Effective Investments under Exclusive Dealing and Common Agency

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Effective Investments under Exclusive Dealing and Common Agency

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1. Introduction

In this article we analyze the difference of contract contents between exclusive dealing and common agency. Under exclusive dealing each principal offers a task to his own agency independently, on the other hand the principals do to a single agency under common agency.

Closer to my present work is Martimort (1996), who shows manufacturer's incentive under exclusive dealing and common agency. In his model, the conclusion depends on whether goods are substitutes or complements. Under exclusive dealing, competition on the market is higher because it increase the equilibrium level of output, and lowers prices up to marginal cost in the case of substitutes. But when goods are substitutes, manufacturers take a common retailer to earn higher profits, so the market outcome condition is inefficient.

We focus on the contract between the manufacturers who produce intermediate goods and a retailer who sells final goods on the downstream market on behalf of the manufacturers. In my model, manufacturers are symmetric about their marginal cost and retailer are also about selling cost of manufacturer's goods, and the manufacturers face as incentive problem in their contract with retailers because retailers observe the cost of selling manufacturer's product (or something about demand for it) that the manufacturers does not observe. We deal with situation in which the manufacturers choose the level of quantity, payment and investment in their contracts. We consider that when common agency is adopted an externality arises from investments made by some manufacturers to a single retailer, and discuss an affect such an externality makes.

Besanko and Perry (1993) provide the affects on investment on a contract and show that; exclusive dealing enhance the incentive to investment but the promotional investments also enhance a competition between manufacturers, thus manufacturers might earn higher profits under common agency that lower promotional investments.

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The remainder of this article is organized as follows. In Section 2 we describe the basic model. Section 3 first derives the complete information symmetric equilibrium and the differentiable equilibrium of the exclusive dealing games under incomplete information. Section 4 analyzes the symmetric equilibrium adoption of common agency by manufactures. Section 5 examines the profits on exclusive dealing and common agency of our model. Finally we summarize and conclude.

2. The model

Consider a contract between manufacturers (principals) and retailers (agents). Manufacturers produce goods and retailers sell a final good on the downstream market on behalf of the manufacturer.

Each agent uses one unit of an intermediate good purchased from his principal to produce one of the final good, and has a piece of private information θ on the final good because of his position on the retailing market. This private information is correlated between agents, and is a random variable drawn from a common knowldege cumulative distribution $F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with destiny function $f(\theta)$. Let us define the hazard rate as $h(\theta)=F(\theta)/f(\theta)$, and we make an assumption that the hazard rate is monotonic as most of the mechanism design does[1].

Assumption 1

$$h'(\theta)=\left(\frac{F}{f}\right)'>0 \quad \textit{for all } \theta\in[\underline{\theta}, \bar{\theta}],$$

$$h'(\theta)=\left(\frac{F}{f}\right)'=1 \quad \textit{for } \theta=\underline{\theta}.$$

We assume that there are two manufacturers, and the marginal cost to make one unit of a good is equal to them with constant value c. In these structure, we analyze the exclusive dealing in which each principal contracts with a different agent and the common agency in that two principals do with a common agent. When exclusive dealing is adopted, a hierarchy (i.e., principal-agent) can not only observe contents of the contract that the other signed but also decide about that in own contract.

After manufacturers choose exclusive dealing or common agency, retailers contracts with manufacturers to carry their goods. Principal i (i=1,2) offers about a wholesale price r_i, a quantity q_i, and an investment t_i as contents of the contract. The investment is a fixed cost by manufacturer that lowers the marginal cost of all goods carried by a retailer who sells goods produced by a principal i. The retailer's marginal cost of selling goods i is equal to $\theta-\hat{t}_i$, where $\hat{t}_i$ represents the impact of an investment by manufacturers on the marginal cost of retailing good i, and θ is a parameter representing on marginal cost without the investment. The impact of manufacturer's investments on marginal cost depends on whether a

1) We assume $f(\theta)>0$ for all θ and $f(\cdot)$ is differentiable with a bounded derivative.

retailer is an exclusive or common agent and is given by

$$\hat{t}_i=\begin{cases} t_i & \text{if the retailer is an exclusive agent of good } i \\ t_i+\delta t_j & \text{if the retailer is a common agent of goods } i,j, \end{cases}$$

where $\delta \in (0,1)$. The scalar δ is a parameter indicating the externality of a manufacturer's investment that impacts upon other good when a retailer carries two kinds of goods. The case in which $\delta=1$, manufacturer's investment has directly the same effects on a rival goods as on the own goods. However we consider the case in which $\delta<1$, in other words, the effects from the other manufacturer's investment is lower than that own manufacturer's investment gives directly. Each manufacturer is assumed to have a quadratic investment cost function $t^2/2w$.

The prices for final goods are assumed to be linear function of both final quantities of q_i and q_j.

$$P_i=a-bq_i-dq_j \ (b>d,\ i\neq j).$$

We make assumptions to guarantee some regularity to our problem:

Assumption 2

$$a>\theta+c \quad \textit{for all } \theta.$$

Assumption 3

$$b+d>2w.$$

3. Exclusive dealing

In this section, we analyze the case that manufacturers adopt exclusive dealing. Under exclusive dealing, the contracts being selected between principal i and agent i can not decide about the contracts between principal j and agent j $(i\neq j)$. Each manufacturer offers a contract to his own retailer a contract independently, we get the following their profit functions;

$$\Pi_i=(r_i-c)q_i-\frac{t_i^2}{2w},$$

$$U_i=(a-bq_i-dq_j)q_j-r_iq_i-(\theta-t_i)q_i.$$

3.1 Complete information

Consider as the benchmark case when the retailer's selling cost is common knowledge, the manufacturers know the retailer's true value θ. Under all contractual arrangements, the agent's payoff must be higher than his reservation payoff, which we normalize to zero:

$$U_i(\theta)=(a-bq_i-dq_j)q_i-r_iq_i-(\theta-t_i)q_i\geq 0. \tag{1}$$

We can write the principal's maximization problem as follows:

$$\max_{q_i, r_i, t_i} \quad (r_i - c)q_i - \frac{t_i^2}{2w}$$

$$\textbf{s.t.} \quad U_i(\theta) \geq 0 \quad for \ \theta \in [\underline{\theta}, \bar{\theta}].$$

For any given incentive scheme (i.e., quantity, investment and whole sale price function) chosen by the other principal, principal i decides a scheme so that the participation constraint (1) binds and induces the agent to choose the output level that maximizes his profit under complete information. We can get the principal's first-order condition as follows;

$$a - 2bq_i - dq_j - \theta + t_i - c = 0, \tag{2}$$

$$q_i - \frac{t_i}{w} = 0. \tag{3}$$

From this equations we can solve for the level of output, investment and wholesale price q^F, t^F, r^F as function of agent's type under complete information for a symmetric equilibrium;

$$q^F(\theta) = \frac{a - \theta - c}{2b + d - w}, \tag{4}$$

$$t^F(\theta) = \frac{(a - \theta - c)w}{2b + d - w}, \tag{5}$$

$$r^F(\theta) = a - \theta - \frac{(a - \theta - c)(b + d - w)}{2b + d - w}. \tag{6}$$

3.2 Incomplete information

With incomplete information the principals offer the agents the incentive scheme that depends on agent's information θ and each agent reports about his information. By the revelation principle, there is no loss of generality in assuming that the incentive scheme selected by the principals is a direct mechanism that induces truth telling. In such a mechanism, the principals choose quantity, investment and wholesale price as function of a single report $\hat{\theta}$ by the agent, subject to the incentive compatibility and participation constraint.

Under exclusive dealing the principals can not observe the level of output selected by the other which makes effects on their own profits in duopoly market. Therefore they have to expect the output level as q^e, that is chosen in the other's contract. Given a truth-telling and direct mechanism, an agent i of type θ who reports his type as $\hat{\theta}_i$ to principal i makes a profit as the follows:

$$U_i(\hat{\theta}_i, \hat{\theta}_j, \theta) = [a - bq_i(\hat{\theta}_i) - dq_j^e(\hat{\theta}_j)]q_i(\hat{\theta}_i) - r_i(\hat{\theta}_i)q_i(\hat{\theta}_i) - [\hat{\theta} - t_i(\hat{\theta}_i)]q_i(\hat{\theta}_i).$$

The agent i reports $\hat{\theta}_i$ to maximize his profits $U_i(\hat{\theta}_i, \hat{\theta}_j, \theta)$, but in direct revelation mechanisms truth telling is the best strategy for each agent, i.e.,

$$\theta \in \arg\max_{\hat{\theta}_i} U_i(\hat{\theta}_i, \theta, \theta). \tag{7}$$

We also restrict the analysis to equilibria for which $q_i(\cdot)$, $r_i(\cdot)$, $t_i(\cdot)$ are continuously differentiable. We can then write the following first-order condition for problem (7):

$$(a - 2bq_i(\theta) - dq_j^e(\theta) - \theta + t_i(\theta) - r_i(\theta))q'_i(\theta) + (t'_i(\theta) - r'_i(\theta))q_i(\theta) = 0. \tag{8}$$

A standard revealer-preference argument shows the indirect profit function $U_i(\theta, \theta, \theta) = U_i(\theta)$ (i.e., information rent made by agent when reporting truthfully), and by the envelope theorem we calculate the derivative of agent i's information rent. We assume that the agent's rent is decreasing with respect to retailing cost θ, and second-order condition is hold[2], i.e.,

$$U'_i(\theta) = -dq^{e\prime}_j(\theta)q_i(\theta) - q_i(\theta) \leq 0, \tag{9}$$

$$\frac{\partial^2 U_i}{\partial \hat{\theta}_i^2} = dq^{e\prime}_j(\theta)q'_i(\theta) - q'_i(\theta) \leq 0, \tag{10}$$

and we will show that after solving this problem and getting the equilibrium.

First we can write a principal i's problem:

$$\max_{q_i, r_i, t_i} \int_{\underline{\theta}}^{\bar{\theta}} \left[r_i(\theta)q_i(\theta) - cq_i(\theta) - \frac{t_i(\theta)^2}{2w} \right] f(\theta)d\theta$$

$$= \max_{q_i, t_i, U_i} \int_{\underline{\theta}}^{\bar{\theta}} \left[(a - bq_i(\theta) - dq_j^e(\theta) - \theta + t_i(\theta) - c)q_i(\theta) - \frac{t_i(\theta)^2}{2w} - U_i(\theta) \right] f(\theta)d\theta$$

$$\text{s.t.} \quad U'_i(\theta) = -(dq^{e\prime}_j(\theta) + 1)q_i(\theta)$$

$$U_i(\theta) \geq 0 \quad \textit{for all } \theta \in [\underline{\theta}, \bar{\theta}]$$

Assuming $U'_i(\theta) \leq 0$, the participation constraint becomes

$$U_i(\bar{\theta}) = 0. \tag{11}$$

Taking into account that $F(\underline{\theta}) = 0$ and (9), we can see

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)f(\theta)d\theta = -\int_{\underline{\theta}}^{\bar{\theta}} U'(\theta)F(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} (dq^{e\prime}_j(\theta)q_i(\theta) + q_i(\theta))F(\theta)d\theta.$$

2) In an equilibrium $\hat{\theta}_i = \theta$, using the first order condition we can write as $\partial^2 U_i / \partial \hat{\theta}_i^2 = \partial^2 U_i / \partial \hat{\theta}_i \partial \theta$.

Then optimizing with respect to q_i, t_i, we find

$$(a-2bq_i-dq_j^e-\theta+t_i-c)f(\theta)-(dq^{e\prime}{}_j+1)F(\theta)=0, \tag{12}$$

$$q_i-\frac{t_i}{w}=0. \tag{13}$$

When seeking principal j's best response, we get similar equations. Lemma 1 below provides a necessary condition satisfied by a symmetric differentiable equilibrium.

Lemma 1 *Under incomplete information a symmetric differentiable equilibrium $q(\theta)$ of exclusive dealing is the solution of the following differential equation;*

$$q'(\theta)=\frac{a-\theta-c-(ab+d-w)q(\theta)-h(\theta)}{d\,h(\theta)}, \tag{14}$$

$$q\,(\underline{\theta})=q^F(\underline{\theta}).$$

The corresponding symmetric investment and wholesale price schedule are given by

$$t(\theta)=wq(\theta), \tag{15}$$

$$r(\theta)=a-(b+d)q(\theta)-\theta+t(\theta)-\frac{1}{q(\theta)}\int_{\theta}^{\bar{\theta}}(dq'(u)+1)q(u)du. \tag{16}$$

proof: (14) and (15) are direct consequence of (12) and (13). Taking into account of $F(\underline{\theta})=0$, (12) becomes

$$(a-2bq_i-dq_i-\theta+t_i-c)f(\theta)=0,$$

for $\theta=\underline{\theta}$. It is equal to equations of (2) that $q^F(\theta)$ satisfies. Then we have $q\,(\underline{\theta})=q^F\,(\underline{\theta})$. When exclusive dealing is adopted a wholesale price is given by

$$r(\theta)=a-(b+d)q(\theta)-\theta+t(\theta)-\frac{1}{q(\theta)}U(\theta),$$

and because of (11)

$$\int_{\theta}^{\bar{\theta}}U'(u)du=[U(u)]_{\theta}^{\bar{\theta}}=-U(\theta),$$

$$-U(\theta)=-\int_{\theta}^{\bar{\theta}}(dq'_j(u)+1)q_i(u)du.$$

Then we obtain (16). Q.E.D.

Proposition 1 *There is a unique solution to (14), and it satisfies*

$$q^P(\theta) \leq q(\theta) \leq q^F(\theta),$$

with both equalities only at $\theta=\underline{\theta}$, and where

$$q^P(\theta)=\frac{a-\theta-c-h(\theta)}{2b+d-w}. \tag{17}$$

This is a necessary-sufficient condition to (9) and (10) in the equilibrium under which the agent's rent is decreasing and the second-order condition of the agent's problem is satisfied.

proof: We first analyze the local behavior of the solution around the singularity $\theta=\underline{\theta}$. From a equations of (14), we obtain as

$$q(\theta)=\frac{a-\theta-c-(dq'(\theta)+1)h(\theta)}{2b+d-w}.$$

Taking into account that $h(\theta)=0$, $h'(\theta)=1$ for $\theta=\underline{\theta}$, we can write the following expression by differentiating above equation.

$$q'(\theta) \to \frac{-2-dq'(\theta)}{2b+d-w}.$$

Then the solution in the neighborhood $\theta=\underline{\theta}$ is given by

$$q'(\underline{\theta}) \to \frac{-2}{2b+d-w}.$$

Let now study the global behavior of the solution to (14). First we prove that $q^P(\theta) < q(\theta)$ for any $\theta > \underline{\theta}$. We have just proved above that this property is locally true around $\theta=\underline{\theta}$, since

$$q^{P\prime}(\theta) \to -\frac{2}{2b+d-w} < q'(\theta) = -\frac{2}{2b+2d-w} < 0.$$

Assume that there exists a lowest value $\theta_1 = min\{\theta; q^P(\theta)=q(\theta)\}$. Then we should have $q'(\theta_1)=0$ because $q(\theta_1)$ satisfies with both of equations (12) and (17).

But using assumption $h'(\theta)=1$ for $\theta=\underline{\theta}$, we can prove that $q^P(\theta)$ is strictly deareasing. Then, we have

$$q(\theta_1-\varepsilon) < q^P(\theta_1-\varepsilon),$$

for small $\varepsilon>0$ which is a contradiction. Then it satisfies $q^P(\theta) < q(\theta)$.

Now we prove that $q(\theta) < q^F(\theta)$ for any $\theta > \underline{\theta}$. This property is also locally true because

$$q'(\theta) = -\frac{2}{2b+d-w} < q^{F\prime}(\theta) = -\frac{1}{2b+2d-w} < 0.$$

Assume now that there exists a lowest value $\theta_2 = min\{\theta; q^F(\theta)=q(\theta)\}$, then

$$q'(\theta_2)=-\frac{1}{d} < q^{F\prime}(\theta_2)=-\frac{1}{2b+2d-w} < 0.$$

because $q(\theta_2)$ satisfies with both of equations (2) and (12). But we check that $q^{F\prime}(\theta)>q'(\theta)$ in the neighborhood of $\theta=\underline{\theta}$. Hence, for sufficiently small $\varepsilon>0$, we could have

$$q^F(\theta_2-\varepsilon) < q(\theta_2-\varepsilon),$$

which is a contradiction. Then it satisfies $q(\theta)<q^F(\theta)$. Q.E.D.

4. Common agency

This section sketches out previous results on competition between manufacturers using a common retailer. Each manufacturer can only write a contract contingent on the sales of his intermediate good. The payoff function of the agent under both contracts is now

$$U=\sum_{i\neq j}[(a-bq_i-dq_j)q_i-r_iq_i-(\theta-t_i-\delta t_j)q_i].$$

We consider two cases that there is a cooperation between the manufacturers and no cooperation below.

4.1 Complete information

Under complete information the participation constraint on agent can be bound as similar to exclusive dealing in 3.1.

(1) cooperative behavior When the manufacturers cooperate, they choose the levels of output, investment and wholesale price q, r, t to maximize their total profits Π which is given by

$$\Pi=\Pi_i+\Pi_j=\sum_{i\neq j}\left[(a-bq_i-dq_j-\theta+t_i+\delta t_j-c)q_i-\frac{t_i^2}{2w}\right].$$

We can write the first-order conditions as

$$a-2bq_i-2dq_j-\theta+t_i+\delta t_j-c=0, \tag{18}$$

$$q_i+\delta q_j-\frac{t_i}{w}=0, \tag{19}$$

from which we can solve for symmetric equilibrium under the complete information, $q_c^F(\theta)$, $t_c^F(\theta)$, $r_c^F(\theta)$ as follows;

$$q_c^F(\theta)=\frac{a-\theta-c}{2(b+d)-(1+\delta)^2w}, \tag{20}$$

$$t_c^F(\theta)=\frac{(1+\delta)(a-\theta-c)}{2(b+d)-(1+\delta)^2w}, \tag{21}$$

$$r_c^F(\theta)=a-\theta-\frac{(a-\theta-c)(b+d-(1+\delta)^2w)}{2(b+d)-(1+\delta)^2w}. \tag{22}$$

(2) noncooperative behavior In the absence of a cooperation between the manufacturers, each manufacturer decides to maximize his own surplus. The principal i's profit function is given by

$$\Pi_i=\sum_{i\neq j}(a-bq_i-dq_j-\theta+t_i+\delta t_j)q_i-r_j t_j-cq_i-\frac{t_i^2}{2w},$$

from which we can solve for symmetric equilibrium $q_c^F(\theta), t_c^F(\theta), r_c^F(\theta)$ under the complete information. We can write the two principal's first-order conditions as

$$a-2bq_i-2dq_j-\theta+t_i+\delta t_j-c=0,$$

$$q_i+\delta q_j+\frac{t_i}{w}=0.$$

These conditions are equal to the equations of (18), (19) and from which, it is easy check that the levels of output, investment and wholesale price $q_c^F(\theta)$, $t_c^F(\theta)$, $r_c^F(\theta)$ are same as above are chosen.

4.2 Incomplete information

Let us consider the contracts under incomplete information. In this case agent's true information θ is not known by the manufacturers θ offer $q(\hat{\theta}), t(\hat{\theta}), r(\hat{\theta})$ as function of the agent's report θ. A common retailer makes a single report to the manufactures and they will receive the same value $\hat{\theta}$ if there exists a cooperation between the manufacturers, but they possibly get the different report from a agent because they contract with him independently in no cooperation.

We are required to a warning before proceeding to analyze the contracts under incomplete information. In the case of a common agency we cannot appeal to the revelation principle, because the available version of it concern only the case of a single principal. Thus, as Mezzetti (1997) notes it is an open question whether there is any loss of generality in assuming that the two principals use direct revelation mechanisms. We assume the revelation principale be adapted to this article that models an environment with two principals as most of the mechanism design literature does.

(1) cooperative behavior Under cooperative behavior there is no loss of generality in assuming that the incentive scheme selected by the manufacturers is a direct mechanism that induces truth telling. In such a mechanism, the manufacturers choose output level, investment and wholesale price as function of a single report $\hat{\theta}$. Given a truth-telling mechanism, an agent of type θ that reports $\hat{\theta}$ makes a profit $U(\hat{\theta}, \theta)$:

$$U(\hat{\theta}, \theta)=\sum_{i\neq j}(a-bq_i(\hat{\theta})-dq_j(\hat{\theta})-\theta+t_i(\hat{\theta})+\delta t_j(\hat{\theta})-r_i(\hat{\theta}))q_i(\hat{\theta}).$$

We can write the principal's problem to maximize their joint payoff as follows:

$$\max_{q_i, t_i, q_j, t_j, U} \int_{\underline{\theta}}^{\bar{\theta}} \left[\sum_{i \neq j} (a - bq_i(\theta) - dq_j(\theta) - \theta + t_i(\theta) + \delta t_j(\hat{\theta}) - c) q_i(\theta) - \frac{t_i(\theta)}{2w} - U(\theta) \right] f(\theta) d\theta$$

$$\text{s.t.} \quad U'(\theta) = -q_i(\theta) - q_j(\theta)$$

$$U(\theta) \leq 0 \qquad \textit{for all } \theta \in [\underline{\theta}, \bar{\theta}]$$

Assuming that quantities q_i, q_j are nonnegative, agent's rent is decreasing, i.e., $U'(\theta) \leq 0$ so participation constraint becomes $U(\bar{\theta}) = 0$. The profit function of the agent is

$$U(\theta) = \int_{\theta}^{\bar{\theta}} (q_i(,u) + q_j(u)) du,$$

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) f(\theta) d\theta = -\int_{\underline{\theta}}^{\bar{\theta}} U'(\theta) F(\theta) d\theta = \int_{\underline{\theta}}^{\bar{\theta}} (q_i(\theta) + q_j(\theta)) F(\theta) d\theta,$$

taking into account that $F(\underline{\theta}) = 0$. We can write the principal's first-order codition as

$$(a - 2bq_i - dq_j - \theta + t_i + \delta t_j - c) f(\theta) - F(\theta) = 0, \tag{23}$$

$$\left(q_i + \delta q_j - \frac{t_i}{w} \right) f(\theta) = 0. \tag{24}$$

Then under cooperation between the manufacturers, the optimal functions are given by

$$q_c^C(\theta) = \frac{a - \theta - c - F(\theta)/f(\theta)}{2(b+d) - (1+\delta)^2 w}, \tag{25}$$

$$t_c^C(\theta) = \frac{(1+\delta)(a - \theta - c - F(\theta)/f(\theta)) w}{2(b+d) - (1+\delta)^2 w}, \tag{26}$$

$$r_c^C(\theta) = a - (b+d) q_c^C(\theta) - \theta + (1+\delta) t_c^C - \frac{1}{q_c^C(\theta)} \int_{\theta}^{\bar{\theta}} q_c^C(u) du. \tag{27}$$

(2) noncooperative behavior　We now analyze the case that the manufacturers independently offer incentive scheme to the common retailer. Each manufacturer observes the agent's report only at his own task, and incentive scheme offered cannot contingent on the level of the output in the other manufacturer's task.

Let $\hat{\theta}_i$ be the type reported to principal i and $\hat{\theta}_j$ be the one reported to principal j. Given the mechanisms chosen by principal i and j respectively, an agent of type θ makes a profit $U(\hat{\theta}_i, \hat{\theta}_j, \theta)$, where

$$U(\hat{\theta}_i, \hat{\theta}_j, \theta) = \sum_{i \neq j} (a - bq_i(\hat{\theta}_i) - dq_j(\hat{\theta}_j) - \theta + t_i(\hat{\theta}_i) + \delta t_j(\hat{\theta}_j) - r_i(\hat{\theta}_i)) q_i(\hat{\theta}_i).$$

Under common agency there exist the externalities between the contracting activities of the two principals. These externalities arise because the contract offered by one principal influences the other principal's contract in the case of duopoly market. This implies that the agent's report to a principal depends on the mechanism chosen by the other principal. In other words, principal i is indirectly affected by the agent's report to principal j, because the level of output in principal j's task is determined by such a report. Taking into account this strategic interaction, each principal finds the agent's future optimal report to the other principal as a function of his own choice of a mechanism when choosing own mechanism. Formally, given principal j's choice of a mechanism, principal i recognizes that the agent's optimal report to principal j, $\hat{\theta}_j$ is a function of his own choice of output, investment and wholesale price $\hat{\theta}_j=\hat{\theta}_j(q_i, r_i, t_i)$, and similarly to the principal i.

The following lemma notes the local affect of a change in a principal's mechanism on the agent's optimal report to the other principal.

Lemma 2 In equilibrium, for any θ those equations are given

$$\frac{\partial \hat{\theta}_j}{\partial q_i}=\frac{2dq'(\theta)-\delta t'(\theta)}{(2dq'(\theta)-2\delta t'(\theta)+1)q'(\theta)},$$

$$\frac{\partial \hat{\theta}_j}{\partial t_i}=-\frac{d}{2dq'(\theta)-2\delta t'(\theta)+1}.$$

proof: For any given pair of direct mechanisms chosen by the principals, the agent reports $\hat{\theta}_i$, $\hat{\theta}_j$ which maximizes $U(\hat{\theta}_i, \hat{\theta}_j, \theta)$. The first-order condition is

$$\frac{\partial U}{\partial \hat{\theta}_j}=(a-2bq_j-2dq_i-\theta+t_j+\delta t_i-r_j)q'_j+(t'_j-r'_j)q_j+\delta t'_j q_i=0. \tag{28}$$

To evaluate the affect on $\hat{\theta}_j$ of a change in q_i, r_i, t_i, we totally differentiate the first-order condition with respect to q_i, r_i, t_i, and $\hat{\theta}_j$:

$$\frac{\partial^2 U}{\partial \hat{\theta}_j^2}d\hat{\theta}_j+(-2dq'_j+\delta t'_j)dq_i+\delta q'_j dt_i+0dr_i=0. \tag{29}$$

In equilibrium the first-order condition must hold for $\hat{\theta}_j=\hat{\theta}_i=\theta$, and totally differentiating (28) with respect to $\hat{\theta}_j=\hat{\theta}_i=\theta$ yields

$$\left(\frac{\partial^2 U}{\partial \hat{\theta}_j^2}-2dq'_i q'_j+\delta t'_i q'_j+\delta t'_j q'_i-q'_j\right)d\hat{\theta}_j=0. \tag{30}$$

Then from equations of (29) and (30) we can write as

$$(2dq'_i q'_j-\delta t'_i q'_j-\delta t'_j q'_i+q'_j)d\hat{\theta}_i+(-2dq'_j+\delta t'_j)dq_i+\delta q'_j dt_i=0,$$

from which, equations of lemma are obtained. Q.E.D.

We can write principal i's maximization problem under independent contracting as

$$\max_{q_i, t_i, U} \int_{\underline{\theta}}^{\bar{\theta}} \left[\sum_{i \neq j} (a - bq_i(\theta) - dq_j(\hat{\theta}_j) - \theta + t_i(\theta) + \delta t_j(\hat{\theta}_j)) q_i(\theta) - r_j(\hat{\theta}_j) q_j(\hat{\theta}_j) - U(\theta) - cq_i(\theta) - \frac{t(\theta)^2}{2w} \right] f(\theta) d\theta,$$

$$\textbf{s.t.} \quad U'(\theta) = -q_i(\theta) - q_j(\theta)$$

$$\hat{\theta}_j \in \arg\max_{\tilde{\theta}_j} U(\theta, \tilde{\theta}_j, \theta)$$

$$U(\theta) \geq 0 \qquad \textit{for all } \theta \in [\underline{\theta}, \bar{\theta}]$$

The incentive scheme says that it must be optimal for the agent to report his true type to both of principal *i, j*, and the optimal incentive mechanisms of the two principals must satisfy the following first -order conditions;

$$(a - 2bq_i - dq_j - \theta + t_i + \delta t_j - c) f(\theta) - \left(1 + q'_j \frac{\partial \hat{\theta}_i}{\partial q_i} \right) F(\theta) = 0, \tag{31}$$

$$\left(q_{ic} + \delta q_{jc} - \frac{t_i}{w} \right) f(\theta) - q'_j \frac{\partial \hat{\theta}_i}{\partial t_i} F(\theta) = 0. \tag{32}$$

Under independent contracting the symmetric equilibrium level of output and investment $q_c(\theta)$, $t_c(\theta)$ are solution to the following differential equation;

$$q'(\theta) = \frac{1}{d} \cdot \frac{(1+\delta)q - \frac{1}{w}t}{2((1+\delta) - 2(b+d))q + 2\left((1+\delta) - \frac{1}{w} \right) t - 5\theta + 2(a-c) + 3\underline{\theta}}, \tag{33}$$

$$t'(\theta) = \frac{1}{\delta} \cdot \frac{2((1+\delta) + (b+d))q - \left((1+\delta) + \frac{2}{w} \right) t - (a - c - 3\underline{\theta})}{2((1+\delta) - 2(b+d))q + 2\left((1+\delta) - \frac{1}{w} \right) t - 5\theta + 2(a-c) + 3\underline{\theta}}, \tag{34}$$

$$r_c(\theta) = a - (b+d) q_c(\theta) - \theta + (1+\delta) t_c(\theta) - \frac{1}{q_c(\theta)} \int_{\theta}^{\bar{\theta}} q_c(u) du. \tag{35}$$

5. Common agency versus exclusive dealing

In this section we compare the two following organizational choices. Firstly manufacturers have a common agent to sell both of their final goods on their behalf. Secondly each chooses an exclusive agent.

We first derive the analysis of the symmetric differentiable equilibrium with complete information. The difference in the level of output and investment between under common agency and exclusive dealing are given by;

$$q^F(\theta)-q_c^F(\theta)=(a-\theta-c)\frac{d-w(\delta^2+2\delta)}{(2b+d-q)(2(b+d)-(1+\delta)^2w)},$$

$$t^F(\theta)-t_c^F(\theta)=(a-\theta-c)\frac{d-w\delta^2-(2b+d+w)\delta}{(2b+d-q)(2(b+d)-(1+\delta)^2w)},$$

then the followings are obtained;

$$\begin{cases} q^F>q_c^F \text{ if } d>d_1(\delta) \\ q^F<q_c^F \text{ if } d<d_1(\delta), \end{cases} \qquad \begin{cases} t^F>t_c^F \text{ if } d>d_2(\delta) \\ t^F<t_c^F \text{ if } d<d_2(\delta), \end{cases}$$

$$\text{where } d_1(\delta)=(\delta^2+2\delta)w,\ d_2(\delta)=\frac{w\delta^2+(2b-w)\delta}{1-\delta}.$$

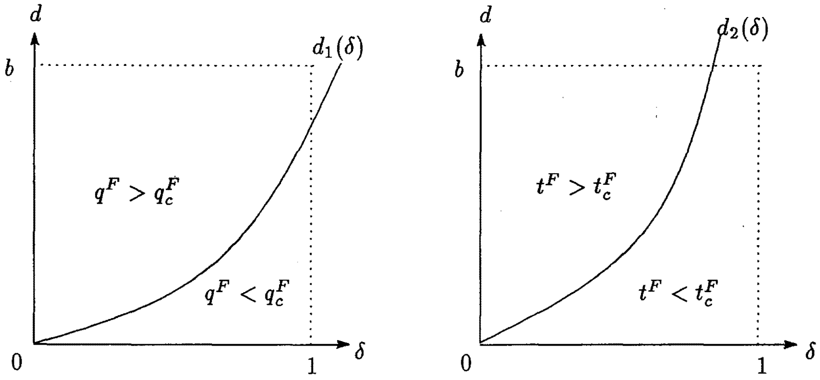

Figure 1

When δ and the product difference are large (i.e., $d \to 0$) $q_c^F(\theta)$ is higher than $q^F(\theta)$, and when δ and the product difference is small (i.e., $d \to b$) $q^F(\theta)$ is higher as figure 1. As to the investment, the similarity is held because the level of investment is proportional to the output level.

Moreover, with complete information the agent's rent is zero and the difference in the level of wholesale price and principal's profit are given by

$$r_c^F(\theta)-r^F(\theta)=(a-\theta-c)\frac{d(b+d-w)+bw(\delta^2+2\delta)}{(2b+d-q)(2(b+d)-(1+\delta)^2w)}>0,$$

$$\Pi_c^F(\theta)-\Pi^F(\theta)=(a-\theta-c)\frac{d^2+w(2b-w)(\delta^2+2\delta)}{2(2b+d-q)^2(2(b+d)-(1+\delta)^2w)}>0.$$

Under common agency both of the level of wholesale price and principal's profit are higher than under exclusive dealing. Why the levels of the price and the profit under common agency are higher? With complete information and under common agency we have the same outcome as above whether the manufacturers cooperate or not. We focus our mind on the case that the manufacturers cooperate, the case under common agency can be considered similar to a monopoly market. Then we can note that common agency dominates exclusive dealing where there are competitions between two manufacturers.

Proposition 2 *With complete information, the level of output and investment are higher under common agency when the externality and the product differentiation are large, and that under exclusive dealing are higher when the externality and the product differentiation are small. Under common agency the principal's payoff is higher than under exclusive dealing and the agent earns no rent.*

Next with incomplete information we formulate that for θ drawn from the uniform distribution, the symmetric equilibrium under common agency with cooperation between manufacturers and exclusive dealing are given by

$$q(\theta)=\frac{a-\underline{\theta}-c}{2b+d-w}-\frac{2(\theta-\underline{\theta})}{2(b+d)-w},$$

$$q_c^C(\theta)=\frac{a-\underline{\theta}-c}{2(b+d)-(1+\delta)^2w}-\frac{2(\theta-\underline{\theta})}{2(b+d)-(1+\delta)^2w}.$$

Then the difference in the level of output and investment are as follows;

$$q(\theta)-q_c^C(\theta)=q^F(\theta)-q_c^F(\theta)+(a-\theta-c)\frac{2w(\delta^2+2\delta)}{(2(b+d)-w)(2(b+d)-(1+\delta)^2w)}(\theta-\underline{\theta}),$$

$$t(\theta)-t_c^C(\theta)=t^F(\theta)-t_c^F(\theta)+\frac{2w\delta^2(2(b+d)+(1+\delta)w)}{(2b+d-w)(2(b+d)-(1+\delta)^2w)}(\theta-\underline{\theta}).$$

With incomplete information the differences are similar to the case of complete information, but the range of $q(\theta)\geq q_c^C(\theta)$, $t(\theta)\geq t_c^C(\theta)$ is larger than with complete information.

We can get $E(\theta)=(\bar{\theta}+\underline{\theta})/2$ when the uniform distribution is assumed, the expected profits of agent that earns from a single task are given by

$$EU(\theta)=\frac{\Delta\theta}{2}\left(\frac{a-\underline{\theta}-c}{2b+d-w}-\frac{\Delta\theta-4\underline{\theta}}{2(2(b+d)-w)}\right)\frac{2b-w}{2(b+d)-w},$$

$$EU_c^C(\theta)=\frac{\Delta\theta}{2}\left(\frac{a-\underline{\theta}-c}{2(b+d)-(1+\delta)^2w}-\frac{\Delta\theta-4\underline{\theta}}{2(2(b+d)-(1+\delta)^2w)}\right),$$

where $\Delta\theta=\bar{\theta}-\underline{\theta}$. Moreover, the principal's expected profit under exclusive dealing and common agency are given by

$$E\Pi(\theta)=\frac{a-\underline{\theta}-c}{2}\left(\frac{a-\underline{\theta}-c}{2b+d-w}-\frac{\Delta\theta}{2(b+d)-w}\right)\frac{2b-w}{2b+d-w}-EU(\theta),$$

$$E\Pi_c^C(\theta)=\frac{a-\underline{\theta}-c}{2}\left(\frac{a-\underline{\theta}-c}{2(b+d)-(1+\delta)^2w}-\frac{\Delta\theta}{2(b+d)-(1+\delta)^2w}\right)-EU_c^C(\theta).$$

From above equations, the difference of $U(\theta)$ and $U_c^C(\theta)$ is given by

$$\Delta U = EU_c^C(\theta) - EU(\theta) = \frac{\Delta\theta}{2}\left[(a-\underline{\theta}-c)H_2 - \frac{\Delta\theta - 4\underline{\theta}}{2}H_3\right],$$

and the difference of $E\Pi$ and $E\Pi_c$ is given by

$$\Delta\Pi = E\Pi_c^C(\theta) - E\Pi(\theta) = \frac{(a-\underline{\theta}-c)^2}{2}H_1 - (a-\underline{\theta}-c)\Delta\theta H_2 + \frac{(\Delta\theta - 4\underline{\theta})\Delta\theta}{4}H_3,$$

where

$$H_1 = \frac{(2b-w)((1+\delta)^2-1)w + d^2}{(2(b+d)-(1+\delta)^2 w)(2b+d-w)^2},$$

$$H_2 = \frac{(2b-w)((1+\delta)^2-1)w + d(2(b+d)-w)}{(2(b+d)-(1+\delta)^2 w)(2b+d-w)(2(b+d)-w)},$$

$$H_3 = \frac{(2b-w)((1+\delta)^2-1)w + 2d(2(b+d)-w)}{(2(b+d)-(1+\delta)^2 w)(2(b+d)-w)^2},$$

and $H_1 < H_2 < H_3$.

We can get

$$\Delta U > \frac{(a-\underline{\theta}-c)\Delta\theta}{4}(2H_2 - H_3) > 0,$$

from which,we find agent's rent under common agency are always higher than under exclusive dealing[3].

With incomplete information the difference of the principal's expected profit under exclusive dealing and common agency is $\Delta\Pi > 0$ when $\Delta\theta$ is small enough, but depends on d and δ when $\Delta\theta$ is largre. From the figure 2, we find $\Delta\Pi < 0$ when $\Delta\theta$ is large enough and $\Delta\theta \to a-\underline{\theta}-c$. When $\Delta\theta$ is a middle value, $\Delta\Pi > 0$ with small d and large δ, and then $\Delta\Pi < 0$ with large d and small δ.

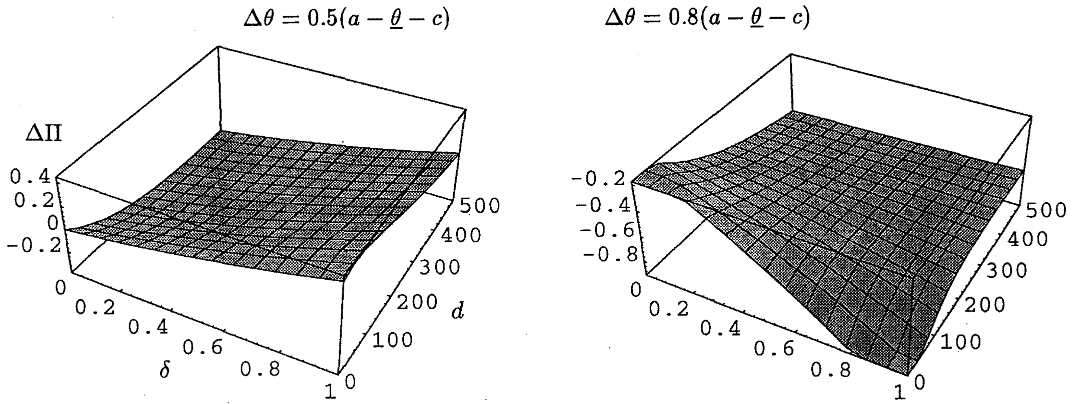

Figure 2

3) We assume $a > \theta + c$ for all θ, then $a - \underline{\theta} - c > \Delta\theta$ holds.

From the above, when $\Delta\theta$ is a middle value we can describe about the principal's expected profits as

- common agency dominates exclusive dealing when the product differentiation and the externality of the investment are large enough,
- exclusive dealing dominates common agency when the product differentiationthe and externality of the investment are small enough.

Proposition 3 *With incomplete information, similar to the case of complete information the level of output and investment are higher under common agency when the externality is large, and higher under exclusive dealing when the product differentiation is small. The range of that the level of under common agency is smaller than with complete information.*

Proposition 4 *With incomplete information the agent's rent under common agency is always higher than under exclusive dealing. The principal's payoff under the common agency dominates when the distribution of agent's information is small or the product differentiation and the externality of the investment are large enough, and under the exclusive dealing dominates when the distribution of agent's information is large or the product differentiation and the externality of the investment are small enough.*

6. Conclusion

This article has attempted to analyze differences in contracts between exclusive dealing and common agency when the manufacturers choose the level of investments in addition to that of output and payment.

My analysis suggests that the level of investment does not depends on the difference of whether exclusive dealing or common agency but the level of output. When the output level increases, the level of investment also increases under common agency in which externalities arise, and becomes higher than that under exclusive dealing. In the case that the manufacturers adopt common agency there is an externality from investment, the increase of which makes the principal's payoff. Under common agency the principals always earn higher profits than under exclusive dealing with complete information. In contast, the principal's payoff under exclusive dealing becomes higher than under common agency with incomplete information when the distribution of agent's private information is large and the externality of investment is small enough.

Several existing works have already compared both of exclusive dealing and common agency. In our model we analyze the difference in the situation that the two principals and agents are assumed to be symmetric. The future direction of this study will be extending model to the case that they are not symmetric and each agent has different information independently.

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