical, non-financial information. Secondly, the financial dimension is measured independently and then combined in a statistical fashion with the physical information. The difference between the traditional and SACT viewpoints on separability may be illustrated as follows. Given a situation where a fixed asset, say a machine, is used to transform variable inputs (materials and labour) into an output, the production relationship is conventionally envisaged as:



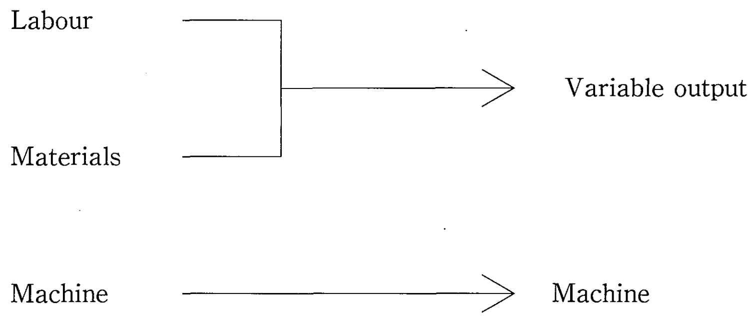
(A1)

where the output is taken to be the thing produced by the machine. From a physical viewpoint this interpretation is incomplete and therefore deficient. It makes no mention of what became of the machine in the process. Physically (assuming the machine is not physically used up to any noticeable extent by the process) a more accurate description is that the output consists of two parts, a variable output (being the product produced) and the physically unchanged machine, i.e.:



(A2)

The axiom of separability allows this composite production relation to be separated into two distinct, smaller production relations.



(3)

The interpretation *then* given to accounting measurement is that a *random* cost is directly associated with each input and output in the separate activities. The distributions of the random costs and their various characteristics (e.g. average costs) are estimated by reference to invoice and other transactions data.

The claim made by SACT is that with this basic structure *any* accounting calculation can be modelled given distributions of invoice costs for particular resources (i.e. goods and services), and of start times and durations of the underlying activities. The properties of conventional measures and averaging processes (with respect to time in the case of depreciation) can then be investigated using statistical criteria. It should be appreciated that once activities have been reduced to their minimum level of aggregation they can be separately costed in the manner described in any combination that may be desired. Therefore although this modeling process avoids Thomas's allocation problem it retains maximum flexibility in the use of accounting data for decision making purposes.

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