

飛来物の高速衝突を受けるコンクリート版の衝撃貫通挙動に関する解析的研究

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An Analytical Study on the
Penetrating Behavior of
Concrete Slab Subjected to
High Velocity Projectiles

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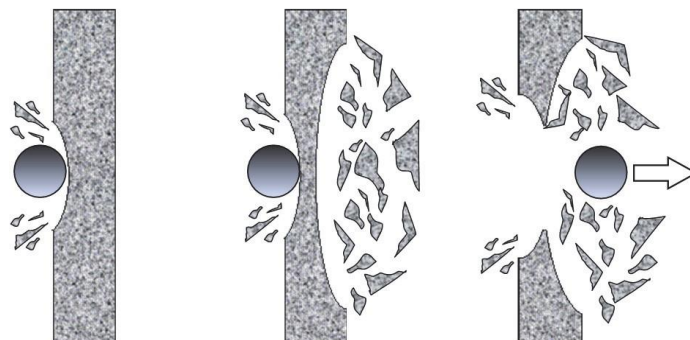
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Chapter 1 Introduction

1.1 background

In recent years, to prevent serious damage from natural disasters such as tornados and volcanic eruptions, the demand for protecting structures against the collision of flying objects is recognized in Japan, and it is important to predict the impact performance (penetration prevention) of these structures. Not only in Japan, the damage done by natural disasters are not a rare case in many oversea countries. U.S. Nuclear Regulatory Commission has already established design code considering the collision of flying objects towards nuclear facilities¹⁾. As a container for nuclear fuels and structures protecting the security system for the nuclear plant, the durability against such disasters is required. On the other hand, to give an example for volcano eruption, the Mt. Ontake erupted on September 27th, 2014. Car accidents were caused by the high-velocity rocks erupted, which raised focus on the necessity of rock sheds. Along with these natural disasters such as tornados and volcano eruptions, structures and sheds are collided with high-velocity rocks and other projectiles, resulting in severe damage. Fig 1.1 shows many damage pattern caused by high-velocity projectiles, and researches aiming the impact phenomena has already started in the 1930s. From these researches, it is also known that there is a possibility of secondary disaster after the primary damage.



(a) surface damage (b) spalling (c) perforation

Fig 1.1 damage pattern of concrete slab

Generally, for the slight plastic deformation of structures, it is confirmed that the prediction can be easily done by universal FEM software. While for the large deformation relating to high-velocity projectiles and penetrations, the prediction is difficult to be conducted, the accuracy of the analysis result is not yet validated, and the demand for the complex design of the structures against such penetration and local damage is growing. On the other hand, the experimental research conducting on the RC slabs for the evaluation of penetration performance consumes lots of time to prepare, and faces the financial problem constraining the actual experiment scales and trial numbers. Based on the given background, it is important to build up a meshless analytical method evaluating the penetration performance of RC concrete slabs.

1.2 Related works

In order to comprehend the current research progress relating to the impact response analysis of the structures in Japan and other countries, here we summarized the researches relating to the penetration and local damage experiments conducted on concrete, the empirical formula, and the analytical researches.

For the existing studies on the local failure prevention measure, Beppu et al.²⁾ performed collision with a mushroom-shaped projectile on concrete plates. Kojima³⁾ conducted a set of tests with various RC slab targets and projectiles, inferring that the local damage of RC slab is the most related to the hardness of projectile's nose and more reinforcement would reduce local damage against hard-nose projectile. M.H Zhang et al.⁴⁾ pointed out that the perforation depth and crater diameter of high-strength concrete due to collision can be reduced by the increase of compressive strength and the presence of coarse granite aggregates.

Formulae have been developed based on experimental results, to predict the perforation depth and the critical slab thickness for perforation and scabbing of reinforced concrete to occur. Kennedy⁵⁾ compared and reviewed formulae of modified Petry, ACE (Army Corps of Engineers), BRL (Ballistic Research Laboratory), modified NDRC (National Defense Research Committee) and Ammann and Whitney and the Ballistic Research Laboratory. There are also formulae proposed by Hughes⁶⁾ to evaluate the perforation depth, critical slab thickness for scabbing, and critical slab thickness against perforation.

For existing studies, the perforation limit of a concrete slab has been examined by experiments, while impact tests were usually performed by small scale specimens due to the limitation of experimental conditions. Furthermore, it is still difficult to predict the accurate local failure of the concrete structure. The use of numerical analysis is a powerful alternative to study impact phenomena in term of scale and cost. In general, FEM is not suitable to simulate the discontinuous displacement field such as perforation process. Therefore, Smoothed Particle Hydrodynamics (SPH) as one of the particle methods that can reproduce local failure of concrete structural members such as crushing or perforation is utilized. SPH is a particle-based Lagrangian method for solving systems of partial differential equations. SPH was originally invented for astrophysical applications^{7,8)}. Since its invention, it has been successfully applied to a vast range of problems such as the dynamic response of material strength^{9,10)}, fluid flow¹¹⁾, etc.

1.3 Objective and composition

1.3.1 Objective

In this study, based on the given background, SPH method, which is one of the meshless analytical methods and suitable for problems relating to the large deformation and damage of solid body, is adopted. Then a mechanical model for the penetration phenomenon is built, and the results of the experiment and the SPH analysis is compared to validate the proposed analysis method. By using the proposed analysis method, the influence of different projectile and impact velocity on the penetration response of concrete slab and the impulse consumed is investigated. Finally, an evaluation method for the penetration/perforation damage of the concrete slab based on the impulse calculation is proposed.

1.3.2 Composition

This paper is composed of 8 chapters. The outline of each chapter is described below.

In *Chapter 1, Introduction*, the background and purpose of this study, and the previous studies, the importance and problems of performance evaluation for penetration prevention are described, and the structure of this paper is shown.

In *Chapter 2, Theory of SPH Method*, the basic concept of SPH (Smoothed Particle Hydrodynamics) method is explained, which is the main analysis method of this research.

In *Chapter 3, Application of SPH in numerical analysis*, the procedure of analysis using SPH method in this study is described, and the contact algorithm is also described.

In *Chapter 4, Material constitutive law in impact analysis*, a mechanical model of concrete material used for impact analysis with penetration is described.

In *Chapter 5, Verification analysis for the proposed SPH impact analysis method*, the validity and the accuracy of the proposed SPH analysis method is verified by comparing analysis results with a falling weight experiment on RC beam and a perforation experiment on concrete slab.

In *Chapter 6, Discussion on penetration fracture by SPH method*, a concrete slab is taken as the analysis object, and the damage of the projectile done to this concrete slab is discussed based on numerical analysis results.

In *Chapter 7, Proposal of evaluation method of penetration damage*, an impulse equation is established to predicted the consumed impulse of the projectile during penetration process.

In *Chapter 8, Conclusions*, the results obtained in this study are summarized and the tasks needed in the future are described.

Chapter 2 Theory of SPH

The SPH method was proposed by Lucy and Monaghan et al. In the late 1970s^{1,2)}, and is an analysis method based on particle representation based on the compressed fluid theory. Basically, hydrodynamic problems are solved in the form of differential equations of the deviation of variables such as density, velocity, energy, etc. Since then, it has been extensively studied and applied to astrophysical problems in three-dimensional space, dynamic response including material strength, and impact analysis of solids with large deformation. In this chapter, the SPH method will be described first, followed by a description of the SPH method applied to general solid problems.

2.1 Concept of SPH

The SPH method is a particle-based mesh-free method, and, like other mesh-free particles, the computational framework is not a grid cell in the finite difference method nor a mesh element in the finite element method, but particles in a 3D space with no need to pre-define connectivity between them. In SPH, the particle approximation described in Section 2.2 is performed for particles in the smoothing length at each time step. However, since it is almost impossible to solve a set of partial differential equations analytically except in very rare cases, as the solution procedure, the problem domain defined by the partial differential equations is first discretized, then method to approximate the region function and its derivative value at random point is needed. Therefore, approximate equations are applied to partial differential equations to create discretized ordinary differential equations that depend only on time increment. This discretized ordinary differential equation can be solved using time integration such as the finite difference method. In order to execute such a solution, the SPH method is performed as follows.

1. First, the problem area is represented by arbitrarily distributed particles. There is no need for binding to these particles. (Mesh free)
2. An integral representation is used to approximate the domain function. This is called kernel approximation in the SPH method. (Integration function display)

3. The kernel approximation is performed using particles. This is called particle approximation in SPH, which is to replace the integral representation of the region function and its derivative with the sum of all the corresponding values of the particles within the local region called the supporting domain. (Compact support)
4. Since particle approximation is performed at each time step, the use of particles depends on the local distribution of particles. (Adaptation)
5. Particle approximation is performed on all terms related to the domain function of the partial differential equation to create a discretized ODE that depends only on time. (Lagrange)
6. Ordinary differential equations are solved using explicit integration to speed up calculations and obtain time history of field variables for all particles. (Dynamic)

Step 1 stipulates the mesh-free properties of the SPH method. It is not difficult to formulate a numerical method without using a mesh, but when Neumann boundary conditions are used for irregular nodes and particles in the region, it is important to ensure the stability of the numerical method.

Step 2 provides the stability necessary for the SPH method mathematically because the integral representation is a weak form with smoothing effect. As long as the numerical integration is done correctly, the weak form is often very stable.

Step 3 creates discretized banded and sparse matrices. Problems with large deformations require a large number of particles to represent the problem area, and it takes a very long time to solve a huge amount of equations. At this point of view, this is a very important point in terms of computational effects.

By performing Step 4, SPH adaptability can be obtained at each time step based on any particles distributed within the smoothing length. Since this adaptable SPH approximation is performed early in the field variable approximation, the SPH formulation is not affected by any time-varying particle partitioning and therefore can handle very large deformation problems. It should be noted that this particle approximation is a method of numerical integration required for Step 2. Also, enough particles must be used to guarantee the accuracy and numerical stability of the integration.

In Step 5, the use of Lagrange will give the SPH method all the features of the Lagrange method.

Step 6 is the standard time integral methods for solving dynamic problems. What the SPH method should do is to determine an appropriate method for determining the time step to guarantee a stable

time integration.

Thus, these six steps guarantee the SPH method as mesh-free, adaptability, stability, and Lagrangian solutions to dynamic problems.

The SPH method can be applied from very small scales to very large scales, from astrophysics to fluid and solid mechanics problems, as well as very irregular and uniform dynamic shape problems. It is believed that deformation problems and impact problems can be easily dealt with. SPH can sometimes be affected by numerical problems such as inaccurate boundaries and tension instability, which can lead to large errors in these situations, but the accuracy can be improved by various corrections. Thus it can be said that SPH is a very versatile method.

2.2 Formulation of SPH

The SPH approximation includes continuous integral representation (kernel approximation) and arbitrary particle approximation, which are the principles of the SPH method. Since this involves complex partial differential equations, the SPH formulation can be divided into two stages. The first step is an integral representation of the field function, or kernel approximation, and the second step is particle approximation. Here we explain these two stages.

2.2.1 Integral representation of functions

In the first step, the integral of the product of the arbitrary function and the kernel function gives a kernel approximation to the integral representation of the function. The integral representation of the function can be approximated by summing the values at the interpolation points. This gives a particle approximation of the function with discrete points and particles. The concept of integral representation of the function $f(x)$ used in the SPH method is expressed by equation (2.2.1).

$$f(x) = \int_{\Omega} f(x') \delta(x - x') dx' \quad (2.2.1)$$

Here f is the function of position vector x , and $\delta(x - x')$ is Dirac delta function, with definitions shown below

$$\delta(x - x') = \begin{cases} 1 & x = x' \\ 0 & x \neq x' \end{cases} \quad (2.2.2)$$

In equation (2.2.1), Ω is the integration range including x . Equation (2.2.1) means that the function

can be expressed in the form of an integral. Since the Dirac delta function is used, as long as $f(x)$ is defined continuously in Ω , the integral representation of Equation (2.2.1) holds. If delta function $\delta(x - x')$ is replaced with the kernel function $W(x - x', h)$, the integral representation of (2.2.1) is given by

$$f(x) \approx \int_{\Omega} f(x')W(x - x', h)dx' \quad (2.2.3)$$

Here, h is the smoothing length defined as the influence range of the kernel function W . As long as W is not a Dirac function, the integral representation of (2.2.3) can only be an approximation. This is the starting point of the kernel approximation. Equation (2.2.3) may be expressed as the following equation (2.2.4).

$$\langle f(x) \rangle = \int_{\Omega} f(x')W(x - x', h)dx' \quad (2.2.4)$$

The kernel function W is generally an even function, but there are some conditions that must be met³⁾. The first condition is that it is standardized.

$$\int_{\Omega} W(x - x', h)dx' = 1 \quad (2.2.5)$$

This condition is called the unity condition because it is the integral of the kernel function that produces 1. The second condition is the characteristic of the delta function can be found when the smoothing length approaches zero.

$$\lim_{h \rightarrow 0} W(x - x', h) = \delta(x - x') \quad (2.2.6)$$

The third condition is compact condition

$$W(x - x', h) = 0 \quad \text{when} \quad |x - x'| > \kappa h \quad (2.2.7)$$

where κ has a certain relationship with the kernel function at the point x and is defined as a valid (non-zero) region of the kernel function. This effective region is called the support domain of the kernel function at point x . Also, by using the compact condition, the integration in the problem domain concentrates on the integration in the support domain of the kernel function. Therefore, the integration region Ω can be treated in the same way as the support domain. Equation (2.2.7) shows that the support domain of the kernel function is $|x - x'| \leq \kappa h$.

The error of SPH integral representation can be estimated by using the Taylor series expansion of the function around the point. Here $f(x)$ is a differentiable function. From equation (2.2.4)

$$\begin{aligned}\langle f(x) \rangle &= \int_{\Omega} [f(x) + f'(x)(x'-x) + r((x'-x)^2)] W(x-x', h) dx' \\ &= f(x) \int_{\Omega} W(x-x', h) dx' + f'(x) \int_{\Omega} (x'-x) W(x-x', h) dx' + r(h^2)\end{aligned}\quad (2.2.8)$$

where r means the remainder. Since W is an even function depending on x , $(x'-x)W(x-x', h)$ becomes an odd function. Therefore, the following equation (2.2.9) holds.

$$\int_{\Omega} (x-x') W(x-x', h) dx' = 0 \quad (2.2.9)$$

Substituting Equation (2.2.5) and Equation (2.2.9) into Equation (2.2.8)

$$\langle f(x) \rangle = f(x) + r(h^2) \quad (2.2.10)$$

From the above equation, it can be seen that the SPH method has second-order accuracy for the integral representation of the function and the kernel approximation. However, the kernel approximation does not require second-order accuracy when the kernel function is not an even function.

2.2.2 Integral representation of derivatives

An approximation to the spatial derivative $\nabla \cdot f(x)$ is obtained by replacing $f(x)$ in equation (2.2.4) with $\nabla \cdot f(x)$.

$$\langle \nabla \cdot f(x) \rangle = \int_{\Omega} [\nabla \cdot f(x')] W(x-x', h) dx' \quad (2.2.11)$$

Here, the divergence in the integral is treated as dependent on the main coordinates.

$$[\nabla \cdot f(x)] W(x-x', h) = \nabla \cdot [f(x') W(x-x', h)] - f(x') \cdot \nabla W(x-x', h) \quad (2.2.12)$$

Therefore, the following equation is obtained from equations (2.2.11) and (2.2.12).

$$\langle \nabla \cdot f(x) \rangle = \int_{\Omega} \nabla \cdot [f(x') W(x-x', h)] dx' - \int_{\Omega} f(x') \cdot \nabla W(x-x', h) dx' \quad (2.2.13)$$

For the first term on the right-hand side of equation (2.2.13), Ω can be replaced with the integral of the integration domain surface S based on the divergence theorem.

$$\langle \nabla \cdot f(x) \rangle = \int_S f(x') W(x-x', h) \cdot \vec{n} dS - \int_{\Omega} f(x') \cdot \nabla W(x-x', h) dx' \quad (2.2.14)$$

where $\vec{n}$ is the unit normal vector of surface S .

When the supporting domain is within the problem domain as shown in Fig. 2.1, the kernel function

W satisfies the compact condition, so the surface integral on the right side of equation (2.2.14) is zero. However, on the other hand, in the case of Figure 2.2 where the supporting domain is partially outside the problem domain, the kernel function W is cut off by the boundary, and the surface integral is no longer zero. Under such circumstances, if the surface integral in equation (2.2.14) is zero, the boundary effect needs to be improved. Assuming that the points in the supporting domain are all in the problem domain, equation (2.2.14) can be simplified as follows.

$$\langle \nabla \cdot f(x) \rangle = - \int_{\Omega} f(x') \cdot \nabla W(x - x', h) dx' \quad (2.2.15)$$

From the above equation, in the integral representation of the derivative of the SPH field function, the spatial gradient is determined from the value of the function and the derivative of the kernel function rather than the derivative of the function itself.

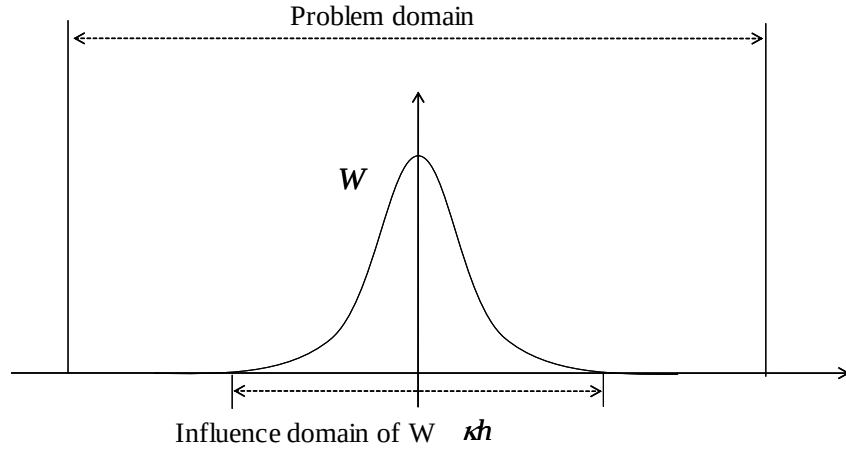


Fig 2.1 The kernel function supporting domain in the problem domain

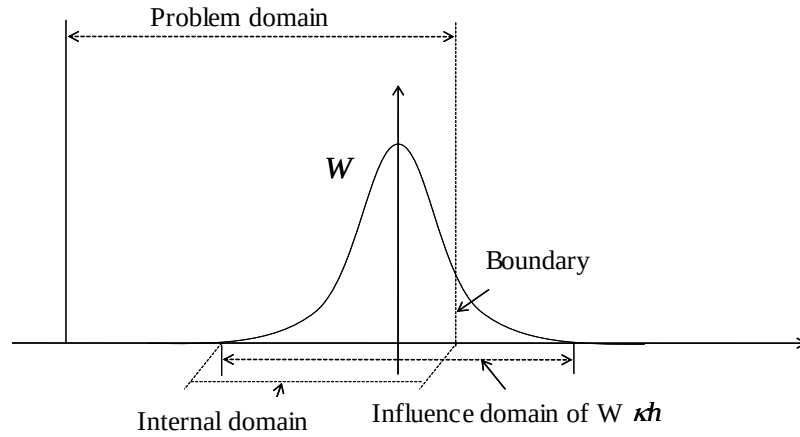


Fig 2.2 The supporting domain of the kernel function intersects with the problem domain

2.2.3 Particle approximation

In the SPH method, the entire system is represented by a finite number of particles with individual mass and volume (area), which is a very important operation in the SPH method. In SPH kernel approximation, the continuous integral representation expressed by equations (2.2.4) and (2.2.15) can be approximated to the sum of all particles in the affected area as shown in Fig. 2.3. In the figure, W : kernel function, Ω : influence region, κh : influence radius, r_{ij} : inter-particle distance.

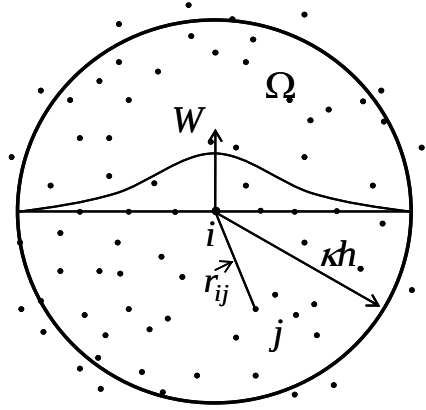


Fig 2.3 The influence domain circle of particle i

The discretization process corresponding to the sum of particles is generally known as particle approximation in SPH papers, and this process is described below.

First, replace the infinitesimal value dx' of the particle j integral with the particle finite volume ΔV_j related to the particle mass m_j . Here, m_j is expressed by the following equation.

$$m_j = \Delta V_j \rho_j \quad (2.2.16)$$

In the above equation, ρ_j is the density of particles j ($= 1, 2, \dots$). N is the number of particles in the affected area of particle j . A continuous SPH integral representation for $f(x)$ and can be written in the form of a discretized particle approximation such as

$$\begin{aligned}
\langle f(x) \rangle &= \int_{\Omega} f(x') W(x - x', h) dx' \\
&= \sum_{j=1}^N f(x_j) W(x - x_j, h) \Delta V_j \\
&= \sum_{j=1}^N f(x_j) W(x - x_j, h) \frac{1}{\rho_j} (\rho_j \Delta V_j) \\
&= \sum_{j=1}^N f(x_j) W(x - x_j, h) \frac{1}{\rho_j} (m_j)
\end{aligned} \tag{2.2.17}$$

Or,

$$\langle f(x) \rangle = \sum_{j=1}^N \frac{m_j}{\rho_j} f(x_j) W(x - x_j, h) \tag{2.2.18}$$

The particle approximation is derived as shown above. Thus, the particle approximation of the function at particle i can be finally expressed as

$$\langle f(x_i) \rangle = \sum_{j=1}^N \frac{m_j}{\rho_j} f(x_j) \cdot W_{ij} \tag{2.2.19}$$

where

$$W_{ij} = W(x_i - x_j, h) \tag{2.2.20}$$

Equation (2.2.19) indicates that the value of the function at particle i is approximated using the average of all particle values within the smoothing length of particle i weighted by the kernel function. Similarly, the particle approximation in the spatial differentiation of the function can be expressed as

$$\langle \nabla \cdot f(x) \rangle = - \sum_{j=1}^N \frac{m_j}{\rho_j} f(x_j) \cdot \nabla W(x - x_j, h) \tag{2.2.21}$$

Here, the gradient ∇W in the above equation depends on the particle j . Therefore, the particle approximation of the function at particle i can be finally expressed as

$$\langle \nabla \cdot f(x_i) \rangle = - \sum_{j=1}^N \frac{m_j}{\rho_j} f(x_j) \cdot \nabla_i W_{ij} \tag{2.2.22}$$

where

$$\nabla_i W_{ij} = \frac{x_i - x_j}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} = \frac{x_{ij}}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} \tag{2.2.23}$$

Equation (2.2.22) shows that the gradient value of the function at particle i is approximated using the average of all particle values in the region of influence of particle i weighted by the gradient of the kernel function.

Particle approximation can be conveniently applied to hydrodynamic problems where density is an

important field variable by introducing particle mass and density into the equation. This is one of the main reasons why the SPH method is particularly used in dynamic flow problems. If the SPH particle approximation can be applied to solid mechanics problems, special handling is considered necessary. In addition, particle approximation is related to numerical problems specific to SPH method such as particle inconsistency and stretchable instability. It is said that the number of reference points for integration should be larger than the field nodes (particles) to eliminate these problems.

For particle approximation, given particle i , the function value and its derivative are approximated at particle i as follows.

$$\langle f(x_i) \rangle = \sum_{j=1}^N \frac{m_j}{\rho_j} f(x_j) W_{ij} \quad (2.2.24)$$

$$\langle \nabla \cdot f(x_i) \rangle = \sum_{j=1}^N \frac{m_j}{\rho_j} f(x_j) \cdot \nabla_i W_{ij} \quad (2.2.25)$$

$$W_{ij} = W(x_i - x_j, h) = W(|x_i - x_j|, h) \quad (2.2.26)$$

$$\nabla_i W_{ij} = \frac{x_i - x_j}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} = \frac{x_{ij}}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} \quad (2.2.27)$$

where r_{ij} is the distance between particle i and particle j . Since $\nabla_i W_{ij}$ is determined by the particle i , the negative sign in Eq. (2.21) is removed in Eq. (2.2.25). If we replace function $f(x)$ with density ρ in equation (2.2.24), the SPH approximation for density is

$$\rho_i = \sum_{j=1}^N m_j W_{ij} \quad (2.2.28)$$

Equation (2.2.28) is the most general formula for obtaining the density with SPH, which is the sum density, and indicates that the particle density is the average of the weighted densities of all particles in the affected area.

2.2.4 Formulation method of SPH

By using the kernel approximation and particle approximation procedures described above, the formulation of SPH by partial differential equations can always be derived. There are many methods for deriving SPH formulation using partial differential equations, but a simpler method is used directly

using equations (2.2.24) and (2.2.25). This method uses the following two expressions to define the density within the gradient operator.

$$\nabla \cdot f(x) = \frac{1}{\rho} [\nabla \cdot (\rho f(x)) - f(x) \cdot \nabla \rho] \quad (2.2.29)$$

$$\nabla \cdot f(x) = \rho \left[\nabla \cdot \left(\frac{f(x)}{\rho} \right) + \frac{f(x)}{\rho^2} \cdot \nabla \rho \right] \quad (2.2.30)$$

The above two equations are substituted into the integral of equation (2.2.11), and the gradient term on the right side of equations (2.2.29) and (2.2.30) is applied as the same procedure as particle approximation of equation (2.2.25). Each equation outside the gradient term uses the particle's own value, and the following equation is obtained from the result of $f(x)$ divergence for particle i obtained from equations (2.2.29) and (2.2.30).

$$\nabla \cdot f(x_i) = \frac{1}{\rho_i} \left[\sum_{j=1}^N m_j [f(x_j) - f(x_i)] \cdot \nabla_i W_{ij} \right] \quad (2.2.31)$$

$$\nabla \cdot f(x_i) = \rho_i \left[\sum_{j=1}^N m_j \left[\left(\frac{f(x_j)}{\rho_j^2} \right) + \left(\frac{f(x_i)}{\rho_i^2} \right) \right] \cdot \nabla_i W_{ij} \right] \quad (2.2.32)$$

One of the advantages of the above two formulas is that the field function appears in the form of a pair of particles.

In addition to the two equations described above, there are other useful operational rules that can be used to derive SPH equations for complex systems. For any two functions of field variables f_1 and f_2 , the following rules exist:

$$\langle f_1 + f_2 \rangle = \langle f_1 \rangle + \langle f_2 \rangle \quad (2.2.33)$$

$$\langle f_1 f_2 \rangle = \langle f_1 \rangle \langle f_2 \rangle \quad (2.2.34)$$

Therefore, the SPH approximation of the sum of the functions and the sum of the SPH approximation of each function is equal, and the SPH approximation of the functions multiplied and the product of the SPH approximation of each function is equal.

$$\langle c f_2 \rangle = c \langle f_2 \rangle \quad (2.2.35)$$

Equations (2.2.33) and (2.2.35) mean that the SPH approximation operator is a linear operator. It is

easy to understand that the SPH approximation operator is commutative, and can be expressed as

$$\langle f_1 + f_2 \rangle = \langle f_2 + f_1 \rangle \quad (2.2.36)$$

$$\langle f_1 f_2 \rangle = \langle f_2 f_1 \rangle \quad (2.2.37)$$

2.3 Kernel function

2.3.1 Conditions of kernel function

Section 2.2 described the basic concept and important formulation of the SPH method. In the SPH approximation, the kernel function plays an important role in determining the accuracy and efficiency of the computation.

Here, a general method for constructing the kernel function in the SPH method is shown. In the method, a well-formed expression using Taylor series expansion is used. A set of conditions is systematically derived and used to construct both kernel functions and point-dependent kernel functions in the analysis given in numerical form. These conditions not only ensure consistency in the SPH approximation, but also represent the compact support condition for the kernel function.

One of the main problems of the mesh-free method is to perform function approximation based on nodes that are scattered efficiently and arbitrarily without pre-defining meshes and grids that provide node connectivity. Methods for approximating functions fall into three categories. The three are 1) integral representation, 2) series representation, and 3) difference representation. The SPH method uses integral representation with the kernel function. The kernel function is a very important factor because it determines the type of function approximation and not only defines the range of the particle influence area, but also the consistency and accuracy of the kernel approximation and particle approximation.

In SPH method, different kernel functions are used, while these kernel functions satisfy the same conditions and are with same characteristics although the function types are different. The main characteristics are described below.

1. The kernel function must be standardized (Unity condition)

$$\int_{\Omega} W(x - x', h) dx' = 1 \quad (2.3.1)$$

2. kernel functions should be represented in a certain region (Compact support),

$$W(x - x') = 0, \text{ for } |x - x'| > \kappa h \quad (2.3.2)$$

The compact support domain is defined by the kernel function h and the coefficient κ .

Where h is the smoothing length and κ determines the spread of a specific kernel function.

3. $W(x - x') \geq 0$ holds for any point x' in the region of influence of the particle at any point x (positive value).

4. The value of the kernel function for a particle generally decreases monotonically as the distance from the particle increases. (Attenuation)

5. The kernel function satisfies the Dirac delta function condition when the smoothing length approaches 0 (the characteristics of the delta function).

$$\lim_{h \rightarrow 0} W(x - x', h) = \delta(x - x') \quad (2.3.3)$$

6. The kernel function is an even function (Symmetric) .

7. The kernel function is smooth enough (Smoothness)

The first “Unity condition” means that the integral value of the kernel function in the support domain is 1.

The second condition is responsible for converting the SPH approximation from global to local. Because this condition leads to an arbitrarily distributed calculation matrix, this condition is largely related to the computing power of the computer.

The third condition means that the kernel function value should not be negative in the support area. This is not necessary for mathematically converging calculations, but for stabilizing physical phenomena.

The fourth condition is based on the physical idea that particles that are close to the particle being considered should have a significant influence to each other. That is, as the distance between two interacting particles increases, the force acting between the two interacting particles decreases. However, this condition is not satisfied for an object in which the influence force increases as the distance between two particles increases.

As for the fifth condition, it can be satisfied as the smoothing length approaches zero. This condition is naturally satisfied by satisfying conditions 1 to 4.

The sixth condition means that particles that are located at different locations for a particle but have the same distance have the same effect on a particle. This is not an absolutely necessary condition and may not be met by the highly invariant mesh-free particle method.

The seventh condition is for obtaining an accurate approximation. To obtain accurate results in function and derivative approximations, the kernel function should always be continuous. A kernel function with a smooth function and derivative value generally yields good results. This is because the kernel function does not affect irregularly arranged particles and the approximation error of the integral interpolation formula is reduced.

A function with the above characteristics is used as the kernel function of the SPH method. Many researchers devise and use different types of kernel functions. Some of the kernel functions used in SPH literature are listed in the following section.

2.3.2 Typical example

Monaghan⁴⁾ states that it is best to use a Gaussian distribution for the kernel function to obtain a physical interpretation in the SPH formulation. Gingold and Monaghan⁵⁾ adopted the following Gaussian kernel shown in Fig. 2.4.

$$W(R, h) = \alpha_d e^{-R^2} \quad (2.3.4)$$

where $\alpha_d = 1/\pi^{1/2}h, 1/\pi h^2, 1/\pi^{3/2}h^3$, in 1D, 2D, 3D space, respectively.

The Gaussian kernel is considered the best kernel function because even higher-order derivatives are sufficiently smooth, and the calculations are accurate and stable for irregularly moving particles. However, even if R approaches infinity, W is not theoretically zero, so it cannot be said to be truly compact. In addition, it must be noted that this function is the most computationally expensive because calculation domain is too large. In particular, this applies to all higher-order derivatives of the kernel function, which requires a broad support domain containing many particles in the particle approximation and requires the ability to process individual computational matrices.

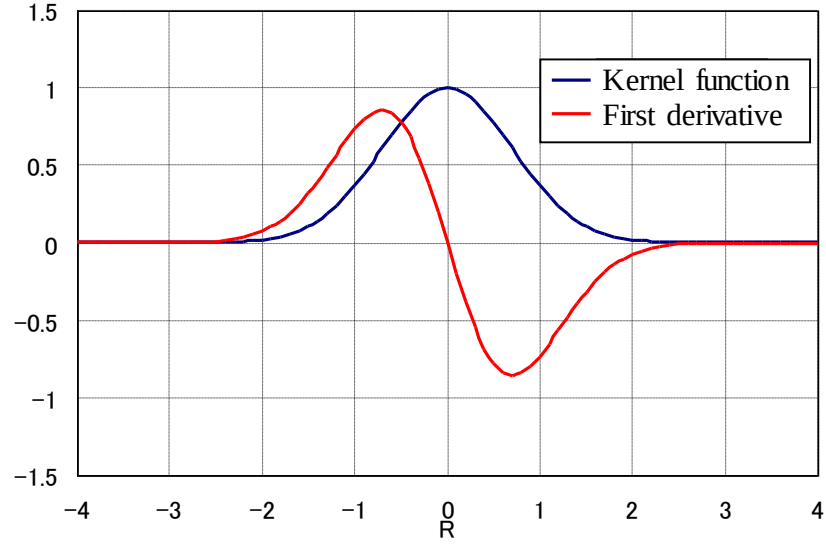


Fig 2.4 Gaussian kernel and first derivative

Based on a cubic spline function, Monaghan and Lattanzio⁶⁾ devised a kernel function called “B-spline function” shown below.

$$W(R, h) = \alpha_d \times \begin{cases} \frac{2}{3} - R^2 + \frac{1}{2}R^3 & 0 \leq R < 1 \\ \frac{1}{6}(2 - R)^3 & 1 \leq R < 2 \\ 0 & 2 \leq R \end{cases} \quad (2.3.5)$$

where $\alpha_d = 1/h, 15/7\pi h^2, 3/2\pi h^3$, in 1D, 2D, 3D space, respectively.

The cubic spline function is similar to the Gaussian kernel and is the most widely used kernel function so far because it can provide a narrower compact support than the Gaussian kernel. However, since the second derivative of the cubic spline function is piecewise linear, the stability characteristics are inferior to other functions. In addition, the kernel function is defined for three ranges, which is slightly more difficult than a kernel function consisting of one function.

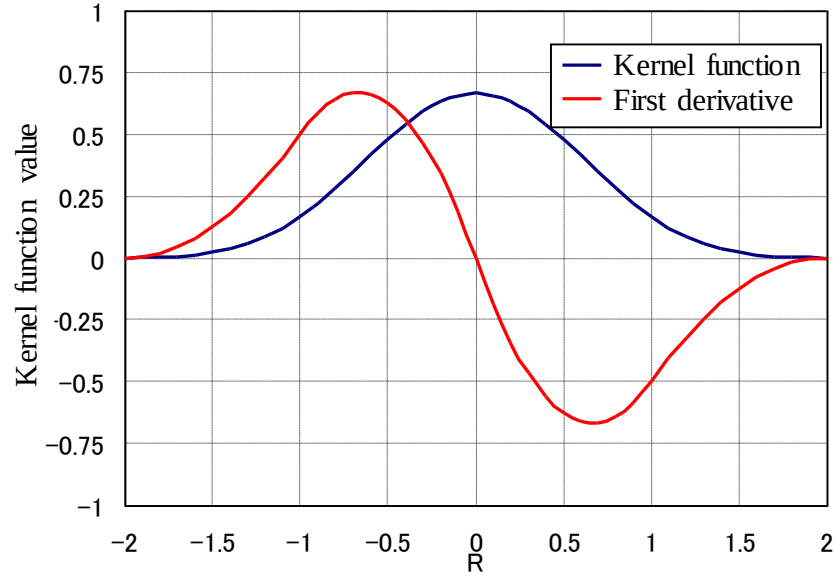


Fig 2.5 Cubic spline function and first derivative

Morris^{7,8)} proposed a higher-order spline function that is closer to the Gaussian kernel and more stable.

Here, the quintic spline function is shown in equation (2.3.6).

$$W(R, h) = \alpha_d \times \begin{cases} (3-R)^5 - 6(2-R)^5 + 15(1-R)^5 & 0 \leq R < 1 \\ (3-R)^5 - 6(2-R)^5 & 1 \leq R < 2 \\ (3-R)^5 & 2 \leq R < 3 \\ 0 & R \geq 3 \end{cases} \quad (2.3.6)$$

where $\alpha_d = 120/h$, $7/478\pi h^2$, $3/359\pi h^3$, in 1D, 2D, 3D space, respectively.

2.4 Numerical aspects

In the SPH method, artificial viscosity is added to stabilize the calculation. The details are explained here. Influence radius directly related to analysis accuracy is also introduced.

2.4.1 Artificial viscosity

To simulate fluid mechanics problems, special measures and methods are needed to allow the modeling of shock waves that amplify non-physical vibrations in numerical results. Although the shock wave is not discontinuous in reality, it is a parallel free path in a very limited displacement region with a thickness of a few molecules. Application of mass, momentum, and energy conservation conditions to the front of the shock wave requires a simulation of conversion of kinetic energy to thermal energy,

and physically this energy conversion is expressed as viscous dissipation. This idea leading to the development of von Neumann-Richtmyer artificial viscosity is expressed by equation (2.4.1).

$$\Pi_1 = \begin{cases} a_1 \Delta x^2 \rho (\nabla \cdot \mathbf{v})^2 & \nabla \cdot \mathbf{v} < 0 \\ 0 & \nabla \cdot \mathbf{v} \geq 0 \end{cases} \quad (2.4.1)$$

where Π_1 is von Neumann-Richtmyer artificial viscosity and is required during material compression. a_1 is an adjustable dimensionless constant. von Neumann-Richtmyer artificial viscosity is the second-order term of velocity derivative.

It was also discovered that the addition of the linear artificial viscosity term Π_2 expressed by Eq. (2.4.2) has the advantage of smoothing vibrations that cannot be suppressed by secondary artificial viscosity.

$$\Pi_2 = \begin{cases} a_2 \Delta x c \rho \nabla \cdot \mathbf{v} & \nabla \cdot \mathbf{v} < 0 \\ 0 & \nabla \cdot \mathbf{v} \geq 0 \end{cases} \quad (2.4.2)$$

where c is the speed of sound and is a_2 an arbitrary constant.

The second-order von Neumann-Richtmyer artificial viscosity Π_1 and linear artificial viscosity Π_2 are widely used in fluid dynamics analysis to eliminate numerical vibrations. The artificial viscosity term serves to normalize numerical instabilities caused by (discontinuous) sharp fluctuations by spreading the shock wave into several meshes. Also, the artificial viscosity term is often added to the physical pressure term to diverge sudden fluctuations at transitions and help diffuse energy at high vibration terms.

The SPH method was first applied to deal with low-loss or no-loss problems, and then developed to simulate impact problems by Monaghan and Gingold⁹⁾. This Monaghan type artificial viscosity Π_{ij} is the most widely used artificial viscosity in SPH papers. This not only defines the loss required to convert kinetic energy into thermal energy in front of the shock wave, but also prevents non-physical, close-up penetration of each other's particles. The detailed formula is described below.

$$\Pi_{ij} = \begin{cases} \frac{-\alpha_{\Pi} \bar{c}_{ij} \phi_{ij} + \beta_{\Pi} \phi_{ij}^2}{\bar{\rho}_{ij}} & \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} < 0 \\ 0 & \mathbf{v}_{ij} \cdot \mathbf{x}_{ij} \geq 0 \end{cases} \quad (2.4.3)$$

where

$$\phi_{ij} = \frac{h_{ij} \mathbf{v}_{ij} \cdot \mathbf{x}_{ij}}{|\mathbf{x}_{ij}|^2 + \varphi^2} \quad (2.4.4)$$

$$\bar{c}_{ij} = \frac{1}{2}(c_i + c_j) \quad (2.4.5)$$

$$\bar{\rho}_{ij} = \frac{1}{2}(\rho_i + \rho_j) \quad (2.4.6)$$

$$h_{ij} = \frac{1}{2}(h_i + h_j) \quad (2.4.7)$$

$$\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j, \quad \mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j \quad (2.4.8)$$

where α_{Π} , β_{Π} are coefficients and are often 1.0

The coefficient $\varphi = 0.1h_{ij}$ is included to prevent numerical divergence when two particles approach each other. c and $\mathbf{v}$ represent the sound velocity and particle velocity vectors, respectively. Viscosity related to α_{Π} gives rise to volume viscosity, while the second term related to β_{Π} to suppress interpenetration of particles at high Mach numbers is similar to the von Neumann-Richtmyer artificial viscosity.

In addition, Monaghan-type artificial viscosity introduced shear viscosity into the flow, especially in the region away from the impact field, and the artificial viscosity based on the velocity field derivative was used by Hernquist and Katz¹⁰.

$$\Pi_{ij} = \frac{q_i}{\rho_i^2} + \frac{q_j}{\rho_j^2} \quad (2.4.9)$$

where

$$q_i = \begin{cases} \alpha_{\Pi} h_i \rho_i c_i |\nabla \cdot \mathbf{v}_i| + \beta_{\Pi} h_i^2 \rho_i |\nabla \cdot \mathbf{v}_i|^2 & \nabla \cdot \mathbf{v}_i < 0 \\ 0 & \nabla \cdot \mathbf{v}_i \geq 0 \end{cases} \quad (2.4.10)$$

2.4.2 Smoothing length

The smoothing length is very important in SPH method because it directly affects the efficiency of the calculation and the accuracy of the solution. If it is too small, there will be insufficient particles within the supporting domain, and the result will be less accurate. On the other hand, if it is too large, the micro-changes and local properties of the particles will probably be smoothed and inaccurate. The particle approximation used in the SPH method depends on the number of sufficient and necessary

particles present in the supporting domain. The time and speed of the calculation also depend on the number of particles. In 1, 2 and 3 dimensions, if the particles are located in a grid and the radius of influence is 1.2 times the distance between the particles, then the number of surrounding particles is about 5, 21, and 57, respectively.

In the early stage of SPH, the overall particle radius was used depending on the initial average density of the system. Later, the influence radius can be solved by assigning individual influence radius determined by small changes in density in each particle, thereby maintaining consistent accuracy throughout the space and solving the problem of local expansion or contraction of the liquid. For anisotropic problems such as impact problems, the radius of influence is required to change with both space and time.

There are many ways to dynamically expand h so that the number of surrounding particles is relatively constant. The simplest method is to update the radius of influence using the average density

$$h = h_0 \left(\frac{\rho_0}{\rho} \right)^{\frac{1}{d}} \quad (2.4.11)$$

where h_0 and ρ_0 are the initial smoothing length and density, respectively. d is the number of dimensions.

Benz¹¹⁾ proposed another method of expanding the smoothing length by incorporating the time derivative of the kernel function into a continuous equation.

$$\frac{dh}{dt} = -\frac{1}{d} \frac{h}{\rho} \frac{d\rho}{dt} \quad (2.4.12)$$

Equation (2.4.12) is discretized using the SPH approximation and computed in parallel with other differential equations

2.4.3 Symmetry of interaction

If the smoothing length varies with both space and time, each particle has its own smoothing length. If h_i is not equal to h_j , the region of influence of particle i may extend to particle j , but not vice versa. In addition, particle i can exert a force on particle j even if particle j does not have a corresponding action on particle i . But this relationship violates Newton's third law. In order to solve this problem, several measures were required to maintain the symmetry of the particle interaction. Here, a method to improve the smoothing length is described. There are different ways to improve the symmetry radius

in order to maintain interaction contrast. One way to calculate a symmetric smoothing length is to take the arithmetic mean or average of a pair of smoothing length of interacting particles.

$$h_{ij} = \frac{h_i + h_j}{2} \quad (2.4.13)$$

In addition, a method of calculating the smoothing length using the geometric mean of the smoothing length of a pair of interacting particles is also used.

$$h_{ij} = \frac{2h_i h_j}{h_i + h_j} \quad (2.4.14)$$

Or take the maximum of h_i and h_j as the smoothing length

$$h_{ij} = \max(h_i, h_j) \quad (2.4.15)$$

Or take the minimum of h_i and h_j as the smoothing length

$$h_{ij} = \min(h_i, h_j) \quad (2.4.16)$$

Furthermore, the kernel function is obtained from using the symmetric smoothing length

$$W_{ij} = W(r_{ij}, h_{ij}) \quad (2.4.17)$$

There are advantages and disadvantages to these different ways of determining the symmetric smoothing length h_{ij} . When using the arithmetic mean of equation (2.4.13) or the maximum smoothing length of equation (2.5.15), there is a tendency to use more surrounding particles and to overly smooth the effects between surrounding particles. In addition, when using the geometric mean of equation (2.5.14) or the minimum value of the smoothing length of equation (2.4.16), it is necessary to be careful because fewer surrounding particles are used. In addition, as an alternative method of maintaining particle interaction, the average of kernel function values is used directly without using the symmetric smoothing length.

$$W_{ij} = \frac{1}{2}(W(h_i) + W(h_j)) \quad (2.4.18)$$

These equations are widely used in SPH that preserves particle interactions.

Chapter 3 Application of SPH in numerical analysis

3.1 Analysis procedure of SPH method

In the SPH method in this study, calculations are conducted according to the flow shown in Fig. 3.1.

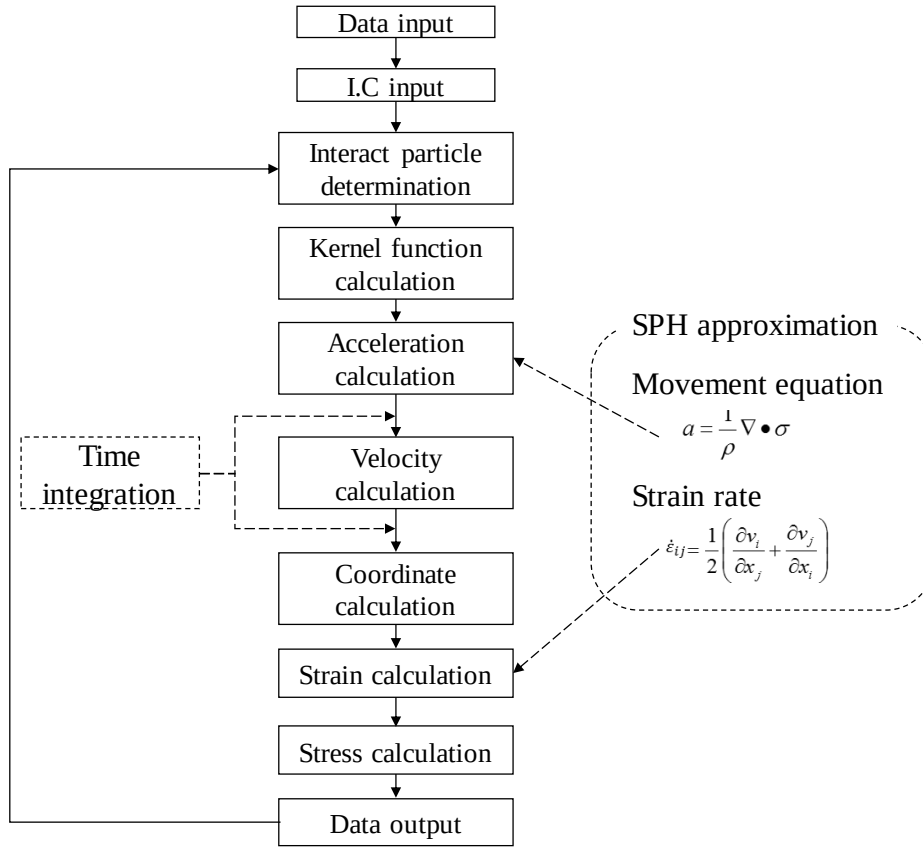


Fig 3.1 Analysis procedure

3.2 Particle approximation

3.2.1 Basic theory and formula of ASPH method

The general SPH method uses a uniform kernel function in all directions, but here we outline the ASPH method using a kernel function that expresses anisotropy. In this study, the crack model with stress release and anisotropic constitutive law is adopted. The crack model is implemented by the effect

reduction rates $\phi_x \phi_y \phi_z$ along the three orthogonal axes of the global coordinate system. The conventional SPH method was extended and the ASPH proposed by Michael Owen et al.¹⁾ was used, which can apply different kernel function to each axis. ASPH defines three independent kernel functions for three orthogonal axes, and ASPH weights and the first derivative of the weights are expressed as in equations (3.1.1) and (3.1.2).

$$W = W_x \times W_y \times W_z \quad (3.1.1)$$

$$\nabla W = \left(\frac{\partial W_x}{\partial x} W_y W_z, W_x \frac{\partial W_y}{\partial y} W_z, W_x W_y \frac{\partial W_z}{\partial z} \right) \quad (3.1.2)$$

Fig 3.2 shows the concept of the SPH kernel function, and Fig 3.3 shows the concept of the ASPH kernel function. In the SPH kernel function, the weight of each inter-particle distance is first calculated from the relationship with the smoothing length and then distributed to each axis with the cosine value. Therefore, the kernel function sets one kind of kernel function no matter the number of dimensions. On the other hand, as shown in Fig 3.3, ASPH's kernel function sets a one-dimensional kernel function in each axial direction, and calculates the weight in each direction from the relationship between the smoothing length and the inter-particle distance. This makes it possible to set three independent kernel functions for three orthogonal axes.

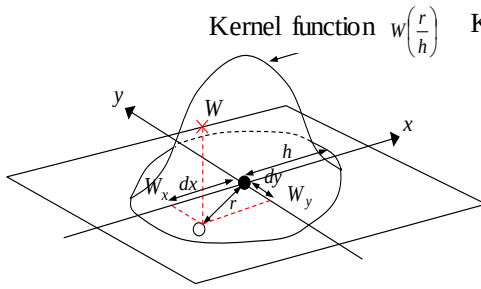


Fig 3.2 Kernel function in SPH

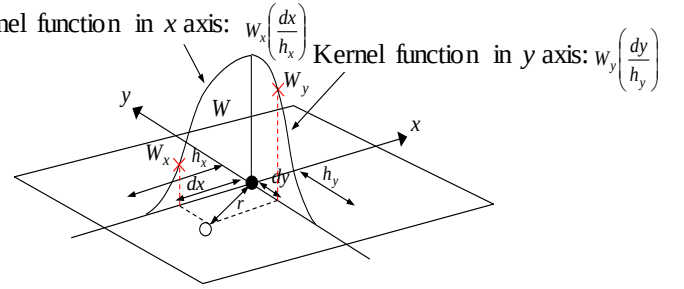


Fig 3.3 Kernel function in ASPH

3.2.2 Kernel function used in this study

As described above, SPH method is a method that calculates the physical quantity of a certain evaluation point by approximating it with the physical quantities of surrounding particles, and therefore the calculation process includes the determination of particles that affect each other. In this study, the distance between particles in 3D space is calculated, and the interaction is considered only between particles that exist within a certain range. The kernel function is used to calculate the weighted average.

In this study, the third-order Spline function in the three-dimensional space shown below is used.

$$W = \frac{3}{2\pi h^3} \times \begin{cases} \frac{2}{3} - \left(\frac{r}{h}\right)^2 + \frac{1}{2}\left(\frac{r}{h}\right)^3 & 0 \leq \frac{r}{h} \leq 1 \\ \frac{1}{6}\left(2 - \frac{r}{h}\right)^3 & 1 \leq \frac{r}{h} \leq 2 \\ 0 & 2 \leq \frac{r}{h} \end{cases} \quad (3.2.3)$$

where r : distance in 3D space ($r = \sqrt{dx^2 + dy^2 + dz^2}$), dx : distance on x axis, dy : distance on y axis, dz : distance on z axis, h : smoothing length. For the its first derivative:

$$\frac{\partial W}{\partial x} = \frac{3}{2\pi h^3} \times \begin{cases} \left(-2 + \frac{3}{2} \times \frac{r}{h}\right) \frac{dx}{h^2} & 0 \leq \frac{r}{h} \leq 1 \\ -\frac{1}{2}\left(2 - \frac{r}{h}\right)^2 \frac{dx}{h \times r} & 1 \leq \frac{r}{h} \leq 2 \\ 0 & 2 \leq \frac{r}{h} \end{cases} \quad (3.2.4)$$

The above is the expression on x axis. For y axis, $\partial x \rightarrow \partial y$, $dx \rightarrow dy$, and for z axis, $\partial x \rightarrow \partial z$, $dx \rightarrow dz$

3.2.3 Smoothing length used in this study

Adaptive smoothing length is adopted in this study to increase the accuracy. There are many ways to dynamically expand h so that the number of surrounding particles is relatively constant. The simplest method is to update the radius of influence using the average density

$$h = h_0 \left(\frac{\rho_0}{\rho} \right)^{\frac{1}{d}} \quad (2.4.11 \text{ repost})$$

where h_0 and ρ_0 are the initial smoothing length and density, respectively. d is the number of dimensions.

In order to obey Newton's law of movement, the symmetric smoothing length is considered. One way to calculate a symmetric smoothing length is to take the arithmetic mean or average of a pair of

smoothing length of interacting particles.

$$h_{ij} = \frac{h_i + h_j}{2} \quad (2.4.13 \text{ repost})$$

3.3 Time integration

3.3.1 Numerical integration method

As a basic equation for calculating and analyzing the response of a structure to an impact load, the equation of motion for a dynamic load can be used as it is. If the function given as the applied load can be integrated, the impact response of the structure can be calculated relatively easily. In actual practice, however, loads are often given as experimental data, and numerical analysis methods must be used for response analysis. In the case of analysis of linear elastic structures subjected to arbitrary dynamic loads, numerical vibration response analysis such as Duhamel integration or Fourier transform analysis can be applied. However, since both methods use the principle of superposition, the application is limited to linear systems only.

On the other hand, in dynamic response analysis such as vibration and impact of a structure, the external force of the structural analysis model is often complex and it is often necessary to obtain the response from local damage to failure, and the process cannot often be summarized as a linear process. In this case, nonlinear (elasto-plastic) response analysis is performed, and it is impossible to obtain a mathematical theoretical answer.

In order to perform elasto-plastic response analysis of structures, it is necessary to solve equations of motion given as differential equations with respect to time. To do so, differential equations must be calculated sequentially by methods called numerical integration, time integration, or direct integration.

There are two methods of time integration: implicit method and explicit method. The implicit method is a method for finding a solution (acceleration, velocity, displacement, etc.) on the condition that the equation of motion at the next time is satisfied, while the explicit method do not need a convergence calculation, and it uses only the information up to the current time to find the response at the next time. In this study, equation of motion of each element is solved by using an improved explicit time integration Euler method.

3.3.2 Time integration in this study

In the time integration used in this study, the displacement $x(t + \Delta t)$ and velocity $\dot{x}(t + \Delta t)$ after a small time increment Δt centered on the time t are expressed by equations (3.3.1) and (3.3.2).

$$x(t + \Delta t) = x(t) + (\dot{x}(t - \Delta t) + \dot{x}(t)) \frac{\Delta t}{2} \quad (3.3.1)$$

$$\dot{x}(t + \Delta t) = \dot{x}(t) + (\ddot{x}(t - \Delta t) + \ddot{x}(t)) \frac{\Delta t}{2} \quad (3.3.2)$$

3.3.3 Stability of explicit methods

In the explicit method, a constraint on the time step Δt to obtain a stable numerical solution is required instead of being conditional stable based on the equation of motion at the next time. That is, the value obtained by dividing the element length $\Delta l_{\min}$ of the smallest dimension in the analysis model by the propagation speed c of the stress wave, that is, the time required for the stress wave to pass through the minimum element $\Delta T_{\min} = \Delta l_{\min} / c$ must be smaller than the time step used for time integration Δt . This is a condition that time step must be small, and is called the Courant condition or CFL condition and is expressed by equation (3.3.3). If this condition is not satisfied, the propagation distance of the stress wave becomes too large in a single increment, and the change of stress that should have occurred within that time is prevented from being reflected in the behavior of each element.

$$\Delta t \leq \Delta T_{\min} = \frac{\Delta l_{\min}}{c} \quad (3.3.3)$$

where $c = \sqrt{\frac{E}{\rho}}$: Stress wave (longitudinal wave) propagation velocity, E : elastic modulus, ρ : density

In this study, the analysis is conducted by applying a time step that satisfies the above equation (3.3.3).

3.4 Calculation of acceleration, velocity, and displacement

In the SPH method in this study, the acceleration is calculated and then the velocity is calculated by integrating acceleration, and the velocity is integrated to calculate the displacement. Here, the

calculation algorithm in acceleration calculation is explained.

In order to use the SPH method for the analysis of elasto-plastic fields of solids, it is common to use the following equation of motion using the stress gradient in a continuum field.

$$a = \frac{1}{\rho} \nabla \bullet \sigma \quad (3.4.1)$$

where σ represents the stress tensor. Since the above equation represents the first-order spatial differential of stress, it is possible to use the basic equation (2.2.21) of the SPH method. Therefore, by applying Eq. (2.2.1) to Eq. (3.3.1), it is possible to calculate the particle acceleration as shown below.

$$a_i^\alpha = -\frac{1}{\rho_i} \sum_{j=1}^N \frac{m_j}{\rho_j} \left(\sum_{\beta=1}^{\dim} \sigma_j^{\alpha\beta} \frac{\partial W_{ij}}{\partial x^\beta} \right) \quad (3.4.2)$$

Or, by the differential of the equation (2.2.21)

$$\langle \nabla \cdot f(x_i) \rangle = -\rho_i \left[\sum_{j=1}^N m_j \left[\frac{f(x_j)}{\rho_j^2} + \frac{f(x_i)}{\rho_i^2} \right] \cdot \nabla W(x_i - x_j, h) \right] \quad (3.4.3)$$

And apply this in equation (3.3.1)

$$a_i^\alpha = -\sum_{j=1}^N m_j \sum_{\beta=1}^{\dim} \left[\left(\frac{\sigma_j^{\alpha\beta}}{\rho_j^2} \right) + \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} \right) \right] \frac{\partial W_{ij}}{\partial x^\beta} \quad (3.4.4)$$

where α and β are components indicating the direction of each orthogonal axis in the global coordinate system.

In Eq. (3.3.2) and Eq. (3.3.4), the latter has the advantage that the forces acting on particle i and particle j are equal, and Newton's third law is satisfied. Therefore, the momentum conservation law is more strictly guaranteed.

Here, the following equation is improved to alleviate the influence of the density difference between particle i and particle j .

$$a_i^\alpha = -\frac{2}{\rho_i} \sum_{j=1}^N \frac{m_j}{\rho_j} \sum_{\beta=1}^{\dim} \left(\frac{\sigma_j^{\alpha\beta} \rho_i + \sigma_i^{\alpha\beta} \rho_j}{\rho_i + \rho_j} \right) \frac{\partial W_{ij}}{\partial x^\beta} \quad (3.4.5)$$

In addition, as mentioned above, in the SPH method, the term artificial viscosity is added to the acceleration calculation formula to stabilize the calculation. In this study, the formula (2.4.3) proposed by Monaghan is used. Unlike the fluid, in the solid analysis, it is necessary to act on both sides of compression and tension. Therefore, it is used as follows without dividing the acting direction as shown in Equation (2.4.3). Therefore, the acceleration calculation formula by SPH method is

$$a_i^\alpha = -\frac{2}{\rho_i} \sum_{j=1}^N \frac{m_j}{\rho_j} \sum_{\beta=1}^{\dim} \left(\frac{\sigma_j^{\alpha\beta} \rho_i + \sigma_i^{\alpha\beta} \rho_j}{\rho_i + \rho_j} + \Pi^{\alpha\beta} \right) \frac{\partial W_{ij}}{\partial x^\beta} \quad (3.4.6)$$

where the artificial viscosity $\Pi^{\alpha\beta}$ is written below

$$\Pi^{\alpha\beta} = \frac{1}{r_{ij}} \frac{\bar{c}_{ij}}{\bar{\rho}_{ij}} v_{ij}^{\alpha\beta} x_{ij}^{\alpha\beta}$$

From equation (3.4.6), the velocity and displacement are calculated by performing the time integration shown in section 3.3 with the acceleration obtained.

3.5 Calculation of strain

In plastic deformation, there is a state in which the deformation direction is reversed even if the length is the same when particles are under tension $\rightarrow$ unloading $\rightarrow$ compression. As shown in Fig. 3.4, a metal rod is pulled shown in (a) and extended to (b), and deformation stops in (c). Here, if the unloading direction is reversed and compression is performed, and the state goes to (d) where the length is the same as (b), and here the direction of deformation is opposite, with tensile stress acting in (b) and compressive stress acting in (d). In order to express such plastic deformation characteristics, the strain increment, which is a very small strain change, is used.

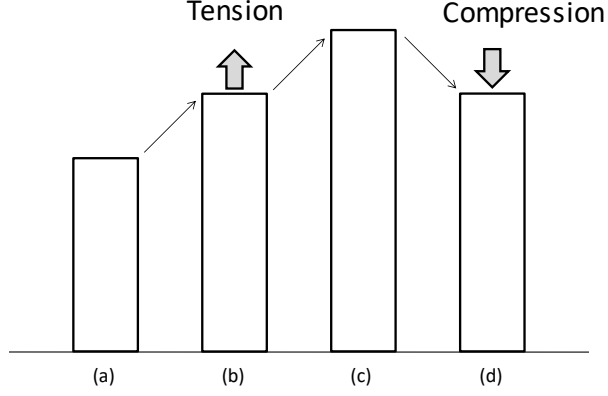


Fig 3.4 Plastic deformation

First, the strain rate tensor is

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (3.5.1)$$

From equation (3.5.1), the velocity strain tensor of the particle is expressed by the spatial gradient of the particle velocity v_i ($i = x, y, z$). Therefore, if the SPH particle approximation is used for this spatial gradient, the strain rate tensor $\dot{\epsilon}_{ij}^I$ of particle i is expressed by the following equation.

$$\dot{\epsilon}_{ij}^I = \frac{1}{2} \sum_{J=1}^N \left(\frac{m^J}{\rho^J} v_i^{JI} \frac{\partial W^{IJ}}{\partial x_j} + \frac{m^J}{\rho^J} v_j^{JI} \frac{\partial W^{IJ}}{\partial x_i} \right) \quad (3.5.2)$$

where

$$v_i^{JI} = v_i^J - v_i^I \quad (3.5.3)$$

In this study, the strain rate tensor of each particle is obtained using equation (3.5.2).

3.6 Calculation of stress

In the elasto-plastic analysis using the SPH method, the constitutive equation of the stress-strain relationship can adopt the same equation used by continuum mechanics such as the finite element method. There are two constitutive equations that give the relationship between stress and strain, the total strain theory and the strain increment theory. The SPH method uses the strain increment theory to solve the time expansion explicitly (the explicit method).

In addition, in this analysis, correction by co-rotational speed is introduced when coordinate transformation is performed. Specifically, when representing deformation of an object, there are two

coordinate systems: a reference coordinate system fixed in space and a local coordinate system fixed in the object point. To calculate interactions between particles and constitutive laws, the equations used in this study are finally converted to the reference coordinate system to calculate interactions between particles and constitutive laws. When the deformation is small, the difference between the reference coordinate system and the local coordinate system is not a problem. However, if the deformation of the member becomes large and the influence of the amount of rotation of the object cannot be ignored, there will be a gap between the reference coordinate system and the local coordinate system. For such deviations, it is necessary to use an algorithm that considers the local coordinate system rotating with the object point. Several methods have been proposed to correct the stress velocity form, such as the embedding speed and co-rotation speed.²⁾ The stress during the process was corrected by the following equation

$$\hat{\sigma}_{ij} = \dot{\sigma}_{ij} - W_{ik}\sigma_{kj} + \sigma_{ik}W_{kj} \quad (3.6.1)$$

where $\hat{\sigma}_{ij}$ is the stress increment in local coordinate system, W_{ik} and W_{kj} are the rotation tensor.

Specific stress calculation and material constitutive law used in this study will be described in the next chapter.

3.7 Calculation of contact force

For the interaction force between objects, the contact force is calculated. Other than SPH method, the impact force is calculated by a penalty method. When particles contact with each other, the overlap volume between the particles is calculated, then the contact force is calculated proportional to the overlap volume.

$$F_{contact} = V_{overlap} \cdot \frac{E_i + E_j}{2} \quad (3.7.1)$$

where $F_{contact}$ is the contact force, $V_{overlap}$ is the overlap volume between the contacting particle i and j , and E_i and E_j are the Young's modulus of particle i and j .

Especially, if two particles are positioned with smaller distance than h_0 at the beginning of the analysis, we defined that the two particles are "SPH-linked", and the SPH-link between them will break

when the distance between them is larger than the adaptive smoothing length h_a , or the particles enter the crushing state which is explained in the next chapter, as shown in Fig 3.5

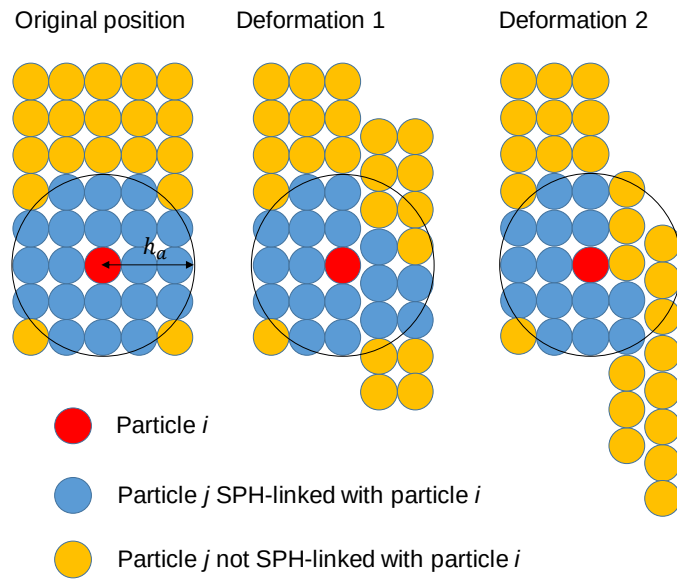


Fig 3.5 “SPH-link” during the analysis process

Since damage in solids is generally unrecoverable, and solid material such as concrete is not able to bond together due to the decrease of distance. Thus, if two particles are not SPH-linked at the beginning of the analysis, or SPH-link breaks by failure, no SPH link will be generated even if the distance between particles become smaller than h_a , and contact forces are calculated when the particles overlap occurs.

Chapter 4 Material constitutive law in impact analysis

4.1 Introduction

Concrete materials are complex with anisotropy and compression dependence, and the results of numerical analysis are likely to be unstable unless the appropriate material properties are introduced in detail. In addition to the elasto-plastic behavior, a crush criterion must be provided to reproduce the penetration phenomenon. Thus, here some of the mechanical model and constitutive law of concrete materials are listed and the appropriate ones are applied in the SPH analysis

4.2 Yielding conditions of materials

In the case of materials such as concrete and soil, it is known that compressive strength and tensile strength are different, and hydrostatic stress affects yield conditions. For such materials, it is necessary to set a yield function that can consider the influence of hydrostatic stress.

In general, von Mises yield conditions (Fig. 4.1) and Drucker-Prager yield conditions are used for isotropic materials such as iron materials that do not depend on compression. First of all, the von Mises material is probably the simplest yielding condition, not depending on the hydrostatic pressure, but only on the shear stress component deviation stress. On the other hand, the Drucker-Prager yield condition is a function that extends the von Mises yield criterion and includes the effect of hydrostatic pressure on the shear resistance of the material, meaning that a volume increase called dilatancy occurs during plastic deformation. Mohr-Coulomb yield condition is similar to Drucker-Prager, but the fracture surface called the π plane in the stress space is different, where the former represents a hexagon (hexagonal spindle), and the latter represents smooth circle (cone). In the existing experimental data, the π plane of Mohr-Coulomb yield condition is considered better, but in order to maintain stability, the Drucker-Prager fracture surface is used in this study. Each yield condition is shown in the followings.

- von Mises

$$f(J_2) = J_2 - k^2 = 0 \quad (4.2.1)$$

$$k = \frac{\sigma_y}{\sqrt{3}} \quad (4.2.2)$$

$$J_2 = \frac{1}{6} \{ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 \} + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2 \quad (4.2.3)$$

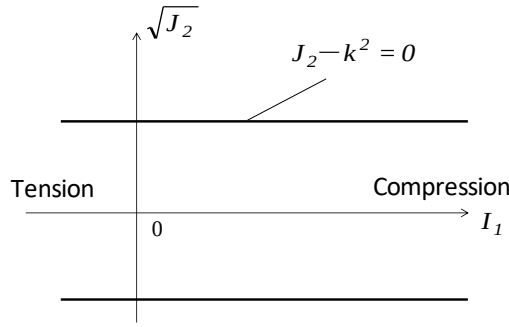


Fig 4.1 Yielding surface of von Mises

- Linear Drucker-Prager

$$f(J_2, I_1) = \sqrt{J_2} + \alpha I_1 - k = 0 \quad (4.2.4)$$

$$\alpha = \frac{(\sigma_c - \sigma_t)}{\sqrt{3}(\sigma_c + \sigma_t)}, \quad k = \frac{2\sigma_c\sigma_t}{\sqrt{3}(\sigma_c + \sigma_t)} \quad (4.2.5)$$

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33} \quad (4.2.6)$$

Where I_1 is the primary invariant of stress, J_2 is the secondary invariant of the deviatoric stress, and α and k are positive material constants. A and k are generally expressed by compressive strength σ_c and tensile strength σ_t . When $\alpha = 0$, Eq. (4.2.4) agrees with Eq. (4.2.1). In this case, σ_c and σ_t are positive values.

- non-linear Drucker-Prager

Also, under the linear Drucker-Prager yield condition, when an excessive hydrostatic pressure is applied, the plastic state of the material may not be evaluated properly. To solve this problem, in general, there are Bresler-Pister's three parameter model in octahedral stress evaluation¹⁾, Ottosen's four parameters model using I_1 , J_2 , $\cos 3\theta$ ²⁾, and Willam-Warke's five parameter models that considers even higher compressive stresses³⁾. The actual material tests are often limited in uniaxial compression

tests and uniaxial tensile tests, which makes it difficult to apply many various parameters, and these parameters are difficult to be introduced into the analysis model. Therefore, in this study, we used a nonlinear Drucker-Prager yield condition that can represent nonlinearity with two parameters, referring to previous studies⁴⁾.

The equation of nonlinear Drucker-Prager is shown below

$$f(J_2, I_1) = \sqrt{J_2} - \sqrt{\frac{\gamma^2 - \alpha\beta I_1}{3}} = 0 \quad (4.2.7)$$

$$\gamma = \sqrt{\sigma_c \sigma_t} \quad , \quad \beta = \sigma_c - \sigma_t \quad (4.2.8)$$

where α is a constant, and the yielding surface of Drucker-Prager is shown in Figure 4.2

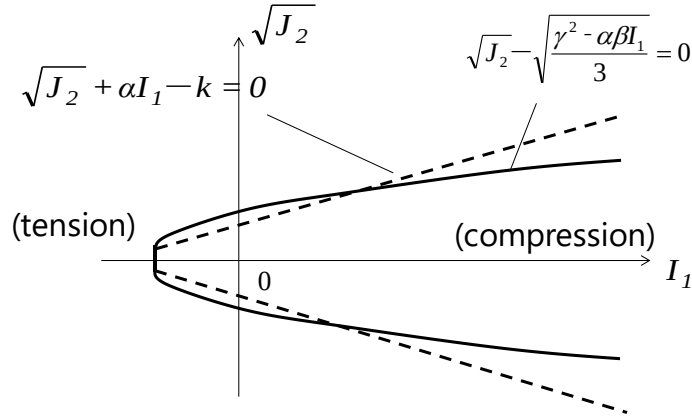


Fig 4.2 Yielding surface of linear and non-linear Drucker-Prager yielding criterion

4.3 Derivation of elastoplastic constitutive law

4.3.1 Derivation of stiffness matrix

The derivation of the elasto-plastic constitutive equation for concrete materials is shown below. The behavior after plasticity is assumed to follow the associated flow rule, and the generally used tensors and vector symbols are adopted here, and the explanation is omitted.

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (\text{Overlapping strain increments}) \quad (4.3.1)$$

$$d\varepsilon_{ij}^p = \Lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (\text{Associated flow rule}) \quad (4.3.2)$$

$$d\sigma_{ij} = D_{ijkl} d\varepsilon_{kl}^e \quad (\text{Hook's law}) \quad (4.3.3)$$

$$f = \sqrt{J_2} - \sqrt{\frac{\gamma^2 - \alpha\beta I_1}{3}} = 0 \quad (\text{Yield condition}) \quad (4.3.4)$$

$$df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \varepsilon_{ij}^p} d\varepsilon_{ij}^p = 0 \quad (\text{Total differential form of the yield function}) \quad (4.3.5)$$

The elastoplastic constitutive law can be derived by using these five equations.

Equation (4.3.4) depends on the yield condition used. In addition, since the contents of equation (4.3.4) do not expand until it is calculated in detail, the flow of calculation is the same for different yield conditions. When strain hardening is taken into account, equation (4.3.5) is used. When hardening is not taken into account, such as a complete elasto-plastic model, the second term is calculated as zero.

$$d\sigma_{ij} = D_{ijkl}(d\varepsilon_{kl} - d\varepsilon_{kl}^p) \quad \text{Substitute (4.3.1) into (4.3.3)} \quad (4.3.6)$$

$$d\sigma_{ij} = D_{ijkl}d\varepsilon_{kl} - D_{ijkl}\left(\Lambda \frac{\partial f}{\partial \sigma_{kl}}\right) \quad \text{Substitute (4.3.2)} \quad (4.3.7)$$

$$\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl}\left(\Lambda \frac{\partial f}{\partial \sigma_{kl}}\right) = \frac{\partial f}{\partial \sigma_{ij}} D_{ijst}d\varepsilon_{st} \quad \text{Transpose and multiply } \frac{\partial f}{\partial \sigma_{ij}} \text{ on both sides} \quad (4.3.8)$$

$$-\frac{\partial f}{\partial \varepsilon_{ij}^p} d\varepsilon_{ij}^p + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl}\left(\Lambda \frac{\partial f}{\partial \sigma_{kl}}\right) = \frac{\partial f}{\partial \sigma_{ij}} D_{ijst}d\varepsilon_{st} \quad \text{From (4.3.5)} \quad (4.3.9)$$

$$-\frac{\partial f}{\partial \varepsilon_{ij}^p} \left(\Lambda \frac{\partial f}{\partial \sigma_{ij}}\right) + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl}\left(\Lambda \frac{\partial f}{\partial \sigma_{kl}}\right) = \frac{\partial f}{\partial \sigma_{ij}} D_{ijst}d\varepsilon_{st} \quad \text{Substitute (4.3.2)} \quad (4.3.10)$$

$$\Lambda \left(-\frac{\partial f}{\partial \varepsilon_{ij}^p} \frac{\partial f}{\partial \sigma_{ij}}\right) + \Lambda \left(\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}\right) = \frac{\partial f}{\partial \sigma_{ij}} D_{ijst}d\varepsilon_{st} \quad (4.3.11)$$

$$\Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} D_{ijst}d\varepsilon_{st}}{-\frac{\partial f}{\partial \varepsilon_{mn}^p} \frac{\partial f}{\partial \sigma_{mn}} + \frac{\partial f}{\partial \sigma_{pq}} D_{pquv} \frac{\partial f}{\partial \sigma_{uv}}} \quad (4.3.12)$$

Substituting (4.3.3) into (4.3.12) and arrange with $d\varepsilon$

$$d\sigma_{ij} = D_{ijkl} \left(\delta_{ks} \delta_{lt} - \frac{\frac{\partial f}{\partial \sigma_{ij}} D_{ijst} \frac{\partial f}{\partial \sigma_{kl}}}{-\frac{\partial f}{\partial \varepsilon_{mn}^p} \frac{\partial f}{\partial \sigma_{mn}} + \frac{\partial f}{\partial \sigma_{pq}} D_{pquv} \frac{\partial f}{\partial \sigma_{uv}}} \right) d\varepsilon_{st} \quad (4.3.13)$$

Therefore, the stress increment formula with elasto-plastic behavior is

$$d\sigma_{ij} = \left(D_{ijst} - \frac{\frac{\partial f}{\partial \sigma_{ij}} D_{ijst} \frac{\partial f}{\partial \sigma_{kl}} D_{ijkl}}{-\frac{\partial f}{\partial \varepsilon_{mn}^p} \frac{\partial f}{\partial \sigma_{mn}} + \frac{\partial f}{\partial \sigma_{pq}} D_{pquv} \frac{\partial f}{\partial \sigma_{uv}}} \right) d\varepsilon_{st} \quad (4.3.14)$$

In other words, when expressed by the elastic stiffness matrix D_{ijst}^e and plastic stiffness matrix D_{ijst}^p , the material stiffness matrix is $\left(D_{ijst}^e - D_{ijst}^p\right)$ as shown in the following equation, showing that and the material stiffness is decreased by the plastic behavior.

$$d\sigma_{ij} = (D_{ijst}^e - D_{ijst}^p) d\varepsilon_{st} \quad (4.3.15)$$

where the elastic matrix D_{ijst}^e is

$$D_{ijst}^e = \mu(\delta_{is}\delta_{jt} + \delta_{it}\delta_{js}) + \lambda\delta_{ij}\delta_{st} \quad (4.3.16)$$

In the case of matrix expression, it is shown by the following equation, where μ and λ are Lamé's constants.

$$[D^e] = \begin{bmatrix} 2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\ & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\ & & 2\mu + \lambda & 0 & 0 & 0 \\ & & & 2\mu & 0 & 0 \\ & & & & 2\mu & 0 \\ \text{Sym.} & & & & & 2\mu \end{bmatrix}$$

For D_{ijst}^p , we have

$$[D_{ijst}]^p = \frac{\frac{\partial f}{\partial \sigma_{mn}} D_{mnst} \frac{\partial f}{\partial \sigma_{kl}} D_{ijkl}}{-\frac{\partial f}{\partial \varepsilon_{ij}^p} \frac{\partial f}{\partial \sigma_{ij}} + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}} \quad (4.3.17a)$$

Calculate the first term in the denominator in Equation (4.3.17a),

$$\begin{aligned} df &= \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \varepsilon_{ij}^p} d\varepsilon_{ij}^p = 0 \\ \left(\frac{\partial f}{\partial \sigma_{mn}} \frac{d\sigma_{mn}}{d\varepsilon_{ij}^p} + \frac{\partial f}{\partial \varepsilon_{ij}^p} \right) d\varepsilon_{ij}^p &= 0 \\ \frac{\partial f}{\partial \varepsilon_{ij}^p} &= -\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial \sigma_{mn}}{d\varepsilon_{ij}^p} \end{aligned}$$

Then the denominator can be written as

$$-\frac{\partial f}{\partial \varepsilon_{ij}^p} \frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial \sigma_{mn}}{d\varepsilon_{ij}^p} \frac{\partial f}{\partial \sigma_{ij}}$$

In plastic cases, $d\sigma_{mn} = D_{mnkl}^{ep} d\varepsilon_{kl}$, so

$$\begin{aligned} \frac{\partial \sigma_{mn}}{d\varepsilon_{ij}^p} &= \frac{\partial}{\partial \varepsilon_{ij}^p} D_{mnkl}^{ep} (d\varepsilon_{kl}^e + d\varepsilon_{kl}^p) \\ &= D_{mnkl}^{ep} \cdot (\delta_{ki}\delta_{lj}) = D_{mni j}^{ep} \end{aligned}$$

Then

$$[D_{ijst}]^p = \frac{\frac{\partial f}{\partial \sigma_{mn}} D_{mnst} \frac{\partial f}{\partial \sigma_{kl}} D_{ijkl}}{\frac{\partial f}{\partial \sigma_{mn}} D_{mni j}^{ep} \frac{\partial f}{\partial \sigma_{ij}} + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}} \quad (4.3.17b)$$

The first term of the denominator can be further simplified,

$$\frac{\partial f}{\partial \sigma_{mn}} D_{mnij}^{ep} \frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial f}{\partial \bar{\sigma}} H \frac{\partial f}{\partial \bar{\sigma}}$$

Then we have

$$[D_{ijst}]^p = \frac{\frac{\partial f}{\partial \sigma_{mn}} D_{mnst} \frac{\partial f}{\partial \sigma_{kl}} D_{ijkl}}{\frac{\partial f}{\partial \bar{\sigma}} H \frac{\partial f}{\partial \bar{\sigma}} + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}} \quad (4.3.17c)$$

Equation (4.3.17c) is the basic equation of the elasto-plastic constitutive law considering the plastic strain hardening. Incidentally, the term of hardening is the first term of the denominator of the plastic stiffness matrix term, and is zero in the case of the perfect elasto-plastic type.

First, we will describe how to calculate specific derivation using three types of yield conditions.

- von Mises

$$f = J_2 - k^2 = \frac{1}{2} \sigma'_{ij} \sigma'_{ij} - k^2 \quad (4.3.18)$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial f}{\partial \sigma'_{mn}} \cdot \frac{\partial \sigma'_{mn}}{\partial \sigma_{ij}} \quad (4.3.19)$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \sigma'_{ij} \quad (4.3.20)$$

where the second invariant of deviatoric stress σ'_{ij} is

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{mm} \delta_{ij} \quad (4.3.21)$$

- linear Drucker-Prager

$$f = \sqrt{J_2} + \alpha I_1 - k = \frac{1}{2} \sigma'_{ij} \sigma'_{ij} - \sigma_{ij} \delta_{ij} - k \quad (4.3.22)$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial f}{\partial J_2} \cdot \frac{\partial J_2}{\partial \sigma'_{mn}} \cdot \frac{\partial \sigma'_{mn}}{\partial \sigma_{ij}} + \frac{\partial f}{\partial I_1} \cdot \frac{\partial I_1}{\partial \sigma_{ij}} \quad (4.3.23)$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\sigma'_{ij}}{2\sqrt{J_2}} + \alpha \delta_{ij} \quad (4.3.24)$$

- non-linear Drucker-Prager

$$f = \sqrt{J_2} - \sqrt{\frac{\gamma^2 - \alpha \beta I_1}{3}} \quad (4.3.25)$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial f}{\partial J_2} \cdot \frac{\partial J_2}{\partial \sigma'_{mn}} \cdot \frac{\partial \sigma'_{mn}}{\partial \sigma_{ij}} + \frac{\partial f}{\partial I_1} \cdot \frac{\partial I_1}{\partial \sigma_{ij}} \quad (4.3.26)$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\sigma'_{ij}}{2\sqrt{J_2}} + \frac{\beta}{2\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{ij} \quad (4.3.27)$$

The detailed calculation process of the material constitutive law is described only for the nonlinear Drucker-Prager used mainly in this study. First, the specific calculation of the plastic stiffness matrix is performed.

$$[D_{ijst}]^p = \frac{\frac{\partial f}{\partial \sigma_{mn}} D_{mnst} \frac{\partial f}{\partial \sigma_{kl}} D_{ijkl}}{\frac{\partial f}{\partial \bar{\sigma}} H \frac{\partial f}{\partial \bar{\sigma}} + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}} \quad (4.3.28)$$

Since the equation is complicated, the calculation will start from the first term $\frac{\partial f}{\partial \sigma_{mn}}$ in numerator

$$\frac{\partial f}{\partial \sigma_{mn}} = \frac{\partial f(I_1, J_2)}{\partial \sigma_{mn}} = \frac{\partial f}{\partial J_2} \frac{\partial J_2}{\partial \sigma'_{kl}} \frac{\partial \sigma'_{kl}}{\partial \sigma_{mn}} + \frac{\partial f}{\partial I_1} \frac{\partial I_1}{\partial \sigma_{mn}} \quad (4.3.29)$$

For $\frac{\partial f}{\partial J_2}$ in Equation (4.3.29), we have

$$\frac{\partial f}{\partial J_2} = \frac{1}{2\sqrt{J_2}} \quad (4.3.30)$$

For $\frac{\partial J_2}{\partial \sigma'_{kl}}$ in Equation (4.3.29), we have

$$\begin{aligned} \frac{\partial J_2}{\partial \sigma'_{kl}} &= \frac{\partial}{\partial \sigma'_{kl}} \left(\frac{\sigma'_{ij} \sigma'_{ij}}{2} \right) \\ &= \frac{1}{2} (\delta_{ik} \delta_{jl} \sigma'_{ij} + \delta_{ik} \delta_{jl} \sigma'_{ij}) \\ &= \sigma'_{kl} \end{aligned} \quad (4.3.31)$$

And for $\frac{\partial \sigma'_{kl}}{\partial \sigma_{mn}}$ in Equation (4.3.29),

$$\begin{aligned} \frac{\partial \sigma'_{kl}}{\partial \sigma_{mn}} &= \frac{\partial}{\partial \sigma_{mn}} \left(\sigma_{kl} - \frac{1}{3} \sigma_{tt} \delta_{kl} \right) \\ &= \delta_{km} \delta_{ln} - \frac{1}{3} \delta_{tm} \delta_{tn} \delta_{kl} \end{aligned} \quad (4.3.32)$$

Then for $\frac{\partial f}{\partial J_2} \frac{\partial J_2}{\partial \sigma'_{kl}} \frac{\partial \sigma'_{kl}}{\partial \sigma_{mn}}$ in Equation (4.3.29), we have

$$\frac{\partial f}{\partial J_2} \frac{\partial J_2}{\partial \sigma'_{kl}} \frac{\partial \sigma'_{kl}}{\partial \sigma_{mn}} = \frac{1}{2\sqrt{J_2}} \left(\sigma'_{kl} \delta_{km} \delta_{ln} - \frac{1}{3} \delta_{tm} \delta_{tn} \delta_{kl} \right) \quad (4.3.33)$$

$$\begin{aligned}
&= \frac{1}{2\sqrt{J_2}} \left(\sigma'_{mn} - \frac{1}{3} \delta_{tm} \delta_{tn} \left(\sigma_{kl} \delta_{kl} - \frac{1}{3} \sigma_{mn} \delta_{tm} \delta_{tn} \right) \right) \\
&= \frac{1}{2\sqrt{J_2}} \left(\sigma'_{mn} - \frac{1}{3} \delta_{tm} \delta_{tn} (\sigma_{mm} - \sigma_{nn}) \right) \\
&= \frac{\sigma'_{mn}}{2\sqrt{J_2}}
\end{aligned}$$

For the second term $\frac{\partial f}{\partial I_1} \frac{\partial I_1}{\partial \sigma_{mn}}$ in the right hand side of Equation (4.3.29), we have

$$\begin{aligned}
\frac{\partial f}{\partial I_1} &= \frac{\partial \left(\sqrt{J_2} - \sqrt{\frac{\gamma^2 - \alpha\beta I_1}{3}} \right)}{\partial I_1} \\
&= -\frac{1}{2} \frac{1}{\sqrt{\frac{\gamma^2 - \alpha\beta I_1}{3}}} \left(\frac{\gamma^2 - \alpha\beta I_1}{3} \right)' \\
&= -\frac{1}{2} \frac{1}{\sqrt{\frac{\gamma^2 - \alpha\beta I_1}{3}}} \left(-\frac{1}{3} \alpha\beta \right) \\
&= \frac{1}{2\sqrt{3}} \frac{\alpha\beta}{\sqrt{\gamma^2 - \alpha\beta I_1}}
\end{aligned} \tag{4.3.34}$$

Then for $\frac{\partial I_1}{\partial \sigma_{mn}}$

$$\frac{\partial I_1}{\partial \sigma_{mn}} = \frac{\partial \sigma_{kk}}{\partial \sigma_{mn}} = \delta_{km} \delta_{kn} = \delta_{mn} \tag{4.3.35}$$

And we have

$$\frac{\partial f}{\partial I_1} \frac{\partial I_1}{\partial \sigma_{mn}} = \frac{1}{2\sqrt{3}} \frac{\alpha\beta}{\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{mn} \tag{4.3.36}$$

Substitute (4.3.33) (4.3.36) into (4.3.29)

$$\begin{aligned}
\frac{\partial f(I_1, J_2)}{\partial \sigma_{mn}} &= \frac{\partial f}{\partial J_2} \frac{\partial J_2}{\partial \sigma'_{kl}} \frac{\partial \sigma'_{mn}}{\partial \sigma_{mn}} + \frac{\partial f}{\partial I_1} \frac{\partial I_1}{\partial \sigma_{mn}} \\
&= \frac{\sigma'_{mn}}{2\sqrt{J_2}} + \frac{1}{2\sqrt{3}} \frac{\alpha\beta}{\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{mn}
\end{aligned} \tag{4.3.37}$$

Finally, Equation (4.3.29) can be arranged as

$$\frac{\partial f(I_1, J_2)}{\partial \sigma_{mn}} = \frac{\sigma'_{mn}}{2\sqrt{J_2}} + \frac{1}{2\sqrt{3}} \frac{\alpha\beta}{\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{mn} \tag{4.3.38}$$

Next, for $\frac{\partial f}{\partial \sigma_{mn}} D_{mnst}$ in the numerator of (4.3.28),

$$\begin{aligned}
\frac{\partial f(I_1, J_2)}{\partial \sigma_{mn}} D_{mnst} &= \left(\frac{\sigma'_{mn}}{2\sqrt{J_2}} + \frac{1}{2\sqrt{3}} \frac{\alpha\beta}{\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{mn} \right) \{ \mu(\delta_{ms}\delta_{nt} + \delta_{mt}\delta_{ns}) + \lambda\delta_{mn}\delta_{st} \} \\
&= \frac{\mu}{2\sqrt{J_2}} (\sigma'_{st} + \sigma'_{ts}) + \frac{\lambda}{2\sqrt{J_2}} \delta_{mn}\delta_{st}\sigma'_{mn} \\
&\quad + \frac{\alpha\beta\mu}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} (\delta_{st} + \delta_{ts}) \\
&\quad + \frac{\alpha\beta\lambda}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{mn}\delta_{mn}\delta_{st} \\
&= \frac{\mu}{\sqrt{J_2}} \sigma'_{st} + \frac{2\alpha\beta\mu}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{st} + \frac{3\alpha\beta\lambda}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{st} \\
&= \frac{\mu}{\sqrt{J_2}} \sigma'_{st} + \frac{\alpha\beta}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} (2\mu + 3\lambda) \delta_{st}
\end{aligned} \tag{4.3.39}$$

For the denominator of (4.3.28), when plastic deformation occurs, $f = \sqrt{J_2} - \sqrt{\frac{\gamma^2 - \alpha\beta I_1}{3}} = 0$

Change the equation to uniaxial space, and use $\bar{\sigma}$ for the equivalent stress

$$\begin{aligned}
\gamma^2 &= \bar{\sigma}^2 + \alpha\beta\bar{\sigma} \\
\frac{\partial f}{\partial \bar{\sigma}} &= \frac{2\sqrt{\gamma^2 - \alpha\beta\bar{\sigma}} + \alpha\beta}{2\sqrt{3} \cdot \sqrt{\gamma^2 - \alpha\beta\bar{\sigma}}} = \frac{2\bar{\sigma} + \alpha\beta}{2\sqrt{3}\bar{\sigma}} \\
\frac{\partial f}{\partial \bar{\sigma}} H \frac{\partial f}{\partial \bar{\sigma}} &= \frac{(2\bar{\sigma} + \alpha\beta)^2}{12\bar{\sigma}^2} \cdot H
\end{aligned} \tag{4.3.40}$$

For the second term of the denominator of (4.3.28),

$$\begin{aligned}
\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} &= \left(\frac{\sigma'_{ij}}{2\sqrt{J_2}} + \frac{\alpha\beta}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{ij} \right) \left\{ \frac{\mu}{\sqrt{J_2}} \sigma'_{ij} \right. \\
&\quad \left. + \frac{\alpha\beta}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} (2\mu + 3\lambda) \delta_{ij} \right\} \\
&= \frac{\mu}{2J_2} \sigma'_{ij} \sigma'_{ij} + \frac{\alpha\beta(2\mu + 3\lambda)}{4\sqrt{3}J_2\sqrt{\gamma^2 - \alpha\beta I_1}} \sigma'_{ij} \delta_{ij} + \frac{\mu\alpha\beta}{2\sqrt{3}J_2\sqrt{\gamma^2 - \alpha\beta I_1}} \sigma'_{ij} \delta_{ij} \\
&\quad + \frac{\alpha^2\beta^2(2\mu + 3\lambda)}{12(\gamma^2 - \alpha\beta I_1)} \delta_{ij} \delta_{ij} \\
&= \mu + \frac{\alpha^2\beta^2(2\mu + 3\lambda)}{4(\gamma^2 - \alpha\beta I_1)}
\end{aligned} \tag{4.3.41}$$

Assemble everything together, we can rearrange (4.3.28) as

$$\begin{aligned}
(\text{numerator}) &= \frac{\partial f}{\partial \sigma_{mn}} D_{mnst} \frac{\partial f}{\partial \sigma_{kl}} D_{ijkl} \\
&= \left\{ \frac{\mu}{\sqrt{J_2}} \sigma'_{st} + \frac{\alpha\beta}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} (2\mu + 3\lambda) \delta_{st} \right\} \left\{ \frac{\mu}{\sqrt{J_2}} \sigma'_{ij} \right. \\
&\quad \left. + \frac{\alpha\beta}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} (2\mu + 3\lambda) \delta_{ij} \right\}
\end{aligned} \tag{4.3.42}$$

$$\begin{aligned}
(\text{denominator}) &= -\frac{\partial f}{\partial \varepsilon_{ij}^p} \frac{\partial f}{\partial \sigma_{ij}} + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} \\
&= \frac{(2\bar{\sigma} + \alpha\beta)^2}{12\bar{\sigma}^2} \cdot H + \mu + \frac{\alpha^2\beta^2(2\mu + 3\lambda)}{4(\gamma^2 - \alpha\beta I_1)}
\end{aligned} \tag{4.3.43}$$

For the tensor form,

$$\begin{aligned}
[D_{ijst}]^p &= \frac{1}{\frac{(2\bar{\sigma} + \alpha\beta)^2}{12\bar{\sigma}^2} \cdot H + \mu + \frac{3\alpha\beta C}{B^2}} \times \\
&\begin{bmatrix} (Q\sigma'_{11} + R)^2 & (Q\sigma'_{11} + R)(Q\sigma'_{22} + R) & (Q\sigma'_{11} + R)(Q\sigma'_{33} + R) & (Q\sigma'_{11} + R)Q\sigma'_{12} & (Q\sigma'_{11} + R)Q\sigma'_{23} & (Q\sigma'_{11} + R)\sigma'_{31} \\ \vdots & (Q\sigma'_{22} + R)^2 & (Q\sigma'_{22} + R)(Q\sigma'_{33} + R) & (Q\sigma'_{22} + R)Q\sigma'_{12} & (Q\sigma'_{22} + R)Q\sigma'_{23} & (Q\sigma'_{22} + R)\sigma'_{31} \\ \vdots & \vdots & (Q\sigma'_{33} + R)^2 & (Q\sigma'_{33} + R)Q\sigma'_{12} & (Q\sigma'_{33} + R)Q\sigma'_{23} & (Q\sigma'_{33} + R)\sigma'_{31} \\ \vdots & \vdots & \vdots & (Q\sigma'_{12})^2 & Q^2\sigma'_{12}\sigma'_{23} & Q^2\sigma'_{12}\sigma'_{31} \\ \vdots & \vdots & \vdots & \vdots & (Q\sigma'_{23})^2 & Q^2\sigma'_{23}\sigma'_{31} \\ \text{Sym.} & \dots & \dots & \dots & \dots & (Q\sigma'_{31})^2 \end{bmatrix}
\end{aligned} \tag{4.3.44}$$

where,

$$B = 2\sqrt{\gamma^2 - \alpha\beta I_1}$$

$$C = \alpha\beta(2\mu + 3\lambda)$$

$$Q = \frac{\mu}{\sqrt{J_2}}$$

$$R = \frac{C}{B}$$

$$\beta = \sigma_c - \sigma_t$$

$$\gamma = \sqrt{\sigma_c \sigma_t}$$

4.3.2 Calculation of associated flow rules and plastic strain increments

First, as the behavior after plasticity, the plastic flow goes in the direction normal to the yield surface as shown in Fig 4.3. In other words, the stress of each component must correspond to the direction of plastic strain, which is called the associated flow rule. Incidentally, if the yield criterion depends on compression, and there is a gradient in the stress space, so volume plastic strain exists. In other words,

volume plastic strain does not exist on von Mises yield surface, but exists on yield surfaces such as Drucker-Prager. Fig 4.3 shows the relationship between plastic strain and yield surface.

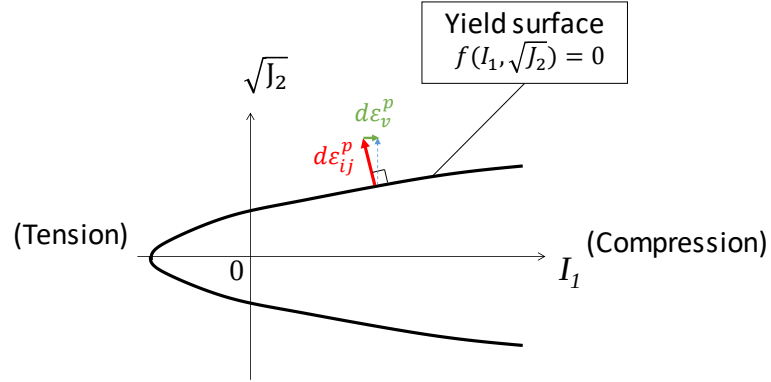


Fig 4.3 Associated flow rule (example)

In addition, as shown in Fig. 4.4, there is a method called non-associated flow rule that calculates plastic strain using a yield function (plastic potential function) g that is different from the yield function f . In this case, the calculation only needs to replace the related flow law function in Eq. (4.3.10) as shown in the following equation.

$$d\epsilon_{ij}^p = \Lambda \frac{\partial g}{\partial \sigma_{ij}} \quad (4.3.45)$$

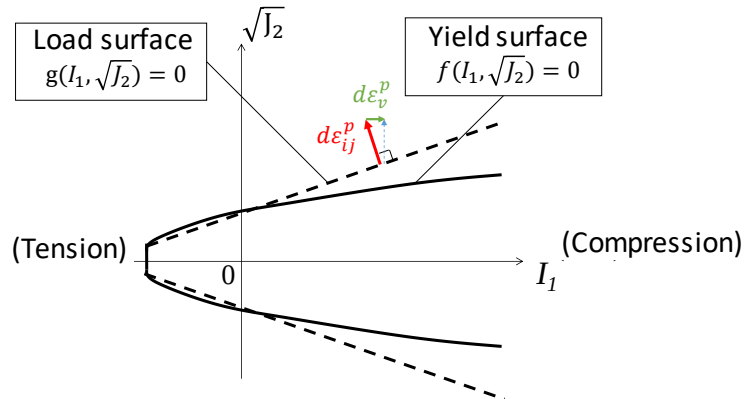


Fig 4.4 Non-associated flow rule (example)

In this study, in order to obtain stable analysis results, steel and concrete are regarded as homogeneous materials, and the related flow law is adopted. The plastic strain increment is also calculated using the associated flow rule mentioned above.

$$d\varepsilon_{ij}^p = \Lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (4.3.2)$$

(repost)

$$\Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} d\varepsilon_{kl}}{-\frac{\partial f}{\partial \varepsilon_{ij}^p} \frac{\partial f}{\partial \sigma_{ij}} + \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}}} \quad (4.3.12)$$

(repost)

Since the denominator in Λ has already been described in equations (4.3.43), the numerator is calculated.

$$(\Lambda \text{ numerator}) = \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} d\varepsilon_{kl} \quad (4.3.46)$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\sigma'_{ij}}{2\sqrt{J_2}} + \frac{1}{2\sqrt{3}} \frac{\alpha\beta}{\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{ij} \quad (4.3.47)$$

$$\begin{aligned} \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} &= \left(\frac{\sigma'_{ij}}{2\sqrt{J_2}} + \frac{\beta}{2\sqrt{3}\sqrt{\gamma^2 - \beta I_1}} \delta_{ij} \right) \{ \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \lambda\delta_{ij}\delta_{kl} \} \\ &= \frac{\mu}{2\sqrt{J_2}} (\sigma'_{kl} + \sigma'_{lk}) + \frac{\lambda}{2\sqrt{J_2}} \delta_{ij}\delta_{kl}\sigma'_{ij} + \frac{\beta\mu}{2\sqrt{3}\sqrt{\gamma^2 - \beta I_1}} (\delta_{kl} + \delta_{lk}) \\ &\quad + \frac{\beta\lambda}{2\sqrt{3}\sqrt{\gamma^2 - \beta I_1}} \delta_{ij}\delta_{ij}\delta_{kl} \\ &= \frac{\mu}{\sqrt{J_2}} \sigma'_{kl} + \frac{2\beta\mu}{2\sqrt{3}\sqrt{\gamma^2 - \beta I_1}} \delta_{kl} + \frac{3\beta\lambda}{2\sqrt{3}\sqrt{\gamma^2 - \beta I_1}} \delta_{kl} \\ &= \frac{\mu}{\sqrt{J_2}} \sigma'_{kl} + \frac{\beta}{2\sqrt{3}\sqrt{\gamma^2 - \beta I_1}} (2\mu + 3\lambda) \delta_{kl} \end{aligned} \quad (4.3.48)$$

Then

$$\begin{aligned} \frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} d\varepsilon_{kl} &= \left\{ \frac{\mu}{\sqrt{J_2}} \sigma'_{kl} + \frac{\alpha\beta}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} (2\mu + 3\lambda) \delta_{kl} \right\} d\varepsilon_{kl} \\ &= \frac{\mu}{\sqrt{J_2}} \sigma'_{kl} d\varepsilon_{kl} + \frac{\alpha\beta}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} (2\mu + 3\lambda) d\varepsilon_v \end{aligned} \quad (4.3.49)$$

$$(d\varepsilon_{kl}\delta_{kl} = d\varepsilon_{kk} = d\varepsilon_v) \text{ (increment of volumetric strain)}$$

Substitute into (4.3.2) and rearrange with the notation in (4.3.44)

$$\begin{aligned} d\varepsilon_{ij}^p &= \Lambda \frac{\partial f}{\partial \sigma_{ij}} \\ &= \frac{Q\sigma'_{kl}d\varepsilon_{kl} + Rd\varepsilon_v}{\frac{(2\bar{\sigma} + \alpha\beta)^2}{12\bar{\sigma}^2} \cdot H + \mu + \frac{3\alpha\beta C}{B^2}} \left(\frac{\sigma'_{ij}}{2\sqrt{J_2}} + \frac{\alpha\beta}{B} \delta_{ij} \right) \end{aligned} \quad (4.3.50)$$

Or in tensor form,

$$d\varepsilon_{ij}^p = \frac{Q\sigma'_{kl}d\varepsilon_{kl} + Rd\varepsilon_v}{\frac{(2\bar{\sigma} + \alpha\beta)^2}{12\bar{\sigma}^2} \cdot H + \mu + \frac{3\alpha\beta C}{B^2}} \begin{bmatrix} \frac{\sigma'_{11}}{2\sqrt{J_2}} + \frac{\alpha\beta}{B} & \frac{\sigma'_{12}}{2\sqrt{J_2}} & \frac{\sigma'_{13}}{2\sqrt{J_2}} \\ \vdots & \frac{\sigma'_{22}}{2\sqrt{J_2}} + \frac{\alpha\beta}{B} & \frac{\sigma'_{23}}{2\sqrt{J_2}} \\ \text{Sym.} & \dots & \frac{\sigma'_{33}}{2\sqrt{J_2}} + \frac{\alpha\beta}{B} \end{bmatrix} \quad (4.3.51)$$

4.3.3 Calculation of equivalent stress and equivalent plastic strain

The calculation of equivalent plastic strain in this study is shown. Equivalent value is a value that can be easily evaluated as a scalar quantity by expressing the stress and strain acting on multiple axes as on a single axis. This value can be used as a simple judgment value for plasticity.

(1) Calculation of equivalent stress

For nonlinear Drucker-Prager yielding criterion, we have,

$$f = \sqrt{J_2} - \sqrt{\frac{\gamma^2 - \alpha\beta I_1}{3}} \quad (4.3.25)$$

(repost)

Substitute with the uniaxial yielding stress,

$$f = \frac{\sigma_y}{\sqrt{3}} - \sqrt{\frac{\gamma^2 - \alpha\beta\sigma_y}{3}} \quad (4.3.25)$$

Since $\gamma^2 = \sigma_y^2 + \alpha\beta\sigma_y$

$$f = \sqrt{J_2} - \sqrt{\frac{\sigma_y^2 \pm \alpha\beta\sigma_y - \alpha\beta I_1}{3}}$$

When yielding

$$f = \sqrt{J_2} - \sqrt{\frac{\sigma_y^2 + \alpha\beta\sigma_y - \alpha\beta I_1}{3}} = 0$$

Rearrange,

$$\sigma_y^2 + \alpha\beta\sigma_y - (3J_2 + \alpha\beta I_1) = 0$$

Solve this equation, we have

$$\sigma_y = \frac{-\alpha\beta \pm \sqrt{\alpha^2\beta^2 + 4(3J_2 + \alpha\beta I_1)}}{2}$$

Adopting the case where the second term of the numerator is positive, the equivalent stress can be shown as

$$\bar{\sigma} = \frac{-\alpha\beta + \sqrt{\alpha^2\beta^2 + 4(3J_2 + \alpha\beta I_1)}}{2}$$

Especially,

$$\bar{\sigma} = \begin{cases} \frac{-\alpha\beta + \sqrt{\alpha^2\beta^2 + 4(3J_2 + \alpha\beta I_1)}}{2}, & I_1 \geq 0 \\ \frac{-\alpha\beta + \sqrt{\alpha^2\beta^2 + 4(3J_2 - \alpha\beta I_1)}}{2}, & I_1 < 0 \end{cases} \quad (4.3.52)$$

(2) Calculation of equivalent plastic strain

To calculate plastic energy, we have

$$\begin{aligned} W^p &= \sigma_{ij} d\varepsilon_{ij}^p = \bar{\sigma} d\bar{\varepsilon}^p \\ \sigma_{ij} \Lambda \frac{\partial f}{\partial \sigma_{ij}} &= \bar{\sigma} d\bar{\varepsilon}^p \\ \Lambda &= \frac{\bar{\sigma} d\bar{\varepsilon}^p}{\sigma_{ij} \frac{\partial f}{\partial \sigma_{ij}}} \end{aligned} \quad (4.3.53)$$

Since $d\varepsilon_{ij}^p = \Lambda \frac{\partial f}{\partial \sigma_{ij}}$,

$$\begin{aligned} d\varepsilon_{ij}^p d\varepsilon_{ij}^p &= \Lambda^2 \frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}} \\ d\varepsilon_{ij}^p d\varepsilon_{ij}^p &= \left(\frac{\bar{\sigma} d\bar{\varepsilon}^p}{\sigma_{mn} \frac{\partial f}{\partial \sigma_{mn}}} \right)^2 \frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}} \\ (d\bar{\varepsilon}^p)^2 &= \left(\frac{\sigma_{mn} \frac{\partial f}{\partial \sigma_{mn}}}{\bar{\sigma}} \right)^2 \frac{d\varepsilon_{ij}^p d\varepsilon_{ij}^p}{\frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}}} \end{aligned}$$

Then

$$d\bar{\varepsilon}^p = \frac{\sigma_{mn} \frac{\partial f}{\partial \sigma_{mn}}}{\bar{\sigma}} \sqrt{\frac{d\varepsilon_{ij}^p d\varepsilon_{ij}^p}{\frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}}}}$$

For $\sigma_{mn} \frac{\partial f}{\partial \sigma_{mn}}$

$$\begin{aligned} \sigma_{mn} \frac{\partial f}{\partial \sigma_{mn}} &= \left(\frac{1}{3} \sigma_{ii} \delta_{mn} + \sigma'_{mn} \right) \left(\frac{\sigma'_{mn}}{2\sqrt{J_2}} + \frac{1}{2\sqrt{3}} \frac{\alpha\beta}{\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{mn} \right) \\ &= \frac{1}{6\sqrt{J_2}} \sigma_{ii} \delta_{mn} \sigma'_{mn} + \frac{\alpha\beta I_1}{6\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{mn} \delta_{mn} + \frac{\sigma'_{mn} \sigma'_{mn}}{2\sqrt{J_2}} \\ &\quad + \frac{\alpha\beta}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} \delta_{mn} \sigma'_{mn} \end{aligned} \quad (4.3.54)$$

$$= \frac{\alpha\beta I_1}{2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}} + \sqrt{J_2}$$

$$\text{Take } B = 2\sqrt{3}\sqrt{\gamma^2 - \alpha\beta I_1}$$

$$\sigma_{mn} \frac{\partial f}{\partial \sigma_{mn}} = \frac{\alpha\beta I_1}{B} + \sqrt{J_2}$$

$$\text{For } \frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}}$$

$$\frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}} = \frac{1}{2} + \frac{\alpha^2 \beta^2}{4(\gamma^2 - \beta I_1)} = \frac{1}{2} + \frac{3\alpha^2 \beta^2}{B^2}$$

To summarize,

$$d\bar{\varepsilon}^p = \frac{\frac{\alpha\beta I_1}{B} + \sqrt{J_2}}{\bar{\sigma}} \sqrt{\frac{d\varepsilon_{ij}^p d\varepsilon_{ij}^p}{\frac{1}{2} + \left(\frac{\sqrt{3}\alpha\beta}{B}\right)^2}} \quad (4.3.55)$$

where

$$B = 2\sqrt{3}\sqrt{\gamma^2 - \beta I_1}$$

$$\beta = \sigma_c - \sigma_t$$

$$\gamma = \sqrt{\sigma_c \sigma_t}$$

4.4 Softening behavior and damage of concrete materials

4.4.1 Tensile softening behavior and tensile damage

In general, on the tensile side of concrete materials, as the strain increases, the stress first reaches the tensile strength, then begin to decrease. This is called tensile softening. To explained this behavior, fine cracks occur in the area of tensile strength, and damage is concentrated in the weakest part to form a fine crack region. Finally, a large crack is generated, and the cross section is reduced and stress transmitted is also reduced. A schematic diagram of the calculation flow of softening and damage is shown in Fig 4.5. This is the same flow on both the compression and tension sides.

In this study, as shown in Fig 4.6a, the stress-strain relationship after tensile strength is forcibly reduced (BC) when the path due to incremental stress (AB) deviates from the path of tensile softening.

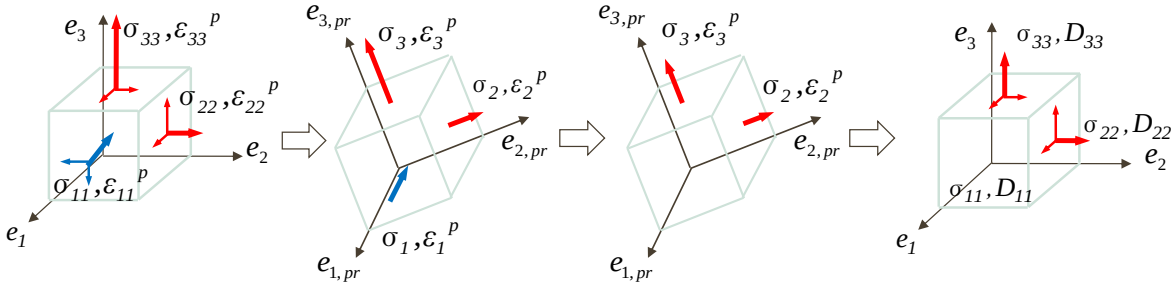


Fig 4.5 Stress softening and damage calculation

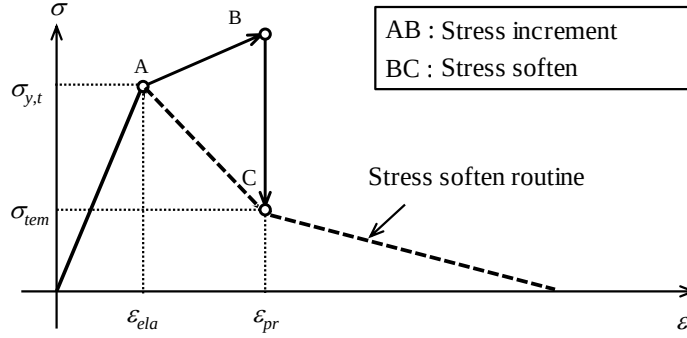


Fig 4.6a Calculation of stress softening

• Tensile stress softening calculation

- Calculate principal value and principal direction of stress and strain tensor of each particle
- When a principal stress exceeds the assumed stress-strain relationship of tensile softening, the temporary principal stress value σ_{tem} is substituted on the assumed path shown in Fig 4.6b.

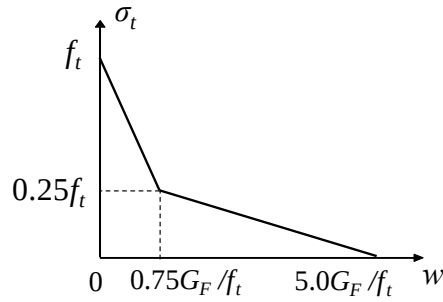


Fig 4.6b Assumed path of stress softening

where σ_t is the tensile stress, w is crack width, f_t is the tensile strength of concrete, and G_F is the fracture energy of concrete.

- The new stress tensor containing modified principal stress is then converted to the global coordinate system using the principal direction.

- Calculation of tensile damage

The tensile softening behavior assumes that micro cracks occur after the tensile strength. Therefore, since the stiffness corresponding to the direction of cracking is reduced compared to a healthy state (no cracking), it is necessary to construct an elasto-plastic matrix that considering the anisotropy due to such damage (cracking). Therefore, in this study, the damage was calculated with a sigmoid function using cumulative plastic strain with reference to damage mechanics⁵⁾, as shown in Fig 4.7.

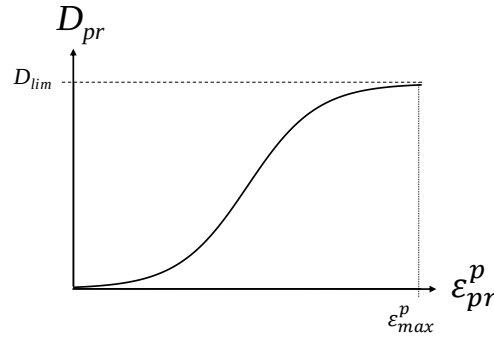


Fig 4.7 Calculation of tension damage

- Add up the plastic strain increments calculated at each step to calculate the cumulative plastic strain ε_{ij}^p
- Calculate the cumulative plastic strain ε_{pr}^p in the principle direction, and calculate the damage in each principle direction according to tensile softening using the following equation.

$$D_{pr} = \frac{D_{lim}}{1 + \exp\left[-\alpha\left(\varepsilon_{pr}^p - \frac{\varepsilon_{max}^p}{2}\right)\right]}, \quad (0 \leq D_{pr} \leq 0.4) \quad (4.4.1)$$

where α : coefficient, D_{lim} : upper limit of tensile damage, ε_{max}^p : limit of cumulative plastic strain. In this study $\alpha = 300$. Also, D_{lim} , which varies depending on the material, is 0.3 to 0.5 for concrete, and is set 0.4 in this study. ε_{max}^p is assumed to be 0.03 where cracks are clearly recognized by tensile failure. D_{pr} is the damage in each principal direction. ε_{pr}^p will not be calculated in compression.

- Calculate the degree of damage on each axis in the global coordinate system using the following formula:

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \left| D_{1,pr} \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix} \right| + \left| D_{2,pr} \begin{Bmatrix} x_2 \\ y_2 \\ z_2 \end{Bmatrix} \right| + \left| D_{3,pr} \begin{Bmatrix} x_3 \\ y_3 \\ z_3 \end{Bmatrix} \right|, \quad e_{i,pr} = \{x_i \ y_i \ z_i\}^t \quad (4.4.2)$$

$$D_i = \sum_{j=1}^3 |D_{i,pr} e_{j,pr}|$$

where, D_i ($i = 1, 2, 3$) is the damage of each axis converted to the global coordinate system, and $e_{i,pr}$ ($i = 1, 2, 3$) is the principal direction vector. The principal direction vector $e_{i,pr}$ is normalized.

(d) The soundness of each direction in the global coordinate system is calculated using the following formula.

$$\phi_i = 1 - D_i, \quad (0.6 \leq \phi_i \leq 1) \quad (4.4.3)$$

Here, ϕ_i ($i = 1, 2, 3$) is the soundness of each axis in the global coordinate system.

(e) Calculate the reduction rate of the elastic matrix due to damage using the following formula

$$\alpha_{ij} = \sqrt{\phi_i \phi_j} \quad (4.4.4)$$

Ignacio Carol, Egidio Rizzi, Kaspar Willam and others have proposed a constitutive equation that considers anisotropy simply by the stiffness reduction in each axial direction in the conventional elastic constitutive law⁽⁶⁾. Also, by applying their method, local material anisotropy near the crack is considered. Equation (4.4.5) shows a simple constitutive equation expressing anisotropy proposed by Ignacio Carol, Egidio Rizzi, Kaspar Willam, and others. This reduction rate is irreversible.

$$[D^e] = \begin{bmatrix} (\lambda + 2\mu) \times \alpha_{11} & \lambda \times \alpha_{12} & \lambda \times \alpha_{31} & 0 & 0 & 0 \\ \lambda \times \alpha_{12} & (\lambda + 2\mu) \times \alpha_{22} & \lambda \times \alpha_{23} & 0 & 0 & 0 \\ \lambda \times \alpha_{31} & \lambda \times \alpha_{23} & (\lambda + 2\mu) \times \alpha_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu \times \alpha_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu \times \alpha_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu \times \alpha_{31} \end{bmatrix} \quad (4.4.5)$$

By the above calculation flow, an elastic matrix considering the damage is created using the reduction rate obtained, and the elastic-plastic matrix in the tensile softening behavior is calculated by subtracting the plastic matrix from the created elastic matrix. This elasto-plastic matrix takes stress softening and damage into account, and by correcting the stiffness with this matrix, the expression of cracks with anisotropy is introduced in this study.

4.4.2 Compressive softening behavior and compression damage

On the compression of the concrete material, just like the tension side, softening and lowering of the cross-sectional stiffness also occurs. However, the compression side is higher in strength than the tension side, so it is not as significant as the tensile behavior.

The calculation procedure for compressive stress softening is the same as that on the tension side described above. Popovics formula⁷⁾ is generally known for compressive softening, but the modified popovics formula⁸⁾ has been expanded to the high compressive strength concrete by conducting experiments. (Fig. 4.8). In the case of compressive failure, unlike the tensile failure, stress transmission is still remains after the failure, so the compression side does not soften as much as the tensile side. Therefore, after softening, a lower limit is set to express compression resistance. Details of the cut-off crushing in Fig. 4.8 will be described in the next section.

- Calculation of compressive stress softening

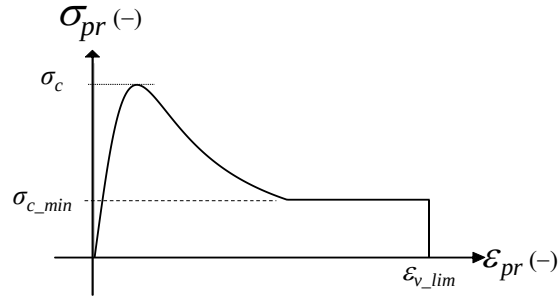


Fig 4.8 Compressive soften

$$\sigma_{pr} = \sigma_c \cdot \frac{n \left(\frac{\varepsilon_{pr}}{\varepsilon_{co}} \right)}{(n-1) + \left(\frac{\varepsilon_{pr}}{\varepsilon_{co}} \right)^n} \quad (4.4.6)$$

$$\varepsilon_{co} = \frac{\sigma_c}{E_c \left(1 - \frac{1}{n} \right)} \quad (4.4.7)$$

$$n = \exp[0.0256 \cdot \sigma_c] \quad (4.4.8)$$

where ε_{co} , n is the constant obtained from experiment, σ_c is the compressive strength of the concrete, ε_{pr} is the compressive principal strain (ignoring the tensile component), E_c is the Young's modulus of the concrete, and ε_{v_lim} is the crushing strain

- Calculation of compressive damage

In addition, the decrease in stiffness due to the damage is also introduced in the compression side just like the tension side. As shown in Fig. 4.9, damage calculation was performed with the rate of decrease in compression softening, and the upper limit was set to a value lower than that on the tensile side.

$$D_{pr} = \frac{n \left(\frac{\varepsilon_{pr}}{\varepsilon_{co}} \right)}{(n-1) + \left(\frac{\varepsilon_{pr}}{\varepsilon_{co}} \right)^n}, \quad (0 \leq D_{pr} \leq 0.2) \quad (4.4.9)$$

where, D_{pr} is the damage in each principal direction, and 0.2 is set as the upper limit in this study.

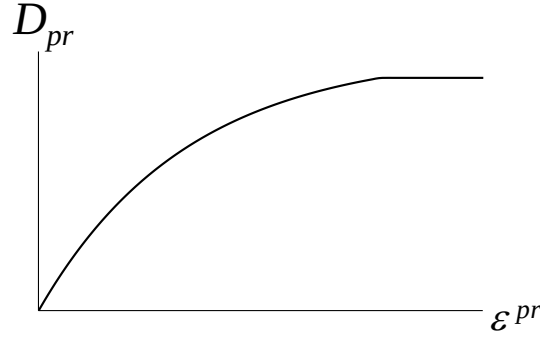


Fig 4.9 Compressive damage

where, $D_{i,pr}$ ($i = 1, 2, 3$) is damage in each principal direction, and $\varepsilon_{i,pr}$ ($i = 1, 2, 3$) is the principal strain. When $\varepsilon_{i,pr}$ is negative (compression), the damage degree $D_{i,pr}$ in each principal direction is set to zero.

Compressive damage was expressed by calculating the reduction rate of the cross-sectional stiffness from the degree of damage obtained by the above calculation flow in the same way as the tension side, then the compressive damage is applied to the elastic matrix. Considering stress softening and damage by this method, this study introduces the expression of behavior after compression side plasticity with anisotropy.

4.5 Crushing

In order to conduct numerical analysis with stable penetration failure, it is necessary to introduce crushing. Crushing is a state in which particles are completely crushed during compression fracture. In

previous studies, Kai Xu et al.⁹⁾ conducted analysis by setting the crushing threshold at the maximum principal strain. Cheng et al.¹⁰⁾ and the manual specification of the impact analysis software AUTODYN¹¹⁾ implied that crushing can be introduced by eliminating elements by judging with the equivalent plastic strain. In the particle method, crushing can be introduced by cutting the influencing radius and interacting pairs¹²⁾.

However, as shown in Fig. 4.10, it has been confirmed from existing experiments^{13~15)} that even if the same load is applied uniaxially, the pressure subjected from other directions influence the compressive strength and strain and stress leading to fracture. It is considered that to judge crushing in only one direction is easier to lead to shear fracture, and it is an overestimation in crushing to delete elements for the analysis stability. Thus, the volume strain ε_v , is used as the crushing criterion.

Specifically, the volumetric strain ε_v is approximately regarded as the sum of the axial strains, which are diagonal terms of the strain tensor, and the concrete particles whose volumetric strain reaches the crushing strain ε_{v_lim} are assumed to be completely compressed and crushed. In order to maintain the law of conservation of mass, the corresponding particles are not erased, but the total stress is forced to zero. These detailed analyzes are shown in Chapter 5.

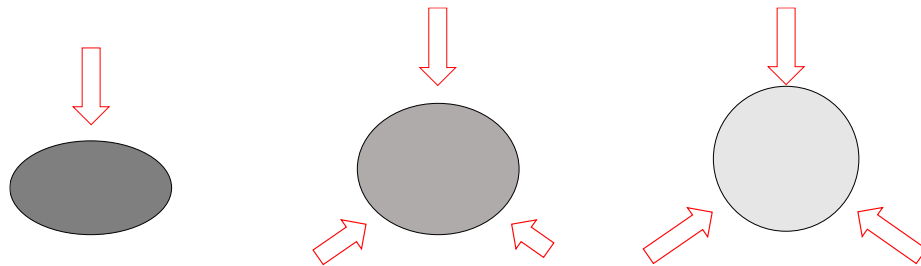


Fig 4.10 Compressive strength change due to lateral compression

4.6 Strain rate effect

This section describes the introduction of the strain rate effect. The strain rate effect is one of the unique behaviors when dealing with impact problems. It is a phenomenon when an object collides with an object at a high speed, the material strength of the object increases as the strain rate increased and propagated by the shock wave. In addition, existing studies have confirmed that this strength increase rate depends on the material and strain rate of the object, and that materials with compression dependence such as concrete have different strength increase rates on the compression side and tension

side^{16~19)}. Therefore, this strain rate effect is introduced in this study.

4.6.1 Calculation of Dynamic Increase Factor

To calculate the strength increase rate DIF (Dynamic Increase Factor), first, the uniaxial elastic strain rate $\dot{\varepsilon}$ of uniaxial elasticity is calculated, and with this value the DIF is calculated from existing formula. And the dynamic strength is calculated by multiplying the static strength of the material with DIF.

$$d\bar{\varepsilon}^e = \frac{(\bar{\sigma} - \bar{\sigma}')}{E} \quad (4.6.1)$$

$$\dot{\varepsilon} = \frac{d\bar{\varepsilon}^e}{dt} \quad (4.6.2)$$

Where $d\bar{\varepsilon}^e$ is the uniaxial elastic equivalent strain, $\bar{\sigma}$ is the equivalent stress, $\bar{\sigma}'$ is the equivalent stress before 100 steps, E is the static elastic modulus, and dt is the time for 100 steps.

First, Fujikake's equation²⁰⁾ is used to calculate the DIF for concrete material. The equation is shown below and the DIF-strain rate relationship is shown in Fig 4.11

$$DIF = \frac{f'_{cd}}{f'_{cs}} = \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s} \right)^{0.006 \left[\log \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s} \right) \right]^{1.05}} \quad (4.6.3)$$

Where $\dot{\varepsilon}_s$ is the strain rate of static load ($= 1.2 \times 10^{-5}$ [1/s]), f'_{cd} is the dynamic compressive strength, f'_{cs} is the static compressive strength.

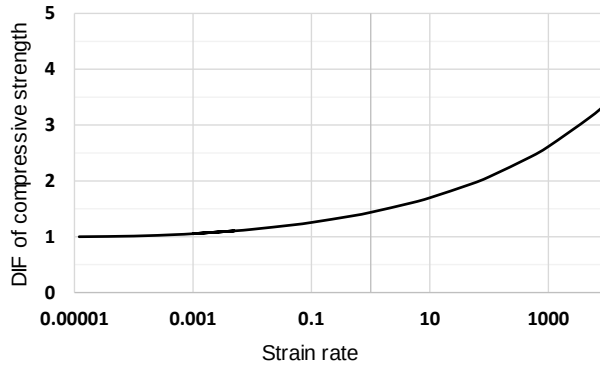


Fig 4.11 DIF of compressive strength (Fujikake's equation)

Then, for the DIF of the tensile strength of concrete, Rose's equation²¹⁾ is used

$$DIF = \frac{f'_{td}}{f'_{ts}} = \exp \left[0.00126 \left(\log \frac{\dot{\varepsilon}}{\dot{\varepsilon}_s} \right)^{3.373} \right] \quad (4.6.4)$$

where $\dot{\epsilon}_s$ is the static strain rate ($= 1.0 \times 10^{-7}$ [1/s]), f'_{td} is the dynamic tensile strength, and f'_{ts} is the static tensile strength.

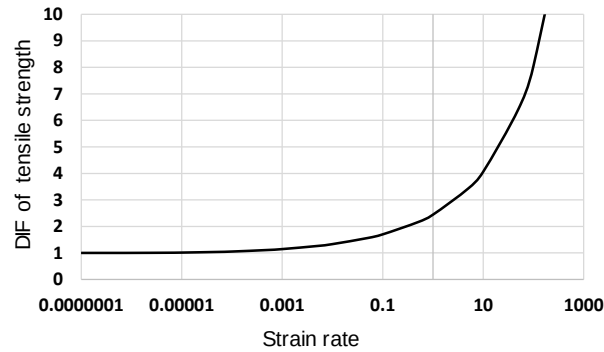


Fig 4.12 DIF of tensile strength (Ross's equation)

Comparing Fig. 4.11 and Fig. 4.12, the rate of increase in tensile strength tends to be steeper as the strain rate increases. For example, when the strain rate is 10 (1/s), the compression strength increases by about 1.7 times compared to static, but the tensile strength is about 4 times. Therefore, it can be seen that the strain rate effect is greater on the tension side than on the compression side.

4.6.2 Verification analysis of strain rate effect

The calculation described above was performed for each particle, and by conducting a simple uniaxial test analysis, whether the strain rate effect is introduced accurately at the member level is verified. In addition, since this analysis is aimed at the response in a single collision, the strain rate effect calculation is conducted within elasticity, and when entering plasticity, the DIF at that time is maintained.

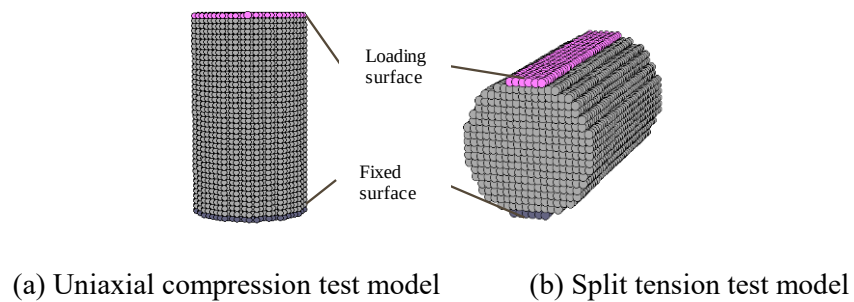


Fig 4.13 Analysis model for uniaxial tests

▪ Analysis model

A cylindrical specimen with a diameter of 100 mm and a height of 200 mm, which is generally used in simple tests, is used as the object of analysis. As shown in Fig 4.13, it is discretized with particles with 50mm diameter, and the analysis model is arranged in the height direction, in Fig 4.13(a) and vertically, in Fig 4.13(b), with upper surface as the loading surface and bottom surface as the fixed surface. The uniaxial compression test, in Fig 4.13(a), and split tension test, in Fig 4.13(b), are reproduced by giving corresponding boundary conditions. Regarding the material properties, the compressive strength is 41.4N/mm^2 , the tensile strength is 4.14N/mm^2 , the static elastic modulus is 23.1N/mm^2 , and the Poisson's ratio is 0.23. The loading speed is set to $10^{-4}\sim 10^2(1/\text{s})$ to verify the strain rate effect. As for the output, the strain is a value obtained by dividing the loading distance by the initial height, and the stress is assumed to be the value of reaction force at the fix bottom divided by the cross-section in uniaxial compression analysis, and the value of the perpendicular force divided by the perpendicular cross-section in the center of the cylinder in split tension analysis.

▪ Analysis result

Fig. 4.14 and Fig. 4.15 show contourmaps of the DIF in compressive strength in the uniaxial compression test and the DIF in tensile strength in the split tensile test, respectively, when loaded at a strain rate of 10/s. As shown in these figures, it can be seen that the DIF in strength corresponding to the strain rate occurs all over the testing member.

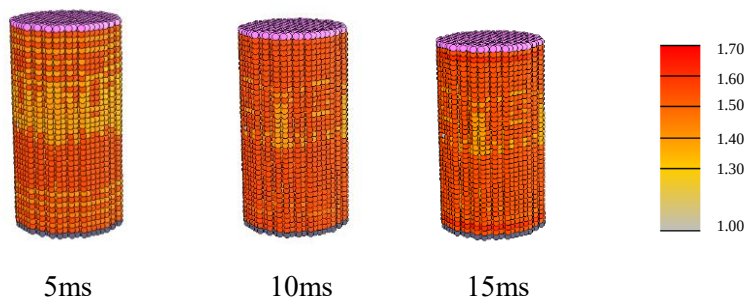


Fig 4.14 Results of uniaxial compression

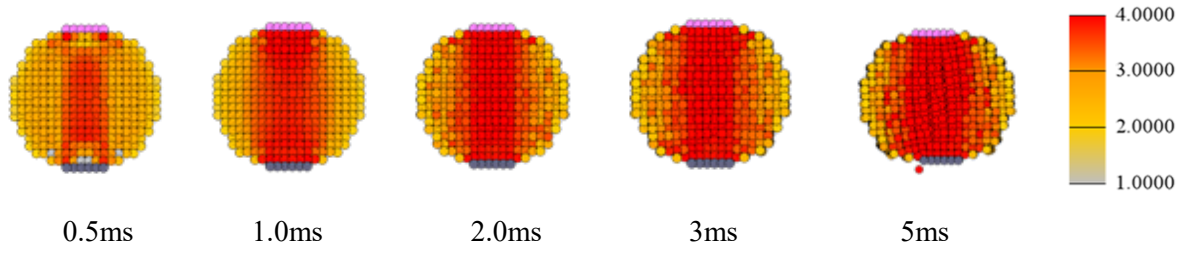


Fig 4.15 Results of split tension

Fig. 4.16 shows the DIF of both compressive and tensile analysis. From this figure, both the compressive and tensile strengths are in good agreement with the theoretical values, and it can be confirmed that the strain rate effect was appropriately introduced at the member level.

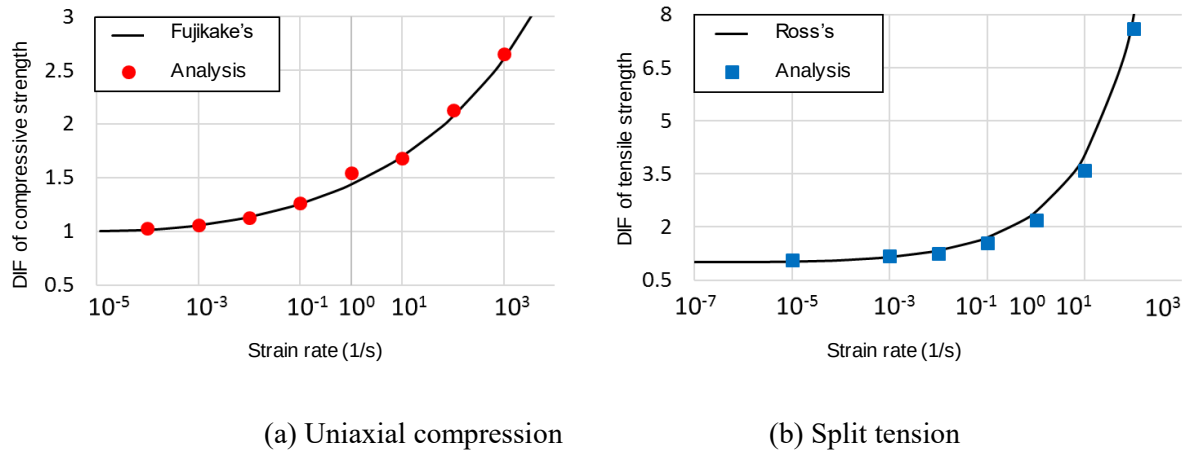


Fig 4.16 Comparison between analysis value and theoretical value

Since the compressive strength and tensile strength change with the strain rate, the compressive softening and tensile softening also change accordingly. Specifically, σ_t and σ_c in Fig. 4.6 and Fig. 4.8 are multiplied by DIF calculated on the tension and compression sides, respectively, to convert to the dynamic value.

In the next chapter, the impact analysis on RC beam member and concrete slab member are conducted with the mechanical model of concrete materials that considers anisotropic stress softening, stiffness damage and impact-specific strain rate effects.

Chapter 5 Verification analysis for the proposed SPH impact analysis method

5.1 Introduction

In this chapter, first, a low-velocity impact experiment on a conventional RC beam is reproduced using the analysis method and dynamic model described in the previous chapter, and the validity of the impact response analysis using the SPH method is compared with the experimental results. After that, in order to focus on penetration failure, the analysis model is changed to a concrete slab, and reproduction analysis of penetration including crushing is performed.

5.2 Validity verification of impact response analysis using RC beam

5.2.1 Previous experimental models and analysis overview

As shown in Fig. 5.1, the size of the test specimen RC beam is 150×250 mm (width $\times$ beam), and the total length is 2400 mm. The thickness of concrete cover is 40mm, and the 4 rebars are arranged in two layers. D6 is arranged as a shear reinforcement bar every 100mm between the supports. The cylinder weight used in the experiment was made of 300kg steel, the length is of 1400 mm, and the diameter of the nose is 90mm. These impact experiments were conducted by Kishi et al. of Muroran Institute of Technology as an activity of the Structural Engineering Committee of the Japan Society of Civil Engineers¹⁾. The material characteristics of rebar and concrete are shown in Table 5.1. In addition, the static shear strength is 143.5kN and the static bending strength is 39.1kN. Therefore, the shear margin is 3.67, and the damage in both static and dynamic cases should be bending damage.

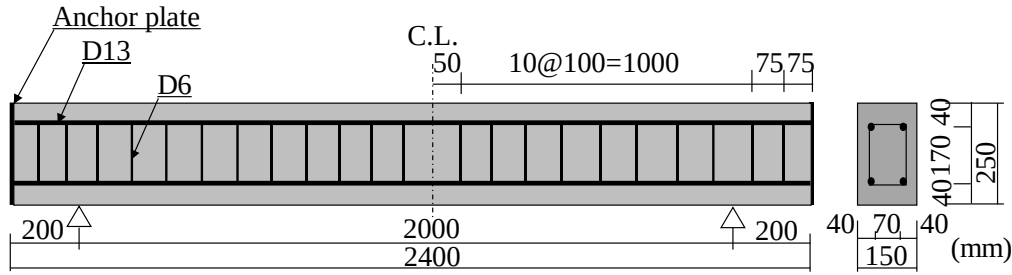


Fig 5.1 Size and rebar distribution of RC beam

Table 5.1 Material characteristics

Material	Rebar		Concrete
	D6	D13	
Density (kg/m ³)	7853	7853	2500
Young' modulus (kN/mm ²)	206	206	23.1
Compressive / Yield strength (N/mm ²)	368	373	25
Tensile strength (N/mm ²)	-	-	2.5
Poisson ratio	0.30	0.30	0.23

In addition, Fig. 5.2 shows the overall view and bar arrangement of the analysis model used in this section. Specifically, the impact response analysis of the RC beam was performed using a 1/2 model with a particle size of 10 mm. The boundary condition is that the vertical displacement of the particles on the supporting position is restrained, and the horizontal (beam axis) displacement of the particles on the symmetry plane is also restrained. In this analysis, the impact response of the first strike is studied as the main target, so modeling of the complex supporting jig is omitted. In addition, for the discontinuous surface in the analysis, anisotropic stiffness considering the damage caused by cracks as described in Chapter 4 is adopted, and a model of complete adhesion between rebar and concrete is adopted. In addition, because the impact object did not enter plasticity in the experiment, it is modeled as a linear elastic body.

Von Mises is adopted as the yield condition for the reinforcing steel material, and exponential hardening is considered on both the compression side and the tension side as shown in Fig. 5.3 (a). Although the influence is relatively small, Takahashi's formula²⁾ is adopted, as shown below

$$DIF = \frac{f'_{yd}}{f'_{ys}} = 1.202 + 0.040 \cdot \log(\dot{\epsilon}) \quad (\dot{\epsilon} \geq 5.0 \times 10^{-5}) \quad (5.2.1)$$

where f'_{yd} and f'_{ys} are the dynamic and static yield strengths, respectively, and $\dot{\epsilon}$ is the strain rate (1/s).

The constitutive laws for concrete materials are shown in Fig 5.3, and represent the stress reduction with anisotropy the decrease in stiffness due to damage as described in the previous section and. As for the yield surface, nonlinear Drucker-Prager yield surfaces as shown in Fig. 5.4 is adopted.

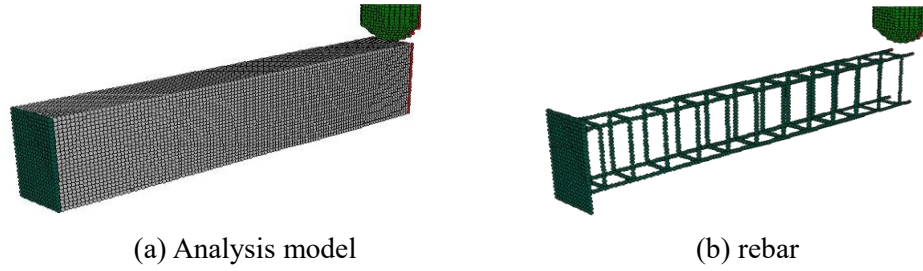


Fig 5.2 1/2 analysis model

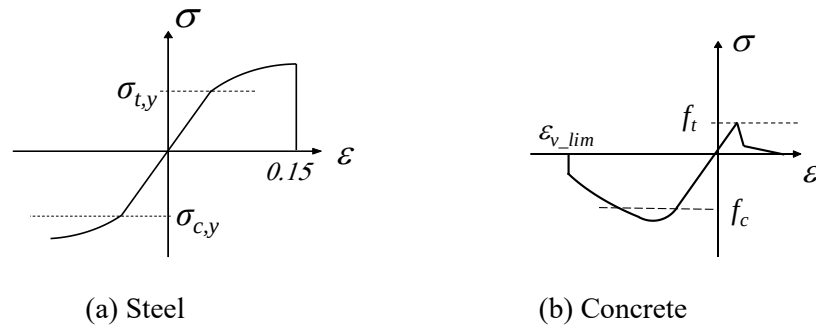


Fig 5.3 Stress-strain relationship

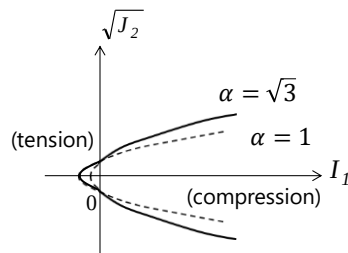


Fig 5.4 non-linear Drucker-Prager yielding criterion

5.2.2 Results and discussion

The displacement response is shown in Fig 5.5, and it can be found that displacement-time history of the analysis results simulated well the experiment results. Although the difference of residual

displacement is a little larger under 4m/s collision velocity, maximum displacements of both cases are almost the same. Fig 5.6 shows the comparison of crack distribution between experiment and analysis under 5.0m/s collision velocity. Maximum principal strain is used to describe concrete cracks (more than 4000μ is expressed by dark color) in this figure. It can be found that, the crack growth path in the central span area agreed well, and minor cracks can be found in other positions. From these verifications, the impact response analysis by SPH method could predict reliable elastic-plastic impact behavior of RC beam in this case (impact velocity is relatively slow and there is no perforation of concrete).

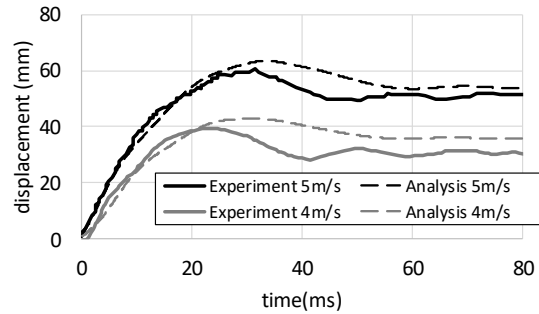
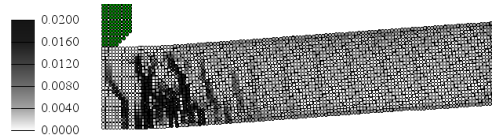


Fig 5.5 Impact responses in analysis and experiment



(a) Experiment



(c) Analysis

Fig 5.6 Crack distribution between analysis and experiment

5.3 RC SLAB PENETRATION ANALYSIS

5.3.1 Analysis object

Fig 5.7 shows the geometry of the concrete slab used in this analysis. The collision object and test specimens (concrete slabs) are modeled from the experiments conducted by Beppu et al.³⁾ The size of concrete slab is 500mm×500mm, and the thickness varies in each test cases. It is assumed that two sides of slab are fixed in out of plane direction. The collision object is 100g in weight, whose nose shape is a hemisphere, and the collision point is the upper center surface of RC slab. The material properties of concrete are set as shown in Table 2. The impact velocity of the projectile varies from

180m/s to 500m/s. The analysis cases are listed in Table 3, with different slab thickness, impact velocity, and erosion limit. The failure mode and penetration depth are investigated from analysis results given by cases with various parameters.

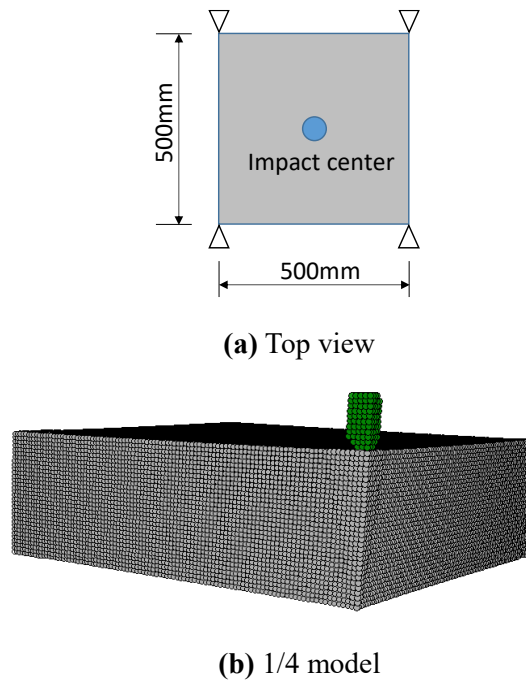


Fig. 5.7 The concrete slab analysis object and analysis model

Table 2 Material property of concrete

Mass density (kg/m ³)	2500
Young's modules (kN/mm ²)	29.0
Compressive strength (N/mm ²)	25.0
Tensile strength (N/mm ²)	2.50
Poisson ratio	0.22

Table 3 Analysis cases

Case	Thickness (mm)	Velocity (m/s)	Erosion limit (%)
30-180-fixh (h fixed length)	30	180	06
30-180-06	30	180	06
30-210-06	30	210	06
30-210-10	30	210	10
30-250-10	30	250	10

60-200-06	60	200	06
60-300-06	60	300	06
⋮	⋮	⋮	⋮
80-450-06	80	450	06
80-500-10	80	500	10

5.3.2 Results and discussion

The results of the first cases with slab thickness of 30mm are shown in Fig.5.8. When the smoothing length h is fixed, the collision object cannot perforate the concrete slab, which is different from the experiment. This is because that the overlap between particles becomes noticeable due to the intrusion of impact objects, and the number of particles within the smoothing domain increases rapidly. While for case with smoothing length h_a changes with the density of the particle, the concrete slab is perforated by collision object. It is found that the influence domain length becomes smaller, since concrete particles reacts only with nearby particles, stress concentration and local failure easily occur. This makes the concrete particles behave more like real behavior under high velocity collision, and the perforation phenomenon observed in the experiment could be reproduced. Since the smoothing length affect analysis results, the proposed adaptive smoothing length h_a was confirmed useful and it is adopted for the following perforation analysis.

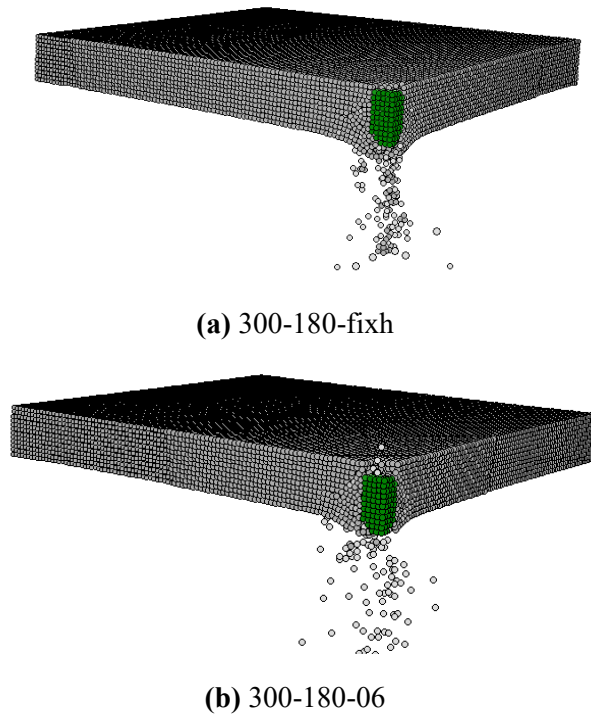


Fig 5.8 Penetration of case 30-180-fixh and 30-180-06

For the other cases, the maximum principal strain distribution after impact of case 60-300-06 and 60-350-06 are shown in the Fig 5.9. It can be found that the projectile with a velocity of 300m/s does not perforate the concrete slab, whereas the projectile with a velocity of 350m/s perforates the slab. Inclined shear cracks can be found in both cases, and the concrete damage is distributed along with the touching-surface of projectile. In case 60-300-06, although the concrete slab is not perforated, concrete at the back surface is damaged, and particles flies from back surface along the impact direction, which is similar to the scabbing phenomenon observed in the experiment. Thus, the perforation could be reproduced by the proposed SPH method.

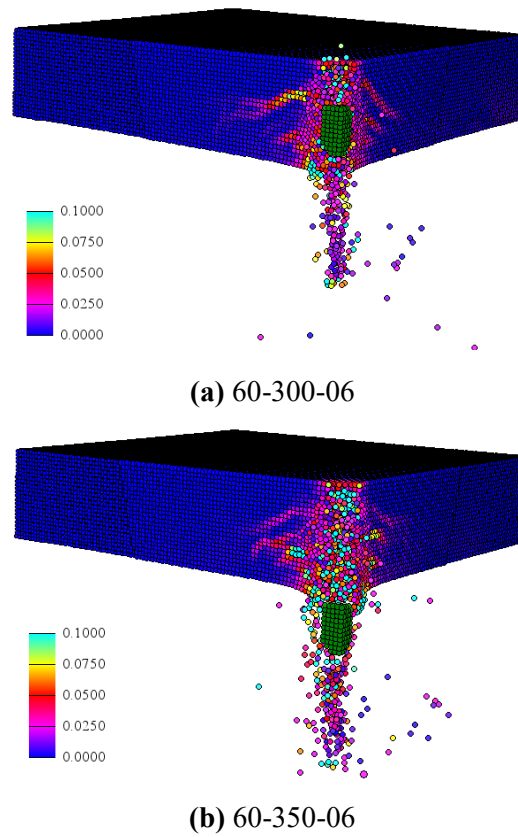


Fig 5.9 Maximum principal strain distribution in analysis

Next, the perforation in the analysis results are compared with experiment results, as shown in Fig 5.10. In this figure, solid circles represent the perforation cases in the experiment, and solid squares show surface damage cases or spalling cases. On one hand, hollow circles represent perforation cases in the analysis, hollow squares express surface damage cases or spalling cases, and the size of the circle represents the erosion limit. For example, when the thickness is 80mm and impact velocity is 400m/s, perforation occurred in the experiment. When erosion limit is set to 6% in the analysis, the slab with 80mm thickness is perforated under 450m/s impact velocity. It can be found that the perforation in the

experiment are almost reproduced by proposed SPH analysis. In this analysis, to achieve accurate predictions on the perforation limit, the erosion limit must be set to 6%. Larger erosion limit provides more impact resistance against projectile, and it gives large error on the perforation limit in this simulation. Although, erosion limit of 6% would provide a good simulation result in this simulation, further consideration (including more detailed yield function of concrete) should be given to establish a highly accurate analysis method and reasonable value of erosion limit.

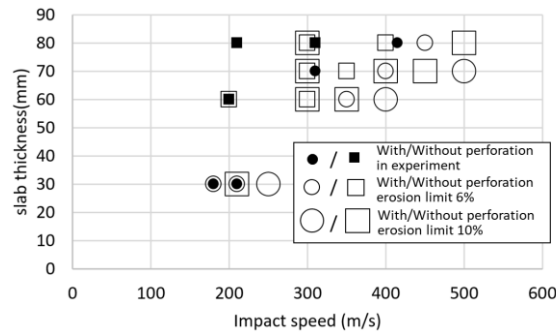


Fig 5.10 Comparison of perforation results between experiment and analysis

Finally, the penetration depth under incomplete perforation cases are simulated. The penetration depths in analysis are compared with experiment results in Fig 5.11. In general, larger impact velocity generates deeper penetration depth in both experiment and analysis. In addition, larger erosion limit gives a smaller penetration depth in SPH analysis. As discussed in the last section, larger erosion limit provides more impact resistance of concrete slab against projectile, which makes perforation process difficult. It is confirmed that the penetration depth by SPH analysis are larger than the experimental results. In this study, the results by 20% erosion limit gives a smaller difference with experimental results. As for the real impact phenomenon, scabbing is often accompanying the impact process. The thickness of the concrete slab is usually decreased by scabbing, and the penetration depth is limited. Since the scabbing of concrete is not exactly reproduced in this analysis, the penetration depth of the projectile is hard to predict. In the next stage of this research, yield criterion and damage mechanism of concrete should be re-examined in order to improve the accuracy of this analysis.

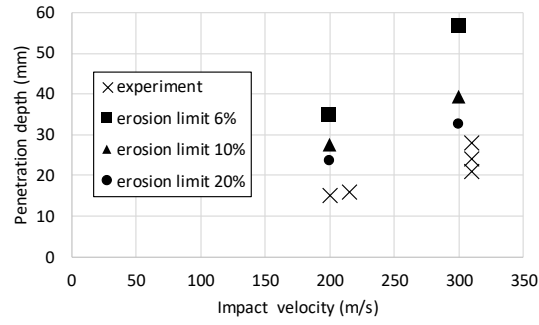


Fig 5.11 Comparison of penetration depth between experiment and analysis

5.4. CONCLUSION

In this chapter, the proposed SPH analysis method is compared with a RC beam falling weight experiment and a concrete slab penetration experiment. It is found that the displacement response and the crack distribution of the RC beam is well matched by the SPH analysis. For the penetration analysis of the concrete slab, the adaptive smoothing length plays a good role to improve the analysis accuracy, and the experiment results are reproduced by the SPH analysis roughly. Since the validity of the proposed SPH method is verified, this method is used in other analysis in the following chapters.

Chapter 6 Discussion on penetration fracture by SPH method

6.1 Introduction

In the analysis, the crushing criterion was introduced into the constitutive law of the concrete material, and the penetration failure of the unreinforced concrete slab subjected to the impact of projectile was simulated using the proposed SPH method. Here, assuming the case where the penetration of the collision speed occurs, the failure mode, impact force, impulse, etc. are investigated when the projectile collides with the unreinforced concrete slab, and the penetration prevention performance of the unreinforced concrete slab is investigated.

6.2 Analysis model and cases

As shown in Fig. 6.1 (a) and (b), the geometric shape of the concrete plate used in the analysis are assumed to be a $2000 \times 2000 \times 200$ mm slab member and the slab is fixed with a four-sided simple support with a distance of 1750 mm. The projectile has a mass of 300 kg and the tip is hemispherical (with a radius of 80 mm), and is collided with the upper surface of the slab center. The material characteristics of the concrete are set as shown in Table 6.1 with reference to the previous impact tests.



Fig 6.1 Size of the concrete model

Table 6.1 Concrete material parameter

Density (kg/m ³)	Static elastic modulus (kN/mm ²)	Compressive strength (N/mm ²)	Tensile strength (N/mm ²)	Poisson
2500	29	34.1	3.41	0.22

As for the analysis model, as shown in Fig. 6.2, impact response analysis is performed using a 1/4 model in which the particle size was discretized with 10 mm. Therefore, the particles at the supporting position restrain the displacement in the out-of-plane direction, and the particles on the 1/4 model symmetry plane constrain the displacement in the direction perpendicular to the symmetry plane. As for the projectile, a linear elastic body is assumed.

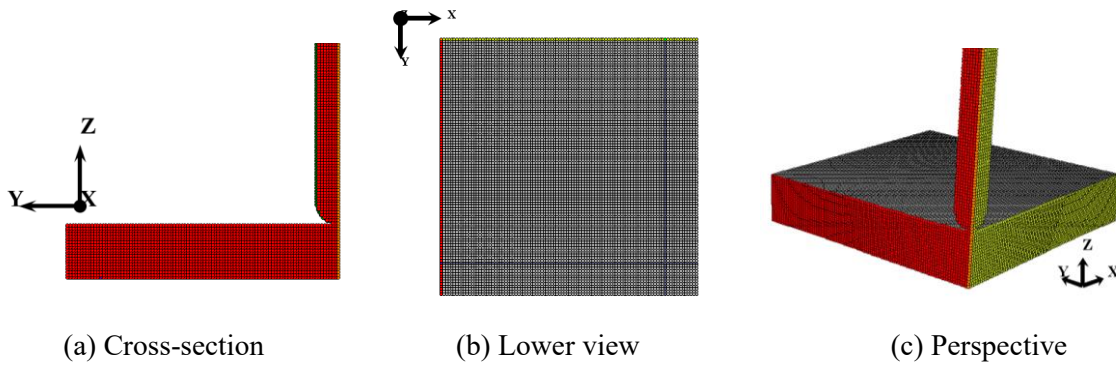


Fig 6.2 1/4 analysis model

6.3 Analysis of penetration failure

Fig. 6.3 and Fig. 6.4 show the calculation results for the process of the penetration of the projectile into the unreinforced concrete slab when the impact speed is 20m/s and 150m/s, respectively. In Fig. 6.3, it can be seen that the projectile is bounced off without penetrating because it collides at a relatively low speed.

In Fig 6.4 (b) and (c), it can be confirmed that the crushing condition and the tensile fracture interacts with each other from the cracks occurred from the bottom. From this result, it can be said that the penetration phenomenon is reproducible by SPH method. Also, in Fig. 6.4, the velocity is very high compared to the previous case, so that the projectile easily penetrates through the concrete slab, and the

penetration phenomenon can be reproduced well. In addition, as the flying object pushes through the concrete slab, the angle of the compression crack increases, confirming that the damage pattern is penetration-shearing damage.

Therefore, it can be seen from these figures that by using the mesh-free SPH method, the process of the penetration and perforation of projectiles with concrete crushing can be reproduced visually and easily.

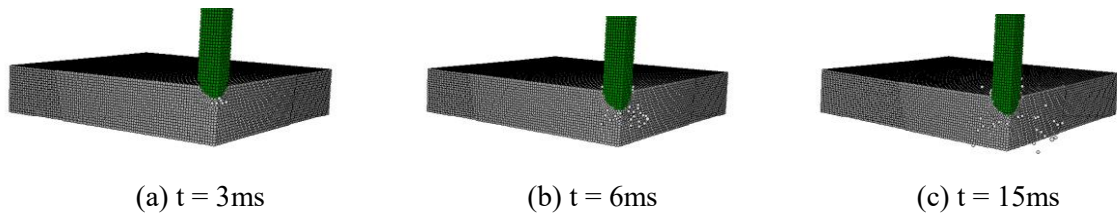


Fig.6.3 Impact velocity 20m/s

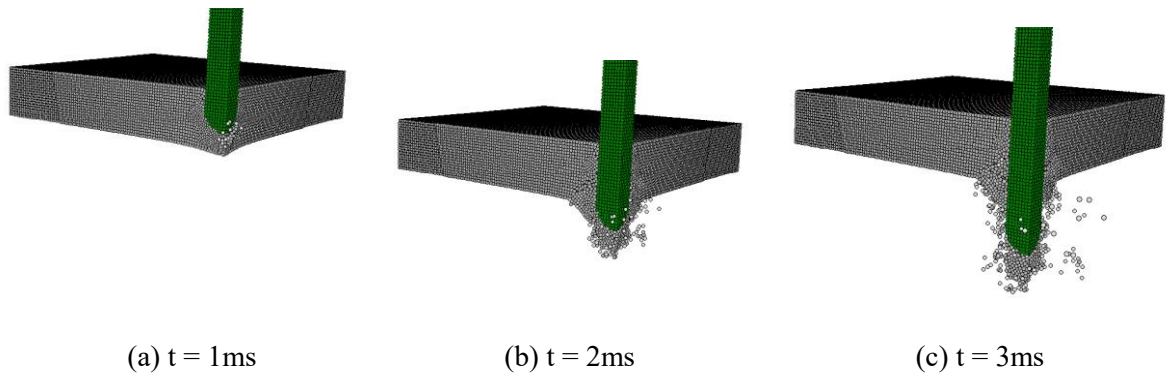


Fig.6.4 Impact velocity 150m/s

6.4 Effect of impact velocity on impact force and impulse

Since penetration analysis became possible, the collision velocity is changed parametrically, and comparisons are made focusing on the impact force which is an important value in the analysis response results.

Fig. 6.5 shows the impact load-time relationship when the collision velocity of the projectile is changed. The impact load shown here was obtained by multiplying the acceleration by the mass, and the impact load waveforms in the analysis results are from cases where the concrete slab is penetrated. Fig 6.5 shows the original waveform, in which the vibrancy is large and the difference among cases is hard to distinguish. Alternatively, Fig 6.6 shows the waveforms processed by moving-average method,

where the vibrancy is mitigated and the difference among cases is obvious. From this figure, it can be clearly seen that the higher the collision velocity is, the larger the maximum impact force and the shorter the duration time becomes.

Next, the relationship between the impulse and the collision velocity is shown in Fig. 6.7. The impulse is obtained by integrating the area of the impact force-time curve in Fig. 6.6. This shows that there is a clear tendency between the impact velocity and the impulse. When the projectile penetrates at a low speed of 10m/s to 50m/s, the concrete slab is not totally penetrated, and the impulse required for penetration increase gradually with increasing speed. When the projectile penetrates at a medium speed of 60 m/s to 120m/s, the impulse required for penetration decreases gradually with increasing speed.

From the figure, it can be confirmed that the impulse required to penetrate the concrete slab was almost constant regardless of the collision velocity when the collision speed was higher than 140m/s. This implies that when the speed of the colliding object exceeds a certain threshold, the impulse required for penetration hardly changes.

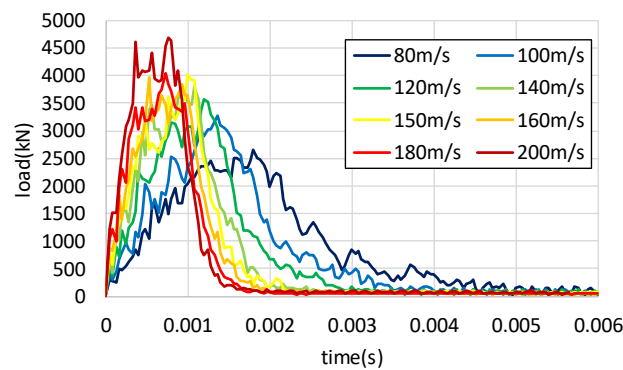


Fig 6.5 Impact force-time relationship

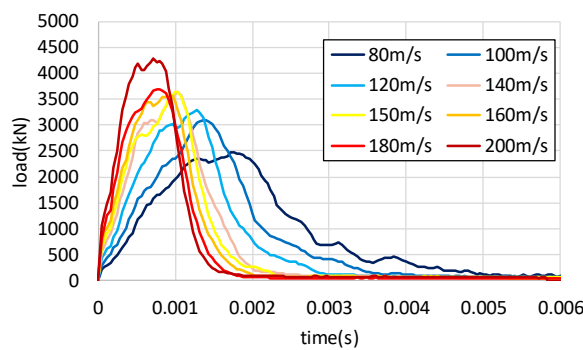


Fig 6.6 Filtered impact force-time relationship

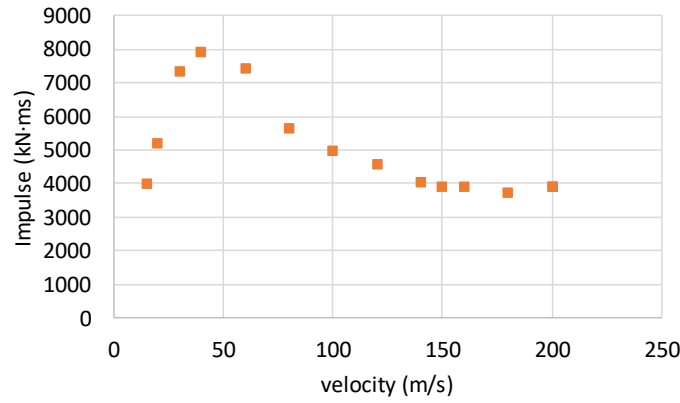


Fig.6.7 Impulse-impact velocity relationship

6.5 Discussion on the penetration failure of different impact velocity

Fig. 6.8 shows the penetration failure mode in the case of the impulse speed (60m/s to 120m/s) when the impulse changes according to the impact velocity, and the impact velocity when the impulse is constant (higher than 140m/s). This contour diagram shows the maximum principal strain distribution in the cross section of the concrete slab of the analysis quarter model, and the area surrounded by the black dotted line in the figure shows the initial position of the slab. From this figure, regarding the deformation problem, it can be confirmed that in (a), the concrete slab deformed much from the initial position and is totally bent. For (b), the deformation is smaller than (a), and compared to the initial position, only small overall deformation can be observed, with local shear deformation dominant near the collision point, indicating that brittle local fracture had occurred.

In addition, there is a clear difference in the strain state between (a) and (b) in Fig. 6.8. Looking at the maximum principal strain distribution in (a), the bending deformation is found not only near the collision area but also on the upper and lower edges of the concrete slab. Large maximum principal strain was observed, indicating that bending fracture and punch-out shear failure were mixed. However, in (b), a smaller maximum principal strain indicating bending fracture is hardly distributed on the upper and lower edges, and a large maximum principal strain value from the front to the back is distributed on the contact surface with the penetrating flying object. Thus, it is inferred from the maximum principal strain distribution that this is a brittle punch-out shear failure mode.

As described above, it can be said that the reason that the impulse at the time of penetration changes depending on the collision velocity is due to the difference in the penetration failure mode. In other words, in the case of penetrating fracture mixing with bending fracture, which is a total fracture, the toughness is increased, so that the impact force acting time is increased, the impulse required for penetration is increased. When the collision velocity is increased, it is thought that the local fracture becomes more dominant than the total fracture, so that the toughness decreases and the impulse required for penetration does not change as the fracture surface converges.

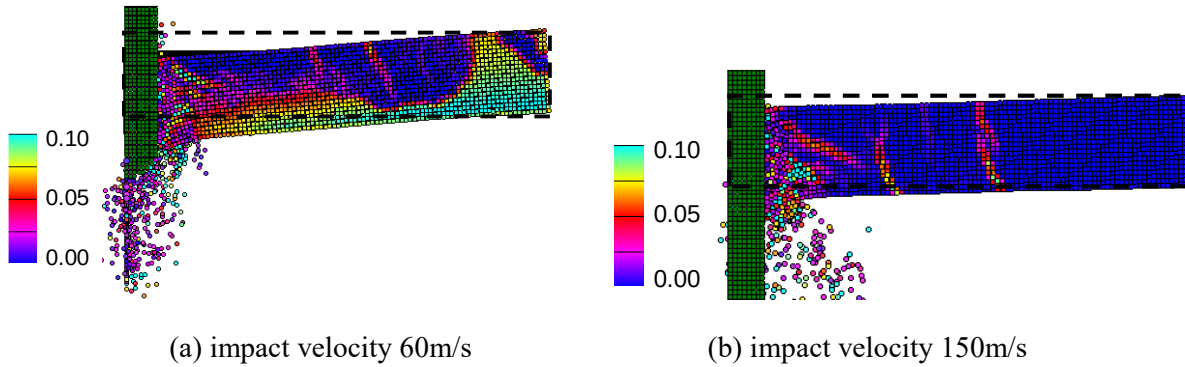


Fig 6.8 Comparison of penetration failure mode

6.6 Effect of the mass of the projectile on penetrating impulse

Fig. 6.9 shows the impulse-collision velocity relationship when the mass of the projectile is changed to 200kg, which is 2/3 of 300kg, and 100kg, which is 1/3 of 300kg. The two modified cases (mass 200kg and 100kg) show only the results of analysis of the collision velocity where the impulse is approximately converged and constant. From this figure, regardless of the mass of the flying object, it can be confirmed that the required minimum impulse does not change.

Fig. 6.10 shows a comparison of the impact force with different projectile mass at a collision speed of 150m/s. From these results, it can be confirmed that the waveform of the impact force changes little with different projectile mass. After all, when the fracture surface formed is the same regardless of the mass, it is considered that there is no significant difference in impulse required for penetration.

Fig.6.11 shows the maximum principal strain distribution of the concrete slab obtained by numerical analysis. From these results, it is clear that in both cases, the modes of penetration fracture are almost the same, such as the predominant local fracture near the collision area and the occurrence of oblique shear cracks due to the punching of projectile. It can also be found that, there is almost no difference in

the angle of the oblique shear crack. Although the maximum principal strain at the bottom of the slab in 100kg case is larger, the overall deformations of the case are almost the same, indicating a similar damage pattern. Therefore, it can be said that the impulse required for penetration at the time of high-speed collision does not change regardless of the mass of the flying object.

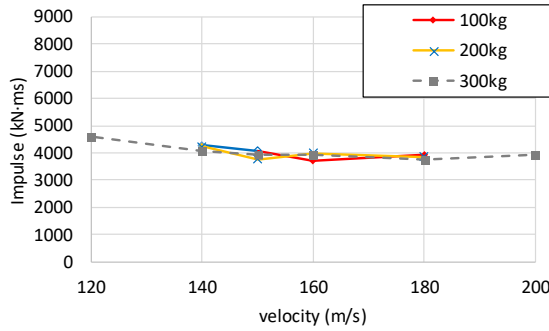


Fig 6.9 Impulse-impact velocity relationship

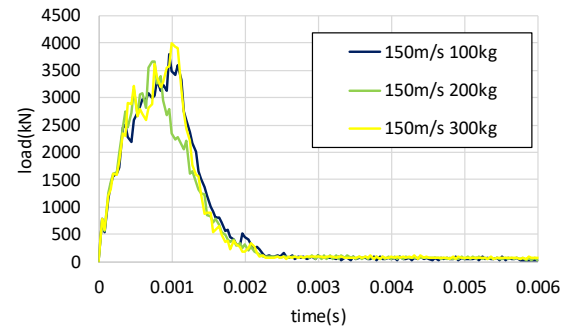
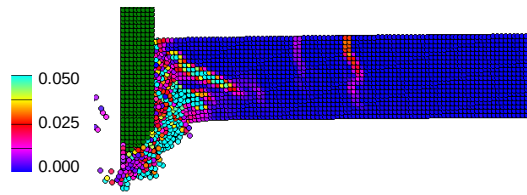
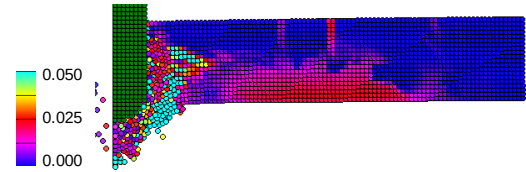


Fig 6.10 Impact force history



(a) 300kg mass



(b) 100kg mass

Fig 6.11 Damage pattern of the concrete slab

6.7 Effect of supporting condition of concrete slab on penetration impulse

Here it is verified whether the impulse remains constant or changes with the supporting condition. First, as the support condition, this analysis assumes four-sided support and two-sided support. Then, it is verified whether the impulse changes with supporting distance.

6.7.1 Comparison between 4-side support and 2-side support

Fig 6.12 shows the comparison of the analysis model in the two case. The impact force time history and impulse obtained by the analysis are shown in Fig 6.13 and Table 6.2. The impact velocity is 150m/s. From these results, it is found that there is no significant difference in the impact force waveform, and that the impulse amount is almost the same, with a relative error of about 1%. In other words, it can be

said that the impulse required for penetration does not change at the collision velocity where local fracture is more dominant than total failure, regardless of the support conditions.

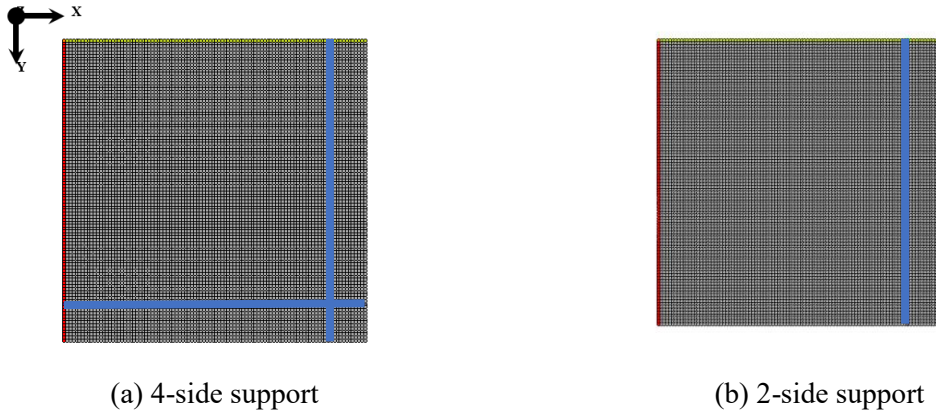


Fig 6.12 1/4 analysis model with difference boundary condition

Table 6.2 Impulse of difference boundary condition

Boundary condition	Impulse (kN · ms)	Relative error
4 side	3949	—
2 side	4148	5%

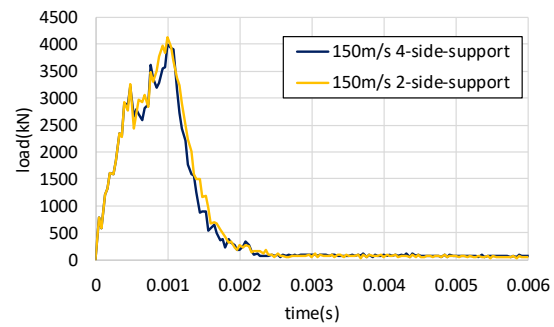


Fig 6.13 Impact force history

6.7.2 Relationship between supporting interval and impulse

Next, the effect of the distance between supporting points on the impulse is investigated. The support condition is four-sided support, and a parametric analysis was performed to determine whether the distance between the supporting points affected the impulse. As shown in Fig. 6.14, it was found that the convergence impulse value was almost unchanged.

This is because, as shown in Fig 6.15, three cases of fractures at different supporting distances are shown: in case (a) 1750mm, and case (b) 800mm, the angle of the oblique shear crack is sharp, and the crack doesn't reach the supporting point when it reaches the bottom from the collision part. Thus the failure mode was local-shear failure, and the impulse value stayed stable.

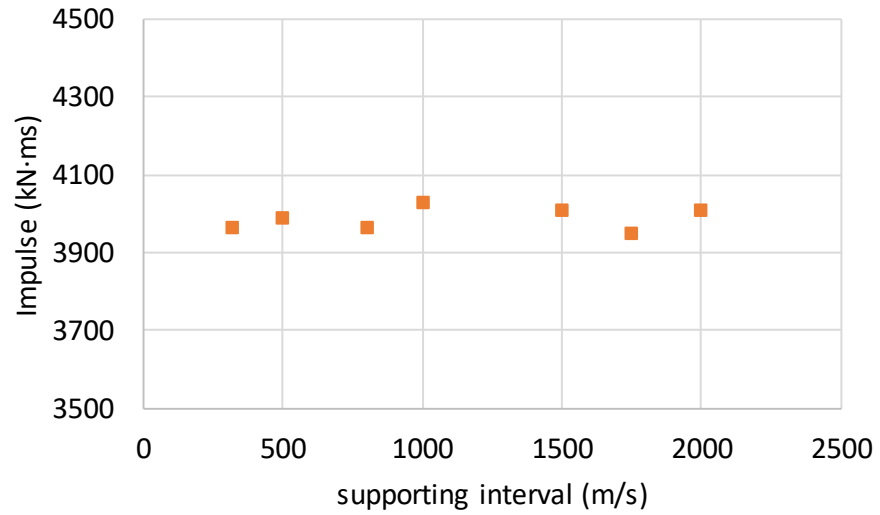


Fig 6.14 Impulse-supporting interval relationship

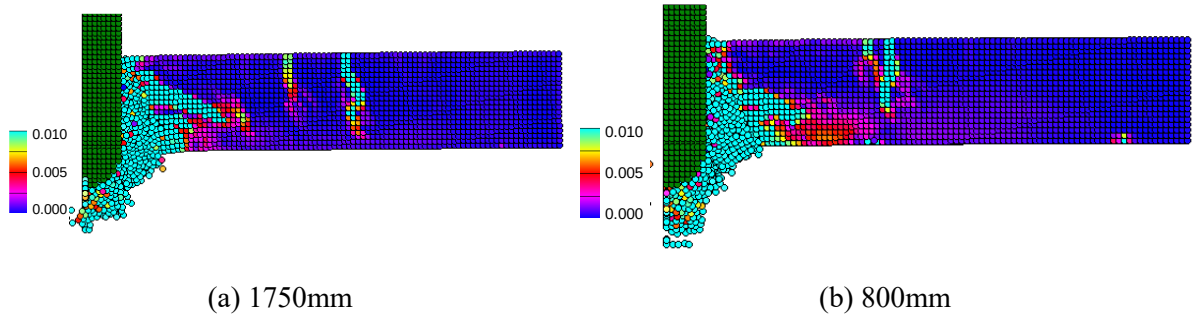


Fig 6.15 Damage of the concrete slab with different support interval

6.8 Conclusion

In this chapter, the failure mode of the concrete slab after penetration and the impulse consumed during the penetration process is discussed. It is found that with the increase of the impact velocity, the failure mode changes from bending failure to local shear failure. During this process, the impulse increases before the slab is perforated, and the impulse decreases after the slab is perforated. Finally, the impulse remains stable when the local shear failure mode is dominating. This stable impulse is influenced little by the mass of the projectile, the supporting conditions, and the supporting interval. This indicates that the impulse is potential to be a criterion to judge the penetration failure of the concrete slab, which is discussed in detail in the next chapter.

Chapter 7 Proposal of evaluation method for penetration damage

7.1 Introduction

From the discussion of the penetration analysis in the previous chapter, the impulse required for penetration is constant at high-velocity collision, and from the results it can be found that the minimum value can be obtained. Therefore, an attempt is made to create a penetration failure criterion using the impulse value. For this purpose, the parameters related to impulse are set as the compressive strength and thickness of the concrete slab, the collision velocity and the circumference of the projectile. By clarifying the correlations of these parameters and the penetration impulse, the evaluation formula predicting the impulse is proposed. Then, we verified whether the impulse evaluation formula had the consistency and validity of the analysis, and examined the usefulness of the impulse formula with an example.

7.2 Penetration damage criterion equation based on impulse

As shown in Fig. 7.1, the impulse area is assumed to be a trapezoid waveform determined by the maximum impact force F_{max} and the impact time Δt , and the impulse equation is established by obtaining the functions of F_{max} and Δt from the analysis results. This impulse equation focuses on the minimum impulse that converges during a high-velocity collision as shown in Figure 7.2. The analysis conditions are the same as in Chapter 6, and the reference values are shown in Table 7.1.

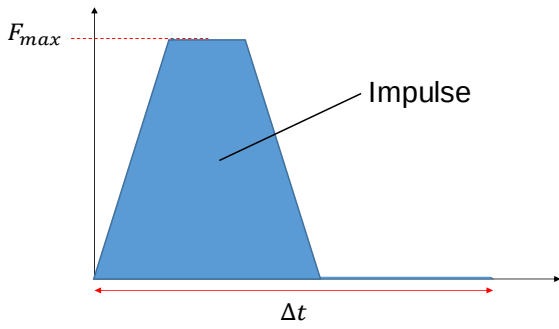


Fig 7.1 Assumption of impact force-time relationship

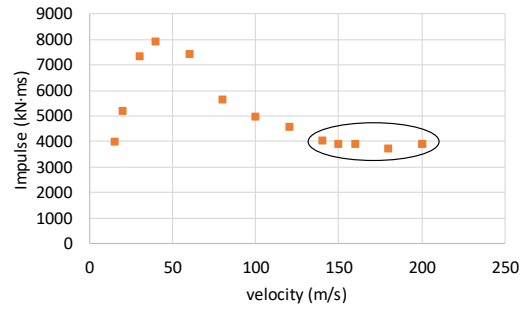


Fig 7.2 Minimum impulse obtained at high-velocity

Table 7.1 Base parameters of the penetration criterion equation

Diameter D_s (mm)	Impact velocity V_0 (m/s)	Compressive strength f_c (MPa)	Slab thickness t_c (mm)
160	150	34.1	200

7.2.1 Determination of the maximum impact force

As shown in Figure 7.3, the maximum impact force function is defined as the maximum load required to form a fracture surface due to the penetration failure that occurs on a concrete slab due to a high-velocity impact of a flying object. Four parameters were used: the diameter D_s of the object (circumference πD_s), the compressive strength f_c of the concrete slab, the slab thickness t_c , and the impact velocity v .

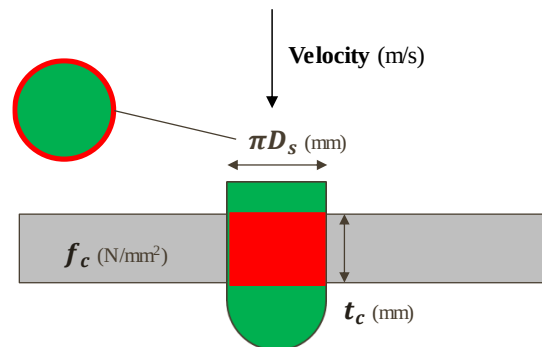
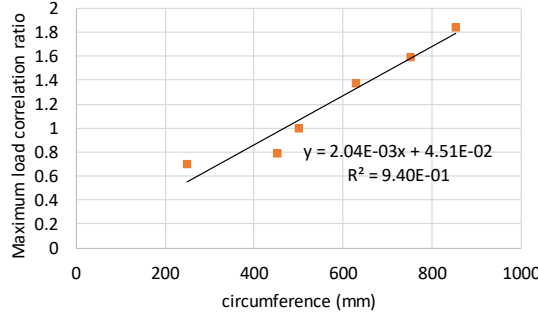


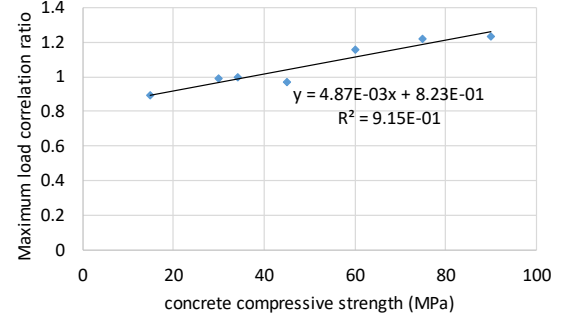
Fig 7.3 Definition of the maximum impact force

The correlation with the maximum impact force was expressed using these parameters. As shown in Fig. 7.4, the circumference of the flying object, the compressive strength of the concrete slab, and the

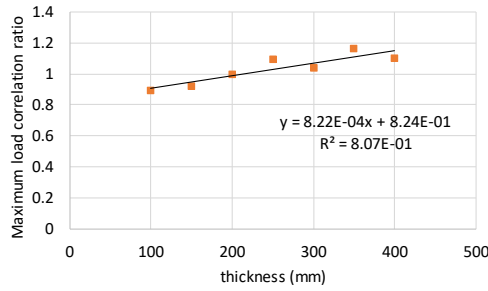
slab thickness are all related to the maximum impact force. The parameter contributing to the maximum impact force the most is the circumference of the projectile. These parameters are obtained by changing the corresponding parameter in the base analysis case, and other parameters remain unchanged to show the influence of a single parameter.



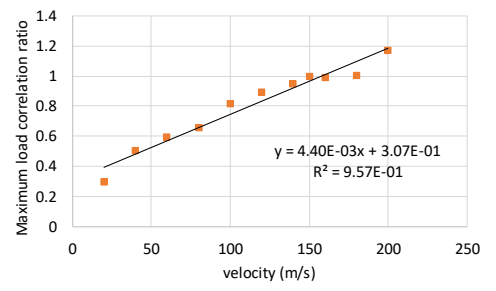
(a) circumference correction



(b) compressive strength correction



(c) Thickness correction



(d) Impact velocity correction

Fig 7.4 Correlation ratio

As shown in these figures, the change due to the parameters is monotonically increasing. Assume that the parameters are independent, the maximum impact force in the analysis can be expressed by the following functional equation.

$$F_{max} = f_{base} \cdot a_D \cdot a_f \cdot a_t \cdot a_v \quad (7.2.1)$$

where f_{base} is the maximum impact force in the base case, a_D is the correlation ratio of circumference, a_f is the correlation ratio of compressive strength, a_t is the correlation ratio of slab thickness, and a_v is the correlation ratio of impact velocity.

As shown in Fig 7.4, the relationship between the parameters and correlation ratios are nearly linear. Thus using these parameters, the correlation ratio can be obtained by a linear regression process. The correlation equations are shown below.

$$a_D = 2.04 \times 10^{-4} \times \pi D_s + 4.51 \times 10^{-2} \quad (7.2.2)$$

$$a_f = 4.87 \times 10^{-3} \times f_c + 8.23 \times 10^{-1} \quad (7.2.3)$$

$$a_t = 8.22 \times 10^{-4} \times t_c + 8.24 \times 10^{-1} \quad (7.2.4)$$

$$a_v = 4.40 \times 10^{-3} \times v + 3.07 \times 10^{-1} \quad (7.2.5)$$

where πD_s , f_c , t_c , v are the circumference (mm), compressive strength (N/mm²), slab thickness (mm), impact velocity (m/s) respectively.

7.2.2 Determination of the impact duration

Now that the function of the maximum impact force has been determined, the formula for calculating the impact duration is determined next. The impact force duration function is defined as the time required for the projectile to pass the distance of the concrete slab thickness at the average speed, as shown in Fig 7.5. Therefore, the parameters that depend on this definition are the average velocity of the projectile V_{ave} (average velocity of the collision velocity V_0 , which is the input condition for penetration analysis, and the residual velocity V_1 obtained from the analysis result), and the thickness t_c of the concrete plate. Therefore, the impact force duration in the analysis is expressed by the following functional equation.

$$\Delta t = g(t_c, V_{ave}) \quad (7.2.6)$$

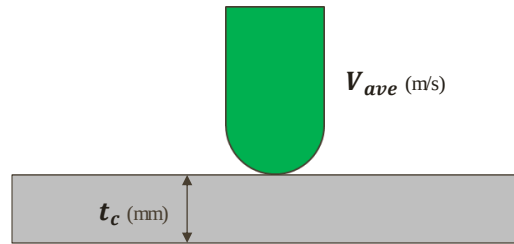
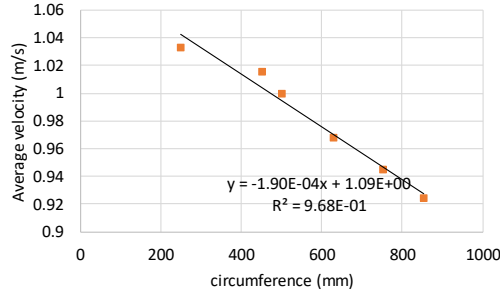
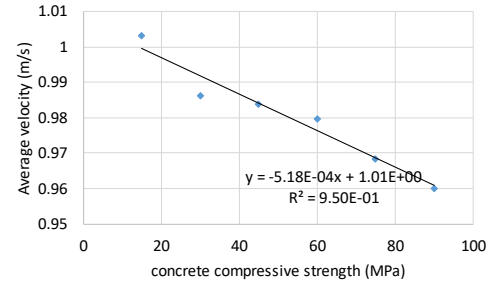


Fig 7.5 Definition of impact force duration

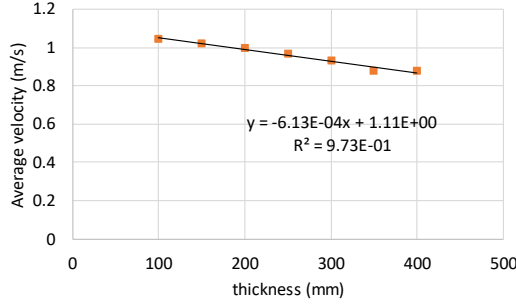
Just like the previous process to determine the maximum impact force, the average velocity is determined by the diameter D_s of the object (circumference πD_s), the compressive strength f_c of the concrete slab, the slab thickness t_c , and the impact velocity v . The relationship of these parameters are shown in the Fig 7.6.



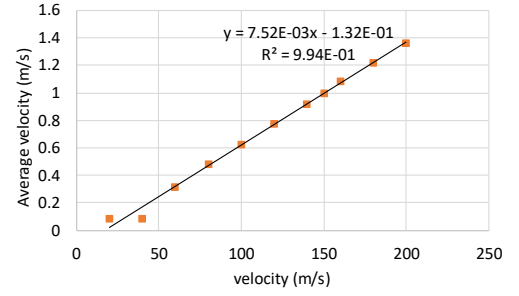
(a) circumference correction



(b) compressive strength correction



(c) Thickness correction



(c) Impact velocity correction

Fig 7.6 Correlation ratio of average velocity

As shown in these figures, the change due to the parameters is monotonically increasing. Assume that the parameters are independent, the average velocity in the analysis can be expressed by the following functional equation.

$$V_{ave} = V_{base} \cdot b_D \cdot b_f \cdot b_t \cdot b_v \quad (7.2.7)$$

where V_{base} is the average velocity in the base case, b_D is the correlation ratio of circumference, b_f is the correlation ratio of compressive strength, b_t is the correlation ratio of slab thickness, and b_v is the correlation ratio of impact velocity.

As shown in Fig 7.6, the relationship between the parameters and correlation ratios are nearly linear. Thus using these parameters, the correlation ratio can be obtained by a linear regression process. The correlation equations are shown below.

$$b_D = -1.90 \times 10^{-4} \times \pi D_s + 1.09 \quad (7.2.8)$$

$$b_f = -5.18 \times 10^{-3} \times f_c + 1.01 \quad (7.2.9)$$

$$b_t = -6.13 \times 10^{-4} \times t_c + 1.11 \quad (7.2.10)$$

$$b_v = 7.52 \times 10^{-3} \times v + 1.32 \times 10^{-1} \quad (7.2.11)$$

where πD_s , f_c , t_c , v are the circumference (mm), compressive strength (N/mm²), slab thickness

(mm), impact velocity (m/s), respectively.

Based on the above definition, the impact force duration can be expressed in the following equation

$$\Delta t = g(t_c, V_{ave}) = t_c / V_{ave} \quad (7.2.12)$$

7.2.3 Determination of the impulse criterion equation

The impulse equation is determined using the above equation. Specifically, it is assumed that the impulse can be roughly estimated by assuming the response of the impact force-time relationship as a trapezoid waveform of which the area can be calculated. The impulse equation is written as following.

$$I_m(kN \cdot ms) = F_{max} \cdot \Delta t / c_{shape} \quad (7.2.13)$$

where the parameters are reposted below

$$F_{max} = f_{base} \cdot a_D \cdot a_f \cdot a_t \cdot a_v \quad (7.2.1)$$

$$a_D = 2.04 \times 10^{-4} \times \pi D_s + 4.51 \times 10^{-2} \quad (7.2.2)$$

$$a_f = 4.87 \times 10^{-3} \times f_c + 8.23 \times 10^{-1} \quad (7.2.3)$$

$$a_t = 8.22 \times 10^{-4} \times t_c + 8.24 \times 10^{-1} \quad (7.2.4)$$

$$a_v = 4.40 \times 10^{-3} \times v + 3.07 \times 10^{-1} \quad (7.2.5)$$

$$\Delta t = t_c / V_{ave} = t_c / (V_{base} \cdot b_D \cdot b_f \cdot b_t \cdot b_v) \quad (7.2.12)$$

$$b_D = -1.90 \times 10^{-4} \times \pi D_s + 1.09 \quad (7.2.8)$$

$$b_f = -5.18 \times 10^{-3} \times f_c + 1.01 \quad (7.2.9)$$

$$b_t = -6.13 \times 10^{-4} \times t_c + 1.11 \quad (7.2.10)$$

$$b_v = 7.52 \times 10^{-3} \times v + 1.32 \times 10^{-1} \quad (7.2.11)$$

For the shape coefficient c_{shape} , this parameter is to determine the area of the trapezoid waveform. The trapezoid shape shows in Fig 7.1. is between triangle shape and rectangle shape, so c_{shape} is set to 1.5, between 2 for triangle and 1 for rectangle.

7.3 Validity verification of the impulse equation

Using the impulse equation determined in section 7.2, it is verified whether the impulse value of the analysis result and the impulse value of the experimental value obtained in the impact experiment with actual penetration can be accurately predicted. And the usefulness of the impulse formula for the

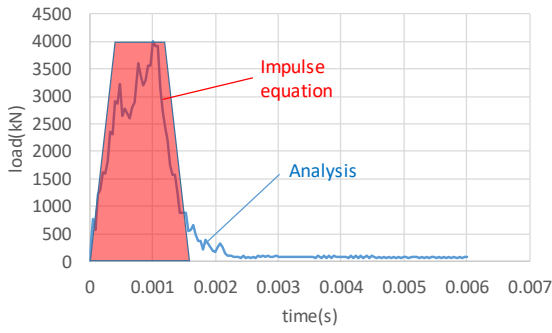
penetration prevention design is examined.

7.3.1 Verification of consistency between analysis response results and impulse equation

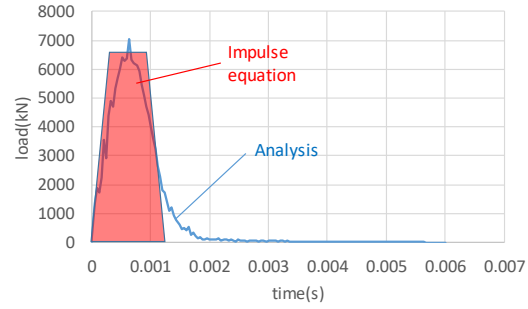
Table 7.2 shows three analysis cases in which each analysis parameter is changed from the standard parameters. The consistency of the impulse equation is shown by comparing the impulse obtained by the analysis of these cases with the impulse value calculated from the impulse equation determined earlier. Case 0 is the based case used to establish the impulse equation, and this case is examined first to verify the accuracy. Fig 7.9 shows the impact force-time relationship of these analysis results and the comparison result of the maximum impact force calculated from the impulse equation and the triangular waveform created from the impact time. From Table 7.2, it can be found that the value calculated by impulse equation is close to the result obtained from numerical analysis, and the error is within 10%, from which it can be said that the accuracy of the proposed impulse equation is good enough to predict the impulse of projectile to penetrate the slab in numerical analysis. In Fig 7.9, The trapezoid shape obtained by impulse equation is similar to the waveform in the analysis, thus the accuracy of this equation can be ensured.

Table 7.2 Analysis parameters for comparing the impulse equation

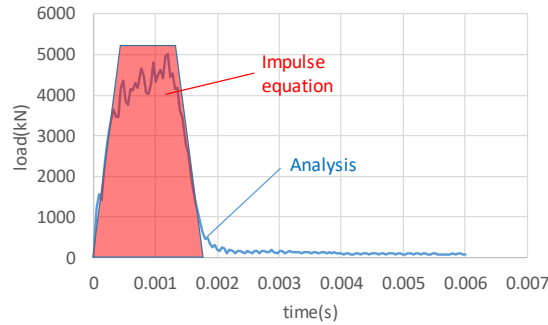
	Diameter (mm)	Compressive strength (N/mm ²)	Slab (mm)	Impact velocity (m/s)	Impulse in analysis (kN·ms)	Impulse by impulse equation (kN·ms)	Error
Case 0	80	34.1	200	150	3950	4038	+2.2%
Case 1	120	25	200	180	5865	5524	-5.8%
Case 2	80	34.1	300	200	6135	6231	+1.6%



(a) Case 0



(a) Case 1



(c) Case 2

Fig 7.9 Impact force — time relationship of analysis value and impulse equation

7.3.2 Comparison with other empirical equations

Finally, the proposed impulse equation is compared with other empirical equations.

Using the previous empirical formula described in Chapter 1, a comparison was made between the proposed impulse equation and the empirical equation on the calculated penetration depth. Specifically, the penetration depth obtained from the existing empirical equation and the impulse equation under the same conditions is compared. For the penetration depth obtained from the impulse equation, the minimum thickness of the slab to consume all the impulse of the projectile is defined as the penetration depth of the impulse equation.

Since the proposed impulse equation is better for projectiles with large mass and high impact velocity, the empirical equation should also be able to be applied to such cases. Here, the calculation example¹⁾ is taken as the comparing object, where the input parameters are listed in Table 7.3, and the used empirical equations are the modified NDRC equation²⁾, the modified Petry equation³⁾, and the modified BRL equation²⁾.

Table 7.3 Assumed impact case

Projectile	Material	Steel
	Nose shape	Hemisphere
	Diameter	200mm
	Mass	100kg
	Impact velocity	500m/s
Concrete	Compressive strength	100MPa
	Thickness	2000mm

(1) modified NDRC equation

The penetration of the NDRC equation can be calculated as

$$G\left(\frac{x}{D}\right) = 3.813 \times 10^{-5} \times \frac{NM}{D\sqrt{f'_c}} \times \left(\frac{V_0}{D}\right)^{1.8} \quad (7.3.1)$$

$$G\left(\frac{x}{D}\right) = \left(\frac{x}{2 \times D}\right)^2, \frac{x}{D} \leq 2 \quad (7.3.2)$$

$$G\left(\frac{x}{D}\right) = \frac{x}{D} - 1, \frac{x}{D} \geq 2 \quad (7.3.3)$$

where G is the function value, x is the penetration depth (m), D is the diameter of the projectile (m), N is the coefficient for the nose shape, and 0.72 for flat nose, M is the mass of the projectile (kg), f'_c is the concrete compressive strength, V_0 is the impact velocity.

First calculate the value of G

$$G = 3.813 \times 10^{-5} \times \frac{0.72 \times 100}{0.2\sqrt{100 \times 10^6}} \times \left(\frac{500}{0.20}\right)^{1.8} = 1.794$$

Then the penetration depth can be calculated

$$1.794 = \left(\frac{x}{2 \times 0.2}\right)^2$$

Solve this equation, $x = 0.535\text{m} = 535\text{mm}$, however $\frac{x}{D} = 2.67 > 2$, the condition of Eq. (7.3.2) is not satisfied, so

$$1.794 = \frac{x}{0.20} - 1$$

Solve this equation, $x = 0.558\text{m} = 558\text{mm}$, and $\frac{x}{D} = 2.67 > 2$, the condition of Eq. (7.3.3) is satisfied, so the penetration depth by modified NDRC equation is 558mm.

(2) modified Petry equation

The penetration depth can be calculated by

$$\frac{x}{D} = 0.0795 K_p \frac{M}{D^3} \log_{10} \left(1 + \frac{V_0^2}{19974} \right)$$

where G is the function value, x is the penetration depth (m), D is the diameter of the projectile (m), M is the mass of the projectile (kg), V_0 is the impact velocity, K_p is 0.00799 for concrete, 0.00426 for RC, and 0.00284 for concrete with more rebar reinforcement.

Substitute all the input parameters,

$$\frac{x}{D} = 0.0795 \times 0.00799 \times \frac{100}{0.2^3} \log_{10} \left(1 + \frac{500^2}{19974} \right) = 8.98$$

Solve this equation, $x = 8.98 \times 0.2 = 1.795\text{m} = 1795\text{mm}$, which is to say the penetration depth calculated by modified Petry equation is 1795mm.

(3) modified BRL equation

The penetration depth can be calculated with

$$\frac{x}{D} = \frac{1.33 \times 10^{-3}}{\sqrt{f_c'}} \left(\frac{M}{D^3} \right) D^{0.2} V_0^{1.33}$$

where G is the function value, x is the penetration depth (m), D is the diameter of the projectile (m), M is the mass of the projectile (kg), f_c' is the concrete compressive strength, V_0 is the impact velocity.

Substitute all the input parameters,

$$\frac{x}{D} = \frac{1.33 \times 10^{-3}}{\sqrt{100 \times 10^6}} \times \left(\frac{100}{0.2^3} \right) \times 0.2^{0.2} \times 500^{1.33} = 4.68$$

Solve this equation, $x = 4.68 \times 0.2 = 0.937\text{m} = 937\text{mm}$, which is to say the penetration depth calculated by modified BRL equation is 937mm.

(4) the proposed impulse equation

For the proposed impulse equation, the impulse needed to penetrate the concrete slab is a function to impact velocity, concrete compressive strength, projectile diameter, and the thickness of the concrete slab. When the impact velocity, concrete compressive strength, and the projectile diameter are fixed as the input parameter, the impulse of the projectile is a function of the thickness of the concrete slab, or the penetration depth. The impulse-penetration depth relationship is shown in Fig 7.10.

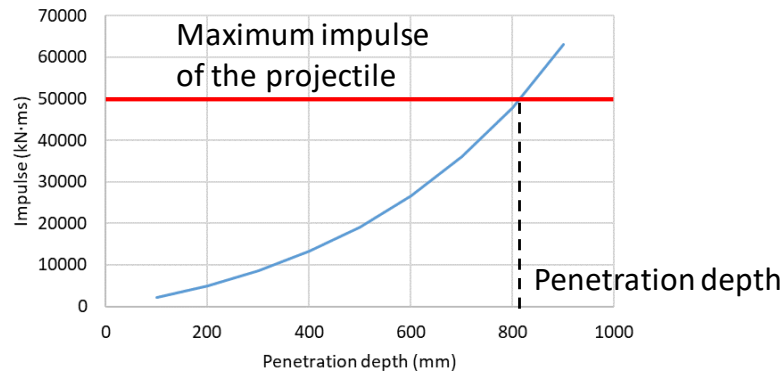


Figure 7.10 Determination of penetration depth by the impulse equation

For the impulse of the projectile, it can be easily calculated that the impulse of the given projectile is $P = mv = 100\text{kg} \times 500\text{m/s} = 50000\text{kg} \cdot \text{m/s} = 50000\text{kN} \cdot \text{ms}$. As shown in Fig 7.10, when the impulse is $50000\text{kN} \cdot \text{ms}$, the corresponding penetration depth is 817mm. This result is very close to the result given by modified BRL equation, and within the range of modified NDRC equation and modified Petry equation. The penetration depths are summarized in Table 7.4. From the results, it can be concluded when the mass and the velocity of the projectile is large, the penetration depth given by the impulse equation falls within a reasonable range.

Table 7.4 Assumed impact case

Equation	Penetration depth (mm)
Modified NDRC	558
Modified Petry	1795
Modified BRL	937
Impulse equation	817

7.4 Conclusion

In this chapter, the impulse consumed during the perforation process is taken as the criterion to judge the perforation limit of the concrete slab. An equation predicting the required impulse to perforate the concrete slab is established, and its accuracy is verified by comparing the equation results with analysis results and results given by other empirical equations. It can be found that, the results given by impulse

equation matches the analysis results in a good accuracy, and the results given by impulse equation falls into a reasonable range compared to other empirical equations. These results indicate that the proposed impulse equation is useful to predict the penetration/perforation failure of a concrete slab subjected to a projectile with large mass and impact velocity.

Chapter 8 Conclusion

8.1 Key conclusions and achievements

In this study, we present an analytical method for evaluating the penetration failure of a concrete slab due to the impact of a projectile. The penetration performance is evaluated by numerical analysis with the proposed method, and a penetration failure criterion is proposed. The results of this study are summarized below.

1. SPH method is adopted in this study to solve the deformation problem of solids. By introducing an adaptive smoothing length and an irreversible inter-particle contact algorithm, the accuracy of the analysis is improved.
2. For the elasto-plastic constitutive law of concrete, the nonlinear Drucker-Prager yield surface is adopted, which tends to cause shear failure as the hydrostatic pressure on the compression side increases. In addition, the strain rate effect, the stress softening, and the damage model, which are indispensable for dealing with the impact problem, are able to be properly introduced in the concrete material model. For the compressive fracture of particles, a crushing criterion is set with the volumetric strain on the compression side of the concrete, and the total stress of the particles is completely released.
3. The results of SPH numerical analysis of the falling weight test on RC beams using the proposed concrete model show that it is possible to accurately reproduce the impact force and displacement response of the falling weight and RC beam, which verified the validity of the proposed analysis method. In addition, by using the proposed SPH method, the penetration/perforation process of a concrete slab is reproduced in good accuracy. This confirmed that the proposed analysis method is effective against problems relating to penetration and perforation.
4. From the results of the penetration analysis on a concrete slab, it is found that with the increase of the impact velocity, the failure mode changes from bending failure to local shear failure. During this process, the impulse increases before the slab is perforated, and the impulse decreases after

the slab is perforated. Finally, the impulse remains stable when the local shear failure mode is dominating. This stable impulse is influenced little by the mass of the projectile, the supporting conditions, and the supporting interval.

5. The impulse consumed during the perforation process is taken as the criterion to judge the perforation limit of the concrete slab. An equation predicting the required impulse to perforate the concrete slab is established, and its accuracy is verified by comparing the equation results with analysis results and results given by other empirical equations. It can be found that, the results given by impulse equation matches the analysis results in a good accuracy, and the results given by impulse equation falls into a reasonable range compared to other empirical equations. These results indicate that the proposed impulse equation is useful to predict the penetration/perforation failure of a concrete slab subjected to a projectile with large mass and impact velocity.

8.2 Future works

As mentioned in Chapter 1, studies on local damage of concrete structures is not an old theme, and the number of researches is not enough so far. Therefore, the fracture mechanism of fracture mode such as spalling is still unclear, and it is necessary to build a concrete mechanical model including spalling damage and reproduce spall phenomenon by analysis. Since the response result is affected by the smoothing length and the kernel function in SPH method, it is recommended that to choose SPH parameters such as cohesion coefficients with great care. In the mechanical model used in this study, the strain rate effect simply increased the material strength in a uniaxial conversion, while according to the reference shown in Chapter 4, in some other concrete models, the strain rate also influences the static elastic modulus. These concrete model may be an alternative and better concrete model in the future work. For the proposed impulse equation, as discussed in Chapter 7, the accuracy is good enough to predict the impulse in the numerical analysis, but it is still uncertain for experiment cases. Since experiments with projectiles with mass of hundreds of kilograms and velocity of hundreds of meter per second is hard to find, the accuracy of the impulse equation should be discussed with other methods. And impulse equations should be established for lower mass and lower impact velocity to increase the applicable range. With the development of building materials and numerical analysis methods, better penetration prediction should be obtained by this study.

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