Note on a Categorical Quantum Field Theory

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Abstract

The categorical quantum field theory by Doplicher and Roberts is extended to describe the local gauge symmetry and the long-range force. These extensions become possible by fixing the observer's point of view.

In the study of the algebraic quantum field theory [1] the categorical description by Doplicher and Roberts was the greatest achievement for decades. However, it only describes the massive fields under the global gauge symmetry so that it cannot describe the QED and the QCD. In this Short Note we remove the obstacles to the description of the QED and the QCD. We employ the Minkowski space-time and neglect the effects of the gravity.

The core [1] of the categorical description by Doplicher and Roberts is the DR category whose **Object** is the localized charge and **Morphism** is the transporter. The element of the DR category is given by

$$\sigma \xrightarrow{T} \rho,$$

where σ and ρ are the **Object**s and T is the **Morphism**. Here σ represents a charged excitation localized around some space-time point. Around another point ρ represents it. The transporter T moves the excitation.

The local gauge symmetry is implemented by Ojima [2] by considering all the possible representations induced from the DR category. This is a huge extension of the description by Doplicher and Roberts. However, it is achieved by using only basic notions of the category theory, [3] the **Functor** and the **Natural Transformation**. A **Functor** V maps the DR category to the representation whose element is given by

$$V(\sigma) \xrightarrow{V(T)} V(\rho).$$

Another **Functor** W maps the DR category to

$$W(\sigma) \xrightarrow{W(T)} W(\rho).$$

The target category of the **Functor**s is the above-mentioned all the possible representations and the representations induced by V and W are sub-categories in the target

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*Department of Physics, Kyushu University category. The target category corresponds to the space of histories which will be discussed later. Each representation in the target category corresponds to a history in the space of histories. The two representations induced by V and W are related by the **Natural Transformation** v as

$$W(\sigma) \xrightarrow{v_\sigma} V(\sigma),$$

and

$$W(\rho) \xrightarrow{v_\rho} V(\rho).$$

This **Natural Transformation** represents the local gauge transformation for the charges if the representations induced by V and W are physically equivalent. This transformation corresponds to the vertical change in the fiber bundle description which will be discussed later. Here the local nature of the gauge transformation is represented by the difference between v_σ and v_ρ. Such a difference was prohibited in the categorical description by Doplicher and Roberts by the requirement of the space-time transportability. [1] In other words, such a difference was an obstacle to the global gauge symmetry.

Consequently the representations of the transporter, $V(T)$ and $W(T)$, are related by

$$W(T) = v_\rho^{-1} \cdot V(T) \cdot v_\sigma.$$

This relation found by Ojima [2] is in accordance with the gauge transformation of the Wilson line. Actually $W(T)$ corresponds to the Wilson line. The transporter corresponding to the Wilson line was expected by Roberts [4] long time ago. Here it has been proven to be a correct way to extend the DR category for the description of the local gauge symmetry.

Since we have assumed that the charged excitations are localized in space-time, the above extension of the DR category to describe the local gauge symmetry is relevant to the QCD. In order to describe the QED we need a further extension taking the long-range nature into account. Namely, we have to modify the definition of the **Object**s, σ and ρ, in the DR category. This modification will be discussed later.

As in the case of the lattice gauge theory we can easily implement the local gauge transformation without ghosts. The reason is as follows.

The gauge theory without ghosts was developed by DeWitt. [5] Here based on its updated formulation [6] we clarify why the local gauge symmetry can be implemented without ghosts.

First of all, we collect all the histories of the relevant fields. A history is a distribution of the relevant fields over the Minkowski space-time. We consider not only on-shell histories but also off-shell histories. Here the relevant fields mean the matter and gauge fields which constitute the QCD. Then we introduce a space of histories Φ. In this space a point represents a history. The introduction of such a large space avoids the use of ghosts.

Next we introduce a metric in this space of histories Φ. The metric represents the observer's point of view. The symmetry under the metric is represented by the Killing vector. Thus the gauge orbit in Φ is generated by the Killing vector.

We can view the space of histories Φ as the total space of a fiber bundle. The connection ϖ in this view defines the horizontal and vertical directions. The connection

ϖ reflects the observer's point of view. The change in the horizontal direction can be seen from the observer and is physical. The change in the vertical direction is that of the gauge transformation. The role of ghosts can be related to the vertical change [6] but we do not have to introduce ghosts in Φ.

The implementation of the local gauge transformation is extremely simplified by employing a large space Φ. The same is true for the case of the categorical description where we consider the categories induced by all the **Functor**s. In the space of histories Φ, the gauge transformation moves a point to another point where each point represents a history. A category induced by a **Functor** from the DR category corresponds to a history in Φ. The **Natural Transformation** maps an induced category to another induced category. The structures of these two schemes are the same.

Now we will discuss the modification of the **Object** in the DR category to describe the QED. In the presence of the long-range force we cannot introduce the charged excitations localized in space-time. Moreover, the physical excitation is a composite of matter and gauge fields as pointed out by Dirac. [7] Since a physical excitation is defined from an observer's point of view, Buchholz and Roberts [8] began their discussion by choosing a specific observer. The observer cannot see the whole Minkowski space-time but only the observer's future light-cone. A physical excitation should be defined in this light cone. Buchholz and Roberts [8] showed that a physical charged excitation is confined in the hyper-cone which is an unbounded part of the future light-cone and that employing such an excitation as the **Object** the categorical description by Doplicher and Roberts is safely applicable to the QED.

Precisely speaking, the super-selection sectors by Doplicher and Roberts is replaced by the charge classes. [8] Although the sectors are distinguished by the number of infrared photons, the observer cannot distinguish them due to the lack of information outside of the observer's future light-cone. Namely, the concept of the sector is not relevant to the observer. Instead the observer should use the charge class which is a class of the sectors with different numbers of infrared photons. The infrared problem is avoided by the use of the class. [8]

The confinement in the hyper-cone is carried out by the standard argument in the algebraic quantum field theory [1] as follows. First an excited state is introduced by creating a pair of matters which is linked by the gauge field and is neutral as a whole. In the pair two matters have opposite charges. Next we focus on one of the matters and throw away the other to infinitely apart space-time region. Then we obtain a charged excitation in the vicinity of the observer. This is the so-called "behind the moon" argument. [1] However, in the usual "behind the moon" argument, the link by the gauge field is not taken into account. The effect of the link is restricted within a hyper-cone which is a part of the future light-cone of the observer. Consequently a charged excitation which is a composite of matter and gauge fields is confined in the hyper-cone. The formation of such a composite (the dressing) is also necessary for the description in the space of histories. [6]

Once the description of the DR category is established, the local gauge transformation in the QED is the same as that in the QCD, since it is a map between categories induced from the DR category.

In this Short Note we have discussed the extensions of the categorical description by Doplicher and Roberts in the algebraic quantum field theory. The wisdom here is to fix the observer's point of view. We can implement the local gauge symmetry for the QED and the QCD by considering all the representations induced from the DR category. We can confine the effect of the long-range force in the hyper-cone so that we can use the description by Doplicher and Roberts even for the case of the QED. We have pointed out the equivalence between an induced category in our scheme and a history in DeWitt's scheme.

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