

## On hydrodynamic characteristics of gap resonance between two fixed bodies in close proximity

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1 On hydrodynamic characteristics of gap resonance between two fixed bodies in close  
2 proximity

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12  
13 **Abstract:**

14 The resonant water motion inside a narrow gap between two identical fixed boxes that are in  
15 side-by-side configuration is investigated using a two-dimensional (2D) numerical wave tank  
16 based on OpenFOAM<sup>®</sup>, an open source CFD package. Gap resonance is excited by regular waves  
17 with various wave heights, ranging from linear waves to strong nonlinear waves. This paper  
18 mainly focuses on the harmonic analyses of the free-surface elevation in the narrow gap and wave  
19 loads (including the horizontal wave forces, the vertical wave forces and the moments) on the  
20 bodies. It is found that the influences of the incident wave height on the higher-order harmonic  
21 components of different physical quantities are quite different. The effects of the incident wave  
22 height on the reflection, transmission and energy loss coefficients are also discussed. Finally,  
23 aiming at the quantitative estimation of the response time and the damping time of gap resonance,  
24 two different methods are proposed and verified for the first time on gap resonance.

25  
26 **Keywords:** Gap resonance; Wave height amplification; Wave force; Harmonic analysis; Response  
27 time and damping time of gap resonance; OpenFOAM<sup>®</sup>

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29 1. Introduction

30 In the past few decades, as the oil and gas industry have moved towards deeper waters and  
31 harsher environments, Floating Production Storage and Offloading (FPSO) platforms have shown  
32 great potential as the most economic ways to process and distribute the hydrocarbon products. One  
33 of the key challenges for FPSO platforms lies in the safe offloading operations from them to a  
34 shuttle tanker when the tanker is positioned side-by-side with them. When multiple floating bodies  
35 are deployed side-by-side in close proximity and are subjected to incident water waves, drastic  
36 water surface oscillations may occur inside the narrow gaps between them at certain frequencies.  
37 This phenomenon is normally referred to as “gap resonance”.

38 The hydrodynamic interactions of multiple bodies with narrow gaps between have been  
39 investigated extensively due to its relevance to offloading facilities for FPSO. The methods used  
40 in these studies include theoretical analyses, physical experiments and numerical simulations. The  
41 theoretical analyses were mainly used in the early studies of the gap resonance problem and were  
42 mainly based on the linear potential flow theory (Miao et al., 2000; Molin, 2001). Subsequently, to  
43 better understand gap resonance and to validate the theoretical analyses, a large number of  
44 physical model tests in 2D and 3D wave basins were also implemented by previous researchers  
45 (e.g., Iwata et al. (2007); Saitoh et al. (2006); Zhao et al. (2017)). The numerical investigations  
46 conducted so far are mainly based on the classical potential flow model employing the boundary  
47 element method and scaled boundary finite element method (e.g., Li et al. (2005); Li and Zhang  
48 (2016); Sun et al. (2010)).

49 Although both theoretical analyses and the numerical simulations based on the potential flow  
50 theory have been shown to predict the resonant frequency well, they were reported to significantly  
51 over-estimate the resonant wave height inside the gap and the wave forces on the floating bodies,  
52 because the physical energy dissipation due to the fluid viscosity, vortex shedding and even  
53 turbulences cannot be considered in the context of potential flow theory. To overcome this  
54 problem, several particular numerical techniques that artificially introduce wave energy  
55 dissipation term into the potential flow model were developed so far (Chen, 2004; Huijsmans et al.,  
56 2001; Lu et al., 2010b; Newman, 2004; Ning et al., 2015a, b). However, the introduction of  
57 artificial damping term seems somewhat arbitrary for the rigorous potential theory, and under  
58 some conditions it was found to be difficult to obtain a unique value of the damping parameter

59 (Pauw et al., 2007; Tan et al., 2014). In recent years, with the fast developments of computing  
60 technology, the CFD simulation has gradually become an alternative method in investigating the  
61 gap resonance problem (Jiang et al. (2018); Lu et al. (2010a); Lu et al. (2011a); Lu et al. (2011b);  
62 Moradi et al. (2015, 2016)). All these studies found that the results obtained by the CFD  
63 simulations agreed well with those from existing experiments.

64 While many research efforts into the gap resonance have been undertaken, the majority have  
65 concentrated on the analyses of the overall resonant wave height in the narrow gap and the overall  
66 wave loads on the boxes under the condition of the linear or weakly nonlinear regular waves (e.g.,  
67 Feng et al. (2017); Jiang et al. (2018); Lu et al. (2010a); Lu et al. (2010b); Lu et al. (2011a); Lu et  
68 al. (2011b); Moradi et al. (2015, 2016)). The investigations on the harmonic analyses of the wave  
69 height and wave loads are relatively rare. By using a semi-analytical formulation of the velocity  
70 potentials, Mavrakos and Chatjigeorgiou (2009) investigated the significance of the second-order  
71 effects to the total wave loading on a cylindrical moonpool, especially in the frequency regions in  
72 which the fluid resonance occurs. Sun et al. (2010) employed a 3D boundary element code  
73 DIFFRACT to investigate the first- and second-order loads and free-surface elevations for two  
74 side-by-side rectangular barges. However, both of their methods are based on the classical  
75 potential flow theory which does not consider the physical energy dissipation due to the viscous  
76 effect. Hence, some of their findings may not reflect real phenomena of the fluid resonance in the  
77 narrow gap or in the moonpool, where the physical energy dissipation plays an important role.  
78 Zhao et al. (2017) investigated experimentally the first and higher harmonic components of the  
79 resonant fluid response in the gap between two identical fixed rectangular boxes excited by the  
80 transient focused wave groups in a 3D wave basin. However, the gap resonance induced by the  
81 regular waves and the harmonic analyses on wave loads were not considered in that paper.

82 To further improve the understanding of related phenomena involved in gap resonance, this  
83 paper mainly focuses on the variations of the first and higher harmonic components of free-surface  
84 elevation inside the gap and wave loads on boxes with respect to the wave height of the incident  
85 regular waves when gap resonance occurs. In this paper, the system of two identical boxes is taken  
86 as the background of this study. For comparison, the same problem is also investigated when the  
87 free-surface elevation in the narrow gap is under non-resonant conditions. Compared to the  
88 previous investigations (i.e., Feng et al. (2017); Jiang et al. (2018); Lu et al. (2010a); Lu et al.

89 (2010b); Lu et al. (2011a); Lu et al. (2011b); Moradi et al. (2015, 2016)), stronger nonlinear  
90 incident waves are used in this paper, which is necessary due to the fact that FPSO platforms are  
91 often exposed to severe wave conditions. Subsequently, the variations of the reflection coefficient  
92  $C_r$ , the transmission coefficient  $C_t$  and the energy loss coefficient  $L_e = 1 - C_r^2 - C_t^2$  with respect  
93 to the frequency of the incident waves with various wave heights are also discussed, because an  
94 integral comprehension of these coefficients may promote the better understanding of the  
95 mechanism essence of the gap resonance (Jiang et al., 2018). Meanwhile, these previous studies  
96 were mainly concerned on the related hydrodynamic phenomena after the free-surface resonance  
97 in the narrow gap reached the steady state, and both the response and the damp phases were paid  
98 little attention to. In the current paper, both the response time and the damping time of gap  
99 resonance are quantitatively evaluated by two different methods. In practical engineering  
100 applications, the fast and accurate estimation of the response time and the damping time is very  
101 important for the safe evacuation of staff and the reasonable arrangement of operation time during  
102 the offloading operations from a FPSO platform to a shuttle tanker under gap resonance  
103 conditions.

104 In Sections 2, 3 and 4, the numerical model employed in this work, numerical experimental  
105 setup and the validations of the numerical model against available experimental and numerical  
106 data are presented, respectively. The numerical results and discussions are presented in Section 5.  
107 Finally, conclusions are drawn in Section 6.

108

## 109 2. Numerical model description

110 To consider the physical energy dissipation near the gap due to the viscous effect, a viscous  
111 flow solver is necessary. In this paper, the numerical wave tank is based on the OpenFOAM®  
112 multiphase solver “interFoam”, and waves are generated and dissipated using the relaxation-based  
113 wave generation toolbox “waves2Foam” proposed by Jacobsen et al. (2012).

### 114 2.1. Governing equations

115 The continuity and Navier-Stokes equations are utilized as the governing equations to solve  
116 the two-phase flow of water and air:

$$117 \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

118 
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}^T) = -\nabla P - (\mathbf{g} \cdot \mathbf{x}) \nabla \rho + \nabla \cdot (\mu \nabla \mathbf{u}) + \sigma_i k_\alpha \nabla \alpha, \quad (2)$$

119 where  $\rho$  is the fluid density,  $\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$  is the gradient operation,  $\mathbf{u} = (u, v, w)$  is the velocity  
 120 vector of the fluid,  $\mathbf{x} = (x, y, z)$  is the Cartesian coordinate vector,  $\mathbf{g}$  is the gravitational acceleration,  
 121  $P$  is the pressure in excess of the hydrostatic part,  $\mu$  is the dynamic viscosity of the fluid,  $\sigma_i$  is  
 122 the surface tension coefficient and  $k_\alpha$  is the surface curvature. The above equations are solved  
 123 for both water and air simultaneously.  $\alpha$  denotes the volume fraction of water in the computational  
 124 cell, which takes a value of 1 for water and 0 for air and intermediate values for a mixture of water  
 125 and air. The distribution of  $\alpha$  is modelled by the following advection transport equation:

126 
$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) + \nabla \cdot [\alpha (1 - \alpha) \mathbf{u}_r] = 0, \quad (3)$$

127 in which  $\mathbf{u}_r = \mathbf{u}_{\text{water}} - \mathbf{u}_{\text{air}}$  is a relative velocity between the water and the air. Using  $\alpha$ , the spatial  
 128 variation of any fluid property  $\varphi$  (e.g., the fluid density  $\rho$  and the dynamic viscosity  $\mu$ ) can be  
 129 expressed through the weighting

130 
$$\varphi = \alpha \varphi_{\text{water}} + (1 - \alpha) \varphi_{\text{air}}, \quad (4)$$

131 where the subscripts “water” and “air” denote the corresponding fluid property of water and air,  
 132 respectively.

## 133 2.2. Boundary conditions and numerical implementations

134 The toolbox “waves2Foam” proposed by [Jacobsen et al. \(2012\)](#) is employed to generate and  
 135 absorb waves at the boundaries (see Fig. 1). At the inlet and the outlet boundaries, the velocities  
 136 are defined as that of a regular incoming wave and as zero, respectively, and the pressure gradients  
 137 are set to zero. Two relaxation zones are deployed at the inlet and the outlet boundaries to absorb  
 138 the reflected and the transmitted waves. At the upper part of the tank, the boundary condition is set  
 139 as “atmosphere”; while at the bottom of the tank and the solid walls of the fixed boxes, “no-slip”  
 140 boundary condition is applied. For a 2D problem, the boundary condition on the walls in the third  
 141 dimension is set to “empty”.

142 The governing equations (1)-(2) and the advection transport equation (3) are solved based on  
 143 the finite volume method. The velocity-pressure coupling is calculated using the PISO (Pressure  
 144 Implicit with Splitting of Operator) algorithm. Gradients are approximated by the Gaussian  
 145 integration method based on a linear interpolation from cell centers to cell faces. The time

146 derivatives are solved by a first-order Euler scheme. The Gauss Convection-specific schemes are  
 147 used for the evaluation of the divergence terms. Identical to Feng et al. (2017), to produce accurate  
 148 and stable results, the largest Courant number is set to 0.25 in all simulations.

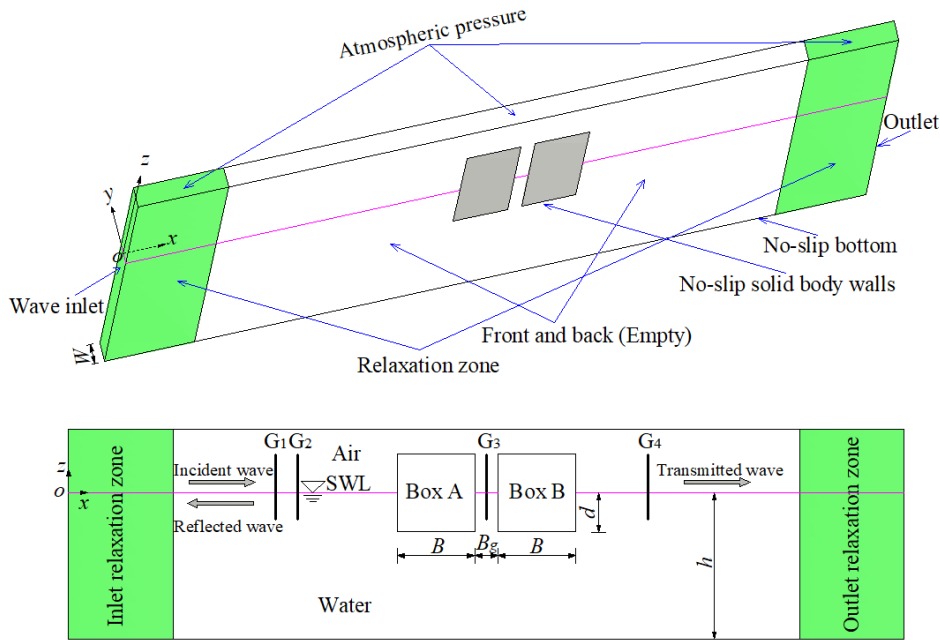
149 Once Eqs. (1)-(3) are solved at each time step, the wave force and the moment on the  
 150 structure can be calculated by the following formulations:

$$151 \quad \mathbf{F} = \int_{\Omega} [P\mathbf{n} + \mu(\partial\mathbf{u}_r/\partial\mathbf{n})] ds, \quad (5)$$

152 and

$$153 \quad \mathbf{M} = \int_{\Omega} \mathbf{r} \times [P\mathbf{n} + \mu(\partial\mathbf{u}_r/\partial\mathbf{n})] ds, \quad (6)$$

154 where  $\mathbf{F}$  and  $\mathbf{M}$  are the vectors of the wave force and the moment, respectively,  $\mathbf{u}_r$  is the tangent  
 155 velocity component,  $\mathbf{n}$  is the unit normal vector,  $ds$  is the surface area differential on the wet solid  
 156 surface  $\Omega$ , and  $\mathbf{r}$  is the position vector of  $ds$  relative to a certain space point. For the gap resonance  
 157 problem that will be described in detail in Section 3, the moments on the two fixed boxes  
 158 correspond to their respective centroids. As for the harmonic analysis for various variables (i.e.,  
 159 the free-surface elevation in the gap, the horizontal and vertical wave forces and the moments on  
 160 the two boxes), they are performed by using the discrete Fourier transform for the time-histories of  
 161 their respective signals.



162  
 163 **Fig. 1.** Sketch of the numerical wave tank: (a) boundary conditions and the definition of the  
 164 coordinate system; (b) positions of wave gauges and the definition of the geometric parameters.

165 3. Numerical wave tank

166 Fig. 1 illustrates the sketch of the 2D numerical wave tank used in the present study. The  
167 wave tank has a length of 18.5 m, a height of 0.8 m and a width of  $W=0.1$  m. The origin of the  
168 coordinate system is located at the still water level (SWL) of the left inlet boundary. The  $x$ -axis is  
169 in the wave propagation direction, and the  $z$ -axis is in the upward direction. The thickness of the  
170 wave tank in  $y$ -direction corresponds to a cell. Two identical fixed boxes are placed at the middle  
171 of the wave tank. The box height is  $H=0.5$  m, the breadth is  $B=0.5$  m, the draft  $d=0.25$  m, the gap  
172 width  $B_g=0.05$  m, the water depth is  $h=0.5$  m, and the air depth is  $h_a=0.3$  m. This configuration is  
173 in accordance with the physical experiments in [Saitoh et al. \(2006\)](#) as well as the numerical  
174 investigations in [Lu et al. \(2008; 2011a; 2011b\)](#).

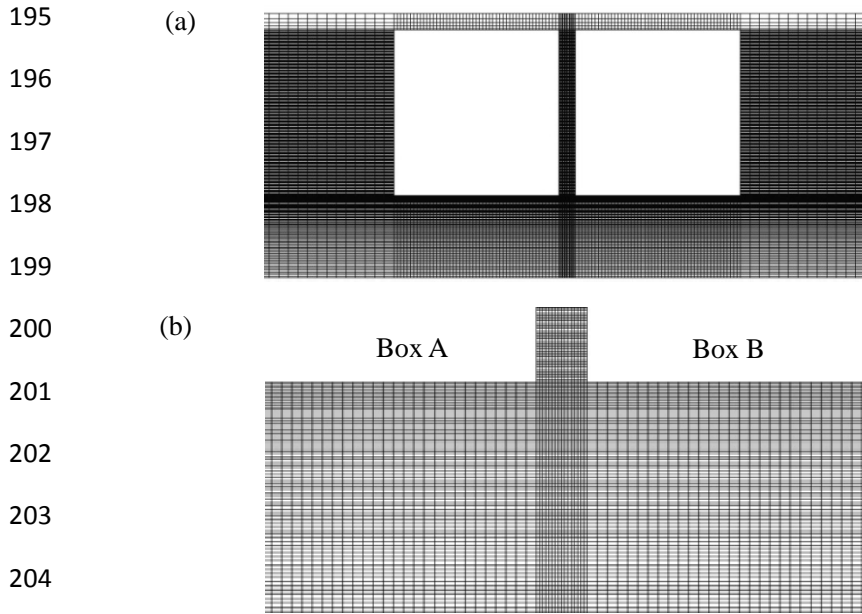
175 Five sets of simulations are implemented, in which the wave heights of the incident regular  
176 waves are set to  $H_0 = 0.010$  m, 0.024 m, 0.050 m, 0.075 m and 0.100 m, respectively. The wave  
177 frequency,  $\omega$ , considered in all the five sets of simulations ranges from 4.456 rad/s to 7.534 rad/s.  
178 Correspondingly, the dimensionless wavenumber,  $kh$ , ranges from 1.210 to 2.910, where  $k=2\pi/L$   
179 denotes the wavenumber and  $L$  denotes the wavelength. Four wave gauges,  $G_1$ - $G_4$ , are arranged to  
180 record the free-surface elevations.  $G_1$  and  $G_2$  are utilized to decompose the incident and reflected  
181 waves, and their distance is set to 0.25 m.  $G_3$  and  $G_4$  are used to obtain the free surfaces inside the  
182 gap and the transmitted waves.  $G_3$  is placed in the middle of the gap; while  $G_2$  and  $G_4$  are  
183 positioned at 1.50 m from the left side of Box A and the right side of Box B, respectively. Two  
184 relaxation zones of 5.50 m long each are placed at the inlet and outlet boundaries of the wave tank  
185 to absorb the reflected and transmitted waves. The length of 5.50 m is approximately 2.11 times of  
186 the maximum wavelength that corresponds to the incident waves with  $\omega=4.456$  rad/s.

187 A built-in mesh generation utility supplied with OpenFOAM®, “blockMesh”, is employed to  
188 generate meshes. A typical computational mesh is shown in Fig. 2. Non-uniform meshes are  
189 adopted for saving the computational time. The fine meshes with higher resolution are used  
190 around the boxes, especially in the vicinity of the narrow gap. To capture the interface between  
191 water and air, the meshes gradually become denser from the bottom and the atmosphere  
192 boundaries to the still water level.

193

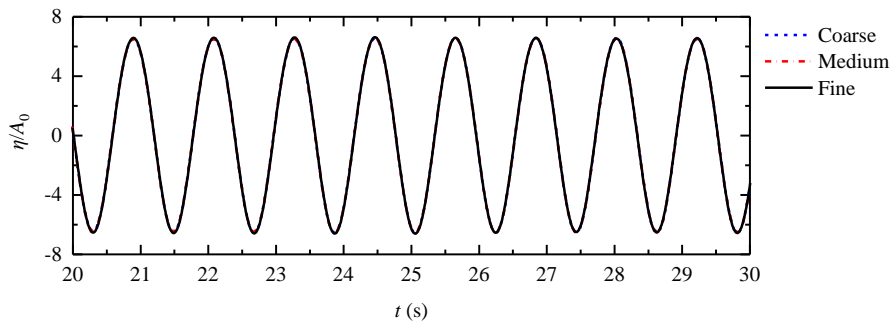
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205 **Fig. 2.** Side view of typical meshes in the computational domain: (a) the meshes around the boxes;  
 206 (b) the meshes close to the gap inlet

207



208

209 **Fig. 3.** Dependence of the free-surface elevation in the gap on the mesh resolution for the incident  
 210 waves with  $kh=1.556$  and  $H_0=0.010$  m, in which  $A_0=H_0/2$  denotes the amplitude of the incident  
 211 waves.

212

213

214 To examine the dependence of the numerical results on the mesh resolution, the free-surface  
 215 response in the narrow gap is simulated using three different meshes, namely the coarse, medium  
 216 and fine meshes. The numbers of the cells for these three meshes are 143600, 224060 and 340880,  
 217 respectively. Based on the numerical results that will be shown in Section 4.1, the free-surface  
 218 resonance in the gap occurs at  $kh=1.556$ . Fig. 3 presents the resonant free surfaces inside the gap  
 219 induced by the incident waves with  $kh=1.556$  and  $H_0=0.010$  m.  $A_0=H_0/2$  in this figure denotes the  
 incident wave amplitude. It is seen that the time histories of the free-surface elevations for the

219

220 three mesh configurations are almost identical to each other. Meanwhile, considering that the  
221 medium mesh can provide more accurate simulations of the wave fields excited by the incident  
222 waves with higher frequencies as compared to the coarse mesh, in all our numerical simulations,  
223 the medium mesh configuration is employed.

224 For most of the simulations, a total time of 40.0 s is considered. However, to study the  
225 damping time of the resonant free surface, for the five cases in which the incident wave frequency  
226 equals the resonant frequency, a total time of 70.0 s is employed. The wave inlet boundary stops  
227 working after 40.0 s, and the numerical model continues to simulate the damping process. It can  
228 be seen from Fig. 3 that the free-surface elevation in the gap has already reached the steady state at  
229  $t=20.0$  s. All the numerical results that will be presented in Section 4.1 and Sections 5.1–5.3 are  
230 based on the simulated steady-state data from 20.0 s to 40.0 s. While in Section 5.4, the time  
231 histories of the free-surface elevation in the gap between 0 – 20.0 s and 40.0 s – 70.0 s are utilized  
232 to investigate the response time and the damping time of gap resonance, respectively.

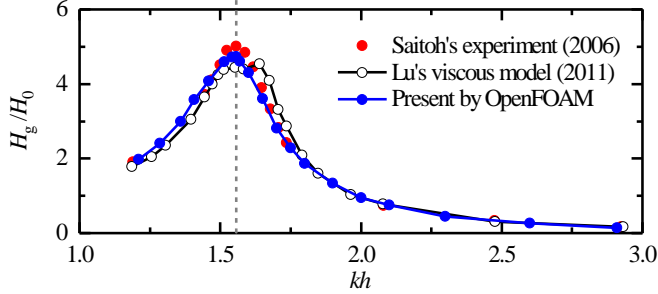
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#### 234 4. Numerical model validations

235 To guarantee the reliability of the model and the accuracy of the numerical results, the  
236 numerical model and the numerical wave tank illustrated in Sections 2 and 3 are first validated by  
237 comparing the present results obtained by OpenFOAM<sup>®</sup> with available experimental data and  
238 numerical results in previous literatures. For the simulations with  $H_0=0.024$  m described in  
239 Section 3, Saitoh et al. (2006) and Lu et al. (2011b) have measured the amplification of the  
240 free-surface elevation inside the gap and the wave forces on boxes by using physical experiments  
241 and a viscous flow model, respectively. Comparisons of the present results with those in the two  
242 papers will be presented in Section 4.1. Because the current research mainly focuses on the  
243 harmonic analysis of the free-surface elevation in the gap and the wave loads on the boxes, it is  
244 essential to further examine the capability of the present model to predict the higher-order  
245 harmonic components of the free-surface elevation or the wave loads. To the best of our  
246 knowledge, for the gap resonance problem, the experimental data on the higher-order harmonic  
247 components of the free-surface elevation or the wave loads are rare. However, Rodríguez et al.  
248 (2016) implemented physical experiments on the interactions between regular waves and one  
249 fixed box, and the experimental data of the vertical wave force on the box (including the first- and

250 second-order harmonic components) were presented in that paper. The numerical reproduction for  
 251 part of their experiments will be implemented in Section 4.2.

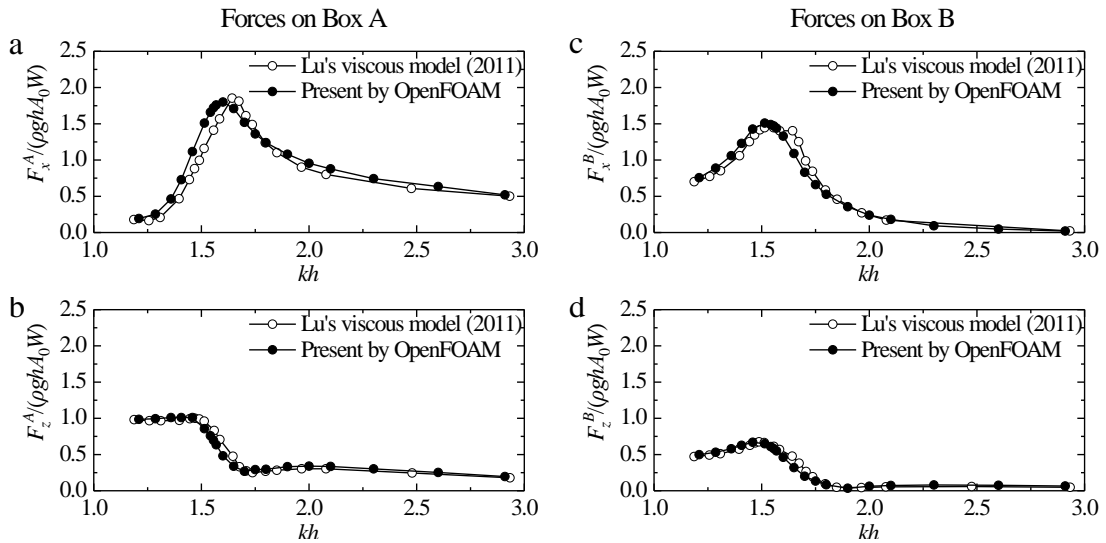
252 4.1. Two-boxes condition



253  
 254 **Fig. 4.** Amplification of the free-surface elevation inside the narrow gap for the cases with  
 255  $H_0=0.024$  m, in which  $H_g$  denotes the wave height inside the narrow gap.

256  
 257 Fig. 4 illustrates the amplification of the free-surface elevation inside the narrow gap excited  
 258 by the incident waves with  $H_0=0.024$  m. It can be seen that the predicted resonant frequency,  
 259  $kh=1.556$ , by the present numerical model is almost identical to those obtained by both the  
 260 laboratory tests of Saitoh et al. (2006) and the CFD results of Lu et al. (2011b). Besides, in general,  
 261 the variation of  $H_g/H_0$  with respect to  $kh$  also agrees well with their results. Fig. 5 further presents  
 262 the comparisons of the horizontal and vertical wave forces on Boxes A and B predicted by  
 263 OpenFOAM® and those by the CFD results in Lu et al. (2011b). Similar to Fig. 4, the overall  
 264 agreement between the present results and those in Lu et al. (2011b) is also observed.

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266  
 267 **Fig. 5.** Variations of the wave forces on the two boxes with respect to the incident wave frequency.

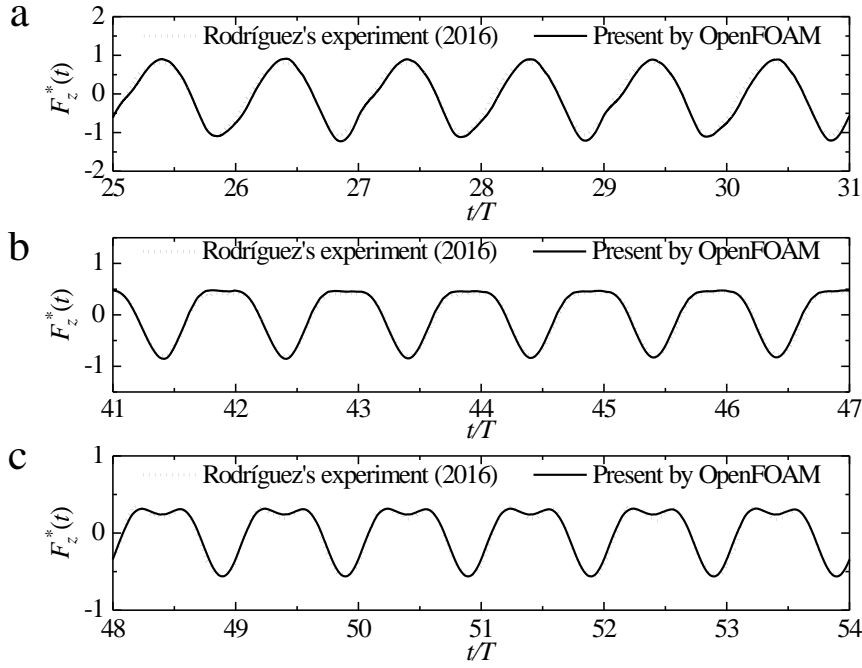
268 (a) and (b) correspond to the horizontal and vertical forces on Box A, respectively; (c) and (d)  
269 correspond to the horizontal and vertical forces on Box B, respectively.

270

271 4.2. One-box condition

272 [Rodríguez et al. \(2016\)](#) performed laboratory experiments in a 2.79 m wide and 63.00 m long  
273 wave tank, and the water depth is  $h=1.25$  m. A rectangular box was placed approximately in the  
274 center of the wave tank, at  $x=29$  m, where  $x=0$  defines the location of the wave-maker. Because  
275 the experimental study sought to achieve 2D flow conditions, the width of the rectangular box was  
276 chosen as 2.76m, leaving only a very small gap of 0.015 m to either of the tank's sidewalls. Single  
277 box geometry with the breadth  $B=0.50$  m and the draught  $d=0.25$  m. The regular incident waves  
278 with  $0.4 \leq kB \leq 2.4$  were considered. Two series of physical experiments were carried out with  
279 two steepnesses of the incident waves  $kA_0 = 0.05$  and 0.10. To examine the performance of the  
280 numerical model for the strongly nonlinear wave conditions, the series of experiments with  $kA_0 =$   
281 0.10 are reproduced by OpenFOAM<sup>®</sup> here. Considering that the box used in [Rodríguez et al.](#)  
282 [\(2016\)](#) has the same breadth and draft with the two boxes shown in Fig. 1, a very similar  
283 numerical wave tank (not shown in this paper for brevity) with that in Fig. 1 is employed to  
284 implement the present simulations. Compared to the wave tank shown in Fig. 1, there only exist  
285 two main differences in the present wave tank. First, there is only a single box located in the  
286 middle of the present wave tank. Second, the water depth is deepened from 0.50 m to 1.25 m. A  
287 mesh configuration that has a similar mesh density with the medium mesh described in Section 3  
288 is utilized. It should be noted that due to the relaxation zone deployed around the inlet and outlet  
289 boundaries, it is not necessary for the numerical wave tank to set the same length, 63.00 m, as the  
290 physical wave tank, and the numerical tank with a length of 18.5 m is already long enough.

291 Fig. 6 presents the simulated and experimental time-histories of the non-dimensional vertical  
292 wave force,  $F_z(t)/(0.5\rho gA_0BW)$ , for three cases with  $kB=0.8$ , 1.4, and 2.0. It can be obviously  
293 seen that significant nonlinearities are present, particularly for  $kB=1.4$  and 2.0, due to the vertical  
294 asymmetry of the force traces. Overall, the agreement between the present numerical results and  
295 the experimental data is good. Fig. 7 further quantitatively compares the first- and second-order  
296 harmonic components of the experimental and numerical vertical forces for all cases with  
297  $kA_0=0.10$ . Good agreement between the experimental and numerical results is also observed.



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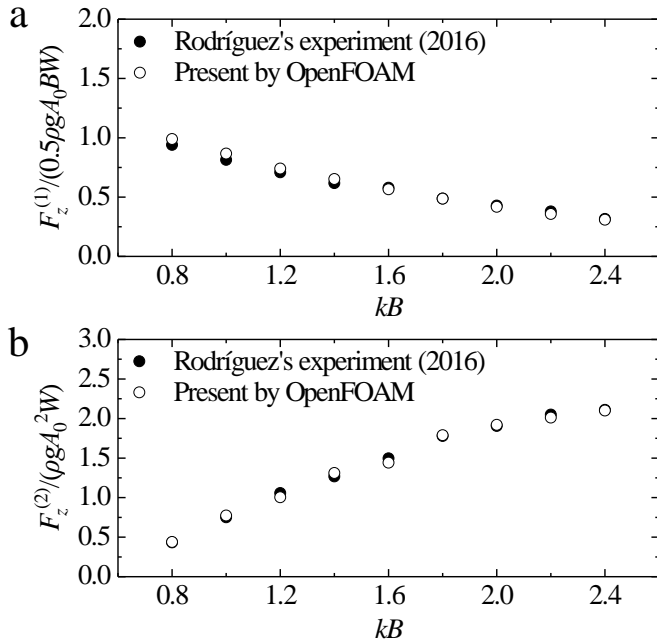
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302

**Fig. 6.** Time histories of the vertical wave forces excited by the regular waves with  $kA_0=0.10$  and (a)  $kB=0.8$ , (b)  $kB=1.4$  and (c)  $kB=2.0$ , in which  $F_z^*(t) = F_z(t)/(0.5\rho gA_0BW)$  denotes the time history of the non-dimensional vertical wave forces.



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**Fig. 7.** Non-dimensional (a) first-order and (b) second-order vertical wave forces excited by the incident regular waves with  $kA_0=0.10$

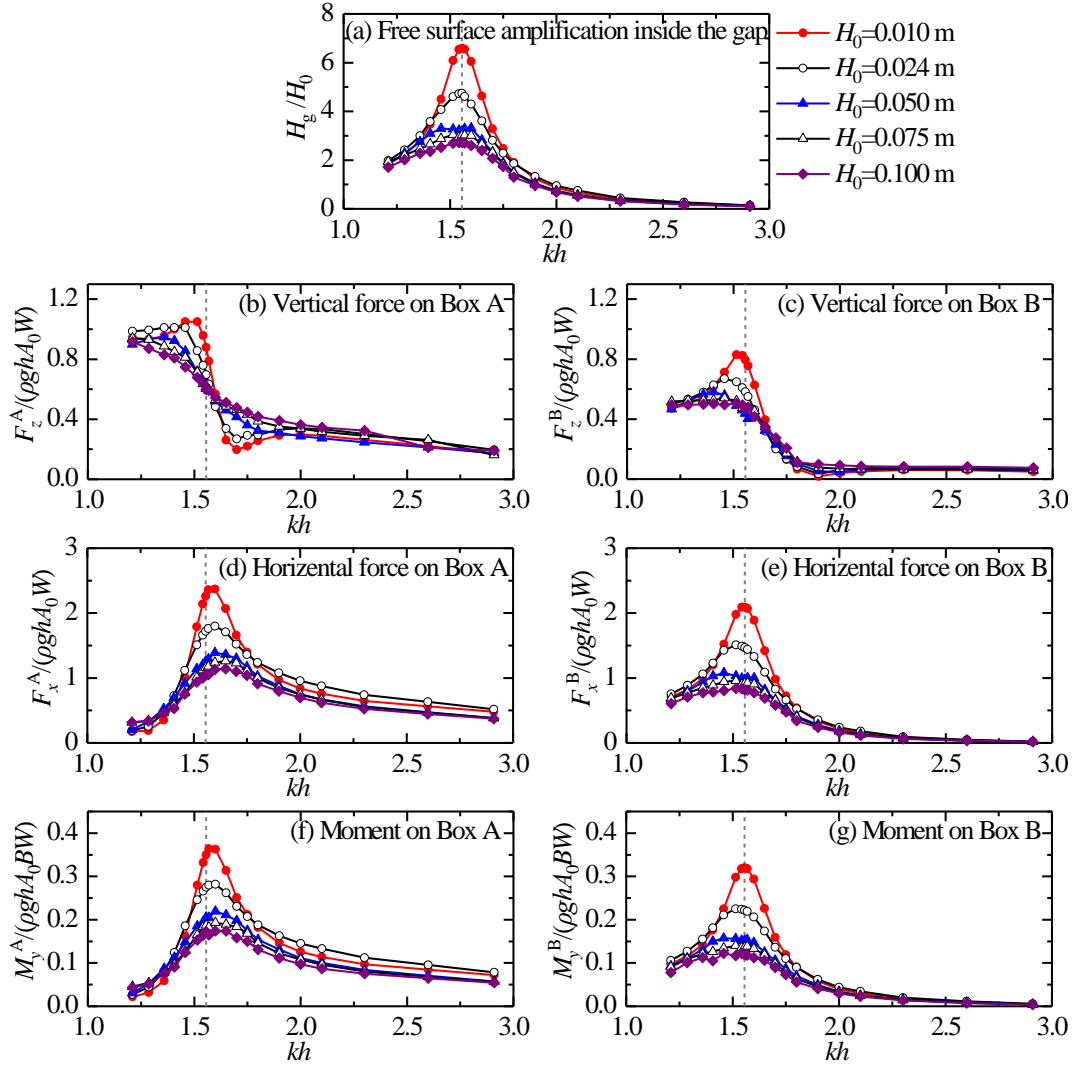
## 308 5. Numerical results and discussion

309 In order to present an overall impression of the hydrodynamic characteristics of gap  
310 resonance under the conditions of various incident wave heights on the reader, the variations of the  
311 overall wave height amplification in the narrow gap and the overall wave loads on boxes with  
312 respect to the incident wave frequency are first illustrated and discussed in Section 5.1.  
313 Subsequently, to find out the relative importance of different harmonic components, the first three  
314 order harmonic components of the free-surface elevation in the narrow gap and the wave loads on  
315 boxes are analyzed in Section 5.2. Then, to better explain some phenomena presented in Sections  
316 5.1 and 5.2 and better understand the mechanism essence of gap resonance, the variations of the  
317 reflection, transmission and energy loss coefficients with respect to the frequency of the incident  
318 waves with various wave heights are discussed in Section 5.3. Finally, considering the importance  
319 of the fast and accurate estimation of the response time and the damping time of gap resonance,  
320 two different estimation methods are proposed and verified in Section 5.4.

321

### 322 5.1 Overall wave height amplifications and overall wave loads

323 Fig. 8 shows the overall free-surface amplification in the narrow gap and the overall wave  
324 forces and moments impacting on Boxes A and B excited by the incident regular waves with  
325 various wave heights. Four obvious phenomena can be easily seen. First, it is seen from Fig. 8a  
326 that the resonant frequency seems not sensitive to the incident wave height. For the cases with  
327  $H_0=0.010$  m,  $0.024$  m and  $0.100$  m, all the three variation curves of the free-surface amplification  
328 with the frequency present perfect single-peak shapes, and the maximum free-surface  
329 amplification in the narrow gap always occurs at the resonant frequency, i.e.,  $kh=1.556$ . However,  
330 for the cases with  $H_0=0.050$  m and  $0.075$  m, the two variation curves of the free-surface  
331 amplification do not show the perfect single-peak shape. The two curves around the resonant  
332 frequency become flat, and the values of free-surface amplification at  $kh=1.556$  are even slightly  
333 less than the ones at its both adjacent sides. The reason for this phenomenon can be attributed to  
334 the almost invariable reflection coefficients around the resonant frequency under the conditions of  
335  $H_0=0.050$  m and  $0.075$  m (it will be shown in Section 5.3).



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**Fig. 8.** The overall free surface amplification in the gap and the overall wave forces and moments on Boxes A and B induced by the incident regular waves with various wave heights. The vertical dash line refers to the position of the resonant frequency.

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Second, for the vertical wave forces on both the two boxes (Fig. 8b and c), there exist obvious deviations between the frequency at which the maximum vertical wave force appears and the resonant frequency. However, there are some different features for the changing trends of the vertical wave forces on the two boxes. For Box A, the difference between the frequency at which the maximum vertical wave force appears and the resonant frequency monotonously increases with the incident wave height. Besides, the vertical wave forces excited by the incident wave with small height tends to increase first, then sharply decrease, then slowly increase, and then slowly decrease with the non-dimensional wavenumber,  $kh$ . However, with the increase of the incident

349 wave height, the vertical wave forces gradually become monotonic decrease with  $kh$ . For Box B,  
350 when the incident wave height is small, the changing trend of the vertical wave forces with  $kh$  is  
351 similar to that for Box A. When the incident wave height becomes large, the value of the vertical  
352 wave force seems insensible to the incident wave frequency at the ranges of  $kh < 1.5$  and  $kh > 1.9$ ,  
353 and its value only decreases sharply with the incident wave frequency at the range of  $1.5 < kh <$   
354  $1.9$ .

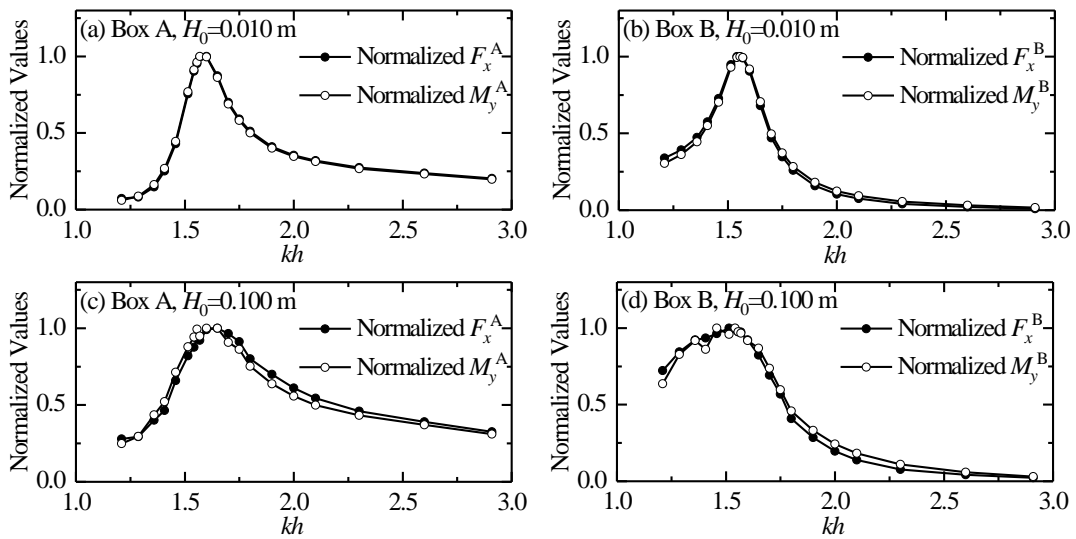
355 Third, for the horizontal wave forces on Box A (Fig. 8d), the frequency at which the  
356 maximum horizontal force occurs is obviously larger than the resonant frequency; the larger the  
357 incident wave height is, the more obvious the deviation becomes. While for the horizontal wave  
358 forces on Box B (Fig. 8e), the frequency at which the maximum horizontal force occurs is equal to  
359 or just slightly less than the resonant frequency. It is due to the fact that the magnitude of the  
360 horizontal force is determined by the free-surface elevation difference between the opposite sides  
361 of the each box (Lu et al., 2011b). The free-surface elevation at the left side of Box A is much  
362 larger than that at the right side of Box B. It leads to that the free-surface elevation difference  
363 between the opposite sides of Box A is more different from the free-surface elevation in the gap,  
364 while the free-surface elevation difference between the opposite sides of Box B is more close to  
365 the free-surface elevation in the gap. As for the reason why the free-surface elevation at the left  
366 side of Box A is much larger than that at the right side of Box B, there are two main reasons: (1)  
367 the reflected wave height is always larger than the transmitted wave height (i.e.,  $C_r > C_t$ , which  
368 will be shown in Section 5.3), and (2) the left side of Box A locates at a antinode of the partially  
369 standing waves composed of the incident and the reflected waves, which causes the wave height at  
370 the left side of Box A is approximately equal to the summation of the incident and the reflected  
371 wave heights.

372 Fourth, for the moments on Boxes A and B (Fig. 8f and g), for all the incident wave heights  
373 considered in this paper, the variation curves of the moment on each box with the frequency is  
374 very similar to those of the horizontal force on the corresponding box. Hence, the phenomena  
375 described above for the horizontal forces are also applicable for the moments. To further examine  
376 the phenomenon that the variation curves of the moment on each box with the frequency are very  
377 similar to those of the horizontal force on the corresponding box for all the incident wave heights  
378 studied in this paper, Fig. 9 presents the comparisons of the normalized curves of the horizontal



379 forces and the moments on Boxes A and B for  $H_0=0.010$  m and 0.100 m. The normalized curve  
 380 refers to the original variation curve divided by the corresponding peak value of the original  
 381 variation curve. Hence, the normalized curve always has a maximum value, 1.0. It can be seen that  
 382 for both the two boxes and for both the two incident wave heights, the normalized curves of the  
 383 horizontal wave forces are almost identical to those of the moments. For the other three incident  
 384 wave heights (i.e.,  $H_0=0.024$  m, 0.050 m and 0.075 m), the similar phenomenon can also be  
 385 clearly observed (their comparisons are not shown in the paper for brevity).

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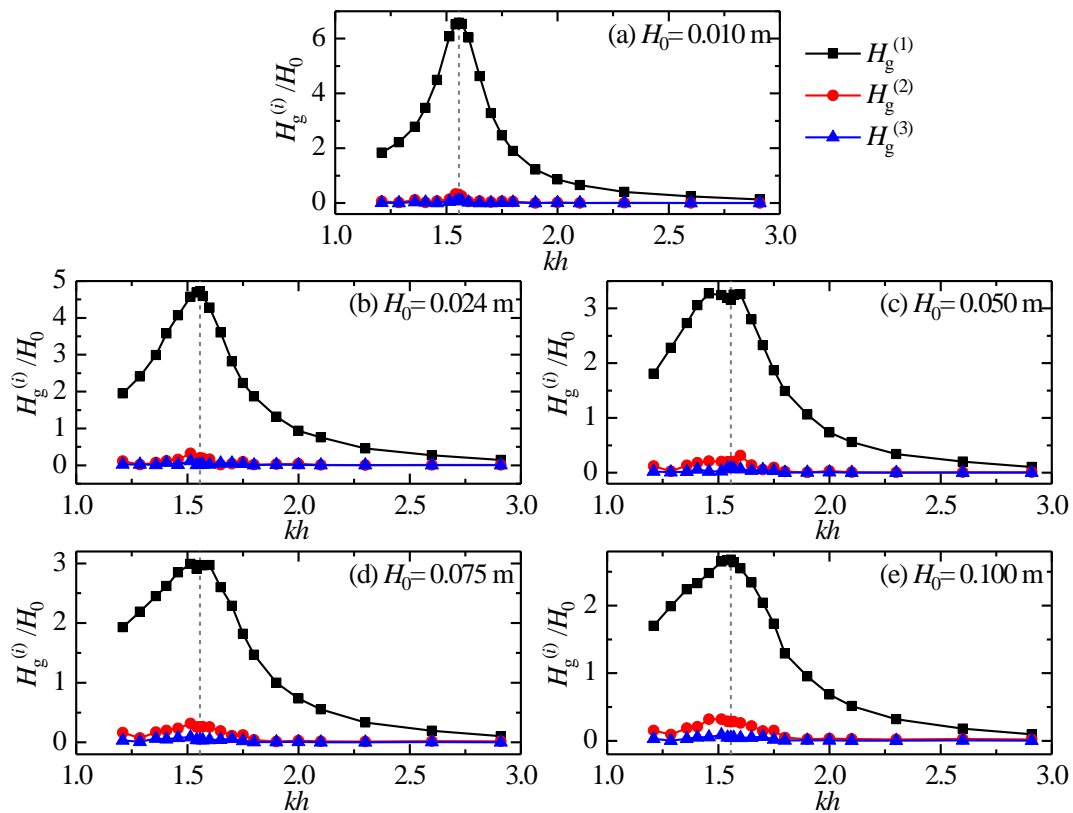
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388 **Fig. 9.** Comparisons of the normalized curves of the horizontal wave forces and the moments on  
 389 Boxes A and B

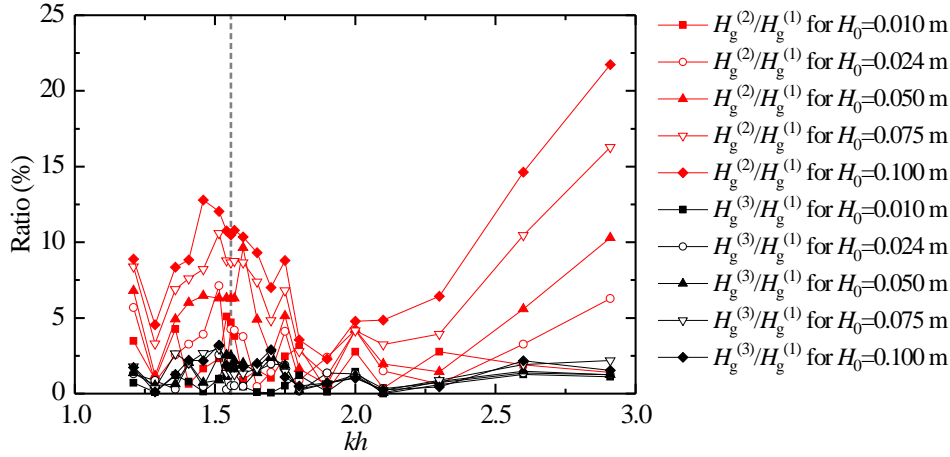
390 5.2 Harmonic analyses of wave height amplifications and wave loads

391 Fig. 10 illustrates the variation of the first three order harmonic components of the  
 392 free-surface elevation in the gap with respect to the frequency under the conditions of various  
 393 incident wave heights.  $H_g^{(i)}$  ( $i=1, 2$  and  $3$ ) in the figure denotes the  $i^{\text{th}}$ -order harmonic component  
 394 of the free-surface elevation in the gap. The following three phenomena can be easily observed.  
 395 First, the first-order component of the free-surface elevation is significantly larger than the second-  
 396 and third-order components. Second, around the resonant frequency, the second-order component  
 397 is larger than the corresponding third-order one; the larger the incident wave height is, the more  
 398 obvious the phenomenon becomes. Third, all the first three order harmonic components around the  
 399 resonant frequency are remarkably larger than the corresponding ones for the non-resonant  
 400 conditions.

401 To quantify the relative importance of higher-order components to the first-order component,  
 402 Fig. 11 further shows the ratios of the second- and third-order harmonic components to the  
 403 first-order harmonic components for the free-surface elevation in the gap under the conditions of  
 404 various incident wave heights. It is seen that at the range of  $1.3 < kh < 1.9$ , there existing obvious  
 405 peak points around the resonant frequency for both the second- and third-order harmonic  
 406 components. For the second-order harmonic components, the maximum of their ratios to the  
 407 first-order harmonic components reaches up to about 13%. It can be attributed to the fact that the  
 408 free-surface elevation around the resonant frequency is remarkably amplified, and naturally the  
 409 higher-order harmonic components of the free-surface elevation are enhanced due to the wave  
 410 nonlinearity. While at the ranges of  $kh < 1.3$  and  $kh > 1.9$ , as the wave frequency becomes far from  
 411 the resonant frequency, the ratios of the second- and third-order components to the first-order  
 412 components tend to gradually increase. It is mainly due to that the value of the first-order  
 413 component significantly decreases as the wave frequency becomes far from the resonant frequency,  
 414 especially for the high-frequency range (i.e.,  $kh > 1.9$ ).



415  
 416 **Fig. 10.** The first three order harmonic components of the free-surface elevation in the gap under  
 417 the conditions of various incident wave heights



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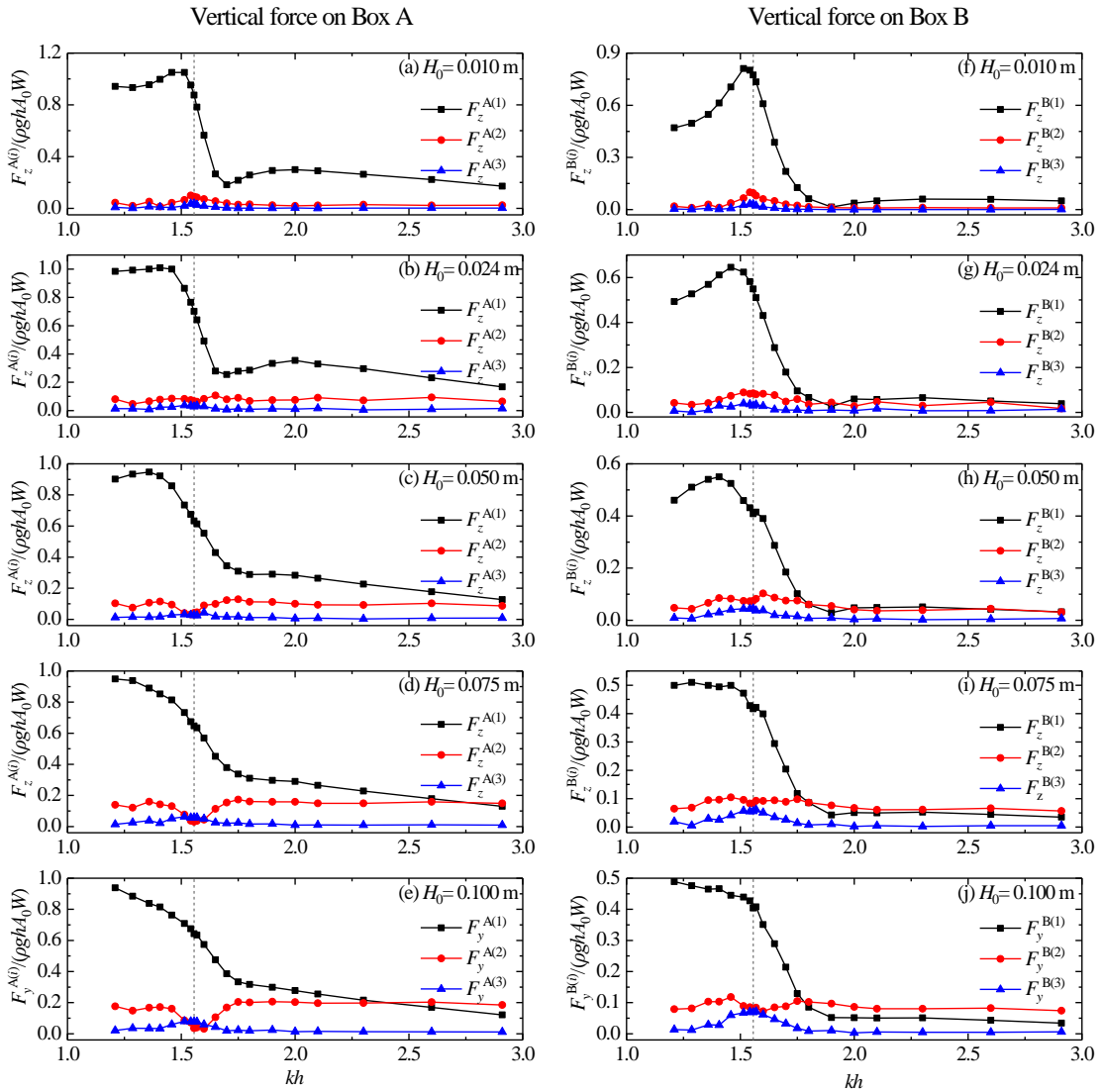
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**Fig. 11.** Ratios of the second- and third-order harmonic components to the first-order harmonic components for the free-surface elevation in the gap under the conditions of various incident wave heights

Fig. 12 illustrates the first three order harmonic components of the vertical wave forces on Boxes A and B for all the simulations.  $F_z^{A(i)}$  and  $F_z^{B(i)}$  ( $i=1, 2$  and  $3$ ) in this figure refer to the  $i^{\text{th}}$ -order harmonic components of the vertical wave forces on Boxes A and B, respectively. The following three phenomena can be easily seen. First, the first-order harmonic components of the vertical wave force are much larger than the higher-order components near the resonant frequency. Second, when the incident wave height is small (Fig. 12a and f), the second-order harmonic component is obviously larger than the third-order ones around the resonant frequency. As the incident wave height increases, the third-order harmonic components around the resonant frequency gradually become obviously larger than those far away from the resonant frequency; on the contrary, the second-order harmonic components around the resonant frequency become smaller and smaller compared with those far away from the resonant frequency. When the incident wave height increases up to  $H_0=0.100$  m (Fig. 12e and j), the third-order harmonic components have approached (for Box B) or even exceeded (for Box A) the second-order ones. Third, for the high-frequency range, because the first-order harmonic components decrease sharply with the wave frequency, the second-order harmonic components approach and even exceed the corresponding first-order ones for both the two boxes.

Fig. 13 further shows the ratios of the second- and third-order harmonic components to the first-order harmonic components for the vertical wave forces on the two boxes for all the

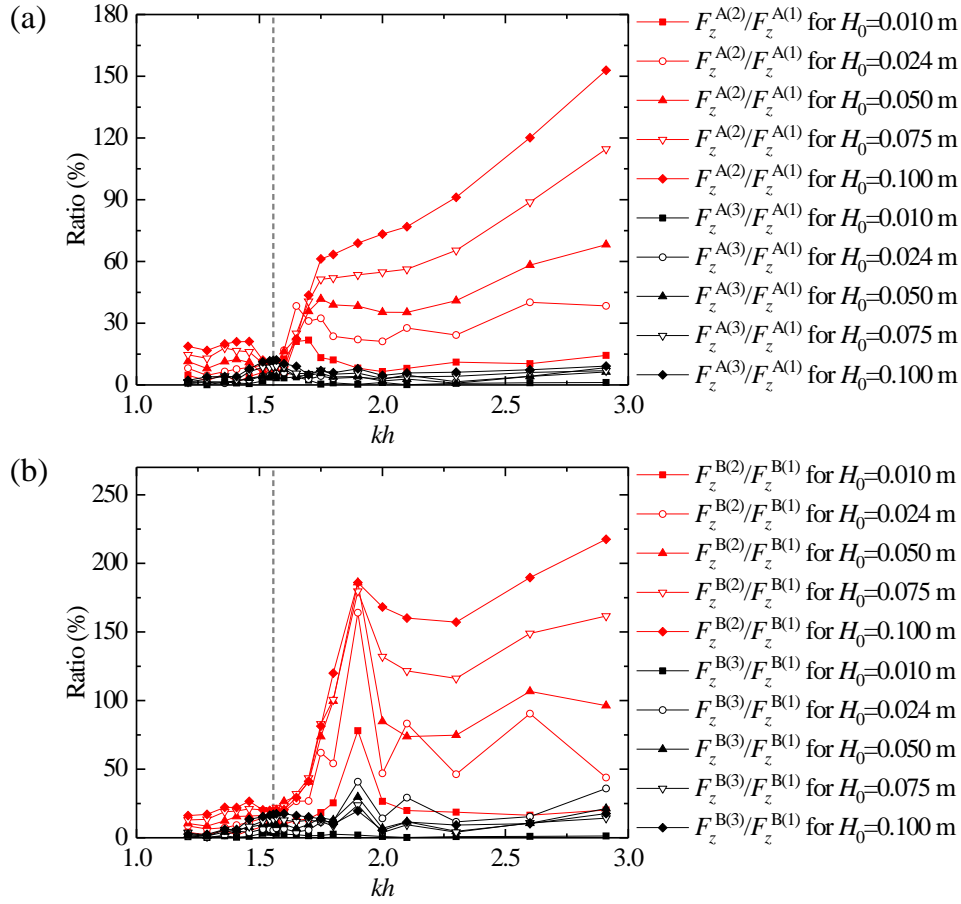
441 simulations. It can be easily observed that for both the two boxes and for the wave frequency far  
 442 away from the resonant frequency, the ratio of the second- to the first-order components is always  
 443 larger than the ratio of the third- to the first-order components, while around the resonant  
 444 frequency, the latter approaches or even exceeds the former. Besides, for the second-order  
 445 components, their ratios near the resonant frequency are less than those far from the resonant  
 446 frequency. While for the third-order components, their ratios near the resonant frequency tend to  
 447 be larger than those far from the resonant frequency (it is valid for the whole frequency range  
 448 considered in this paper for Box A, and for  $kh < 1.800$  for Box B).  
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451 **Fig. 12.** The first three order harmonic components of the vertical wave forces on Boxes A and B

452 under the conditions of various incident wave heights.



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**Fig. 13.** Ratios of the second- and third-order harmonic components to the first-order harmonic components for the vertical wave forces on (a) Box A and (b) Box B under the conditions of various incident wave heights

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Fig. 14 presents the first three order harmonic components of the horizontal wave forces on

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Boxes A and B for all the simulations.  $F_x^{A(i)}$  and  $F_x^{B(i)}$  ( $i=1, 2$  and  $3$ ) in this figure refer to the

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$i^{\text{th}}$ -order harmonic components of the horizontal wave forces on Boxes A and B, respectively.

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Because both the second- and third-order harmonic components of the horizontal wave forces

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around the resonant frequency are extremely small compared to the corresponding first-order

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components, in order to better show the variations of all these three harmonic components with the

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incident wave frequency, the values of both the second- and third-order harmonic components

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shown in this figure are enlarged five times. In general, the above three phenomena for the vertical

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wave forces shown in Fig. 12 can also be observed in this figure.

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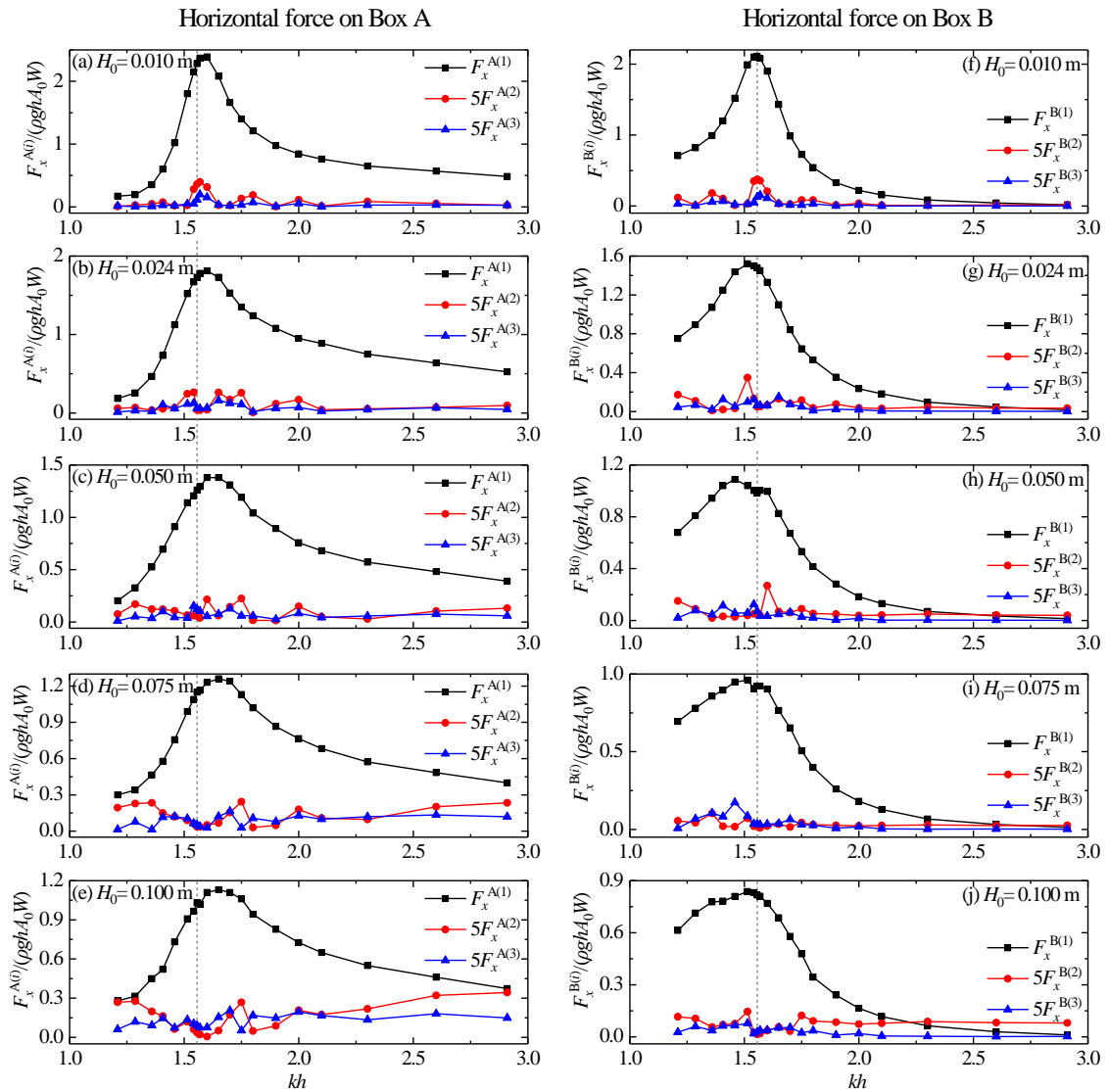
Fig. 15 further presents the ratios of the second- and third-order harmonic components to the

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first-order harmonic components for the horizontal wave forces on the two boxes for all the

469 simulations. It should be noted that, for Box B (Fig. 15b), when  $kh=2.910$  and  $H_0=0.100$  m, the  
 470 value of  $F_x^{B(2)} / F_x^{B(1)}$  has already exceeded 140%. However, to better show the variation  
 471 characteristics of the ratios around the resonant frequency, the maximum changing range of the  
 472 y-axis is only set to 20%. Again, in general, all the phenomena for the vertical wave forces  
 473 presented in Fig. 13 can also be observed in this figure, except that the ratios of the third- to the  
 474 first-order components near the resonant frequency shown in this figure tend to be smaller than  
 475 those far from the resonant frequency.

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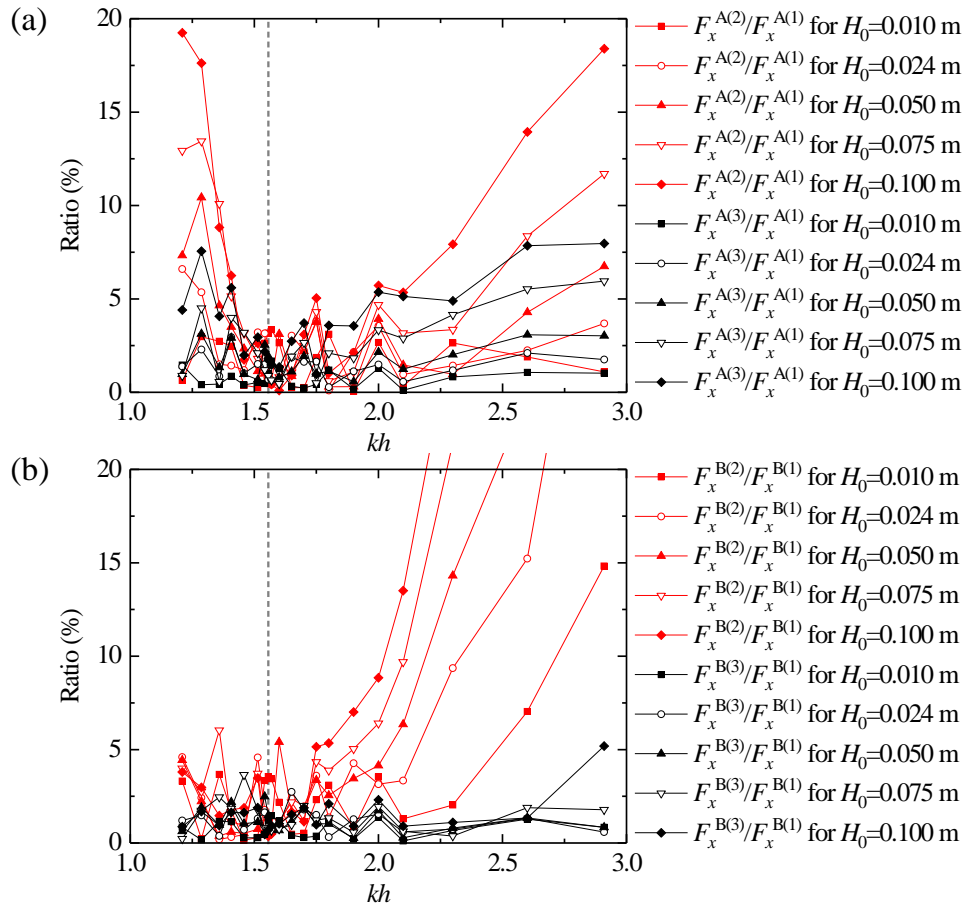
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478 **Fig. 14.** The first three order harmonic components of the horizontal wave forces on Boxes A and

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Box B under the conditions of various incident wave heights.

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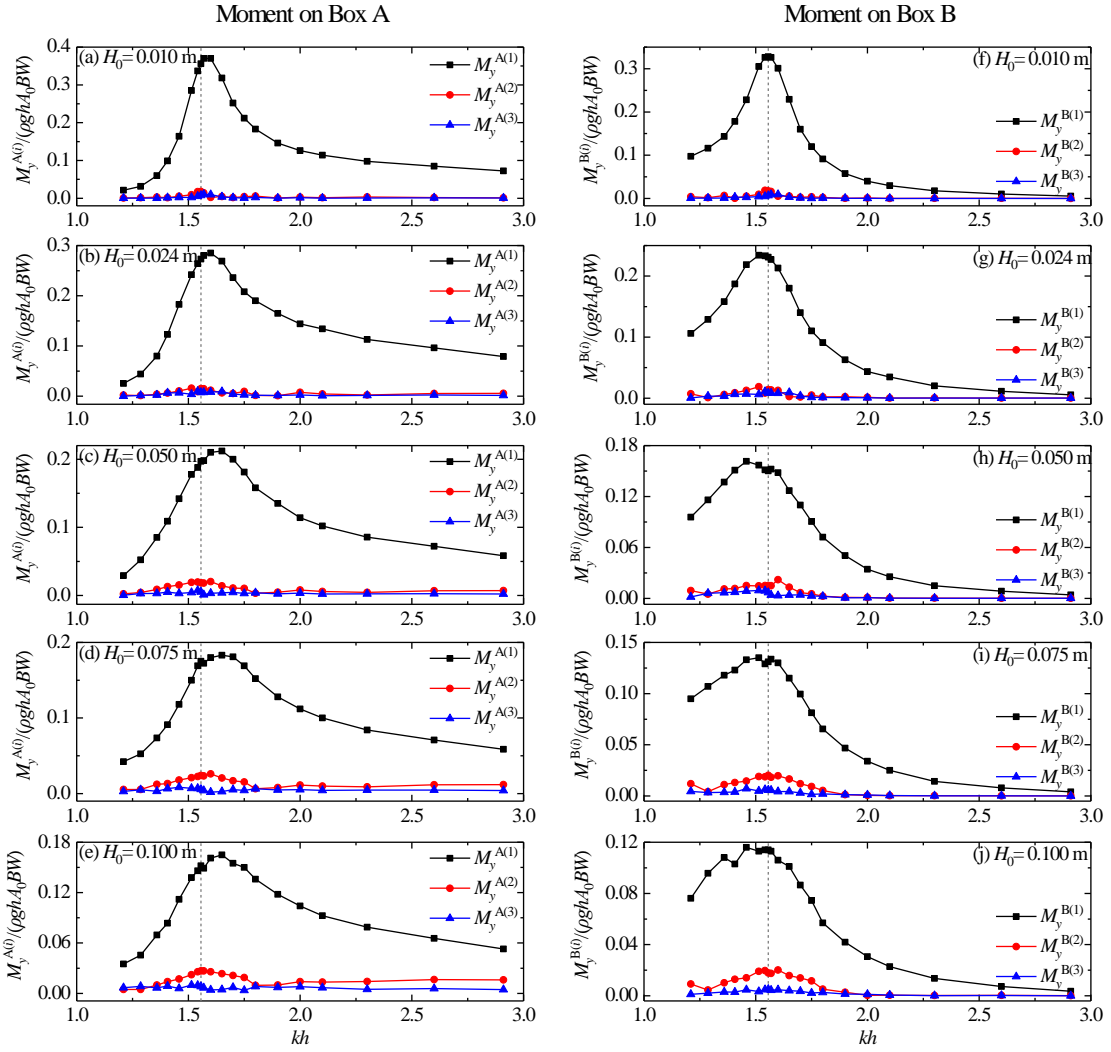


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**Fig. 15.** Ratios of the second- and third-order harmonic components to the first-order harmonic components for the horizontal wave forces on (a) Box A and (b) Box B under the conditions of various incident wave heights

497 Fig. 16 presents the first three order harmonic components of the moments on Boxes A and B  
498 for all the simulations, in which  $M_y^{A(i)}$  and  $M_y^{B(i)}$  ( $i=1, 2$  and  $3$ ) refer to the  $i^{\text{th}}$ -order harmonic  
499 components of the moments on Boxes A and B, respectively. It is obviously seen that when the  
500 incident wave height is small (refer to Fig. 16a and f), both the second- and third-order harmonic  
501 components are much smaller than the corresponding first-order ones around the resonant  
502 frequency for both the two boxes. However, as the incident wave height increases, the values of  
503 the second-order harmonic components around the resonant frequency gradually increase.  
504 Compared to the first-order harmonic components, the second-order harmonic components have  
505 reached a considerable values when  $H_0=0.100$  m (refer to Fig. 16e and j). To better illustrate this  
506 point, the ratios of the second- and third-order harmonic components to the first-order harmonic  
507 components for the moments on both the two boxes under the conditions of various incident wave  
508 heights are presented in Fig. 17. It can be seen that when the incident wave height is small (i.e.,  
509  $H_0=0.010$  m), both the values of  $M_y^{A(2)}/M_y^{A(1)}$  and  $M_y^{B(2)}/M_y^{B(1)}$  are approximately 5%.  
510 However, when the incident wave height increases to  $H_0=0.100$  m, both their values reach up to  
511 near 20%.





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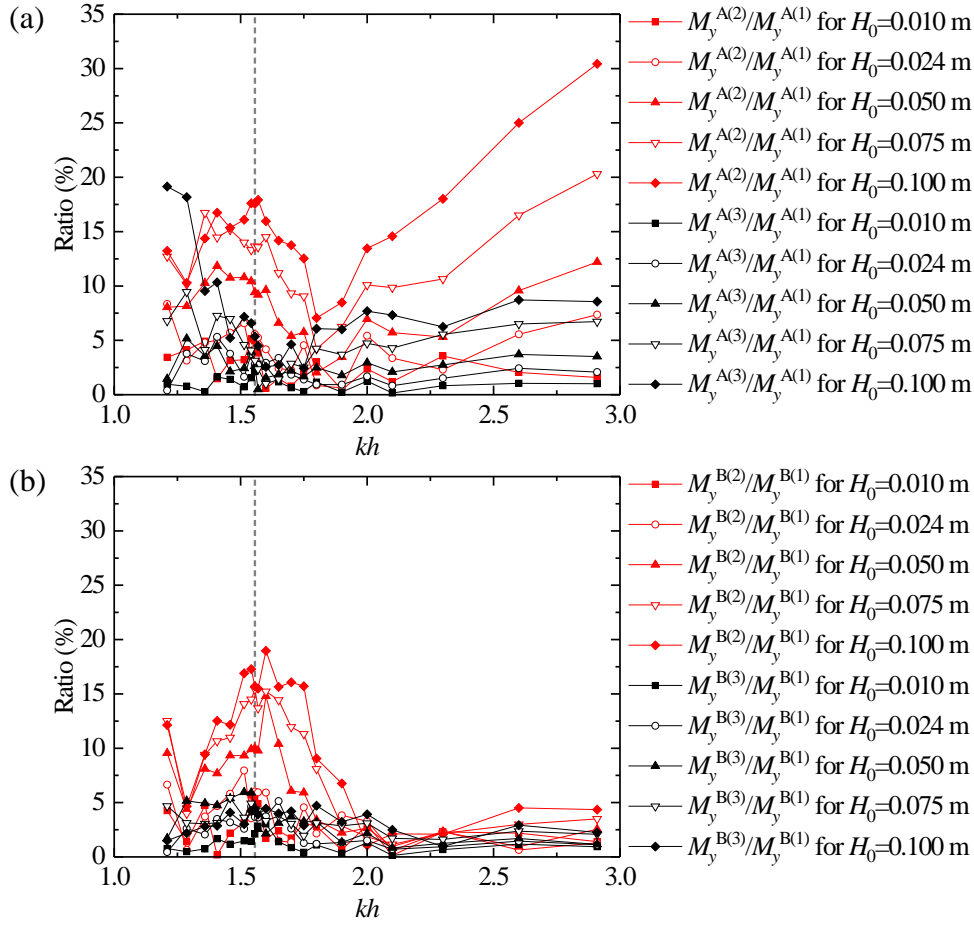
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**Fig. 16.** The first three order harmonic components of the moments on Boxes A and B under the conditions of various incident wave heights.



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**Fig. 17.** Ratios of the second- and third-order harmonic components to the first-order harmonic components for the moments on (a) Box A and (b) Box B under the conditions of various incident wave heights

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### 5.3 Reflection, transmission and energy loss coefficients

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Based on the wave analysis technique in [Goda and Suzuki \(1976\)](#), the wave height of the reflected waves from the two-box system can be obtained by using the free-surface elevations at  $G_1$  and  $G_2$  (refer to Fig. 1). The reflection coefficient  $C_r$  is further calculated as the ratio of the reflected wave height to the incident wave height  $H_0$ . The transmission coefficient  $C_t$  is defined as the ratio of the transmitted wave height to  $H_0$ , and the transmitted wave height can be obtained by the free-surface elevation at  $G_4$ . Then, the energy loss coefficient  $L_e = 1 - C_r^2 - C_t^2$  is calculated. The effects of the incident wave height on the reflection, transmission and energy loss coefficients,  $C_r$ ,  $C_t$  and  $L_e$ , are illustrated in Fig. 18. For the reflection coefficient (Fig. 18a), the following three phenomena can be easily observed. First, the reflection coefficients  $C_r$  near the resonant frequency

532 are always less than those away from the resonant frequency. Second, the reflection coefficient at  
533 the resonant frequency increases with the increasing of the incident wave height. Third, under the  
534 conditions of  $H_0=0.050$  m and  $0.075$  m, both the two variation curves of the reflection coefficient  
535 around the resonant frequency almost become flat, which indicates that the similar wave energy  
536 can propagate into the gap. Hence, this leads to the relatively flat variation curves of  $H_g/H_0$  around  
537 the resonant frequency for  $H_0=0.050$  m and  $0.075$  m shown in Fig. 8a.

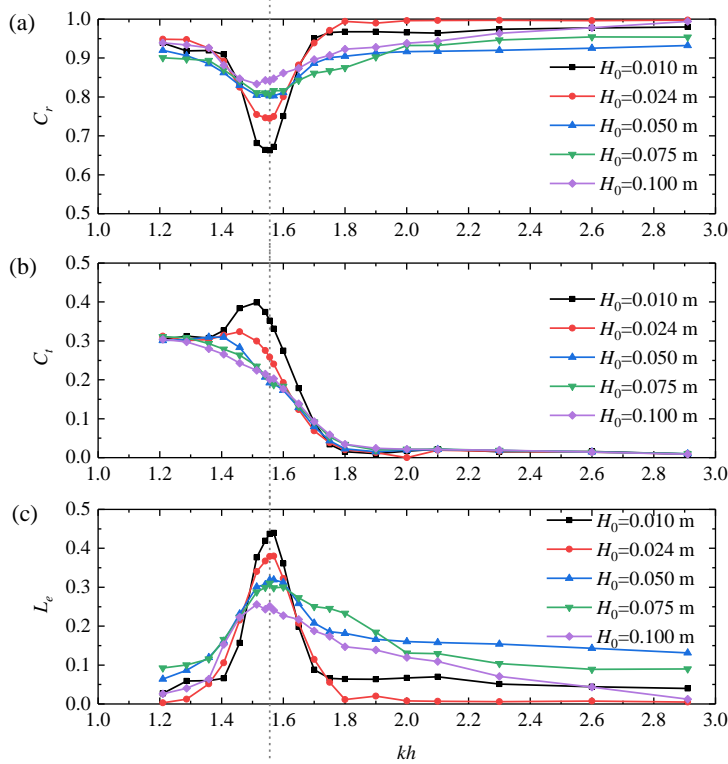
538 For the transmission coefficient (Fig. 18b), the frequency at which the maximum  
539 transmission coefficient  $C_t$  occurs is always less than the resonant frequency. The larger the  
540 incident wave height is, the more obvious their difference becomes. When the incident wave  
541 height is small, the transmission coefficient first increases, then sharply decreases, then slowly  
542 increases, and then slowly decreases with the non-dimensional wavenumber,  $kh$ . However, with  
543 the increase of the incident wave height, the vertical wave forces gradually become monotonic  
544 decrease with  $kh$ . These effects of the incident wave height on the variation characteristics of the  
545 transmission coefficient are very similar to its effects on those of the vertical wave forces on  
546 Boxes A and B (see Fig. 8b and c).

547 By carefully comparing Fig. 18a and b, it can be found that for all the incident wave heights  
548 considered in this paper, the reflection coefficients are always larger than the transmission  
549 coefficient, no matter whether the gap resonance occurs or not. The larger the incident wave height  
550 is, the more obvious the difference between  $C_r$  and  $C_t$  becomes. This explains that phenomenon  
551 shown in Fig. 8d and e that for the horizontal wave forces on Box A, the frequency at which the  
552 maximum horizontal force occurs obviously deviates from the resonant frequency; while for the  
553 horizontal wave forces on Box B, the frequency at which the maximum horizontal force occurs is  
554 approximately equal to the resonant frequency.

555 For the energy loss coefficient (Fig. 18c), it is seen that for all the incident wave heights  
556 considered in this paper, almost all the maximum energy loss coefficients appear at (or very close  
557 to) the resonant frequency. Besides, with the increase of the incident wave height, the energy loss  
558 coefficient at the resonant frequency becomes smaller and smaller. It should be noted that,  
559 intuitively, this finding seems to be incompatible with that phenomenon shown in Fig. 8a that  
560 larger incident wave height leads to smaller amplification of the free-surface elevation in the gap.  
561 In fact, these two findings are compatible with each other. The reason lies on that the increase of

562 the incident wave height tends to remarkably augment the reflection coefficient at the resonant  
 563 frequency, and hence relatively less wave energy can propagate into the gap. Therefore, less  
 564 energy dissipation and smaller free-surface amplification in the narrow gap can be observed at the  
 565 resonant frequency, which agrees with the related findings in Jiang et al. (2018).

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568 **Fig. 18.** Variations of (a) the reflection coefficient, (b) the transmission coefficient and (c) the  
 569 energy loss coefficient with respect to the wave frequency under the conditions of various incident  
 570 wave heights

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#### 572 5.4 Response time and damping time of gap resonance

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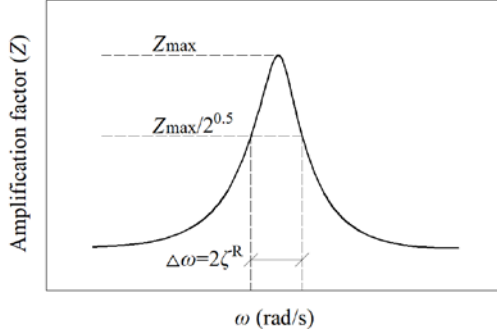
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In practical engineering applications, the fast and accurate estimation of the response time and the damping time is crucial for the safe evacuation of staff and the reasonable arrangement of operation time during the offloading operations from FPSO platforms to a shuttle tanker under gap resonance conditions. In this section, a general method is proposed for accurately estimating both the response time and the damping time of free-surface elevation in the gap under resonance. The general method is based on fitting the envelope of the free-surface elevation. For the evaluation of the response time, there is an alternative method which is based on the amplification curve of the

580 free-surface elevation in the gap.

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**Fig. 19.** Generic amplification curve;  $\Delta\omega$  is the half-power spectral bandwidth

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These methods are inspired by [Bellotti \(2007\)](#) and [Dong et al. \(2010\)](#) who investigated the response time and the damping time of the harbor to long waves under the condition of harbor resonance. In order to facilitate the understanding of the reader, the basic principle of these methods is briefly explained here. To illustrate the basic principle, Fig. 19 shows a generic amplification curve, in which  $Z$  is the amplification factor. The free-surface elevation in the narrow gap can be considered as typical of a 1D system like a mass-spring system, moving along a line, connected to a damper, forced by a periodically unit force. If the considered mass starts from rest (i.e., from the position  $z=0$ ), when the frequency of the force equals the natural frequency of the system, its position along the axis ( $z$ ) can be formulated as

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$$z = Z_{\max} \cos(-\omega \cdot t^*) (1 - e^{-\zeta^R \cdot t^*}), \quad (7)$$

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in which  $\zeta^R$  is a parameter governing the response time of the resonator,  $Z_{\max}$  is the maximum amplification factor and  $t^*$  denotes the relative time with respect to the moment that the mass just begins to move from rest. It requires infinite time for the fluctuation to reach its maximum, following Eq. (7). The time  $t_{\alpha\%}^*$  needed for the waves to reach  $\alpha\%$  of the maximum can be formulated as

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$$t_{\alpha\%}^* = -\frac{\ln(1-\alpha\%)}{\zeta^R}. \quad (8)$$

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Similarity, if the mass damps from the steady-state maximum to the rest state, its position can be expressed as

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$$z = Z_{\max} \cos(-\omega \cdot \tau) e^{-\zeta^D \cdot \tau}, \quad (9)$$

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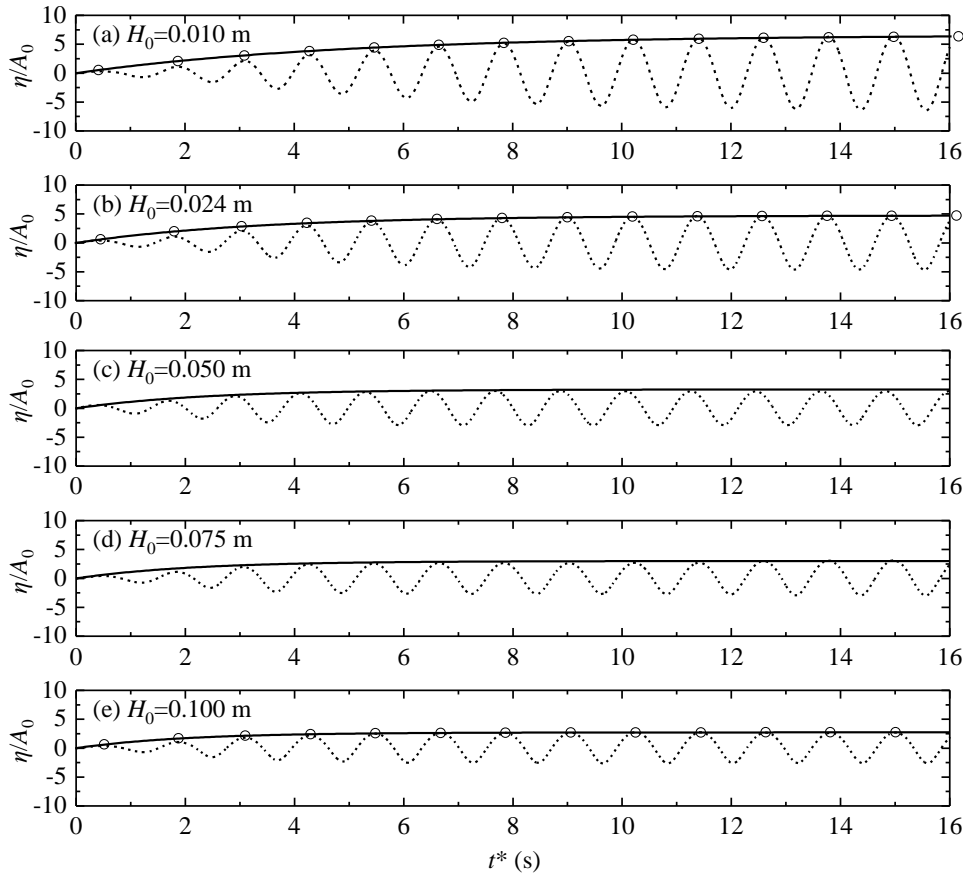
where  $\zeta^D$  is a parameter controlling the damping time of the resonator and  $\tau$  denotes the relative

605 time with respect to the moment that the mass just begins to damp from the steady-state maximum.

606 The time  $\tau_{\beta\%}$  needed by the wave to decrease to  $\beta\%$  of the maximum can be expressed as

$$607 \quad \tau_{\beta\%} = -\frac{\ln(\beta\%)}{\zeta^D}. \quad (10)$$

608 It can be found from Eqs. (8) and (10) that the key step to quantitatively evaluate the  
 609 response time and the damping time lies on how to find the values of  $\zeta^R$  and  $\zeta^D$ . A general method  
 610 to obtain their values is to directly fit the measured (or simulated) envelopes of the displacement  
 611 of the mass with the theoretical ones formulated by Eqs. (7) and (9). It can be demonstrated that  
 612 for 1D resonators, the value of  $\zeta^R$  can also be evaluated from the amplification curve. More  
 613 specifically,  $\zeta^R = \Delta\omega / 2$ , in which  $\Delta\omega$  is the half-power spectral bandwidth (i.e., the width of the  
 614 part of the amplification curve with values larger than  $Z_{\max} / 2^{0.5}$ ). Identical to [Bellotti \(2007\)](#) and  
 615 [Dong et al. \(2010\)](#),  $t_{95\%}^*$  and  $\tau_{5\%}$  are selected in this article to represent the response time and  
 616 the damping time of the resonant free-surface elevations, respectively.



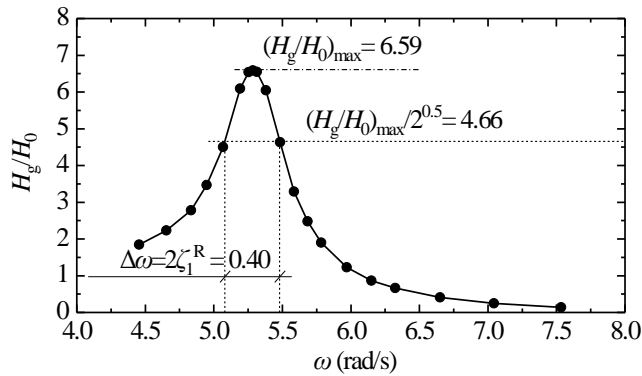
617  
 618 **Fig. 20.** The response process of the free-surface elevation ( $\eta/A_0$ ) in the narrow gap excited by the  
 619 incident regular waves with the resonant frequency (i.e.,  $kh=1.556$ , or equivalently  $\omega=5.285$  rad/s)

620 and various heights. Dashed lines denote the time histories of the simulated free-surface elevations  
 621 obtain by the numerical model. Solid lines refer to the fitted envelope of  $\eta/A_0$  obtained by directly  
 622 fitting the simulated envelopes with the theoretical ones formulated by Eq. (7). Small circles  
 623 represent the envelope of  $\eta/A_0$  as obtained by Eq. (7) using the amplification curve method for  
 624 estimating  $\zeta^R$ .

625

626 Fig. 20 shows the time histories of the free-surface elevations in the narrow gap, obtained by  
 627 using the time-resolving numerical model, from the calm to the steady state. The frequency of all  
 628 the incident regular waves corresponds to the resonant frequency (i.e.,  $kh=1.556$ , or equivalently  
 629  $\omega=5.285$  rad/s). By directly fitting the simulated envelopes with the theoretical ones formulated  
 630 by Eq. (7), the numerical values of  $\zeta^R$  can be obtained. Besides, by measuring the half-power  
 631 spectral bandwidth of the amplification curve as shown in Fig. 19, the values of  $\zeta^R$  can also be  
 632 calculated. It is noted here that, to facilitate comparing the values of  $\zeta^R$  obtained by these two  
 633 different methods, two different symbols,  $\zeta_1^R$  and  $\zeta_2^R$ , are used separately to represent the values  
 634 of  $\zeta^R$  obtained by the amplification curve method and by the direct envelope-fitting method in the  
 635 following. As a concrete example of employing the amplification curve method to evaluate the  
 636 value of  $\zeta_1^R$ , Fig. 21 illustrates the amplification curve of the free-surface elevation in the narrow  
 637 gap under the condition of  $H_0=0.010$  m. It can be seen that the value of  $\zeta_1^R$  under the condition  
 638 of  $H_0=0.010$  m is equal to 0.20.

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641 **Fig. 21.** Amplification curve of the free-surface elevation in the narrow gap under the condition of  
 642  $H_0=0.010$  m

643 Table 1 further lists all the values of  $\zeta_1^R$  and  $\zeta_2^R$ , their relative percentage errors,  $Err$ , and  
644 the response time,  $t_{95\%}^*$  for the free-surface elevations shown in Fig. 20. The response time,  
645  $t_{95\%}^*$ , in this table is calculated by employing Eq. (8) and the value of  $\zeta_2^R$ . As mentioned in  
646 Section 5.1, unlike the typical amplification curves shown in Figs. 19 and 21, the two free-surface  
647 amplification curves for  $H_0=0.050$  m and 0.075 m do not present the perfect single-peak shape; the  
648 two curves around the resonant frequency become flat, and the values of the amplification factor  
649 at  $kh=1.556$  are even slightly less than the ones at its both adjacent sides (refer to Fig. 8a). Hence,  
650 the values of  $\zeta_1^R$  for  $H_0=0.050$  m and 0.075 m are absent. For the other three wave heights, the  
651 relative percentage errors between  $\zeta_1^R$  and  $\zeta_2^R$  are shown to be extremely small. Besides,  
652 observing Fig. 20 can easily find that for all the incident wave heights considered in this paper,  
653 both the two envelopes of the free-surface elevations obtained by  $\zeta_1^R$  and  $\zeta_2^R$  agree well with the  
654 corresponding simulated free-surface elevations by using the time-resolving numerical model.  
655 These phenomena indicate that both the two above-mentioned methods for evaluating the response  
656 time of gap resonance are accurate and reliable.

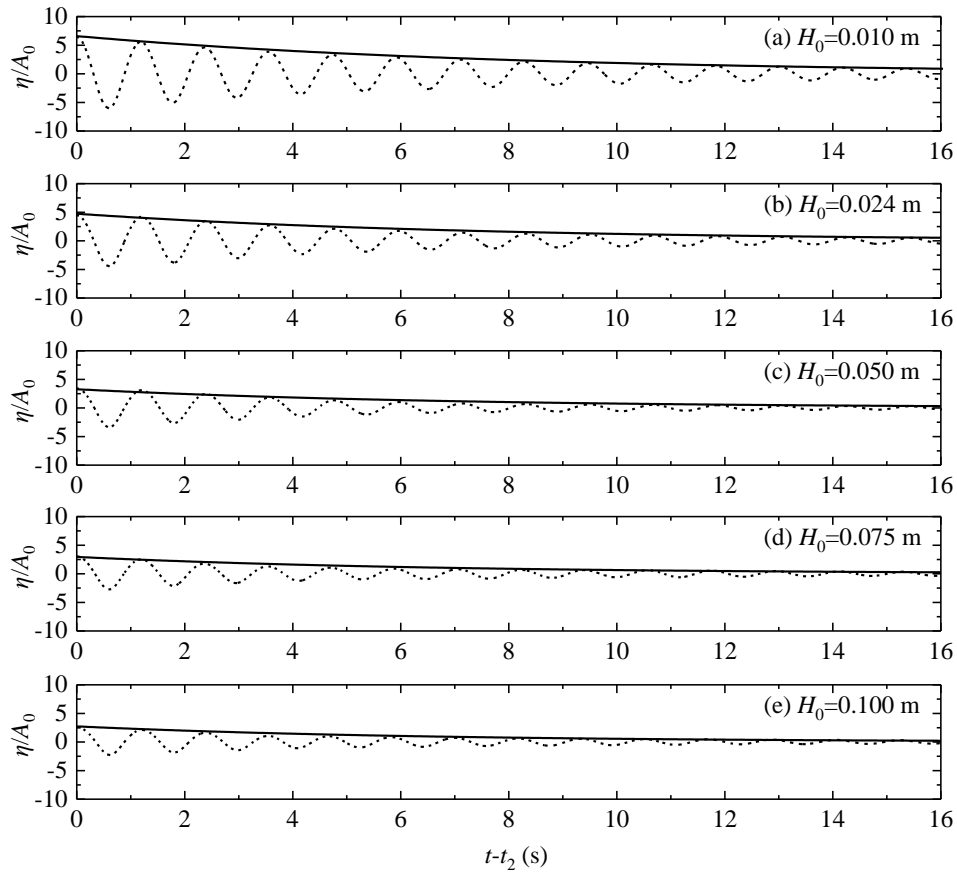
657  
658 **Table 1.** All the parameters related to the response time and the damping time of the resonant  
659 free-surface elevations shown in Figs. 20 and 22.  $Err$  denotes the relative percentage error  
660 between  $\zeta_1^R$  and  $\zeta_2^R$ .  $t_{95\%}^*$  and  $\tau_{5\%}$  refers to the response time and the damping time of the  
661 free-surface elevations, respectively. The evaluation of  $t_{95\%}^*$  is based on Eq. (8) and the value of  
662  $\zeta_2^R$ .

$H_0$ (m)	$\zeta_1^R$	$\zeta_2^R$	$Err$ (%)	$t_{95\%}^*$ (s)	$\zeta^D$	$\tau_{5\%}$ (s)	$\tau_{5\%}/t_{95\%}^*$
0.010	0.200	0.202	0.99	14.83	0.125	23.97	1.62
0.024	0.306	0.299	2.34	10.02	0.136	22.03	2.20
0.050	-	0.421	-	7.12	0.145	20.66	2.91
0.075	-	0.467	-	6.41	0.151	19.84	3.09
0.100	0.512	0.499	2.60	6.00	0.158	18.96	3.16

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**Fig. 22.** The damping process of the free-surface elevation ( $\eta/A_0$ ) in the narrow gap excited by the incident regular waves with the resonant frequency (i.e.,  $kh=1.556$ , or equivalently  $\omega=5.285$  rad/s) and various heights. Dashed lines denote the time histories of the simulated free-surface elevations obtain by the numerical model. Solid lines represent the fitted envelope of  $\eta/A_0$  obtained by directly fitting the simulated envelopes with the theoretical ones formulated by Eq. (9).

Fig. 22 illustrates the time histories of the simulated free-surface elevations in the narrow gap and their fitted envelopes obtained by directly fitting the simulated envelopes with the theoretical ones formulated by Eq. (9) during their damping processes. It is seen that for all the incident wave heights considered in this paper, Eq. (9) can describe the damping process of the resonant free-surface elevation in the gap very well. All the values of  $\zeta^D$  gained by the direct envelope-fitting method and the damping time  $\tau_{5\%}$  under the conditions of various wave heights are also presented in Table 1.

According to the response time and the damping time presented in Table 1, the following two phenomena can be easily observed. First, for all the incident wave heights, the damping time is always significantly larger than the corresponding response time. The ratio of the damping time to

682 the response time gradually increases from 1.62 for  $H_0=0.010$  m to 3.16 for  $H_0=0.100$  m. This  
683 indicates that once the gap resonance is excited, it will persist for a rather long time. Second, both  
684 the response time and the damping time decrease with the incident wave height, and the  
685 decreasing degree of the response time is obviously larger than that of the damping time. The  
686 response time falls up to 60 % from  $H_0=0.010$  m to  $H_0=0.100$  m, while the damping time reduces  
687 only about 21%.

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## 689 6. Conclusions

690 The CFD numerical model, OpenFOAM<sup>®</sup>, together with the wave generation toolbox  
691 “waves2Foam” proposed by [Jacobsen et al. \(2012\)](#), is adopted for investigating the hydrodynamic  
692 behaviors of water resonance in a narrow gap formed by two side-by-side identical boxes excited  
693 by incident regular waves with various wave heights. The overall free-surface amplification in the  
694 gap and the overall wave loads on the boxes are firstly presented. Then, the harmonic analyses of  
695 free-surface elevation and wave loads are mainly investigated. Next, the reflection, transmission  
696 and energy loss coefficients of the two-box system are discussed. Finally, two different methods to  
697 evaluate the response time and the damping time of gap resonance are proposed. The results of  
698 this study have provided new insights of the hydrodynamic characteristics involved in the gap  
699 resonance.

700 The following conclusions can be drawn from the results of the present study:

701 (1) The frequencies at which the maximum vertical wave forces on both boxes and the maximum  
702 horizontal wave force on Box A occur appear to obviously deviate from the resonant  
703 frequency, and a larger incident wave height tends to cause more obvious differences between  
704 them. While the frequency at which the maximum horizontal force on Box B occurs is equal  
705 or very close to the resonant frequency.

706 (2) For the free-surface elevation in the gap and the moments on boxes, the ratios of their  
707 second-order components to the corresponding first-order ones around the resonant frequency  
708 are normally larger than those at the frequencies far from the resonant frequency (except those  
709 at the very high frequency band). The larger the incident wave height is, the larger the ratios  
710 of the second- to the first-order components around the resonant frequency becomes.

711 (3) For both the vertical and horizontal wave forces on both boxes, the ratios of their second- to

712 the first-order components near the resonant frequency are less than those far away from the  
713 resonant frequency. Besides, when the incident wave height is small, their second-order  
714 components are obviously larger than the corresponding third-order ones around the resonant  
715 frequency. However, as the incident wave height increases, the third-order components around  
716 the resonant frequency gradually approach and even exceed the second-order ones.

717 (4) Both the minimum reflection coefficient and the maximum energy loss coefficient always  
718 appear at (or very close to) the resonant frequency, while the frequency at which the  
719 maximum transmission coefficient appears is obvious less than the resonant frequency. The  
720 reflection coefficient is always larger than the transmission coefficient, and the larger the  
721 incident wave height is, the more obvious their difference becomes. Besides, the energy loss  
722 coefficient under the gap resonance condition gradually decreases with the increase of the  
723 incident wave height.

724 (5) Both the amplification curve method and the direct envelope-fitting method are able to  
725 accurately evaluate the response time and the damping time of the resonant free-surface  
726 elevation in the gap, and it is shown that the damping time is always significantly larger than  
727 the corresponding response time. Besides, with the increase of the incident wave height, both  
728 the response time and the damping time decrease, and the decreasing degree of the former is  
729 obviously larger than that of the latter.

730 Finally, we reaffirm here that these conclusions are only valid for the given geometric layout  
731 (including the size and draft of the two boxes, the gap width and the water depth) and the ranges  
732 of the incident wave height and the incident wave frequency studied in this paper.

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