Optimal Reservation Price in Public Biddings : Competition versus Collusion

Miura, Isao
Faculty of Economics, Kyushu University : Associate Professor

https://doi.org/10.15017/4363009

出版情報：經濟學研究. 66 (1), pp.131-144, 1999-06-30. 九州大学経済学会

Optimal Reservation Price in Public Biddings

— Competition versus Collusion* —

Isao Miura

1. Introduction

Recently the way of the biddings for public projects has been changing from the nominated competitive bidding to the general competitive one in Japan. Nevertheless, the collusions among bidders and between bidders and a government have been occuring frequently. One reason for this fact can be considered that the number of bidders hardly increase in spite of the modification of the way of bidding. Actually the Administrative Inspection Bureau in General Affair Agency (1996) reported that in most of the general competitive biddings in Japan the number of the bidders was less than ten as it was.

Considering the present situation of the public biddings in Japan as the above mentioned, it seems to be important to examine what kind of effects do the public biddings bring to the economic environment supposing that a collusion in it occurs. So in this paper we cope with this question by focusing our concern on the reservation price which implies the upper limit contract price in the public biddings and calculating the reservation prices in the biddings with collusion and without collusion respectively and comparing them from various viewpoints.

Concretely we consider the following two extreme cases in public biddings. In the first case the all firms which will join the bidding behave competitively. On the contrary, in the second case they act as cartel members and the winning bidder guarantees the other bidders equal sidepayments. Hereafter we call the former the competitive bidding and the latter the collusive bidding. Then we calculated socially optimal reservation price in each case and examined the property of it in detail. We can summarize the main results of this analysis as follows.

[1] The case of the competitive bidding

[1-1] The reservation price is so determined that all firms have a positive probability of winning if the net social benefit is relatively high, otherwise inefficient firms have no chance to win.

---

* I am grateful to professor Eric Rasmusen for valuable comments and suggestions.

[1-2] The higher the value of public project is, the more reservation price rises.

[1-3] The higher the shadow cost of public funds is, the lower the reservation price is.

[1-4] The reservation price is independent of the number of the bidders.

[2] The case of the collusive bidding

[2-1] The reservation price is so determined that inefficient firms have no chance to win.

[2-2] The effect of the value of public project and the shadow cost of public funds on the reservation price is as same as the case of the competitive bidding.

[2-3] An increase in the number of the bidders lowers the reservation price.

[3] The comparison of the two types of biddings

[3-1] The reservation price in the competitive bidding is higher than that in the collusive one.

[3-2] Compared with the collusive bidding, the competitive bidding enhances the social welfare.

[3-3] The sign of the inequality of the expected contract prices varies depending on the parameters.

The paper is organized as follows. Section 2 formulates the competitive bidding model in a public project. Section 3 considers the collusive bidding. In Section 4 the main results obtained by the analysis of Section 2 are compared with those of Section 3. Finally we discuss our analysis briefly and remark our further work.

2. Competitive Biddings Model

We assume that there are n (>1) firms capable of performing a public project. In this public bidding, the firm which bids the lowest price is succesful if its price is not higher than the reservation price. Given that the firm i is selected, c_i denotes its cost of the project. The value of c_i, which is known to firm i but not to the government nor the other firms, is an independent realization of a distribution function $F(c_i)$ on an interval $[c_L, c_H]$. Let $f(c_i)$ denotes its density function. This informational structure is common knowledge for the government and all firms. Hereafter we assume that $F(c_i)/f(c_i)$ is weakly increasing in c_i[1]. This assumption plays an essential role in our analysis. Figure 1 shows the time procedure of the public bidding.

The government announces a reservation price r.
↓
Each firm determines a bidding price.
↓
Sealed bid first price auction is performed.
↓
The firm which bids the lowest price is successful if its price is not higher than the reservation price.

Figure 1

1) This monotonicity condition is called monotonic hazard rate property and satisfied by the uniform distribution and the normal distribution, etc.

We assume that that the firms which join the bidding act competitively, i.e., there is no collusion among the firms. Then we derive the optimal bidding strategy for each firm and examine its properties. Let π^i denote the *(ex post)* profit of the successful bidder i. Then π^i is

$$\pi^i = b_i - c_i \tag{1}$$

where b_i is the ith firm's bidding price. Assume the each firm chooses the symmetric Nash equilibrium strategy as its bidding price. Then the firm i's bidding price, b_i, can be written as follows:

$$b_i = B(c_i; r). \tag{2}$$

Thus the symmetric Nash equilibrium strategy implies that the firms which have homogenous cost structure bid the same price. In our anlysis, we will assume that there exists a symmetric Nash equilibrium. Futhermore assume that $B(c_i;r)$ is strictly increasing in c_i. Under these assumptions, we can calculate the probability which the firm i bids the lowest price, i.e., it wins the bidding if $B(c_i;r)$ is not higher than the reservation price, and its probability is $(1-F(B^{-1}(b_i))^{n-1}$, where B^{-1} denotes the inverce function of $B(c_i;r)$ when the reservation price is fixed. Thus firm i's expected profit at the time before the bidding, $E\pi^i$, is

$$E\pi^i = (b_i - c_i)(1-F(B^{-1}(b_i)))^{n-1}. \tag{3}$$

Let $E\pi(x;c_i)$ denote firm i's expected profit when it deviates from the symmetric Nash equilibrium strategy and chooses $B(x;r)$ as its bidding strategy while the other firms choose their equilibrium strategies. Then $E\pi(x;c_i)$ is

$$E\pi(x;c_i) = (B(x;r) - c_i)(1-F(B^{-1}(B(x;r))))^{n-1}.$$

The definition of the symmetric Nash equilibrium is as follows:

$$E\pi(c_i;c_i) \geq E\pi(x;c_i) \quad \textit{for all } i \in \{1,2,3,\ldots,n\} \textit{ and all } c_i,\ x \in [c_L, c_H].$$

The symmetric Nash equilibrium strategy was solved by Riley and Samuelson (1981).

Lemma 1 [Riley and Samuelson (1981)[2)]] *The symmetric Nash equilibrium strategy for the firm i is as follows:*

$$B(c_i;r) = c_i + \int_{c_i}^{r} \frac{(1-F(x))^{n-1}}{(1-F(c_i)^{n-1})} dx \quad (c_i \leq r).$$

(Proof) See Riley and Samuelson (1981).

From lemma 1 we can see immedately that $B(c_i;r)$ increases in r. Let π^i_{comp} denote firm i's *ex post* profit and $E\pi^i_{comp}$ denote its expected profit in the symmetric Nash equilibrium respectively. Then π^i_{comp} and $E\pi^i_{comp}$ are

$$\pi^i_{comp} = \begin{cases} B(c_i;r) - c_i & \text{if } c_i = \min\{c_1, c_2, \ldots, c_n, r\} \\ 0 & \text{otherwise,} \end{cases}$$

2) Here we exchange the buyer into the seller in Riley and Samuelson (1981) as they consider the usual auction for private goods, being different from public biddings.

$$E\pi^i_{comp}=(B(c_i;r)-c_i)(1-F(c_i))^{n-1}.$$

Next, we define the social welfare function which represents the government's objective function. Let W_{comp} denote the social welfare at the time after the bidder i wins and S denote the benefit of the project. Then W_{comp} is

$$W_{comp}=S-(1+\lambda)B(c_i;r)+\pi^i_{comp}, \tag{4}$$

where λ (>0) denotes a shadow cost of public funds.[3] The right-hand side in the above equation can be decomposed into the consumer surplus, $S-(1+\lambda)B(c_i;r)$, and the producer surplus, π^i_{comp}. Assume that $S-(1+\lambda)c_L>0$. We consider the expected social welfare function which means the social welfare at the time before bidder i wins. Let EW_{comp} denote the expected social welfare. We assume that the government treats each firm equally before the bidding, which is usually called an assumption of anonymity. Hereafter as long as there is no confusion, we will call EW_{comp} the social welfare. Then we have

$$EW_{comp}=n\int_{c_L}^{r}[S-(1+\lambda)B(c_i;r)+\pi^i_{comp}](1-F(c_i))^{n-1}f(c_i)dc_i.$$

Using Lemma 1 and partial integration, EW_{comp} can be rewritten as follows:

$$EW_{comp}=n\int_{c_L}^{r}\left\{S-(1+\lambda)c_i-\frac{\lambda F(c_i)}{f(c_i)}\right\}(1-F(c_i))^{n-1}f(c_i)dc_i. \tag{5}$$

We formulate a problem for the government.

Problem　$\max_r EW_{comp}$

To solve this problem, we differentiate EW_{comp} as to the reservation price r.

$$\frac{dEW_{comp}}{dr}=nf(r)(1-F(r))^{n-1}\left\{S-(1+\lambda)r-\frac{\lambda F(r)}{f(r)}\right\}. \tag{6}$$

Note the following two equations.

$$\lim_{r\to c_L+0}\frac{dEW_{comp}}{dr}=nf(c_L)[S-(1+\lambda)c_L]>0,$$

$$\lim_{r\to c_L-0}\frac{dEW_{comp}}{dr}=0.$$

Considering the monotonicity of F/f, the solution of this problem is as follows:

*Case*1 if $S-(1+\lambda)c_H-\dfrac{\lambda F(c_H)}{f(c_H)}\geq 0,$

then $r=c_H$.

*Case*2 if $S-(1+\lambda)c_H-\dfrac{\lambda F(c_H)}{f(c_H)}<0,$

3) We can see such a formulation in Laffont and Tirole (1993).

then r satisfies that $S-(1+\lambda)r-\frac{\lambda F(r)}{f(r)}=0$.

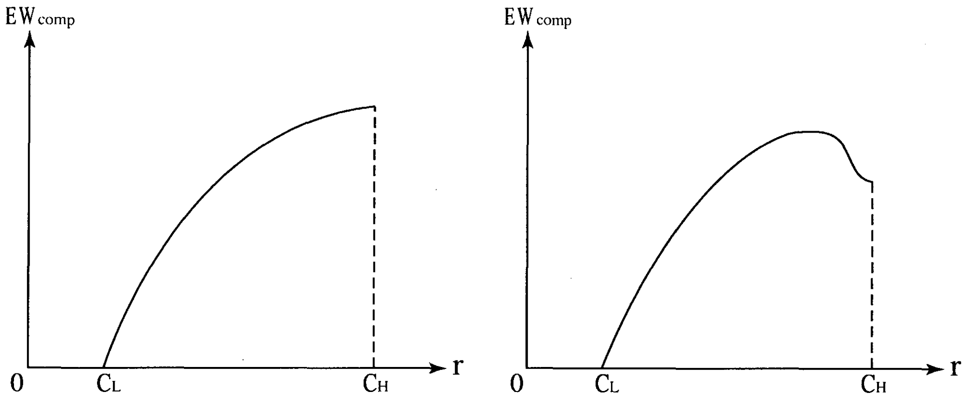

Figure 2a ⟨Case 1⟩ Figure 2b ⟨Case 2⟩

This result can be summarized as the following proposition:

Proposition 1 *The optimal reservation price in the competitive bidding is so determined that all firms have a positive probability of winning if the net social benefit is relatively high, otherwise inefficent firms have no chance to win.*

We state two remarks about Proposition 1. First, in the case 1, it is indifferent for the government whether setting up the reservation price or not. Secondly, the Revenue Equivalence Theorem of Vickrey (1961) holds in the case 1, but not in the case 2, i.e., the expected contract price from this first price auction is the same as if a second price auction were used instead, in the case 1 but not in the case 2.

Now we examine the property of the optimal reservation price in the competitive bidding. Let r_{comp} denote the optimal reservation price in it. We see immediately that r_{comp} is independent of the number of bidders. The reason is conjectured as follows. Suppose that the number of bidders increases. Then the contract price falls by a drop of each firm's bidding price. If the government lowers the reservation price, the risk that the project is not completed increases and if the government raises it, it offsets the welfare improving effect induced by the increase in the number of bidders.

Next we try comparative statics to examine the property of r_{comp} in the case 2. The results can be sammarized as follows:

Proposition 2 *In the Case 2,*

$$(1)\frac{\partial r_{comp}}{\partial S}>0, \quad (2)\frac{\partial r_{comp}}{\partial \lambda}<0.$$

[Proof] The optimal reservation price, r_{comp}, satisfies

$$S-(1+\lambda)r_{comp}-\frac{\lambda F(r_{comp})}{f(r_{comp})}=0.$$

Applying the implicit function theorem to the above equation, we have

$$\frac{\partial r_{comp}}{\partial S}=\frac{1}{1+\lambda\left(1+\frac{\partial}{\partial r_{comp}}\left(\frac{F(r_{comp})}{f(r_{comp})}\right)\right)}=0,$$

and

$$\frac{\partial r_{comp}}{\partial \lambda}=\frac{-r_{comp}-\frac{F(r_{comp})}{f(r_{comp})}}{1+\lambda\left(1+\frac{\partial}{\partial r_{comp}}\left(\frac{F(r_{comp})}{f(r_{comp})}\right)\right)}.$$

By the monotonicity of F/f, we have the desired sign condition. (Q.E.D.)

Proposition 2 can be interpreted as follows: As a rise of the value of the project makes the government strengthen an incentive to complete it, it raises r_{comp}. But when the shadow cost λ raises, it lowers r_{comp} to avoid a high priced order.

3. Collusion among All Bidders

In this section we consider the case where all bidders join a cartel with a sidepayment. The collusion corresponds to the "strong cartel" which is devised in McAfee and McMillan (1992). The sidepayment is so used that a winning bidder gives equal monetary transfer for the other collusion members. According to McAfee and McMillan (1992), the optimal collusion mechanism is expressed as the following lemma.

Lemma 2 [McAfee and McMillan (1992)[4]]

Before the bidding, the collusion members report their costs truthfully. If all reports exceed the reservation price r, the collusion group doesn't bid. If at least one report is not higher than r, the bidder with the lowest report c_i is chosen as a winner and pays the other bidders for $T(c_i)$ totally,

$$T(c_i)=\frac{n-1}{n}\left(r-c_i-\int_{c_i}^{r}\frac{(1-F(x))^n}{(1-F(c_i))^n}dx\right). \tag{7}$$

4) Here we exchange the buyer into the seller in McAfee and McMillan (1992) as footnote 2.

The winner bids r, while the others bid any prices that exceeds r.

(Proof) See the Appendix A.

McAfee and McMillan (1992) insists that the collusion's optimal mechanism is not unique. For example, we can use the competitive bidding in the previous section to choose a winner before the true bidding. We consider the profit of the collusion's member. Let π^i_{coll} denote the firm i's profit (*ex post* profit) at the time after the bidding and $E\pi^i_{coll}$ denote the firm i's expected profit (*ex ante* profit) at the time before the bidding. Then π^i_{coll} and $E\pi^i_{coll}$ can be expressed as follows:

$$\pi^i_{coll}=\begin{cases} r-c_i-T(c_i) & \text{if } c_i=\min\{c_1,c_2,\dots,c_n,r\} \\ \dfrac{T(c_j)}{n-1} & \text{if } c_j=\min\{c_1,c_2,\dots,c_n,r\}\ (i\neq j) \\ 0 & \text{if } r=\min\{c_1,c_2,\dots,c_n,r\}, \end{cases}$$

$$E\pi^i_{coll}=(r-c_i-T(c_i))(1-F(c_i))^{n-1}$$

$$+(n-1)(1-(1-F(c_i))^{n-1})\int_{c_L}^{c_i}\frac{\frac{T(x)}{n-1}(1-F(x))^{n-2}}{1-(1-F(c_i))^{n-1}}f(x)dx$$

$$=(r-c_i-T(c_i))(1-F(c_i))^{n-1}+\int_{c_L}^{c_i}T(x)(1-F(x))^{n-2}f(x)dx.$$

Let EW_{coll} denote the social welfare when the optimal collusion mechanism operates. Then EW_{coll} is

$$EW_{coll}=n\int_{c_L}^{r}(S-\lambda r-c_i)(1-F(c_i))^{n-1}f(c_i)dc_i.$$

To caluculate r that maximizes EW_{coll}, we differentiate EW_{coll} as to r.

$$\frac{dEW_{coll}}{dr}=-\lambda[1-(1-F(r))^n]+nf(r)(1-F(r))^{n-1}[S-(1+\lambda)r]. \tag{8}$$

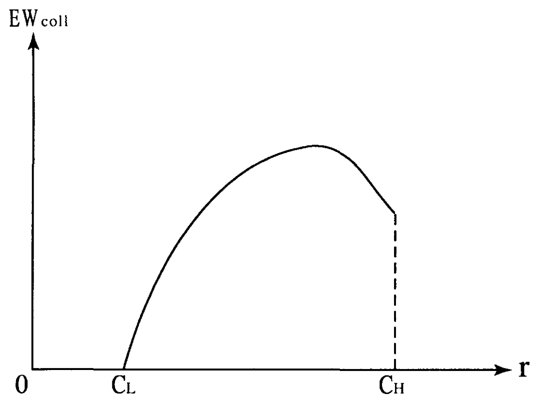

Figure 3

Note the following two equations.

$$\lim_{r \to c_L+0} \frac{dEW_{coll}}{dr} = nf(c_L)[S-(1+\lambda)c_L] > 0,$$

$$\lim_{r \to c_H-0} \frac{dEW_{coll}}{dr} = -\lambda < 0.$$

Hence r that satisfies $dEW_{coll}/dr=0$ belongs to the open interval (c_L, c_H) and is optimal solution. So we obtain the following proposition:

Proposition 3 *The optimal reservation price in the collusive bidding is so determined that inefficient firms have no chance to win.*

Proposition 3 implies that there exists the possibility that the project is not completed. Let r_{coll} denote the optimal reservation price in the collusive bidding. Next we try comparative statics to examine the property of r_{coll}. The results can be summarized as follows:

Proposition 4

$$(1)\frac{\partial r_{coll}}{\partial S} > 0, \quad (2)\frac{\partial r_{coll}}{\partial \lambda} < 0, \quad (3)\frac{\partial r_{coll}}{\partial n} < 0.$$

(Proof) See the Appendix B.

We ommit the interpretation of (1) and (2) in the above proposition because we can explain as the case of Proposition 2. Here we comment only (3). In contrast to the competitive biddings in the previous section, the reservation price r_{coll} depends on n and decreases in n. This is due to the fact that the contract price in the collusive bidding is unchangeable unless the government adjusts r_{coll} even if the number of bidders varies.

4. Comparative Analysis

In this section the main results obtained by the analysis of the competitive bidding in Section 2 are compared with those of the collusive bidding in the previous section. Firstly, we compare the bidder's *ex ante* and *ex post* profit and the social welfare. The next lemma holds when a common reservation price is used in both cases.

Lemma 3

(1) $\pi^i_{comp} \leq \pi^i_{coll}$ *for all* $r \in (c_L, c_H]$

(2) $E\pi^i_{comp} < E\pi^i_{coll}$ *for all* $r \in (c_L, c_H]$

(3) $EW_{comp} > EW_{coll}$ *for all* $r \in (c_L, c_H]$

(Proof) See the Appendix C.

Lemma 3 implies that though the competitive bidding is socially more desirable than the collusive bidding, the government may not be able to implement the competitive bidding because each bidder has always an incentive to collude. Next we examine the reservation prices and social welfare. Let EW^*_{comp} and EW^*_{coll} denote the maximum social welfare in each bidding respectively.

$$EW^*_{comp} \equiv n\int_{c_L}^{r_{comp}} [S - \lambda B(c_i; r_{comp}) - c_i](1-F(c_i))^{n-1} f(c_i)dc_i,$$

$$EW^*_{coll} \equiv n\int_{c_L}^{r_{coll}} [S - \lambda r_{coll} - c_i](1-F(c_i))^{n-1} f(c_i)dc_i.$$

Using Lemma 3 (3), we have the following proposition:

Proposition 5

(1) $r_{comp} > r_{coll}$

(2) $EW^*_{comp} > EW^*_{coll}$

Finally, we compare the expected contract prices in both cases. Let EP^*_{comp} and EP^*_{coll} denote the expected contract prices when the reservation price in each case is evaluated optimally, i.e.,

$$EP^*_{comp} \equiv n\int_{c_L}^{r_{comp}} B(c_i; r_{comp})(1-F(c_i))^{n-1} f(c_i)dc_i,$$

$$EP^*_{coll} \equiv n\int_{c_L}^{r_{coll}} r_{coll}(1-F(c_i))^{n-1} f(c_i)dc_i.$$

We verify through a following numerical example that the sign of inequality between EP^*_{comp} and EP^*_{coll} varies depending on the parameters.

Numerical Example

$n=2$,

$S=150$,

$c_L=100$,

$c_H=110$,

$F(c_i)$; uniform distribution

Table 1

	r_{comp}	EP^*_{comp}	r_{coll}	EP^*_{coll}
$\lambda=0.1$	110	106.67	109.83	109.8
$\lambda=0.2$	110	106.67	109.47	109.15
$\lambda=0.3$	110	106.67	108.39	105.59
$\lambda=0.4$	105.56	84.13	105	78.75

In table 1 r_{comp}, EP^*_{comp}, r_{coll} and EP^*_{coll} are calculated respectively when $\lambda=0.1\sim0.4$. Note in the competitive bidding that the first three rows in Table 1 are case 1 and the last row is case 2. From the table we see that $EP^*_{comp}<EP^*_{coll}$ when $\lambda=0.1$ or 0.2, while $EP^*_{comp}>EP^*_{coll}$ when $\lambda=0.3$ or 0.4. So the sign of the inequality between EP^*_{comp} and EP^*_{coll} cannot be uniquely determined.

5. Concluding Remarks

We state a few complements about the results in our analysis. First is about Propositions 1 and 2. In the competitive bidding the government dosen't have to set up the reservation price when the net social benefit is relatively high. This implies that it is socially desirable to make a bid with free entry. On the contrary, when it is relatively low, the government must so set up the reservation price that inefficient firms have no chance to win. However in calculating it, it may neglect the number of bidders being different from the case of the collusive bidding.

Second is related to Lemma 3. LaCasse (1995) considers the auction game where the buyers decide whether to collude before an auction while the legal authority chooses whether to prosecute the buyers on the basis of the bids tenderd. He shows that in the unique sequential equilibrium of the game, buyers rig their bids with positive probability. By (1) and (2) in Lemma 3, we see that the collusive bidding dominates the competitive one from the point of the bidders' profit. So our analysis supports LaCasse (1995).

Next we remark further work. Three of them in relation to our analysis are as follows:

(1) A partil collusion,

(2) The mechanism design for collusion-proof,

(3) Endogenousty of the quality of the public project.

(1) implies that some firms form a cartel while the others behave competitively one another. In the partial collusion, it may be difficult to calculate the equillibrium bidding strategy considering the strategic interaction between the cartel group and the others.

Laffont and Tirole (1993) develops the mechanism design for collusion-proof between the government and a bidder using three tier agency model that consists of congress (principal), government (agency) and firms (agency). However we haven't found (2) among bidders yet. So we need consider it.

As for (3), in this paper we treated the quality of the public project, S, as given, but in fact it is probable that there is a trade-off between the quality and bidding strategy, i.e., the lower the contract price is, the inferior the quality is. Thus we need reexamine our analysis considering such a trade-off.

Appendix A (The proof of Lemma 2)

If all firms' reports exceed r, the collusion will break up. So we consider the case where at least one report isn't higher than r. It is apparent that the successful bidding price is equal to r because all firms are collusion members. Furthermore the firm who reported the lowest cost becomes the winner as such a case maximizes the total profit of collusion members.

Next we show that each firm reports its production cost truthfully when a transfer mechanism is given as equation (7). It is sufficient to show that the transfer mechanism which satisfies the incentive compatibility condition (hereafter (IC) condition) is determined as equation (7). Let $E\pi_{coll}(u;c_i)$ denote the expected profit when the firm i tells a lie u as his production cost. Then $E\pi_{coll}(u;c_i)$ is

$$E\pi_{coll}(u;c_i)=(r-c_i-T(u))(1-F(u))^{n-1}+\int_{c_L}^{u}T(x)(1-F(x))^{n-2}f(x)dx.$$

The (IC) condition for the firm i can be expressed as follows:

$$E\pi_{coll}(c_i;c_i)\geq E\pi_{coll}(u;c_i)\ for\ all\ u,\ c_i\in[c_L,c_H].$$

It is well known that the (IC) condition for the firm i is equivalent to

$$\left.\frac{\partial E\pi_{coll}(u;c_i)}{\partial u}\right|_{u=c_i}=0, \qquad (A1)$$

and

$$\left.\frac{\partial^2 E\pi_{coll}(u;c_i)}{\partial c_i\partial u}\right|_{u=c_i}=0. \qquad (A2)$$

Note that Guesnerie and Laffont (1987) develops the exact discussion for the equivalence. Now we have

$$\left.\frac{\partial^2 E\pi_{coll}(u;c_i)}{\partial c_i\partial u}\right|_{u=c_i}=(n-1)(1-F(c_i))^2f(c_i)>0.$$

So we see that (A2) holds without depending on $T(c_i)$. Hence we have only to be concerned the derivation of $T(c_i)$ that satisfies (A1). From (A1), we obtain the following linear differential equation:

$$\frac{dT(c_i)}{dc_i}-\frac{nf(c_i)T(c_i)}{1-F(c_i)}=-\frac{(n-1)f(c_i)(r-c_i)}{1-F(c_i)}.$$

Applying the method of variation of constants to the above equation, we have

$$T(c_i)=\frac{1}{(1-F(c_i))^n}\left(\int-(n-1)(1-F(c_i))^{n-1}f(c_i)(r-c_i)dc_i+K\right), \qquad (A3)$$

where K means an integral constant. Using $T(r)=0$, we can calculate the value of K. Futhermore partially integrating the right-hand side in (A3), we have

$$T(c_i)=\frac{n-1}{n}\left[r-c_i-\int_{c_i}^{r}\left(\frac{1-F(x)}{1-F(c_i)}\right)^n dx\right].$$

Therefore the transfer mechanism, $T(c_i)$, satisfies the (IC) condition for the firm i. So we proved Lemma 2. (Q.E.D.)

Appendix B (The proof of Proposition 4)

Firstly, we show (1) and (2). Let $G(r_{coll}, S, \lambda, n)$ define as follows:

$$G(r_{coll},S,\lambda,n)\equiv-\lambda(1-(1-F(r_{coll}))^n)+nf(r_{coll})(1-F(r_{coll}))^{n-1}[S-(1+\lambda)r_{coll}].$$

Applying the implicit function theorem to $G(r_{coll}, S, \lambda, n)=0$, we have

$$\frac{\partial r_{coll}}{\partial S}=-\frac{nf(r_{coll})(1-F(r_{coll}))^{n-1}}{\frac{\partial}{\partial r_{coll}}G(r_{coll},S,\lambda,n)},$$

$$\frac{\partial r_{coll}}{\partial \lambda}=\frac{1-(1-F(r_{coll}))^n+nf(r_{coll})(1-F(r_{coll}))^{n-1}r_{coll}}{\frac{\partial}{\partial r_{coll}}G(r_{coll},S,\lambda,n)}.$$

As EW_{coll} is strictly concave in the neighborhood of r_{coll}, we have

$$\frac{\partial}{\partial r_{coll}}G(r_{coll},S,\lambda,n)<0.$$

Therfore we have

$$\frac{\partial r_{coll}}{\partial S}>0,\quad \frac{\partial r_{coll}}{\partial \lambda}<0.$$

Next we prove (3). Assume that n is a continuous variable. Then by the implicit function theorem,

$$\frac{\partial r_{coll}}{\partial n}=-\frac{\frac{\partial}{\partial n}G(r_{coll},S,\lambda,n)}{\frac{\partial}{\partial r_{coll}}G(r_{coll},S,\lambda,n)}.$$

Here we calculate $\frac{\partial}{\partial n}G(r_{coll},S,\lambda,n)$.

$$\frac{\partial}{\partial n}G(r_{coll},S,\partial,n)=\lambda(1-F(r_{coll}))^n\log(1-F(r_{coll}))$$
$$+f(r_{coll})[S-(1+\lambda)r_{coll}][(1-F(r_{coll}))^{n-1}+n(1-F(r_{coll}))^{n-1}\log(1-F(r_{coll}))].$$

Substituting $G(r_{coll}, S, \lambda, n)=0$ into the above equation and arranging it, we have

$$\frac{\partial}{\partial n}G(r_{coll},S,\lambda,n)=\lambda\left[\log(1-F(r_{coll}))+\frac{1-(1-F(r_{coll}))^n}{n}\right].$$

Note that $\frac{\partial}{\partial n}G(c_L, S, \lambda, n)=0$ and $\frac{\partial^2}{\partial r_{coll}\partial n}G(r_L, S, \lambda, n)<0$. Hence $\frac{\partial}{\partial n}G(r_{coll}, S, \lambda, n)$ is negative. So $\frac{\partial r_{coll}}{\partial n}<0$. (Q.E.D.)

Appendix C (The proof of Lemma 3)

Firstly, we prove (1). If all firms' production costs exceed r, $\pi^i_{comp}=\pi^i_{coll}=0$. The probability which the firm i wins in the competitive bidding is the same as that in the collusive one. So the firm i wins in the former when it does in the latter and vice versa. Now consider the case where the firm i loses. Then its *ex post* profit in the competitive bidding, π^i_{comp}, is of course zero. In the collusive bidding

$$\pi^i_{coll}=\frac{T(c_j)}{n-1},$$

where it is supposed that the firm j wins. Hence $\pi^i_{comp}<\pi^i_{coll}$.

Next we consider the case where the firm i wins. Using Lemma 1 and Lemma 2, π^i_{comp} and π^i_{coll} are respectively as follows:

$$\pi^i_{comp}=\frac{\int_{c_i}^{r}(1-F(x))^{n-1}dx}{(1-F(c_i))^{n-1}},$$

$$\pi^i_{coll}=\frac{r-c_i}{n}+\frac{(n-1)\int_{c_i}^{r}(1-F(x))^{n}dx}{n(1-F(c_i))^{n}}.$$

Differentiating the above equations as to r, we have

$$\frac{d\pi^i_{comp}}{dr}=\left(\frac{1-F(r)}{1-F(c_i)}\right)^{n-1},$$

$$\frac{d\pi^i_{coll}}{dr}=\frac{1}{n}+\frac{n-1}{n}\left(\frac{1-F(r)}{1-F(c_i)}\right)^{n}.$$

By a simple calculation,

$$\frac{d\pi^i_{coll}}{dr}-\frac{d\pi^i_{comp}}{dr}=\frac{(F(r)-F(c_i))(\Sigma^n_{k=1}(1-F(c_i))^{n-k}(1-F(r))^{k-1}-n(1-F(r))^{n-1})}{n(1-F(c_i))^{n}}.$$

As $c_i<r$, $F(c_i)<F(r)$. Therefore the right hand side in the above equation is positive. Considering that $\pi^i_{comp}=\pi^i_{coll}$ if $c_i=r$, we have

$$\pi^i_{comp}<\pi^i_{coll}.$$

(2) follows from (1).

Next we show (3). From equations (6) and (8), we have

$$\frac{dEW_{comp}}{dr}-\frac{dEW_{coll}}{dr}=\lambda F(r)[1+(1-F(r))+(1-F(r))^2+...+(1-F(r))^{n-1}-n(1-F(r))^{n-1}]>0.$$

Considering that $EW_{comp}=EW_{coll}$ if $c_i=r$, we have

$$EW_{comp} > EW_{coll} .$$

(Q.E.D.)

References

[1] Guesnerie, R., and J. J Laffont (1987), "A Complete Solution to a Class of Principal-Agent Ploblems with an Application to the Control of a Self-Managed Firm," *Journal of Public Economics,* Vol. 25, pp329-370.

[2] Laffont, J. J., and J. Tirole (1993), *A THOREY OF INCENTIVE PROCUREMENT AND RENEGOTIATION,* MIT press.

[3] McAfee, P. R., and J. McMillan (1986), "Bidding for contract: a principal-agent analysis," *Rand Journal of Economics,* Vol. 17, pp326-338.

[4] McAfee, P. R., and J. McMillan (1992), "Bidding Rings," *American Economic Review,* Vol. 82, pp579-599.

[5] Miura, I, (1997), "Optimal Payment Contract in Public Biddings," *CONTRIBUTION TO PUBLIC POLICY AND ECONOMIC ANALYSIS,* PROCEEDINGS IN 1996, pp47-62.

[6] LaCasse, C., (1995), "Bid Rigging and the Threat of Government Prosecution," *Rand Journal of Economics,* Vol.26, pp398-417.

[7] Riley, J. G., and W. Samuelson (1981), "Optimal Auctions," *American Economic Review,* Vol. 71, pp381-392.

[8] Vicrey, W., (1961), "Counterspeculation, Auctions and Competitive Sealed Tenders," *Journal of Finance,* Vol. 16, pp8-37.

[9] Administrative Inspection Bureau in General Affair Agency, (eds) (1996), "Nyusatsu Keiyaku Seido no Genjo to Kadai," Printing Bureau in Department of Finance, (In Japanese).

〔Associate Professor of Faculty of Economics, Kyushu University〕