Real Options with Risk Aversion Using Tradable Asset in Project Management

陳, 暁栄
上海交通大学安泰管理学院

時永, 祥三
九州大学大学院経済学研究院

https://doi.org/10.15017/3755

出版情報：經濟學研究. 70 (6), pp.1-17, 2004-06-15. 九州大学経済学会

Real Options with Risk Aversion Using Tradable Asset in Project Management

Xiaorong Chen and Shozo Tokinaga

1 Introduction

In finance, the term real option covers a wider range of problems, and has recently attracted much attention [1]-[7]. As recent articles and book by Dixit and Pindyck describe real options utilized to managerial flexibility in handling real asset investment. Some examples of real option problems include extraction rights to an oil reserve or the option to start up a R&D venture [1]-[7].

In the investment decision, often development can be immediate, delayed or abandoned, until either a fixed date, or an open period in the future . The scheme is the same as the options in stock exchanges, and the investor can preserve the right to possess the real asset by limiting the bearing risks.

This paper deals with introducing a second asset into usual real option scenario, on which no trading is allowed [8]-[10]. Suppose the investor has an option on this second asset, payable at time T. The problem is how to price and hedge this random payoff when trading in the second asset is not permissible.

Formulation of the theory of real option was begun by Dixit by introducing the entry and exit for investment [6][7]. Theoretical results for entry and exit decision are simple enough to understand the effectiveness of real options. The results suggest the optimal value of asset and time for entry and exit for the project by comparing the sunk cost for investment .

However, we must also notice that in real world, the investors are not allowed easily to exit from the project once they joined the project. For example, in cases where the investor is an important contributor to the project such as a consortium, or a firm is expected to contribute the development in a rural area . Therefore, the factor of risk aversion for the non-traded asset is necessary to discuss rather than the simple valuation of sunk cost. We are facing with an incomplete markets situation and replication is not possible. The risk arising form unhedgeable asset is often referred to as "basic risk".

Another example in executive stock options which are given to executives as part of their compensation package frequently executives are not permitted to trade away the risk using the stock or the derivatives on the stock, so that they are essentially receiving options on non-traded asset [8]-[10].

In principle, the two assets in hedging are specified in reverse and the agent expects to receive an unhedgeable claim on an asset by choosing a correlated asset with which to hedge.

This paper will consider the specific real options problems where an option on an investment for a project is assumed from which agent is not allowed to exit. We explore the evaluation of option premium on the basis of real options approach on non-traded assets. We consider agents with constant risk aversion or equivalently a Cobb-Douglas or power-law utility function

Since it seems there is no closed form solution for the general maximization problem facing the agent on the model, and we make several assumption and take a numerical approach. We assume that the option on the non-traded asset is a multiple λ units of the share, and lambda is small, or in another expression, the position in the non-traded asset is small compared with wealth.

In the following, Section 2 shows the evaluation of investment flexbility based on the dynamic programming. In Section 3, we describe the repayment phase of project focusing on the demand uncertainty. Section 4 treats the risk aversion with traded asset for non-traded project.

2 Evaluation of investment flexibility in investment phase

2.1 Investment flexibility

The technique of project finance is being applied to various projects all over the world in the 1990's, which is characterized by the concept of a lender who is looking principally at the cash flow generated by a specific project to recover incurred debt [17]-[20]. A company may decide to fund a project through project finance rather than from its own resources mainly for the purpose to share risk in spite of the damage which may happened from the project if the project ended in failure [17]-[20].

In practice, project manager can repeatedly gather information about risk factor holding potential uncertainty, and based on the information, he may change the course of action taken. Using the analogy with options on financial assets, such investment flexibility is often called a real option of investment flexibility . The real option may significantly enhance the value of investment and this value is often referred as real option value.

The new view of invest opportunities as options is the product of over a decade of research and still an active topic in today's journal articles[1]-[7]. Dixit showed the optimal schema of a firm's entry and exit decisions under uncertainty where the output price follows a random walk [6][7]. Huchzermeier and Loch treated valuing the managerial flexibility in the context of uncertain R&D projects examining several sources of uncertainty, such as market payoff variability, budget variability and etc [14].

In this paper, we take the real option of investment flexibility into consideration in the context of project finance [12][13][16]. Many projects supported by project finance approach are characterized by large scale, long construction horizon, a huge investment fund and with high uncertainties in many aspects. Therefore, for this type of projects, the value of investment flexibility will be more substantial. Since the term "project finance" includes a variety of meanings ranging from the construction of power plants, amusement parks, and even more the national project such as the construction of highways and metros on the basis of the BOT (Built Operate and Transfer) or the BTO (Built Transfer and Operate), it is a hard problem to model and analyze these various types of project finance. Therefore, we restricted this paper to treat simple cases where the projects can be evaluated with a single indicator, however, we think we can extend the result to more general cases including several indicators by introducing appropriate transformation functions.

The object of this paper is to demonstrate the effect to increase the value of project and at the same time decrease credit risk by applying real option approach in the context of project finance [12][13]. Until now, there exist some researches, having demonstrated the effect to increase the value of project by applying real option approach in project management .

But because of the characteristic of project finance, the problem how to lower credit risk is a very crucial. The unified approach evaluating the effect of real option of investment flexibility on both project value and credit risk has never been studied by the conventional research works so far.

Specifically, we divide the project management into two phases consisting of the investment phase and the repayment phase from the viewpoints of finance, although usually it is divided into construction phase and implementation phase [13]. Certainly, from the definition of real option, real option approach is applied in the investment phase of project management. Therefore, at first, the evaluation schema of a project with investment flexibility is developed in the investment phase by applying the approach of DP (Dynamic Programming). The project manager utilizes the investment flexibility to respond to the uncertainty by choosing one of four possible investment decisions at every period.

For simplicity, we only consider that there is uncertainty with expected attainable capacity of the facility to be constructed and it will drift following a specific stochastic process. Through this simplified DP model, we also demonstrated that utilization of real option of investment flexibility can enhance the expected project value substantially.

2.2 Investment and state transition of project

Through the simulation study, we found out that the effect of real option of investment flexibility is favorable because in the repayment phase the credit risk can be lowered and revenue can be increased to some extent, compared with the case where real option is not taken into consideration [12][14].

As a simplified model, it is assumed that at period t and $t+1$ the expected attainable capacity are denoted by i and j, where i and j are integers. The state exhibits a fluctuation over stages of proceeding and at every period takes a jump of size ε_t. Performance uncertainty manifests itself in the variability of a probability distribution of ε_t.

$$j = i + \varepsilon_t \tag{1}$$

Letting the size of each jump ε_t be normally distributed with mean μ_c and standard deviation σ_c. For simplicity, we assume that $\mu = 0, \sigma = 1$. Since we discretize the range of possible values of State i out of $2N+1$ possible numbers, then the value of ε_t is also discretized into $2N+1$ possible values. Namely, letting $\Delta\varepsilon = 1$, if ε_t takes kth or $-k$th value of possible $2N+1$, then $\varepsilon_t = k\Delta\varepsilon$ or $\varepsilon_t = -k\Delta\varepsilon$. Then, for $k = -N, -N+1, ..., -1, 0, 1, ..., N$, the size of state transition from i to j subjects to the normal distribution. The assumption for ε_t is partly generalized compared to the assumption in Reference [14], and if the absolute value of ε_t is small, then the inherent probability becomes small.

Given a value of the state i from one of $2N+1$ possible values. The state space of expected attainable capacity over two adjacent periods is illustrated in Fig.1(a). We must carefully treat the cases with the end points of ε_t . Lumping all exterior values to the boundary we obtain the transition probabilities, as shown in equations in following sections.

We assume that the state of the project is also affected by the flexible investment throughout the construction from time $t = 0$ till time $t = T$ besides the fluctuation and drift defined in equation (1). At each period t, the manager can take any one of four possible investment decisions, namely the manager can abandon, continue, improve or shrink investment based on available information.

(1) abandon (abandonment)
(2) shrink (shrinking)
(3) continue (continuation)
(4) improve (improvement)

The abandonment option terminates the project immediately and any further investments are cut. The continuation option means that the project will be proceeded to the next stage $t+1$ at a continuation investment of $c(t)$. In addition to these two possible alternatives, the manager can also take corrective action in order to inject additional resources to improve mean expected attainable capacity by one level, or cut redundant resources to bring down mean expected attainable capacity by one level as shown in Fig.1 (a), (b) and (c).

We also assume that corrective actions can be carried out purely dealing with resources without additional time delay. For improvement option, an improvement cost $a(t)$ is imposed in addition to continuation cost $c(t)$. And for contraction option, a redundant cost $d(t)$ will be deducted from continuation cost. Moreover, we assume investment is made at the beginning of each period.

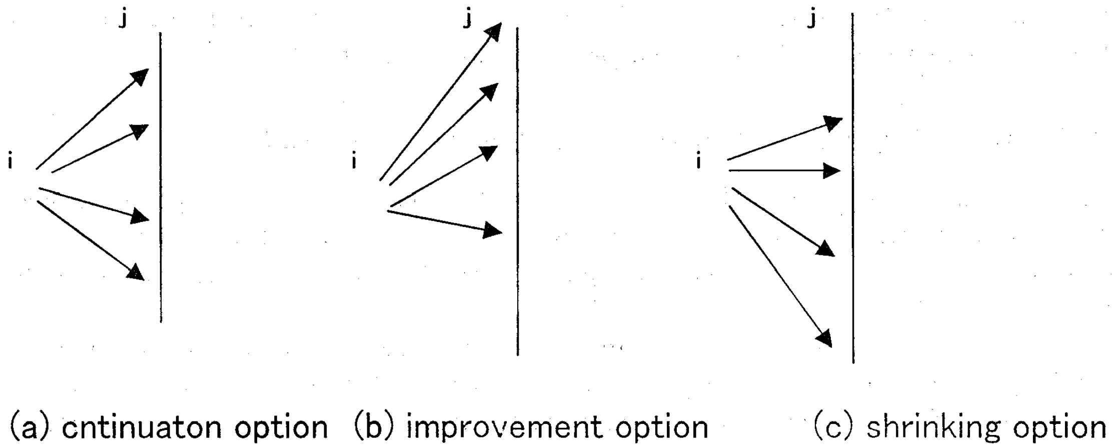

Fig.1-State transition under different investment decision

Under the improvement option , the state transition of project (capacity) is as denoted by equation (2).

$$j = i + 1 + \varepsilon_t \tag{2}$$

Similarly, given a value of i taking shrinking option may cause the mean of project performance to move as follows:

$$j = i - 1 + \varepsilon_t \tag{3}$$

2.3 Evaluation of Real Option Value by Dynamic Programming

To evaluate the value of project with real options of investment flexibility, there are two approaches, namely the Dynamic Programming and the Contingent Claims approach [14]. In this paper, we let project managers have investment flexibility, which means that they can select among four investment options, namely abandonment, continuation, improvement and shrinking option or the complexity raised by the investment flexibility, DP seems to be a very convenient approach to be applied for evaluating the value of project with real options of investment flexibility than Contingent Claims approach.

Here, we denote the state of expected attainable capacity at period t by using i which is an integer.

In fact, in order to evaluate the value of project with real option of investment flexibility, we must solve the sequential optimal investment decision problem to maximize the project value first, which can be formulated as a dynamic programming problem as equation (4) and equation

(5). In these equations, $c(t)$, $c(t)+a(t)$ and $c(t)-d(t)$ denote the cost for three alternatives in period t for continuation, improvement and shrinking, respectively.

$$V_i(T) = \Pi_i \tag{4}$$

$$V_i(t) = \max \begin{cases} 0, & \text{abandon ;} \\ -c(t) + d(t) + \eta \sum_{j=-N}^{N} p_{ij} V_j(t+1) & \text{shrink;} \\ -c(t) + \eta \sum_{j=-N}^{N} q_{ij} V_j(t+1) & \text{continue;} \\ -c(t) - a(t) + \eta \sum_{j=-N}^{N} r_{ij} V_j(t+1) & \text{imporve ;} \end{cases} \tag{5}$$

where $\eta = 1/(1+\rho)$, and $1+\rho$ is the discount rate. The probabilities p_{ij}, q_{ij}, r_{ij} are the state transition probabilities correspond to shrinking, continuation and improvement option in investment for equations (3), (1), and (2). Π_i is the evaluation for project at the end of construction (terminal period) representing the payoff of project if the project is in state i.

We denote $V_i(t)$ $(0 \le t \le T_1, 1 \le i \le 2t+1)$ to be the maximum project value attainable through the optimal investment decision, given that the current performance state is i. The right hand of equation (5) means four project values corresponding to four investment decisions respectively.

Through the formula, we can calculate as the difference between the PV of expected project value at time $t+1$ discounted back to time t and the investment made at period t. By comparison of these four project values, we can select the favorable project value $V_i(t)$ and make the corresponding invest decision as the optimal decision to be chosen without any hesitation.

In order to solve this DP problem using standard backward recursion, we must know the terminal value of $V_i(T)$ $(1 \le i \le 2T_1+1)$, which is very simple because it is equal to project payoff Π_i as shown in equation (4), in which $1+\rho$ is the discount rate which we consider to be exogenously specified.

With all the optimal investment decisions at every period having been determined, , we can derive $V_i(0)$ as the value of project with real option of investment flexibility. Finally, the value of real option of investment flexibility can be calculated as the difference between the project value with and without real option of investment flexibility.

2.4 Numerical examples

We will demonstrate the valuable effect of application of real options of investment flexibility through a numerical example, based on the models. In the following, we set the amount of $c(t)$, $a(t)$ and $d(t)$in equation (4) and (5) as shown in Table 1.

Table 1 -Cost for various investment in each period

t	1	2	3	4	5	6
$c(t)$	100	100	150	200	300	300
$a(t)$	50	50	30	50	50	50
$d(t)$	20	20	30	50	50	50

And we set the value of other primary parameters as follows:
$T_1 = 6, \rho = 0.1, N = 2$
initial state of project $i = 26$
Π_i:given by distribution function (integration of probability density of normal distribution of N(0,0.1), its minimum is 0,and its maximum is 280.

Through solving the dynamic programming problem described in Section 2.3, we can attain the expected project value through effectively utilizing the investment flexibility. And at the same time we find optimal investment plans. The result of optimal selection of alternative depending on the performance state at the time period t is shown in Fig.2. As demonstrated in Fig.2, the lattice tree corresponds to the increasing number of possible performance states over time and below each node in the tree, the maximum project value and the corresponding optimal investment option is shown. Symbol A, C, D, E denote abandonment option, continuation option, shrinking option and improvement option, respectively. We can see from Fig.2 that NPV (Net Present Value) of expected project value is 226.4, if we manage the flexibility effectively.

Fig.3 shows a pass for investment to attain the payoff at time T_1.

For comparison, we show the lattice structure for the conventional strategy in Fig.4, where we can select only continuation of investment. As shown in Fig.4 the NPV of project value is −27.8. Then the difference between these two project values, which is defined as the value of real option, is 254.2 which can be derived straightforward.

The result demonstrates that the value of investment flexibility can be substantial. In order to explain the result clearly, we denote the project management case with investment flexibility to be case A and on the other hand, the case without investment flexibility to be case B.

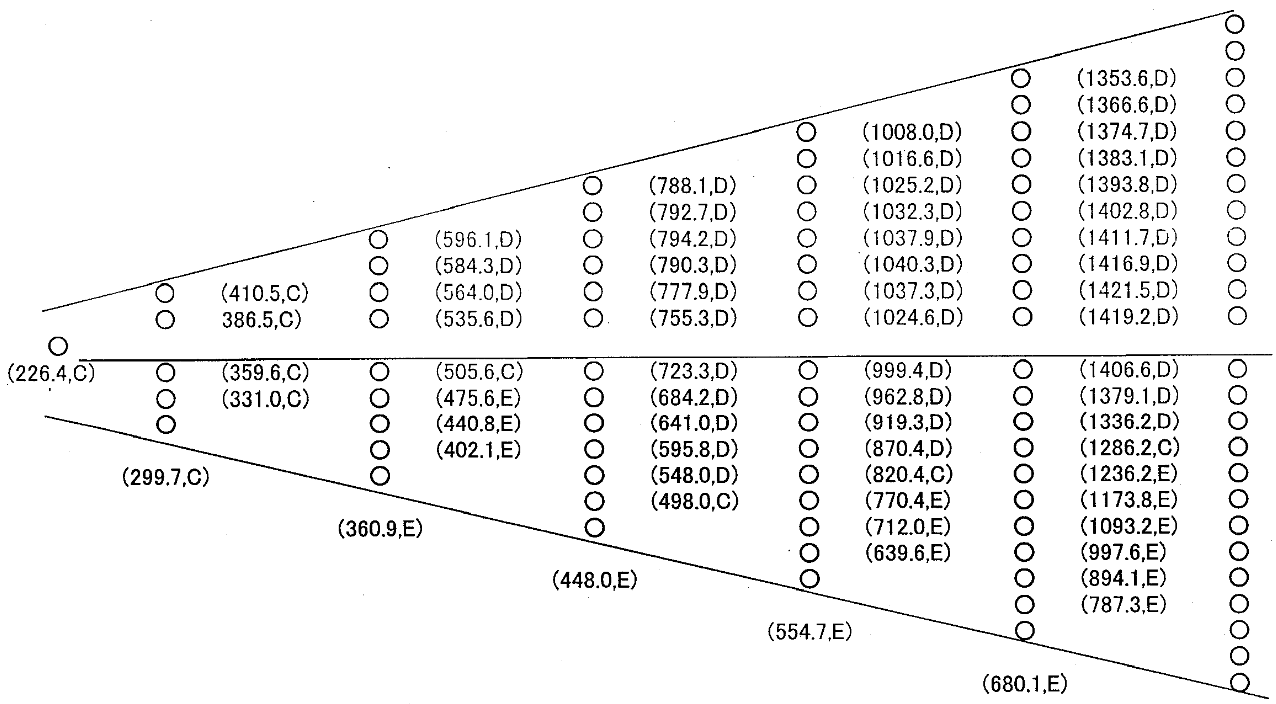

Fig.2- Case A with investment flexibility

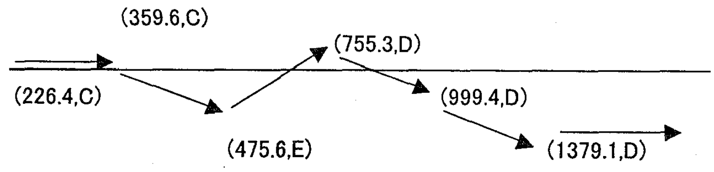

Fig.3-Example of optimal pass of investment

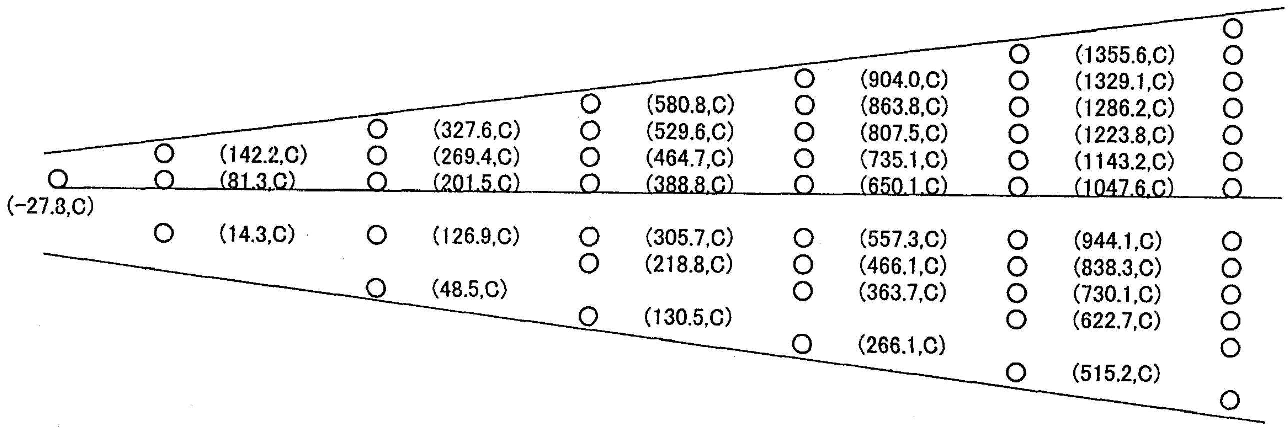

Fig.4- Case B without investment flexibility

Moreover, in terms of the realizable level of capacity and probability through exploration of investment flexibility, we can derive interesting result. As shown in Table 2, finally realizable capacity of facilities constructed at the last stage of investment phase (construction phase) will be one of eight states which are characterized by the corresponding probability and overall investment in order to reach specific state.

Table 2-Result of case A

capacity	21	22	23	24	25	26	27	28
probability	0.01	0.20	0.31	0.30	0.12	0.05	0.01	0.009
overall investment	335.9	318.0	315.0	312.4	305.3	304.3	302.8	296.4

In contrast to the case with investment flexibility, in the case B where we do not use other alternatives than continuation, expected realizable capacity set consists of 13 states. Then, we can draw a conclusion that investment flexibility can make the project performance drift to favorable level with higher market payoff and simultaneously decrease variability of reachable project performance, which then leads to lower variability of market payoff.

Because of this advantage, we see that under investment flexibility expected project value can be enhanced so that the profitability of the project can be enhanced to some extent. The expected overall investment will be used later to calculate repayment M in each period of repayment phase.

3 Cash flow uncertainty and capacity

3.1 Demand and capacity in repayment phase

Then we consider the repayment phase based on the cash flow obtained by operating the facilities constructed through the underlying investment. We assume that the demand is time dependent and follows Geometric Brownian Motion and the cash flow involves potential uncertainty also.

Although there are different uncertainties in many aspects, in this paper for simplicity, we only focus on the uncertainty of expected attainable capacity, which is modeled by a one-dimensional parameter i (final capacity of project). Capacity of facilities can be considered to be a key decision parameter for some projects, such as the project to construct a communications network in the communication industry.

Generally, at the beginning of the project, optimal capacity of facilities to be constructed is determined on the basis of market demand estimated, the attainable investment fund and etc. But in fact, perhaps it will take many years for some projects to be finished and the conditions that decisions were based on will change continually. Consider the investment on a project proceeding in T_1 discrete stages towards the completion of facilities. At the beginning of the project, project managers want to know the relationship between the various capacities and the corresponding expected market payoff and then intend to find the optimal capacity to be constructed.

In the following, a model will be provided, through which project managers can determine optimal capacity according to the information they own about the estimation of the market demand, the necessary investment fund, the operation cost and etc.

Firstly, assume that if project is launched at time T_1 with a capacity level i, namely facilities holding capacity of i are completed, it will generate an expected market payoff Π_i. In general, intuitively the higher the level of capacity of facilities constructed, the higher the payoff or the expected cash flow from the market is. However, in this paper, through a simplified model, we derive a conclusion somewhat different.

Here, we define the expected cumulative cash flow obtained by operating the facility from time T_1+1 through T_1+1+T_2 as follows:

$$\Gamma(i)=\sum_{t=T_1+1}^{T_1+1+T_2}\frac{E[C_k(t)]}{(1+\rho)^{t-T_1}} \tag{6}$$

In equation (6), symbol E denotes the expectation of cash flow, while $C_i(t)$ denotes the cash flow generated in period t, given the capacity of facilities constructed is i. Risk-adjusted discount rate ρ is the project manager's demanding rate of return, reflecting the capital cost and his subjective thought. Theoretically, we can find the adequate demanding rate of return through financial market.

The quantity of cash flow $C_i(t)$ may be affected by many factors, such as market demand, the operation cost and etc. Here, we assume that the higher the capacity of facilities constructed, the high the operation cost will be. In general market demand $D(t)$ is impossible to be constant and it is reasonable to assume that $D(t)$ follows a specific stochastic process.

In this paper, we assumed demand to follow a Geometric Brownian Motion as follows, which will expand eventually, supposing the example that demand to utilize the communication network, not too fast, but will expand eventually. Surely, the change of diffusion process of demand will not influence the result in this paper.

In equation (7), dZ is the incremental of a Wiener process, μ_D is called the drift parameter and σ_D the variance parameter.

$$dD(t) = \mu_D D(t)dt + \sigma_D D(t)dZ \tag{7}$$

In this paper, we apply Monte Carlo simulation method to estimate the expected cash flow. By simulation study, we can attain the function form of Π_i to be concave increasing with k as shown in Fig.5, supposing that $D_0 = 20$, $\mu_D = 0.05$, $\sigma_D = 0.05, 0.1, 0.2$ respectively. We also can obtain similar curve supposing $\mu_D = 0.1$ or 0.2. .

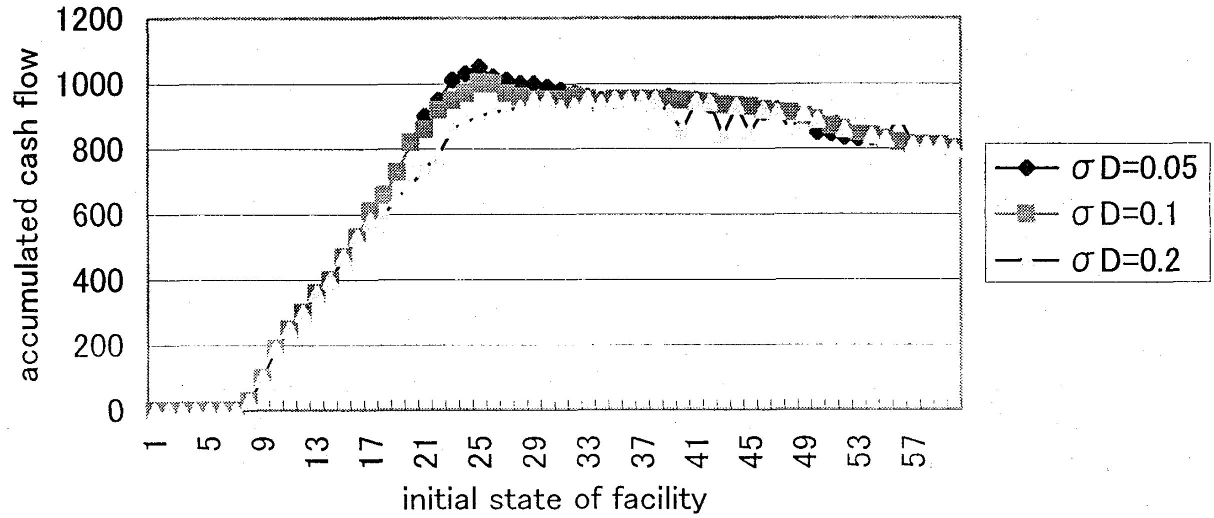

Fig.5-Relationship between various capacities and accumulated cash flow

As shown in Fig.5, there exists a specific optimal capacity with which maximal market payoff can be obtained. The part of the curve left of the optimal point is increasing with capacity. If capacity is very low, the demand cannot be satisfied entirely, which leads to lower overall profit. And with a higher capacity, demand can be satisfied, which contributes to overall profit very well.

But the part of curve right of the optimal point is different and decreasing with capacity, since maintaining of excess capacity will contribute nothing to the overall profit and on the contrary decrease the overall profit. Moreover, considering the variability of demand, we can draw the conclusion that the higher the variability of demand (σ_D) is, the lower the payoff is and the higher the optimal capacity (corresponding to maximum payoff) is. From this simulation result, we found out the optimal capacity to be the appropriate value of final state of facility.

3.2 Evaluation of Credit Risk

In this section, we consider a bank lending a loan to a project company. In the context of project finance, the borrower repays the loan by using the cash flow generated through operating the facilities constructed over the investment phase. The amount of repayment at every period is calculated according to the amount used in the investment phase really.

Usually, it is expected that the obtainable cash flow at time t is large than determined repayment amount. However, it may occur the cases where obtained cash flow is not sufficient to repay, then the firm may default for some reasons. For example, the demands for a high-tech device may suddenly decrease due to the stagnation of national economy.

In the worst cases, the firm bankrupts and the bank is exposed to substantial loan default risk. From the viewpoints of the bank, the task to lower or hedge against the credit risk exposure seems to become more and more important in the world full of various uncertainties now.

In practice, not only bank but also firm must pay a special attention to this credit risk, since no concern about this type of risk may worsen the financing condition in future.

One way to hedge credit risk is through guarantees or undertaking of sponsors or a third party. Indeed, guarantees are the life-blood of most project financings because project companies have high debt to equity ratios [16]-[20]. Moreover, besides the traditional guarantees, in recent years, there has been an explosive growth in the use of credit derivatives as credit risk management tools.

Risk exposure

As is mentioned above, in this paper we consider two-phase project management namely investment phase and repayment phase. Based on the production capacity obtained at the final stage of investment phase through exploration of investment flexibility contingent on project performance realization, the firm will begin to provide service in response to market demand.

We assume that the firm must repay a determined amount M every period to the bank, utilizing cash flow generated from the operation of facilities over the repayment horizon. Let T_2 denote the length of repayment horizon. Variable $C(t) = C_i(t), (T_1 + 1 \leq t \leq T_1 + 1 + T_2)$ for fixed i accounts for the amount of cash flow obtained in period t and $g(t)$ denotes the gap between the amount of cash flow and repayment in period t, which can be calculated by equation (6).

The implication of equation (8) is that if cash flow $C(t)$ is plus and not sufficient to repay M, the gap will equal to $M - C(t)$ and otherwise when there is no problem to repay M, the gap will be set to zero. On the other hand, although cash flow may be minus, the gap will be set to an upper limit M.

Overall, the possible value of $g(t)$ will satisfy $0 \leq g(t) \leq M$. Variable P denotes the credit risk exposure of the project. We can derive the value of P by equation (9), which means that the credit risk exposure should equal to the sum of PV of expected gap between cash flow and repayment in every period, discounted back to time $t = 0$ by discount rate ρ given exogenously.

$$g(t) = \begin{cases} \max[M - C(t), 0] & (C(t) \geq 0) \\ M & (C(t) < 0) \end{cases} \tag{8}$$

$$G = \sum_{t=T_1+1}^{T_1+1+T_2} e^{-\rho t} E[g(t)] \tag{9}$$

Then, the value G denotes the risk exposure of the project management.

Surplus after repayment

Letting variable $h(t)$ denote the surplus of cash flow deducted repayment M in period t, which can be calculated by equation(10).

$$h(t) = \begin{cases} max[C(t) - M, 0] & (C(t) \geq 0) \\ C(t) & (C(t) < 0) \end{cases} \tag{10}$$

$$R = \sum_{t=T_1+1}^{T_1+1+T_2} e^{-\rho t} E[h(t)] \tag{11}$$

The implication of equation (10) is that, if cash flow $C(t)$ is positive and sufficient to repay M, the surplus of cash flow will equal to $C(t) - M$, and if there is problem to repay M entirely, $h(t)$ will be set to zero.

On the other hand, if cash flow is deficit, $h(t)$ will be set equal to the deficit. Overall, the surplus of cash flow will have no upper limit and down limit. Then we let variable R denote the overall revenue after repayment from period $T_1 + 1$ to period $T_1 + 1 + T_2$ and we can derive value of revenue R by equation (11).

Equation(11) means that the value of revenue should equal to the sum of PV of expected surplus of cash flow in every period, discounted back to time $t = 0$ by discount rate $1 + \rho$ given exogenously.

3.3 Examples

Now, we investigate the variation of real option value under different demand variability. We consider following two cases denoted as Case 1 and Case 2.

Case 1:$\sigma_D = 0.05$, initial state of project $i = 26$

Case 2:$\sigma_D = 0.2$, initial state of project $i = 35$

Fig.6 shows the diagram for G in equation (9) depending on the variances σ_D. In the figure, the notation Case A(1) and Case A(2) correspond to the case with investment flexibility where the assumption for the variance σ_D is Case 1 and Case 2, respectively. In a similar manner, the notations Case B(1) and Case B(2) stand for the cases without investment flexibility under the assumption Case 1 and Case 2, respectively.

From the result, we can conclude that investment flexibility can lead to a comparatively lower credit risk (reflected by credit risk exposure) as shown in Fig.6, whenever the value of σ_D varied. The fact is based on the same reason discussed above, and we see that investment flexibility can make the project performance to drift to favorite level with higher market payoff, or can make lower overall investment to reach a performance level without high payoff.

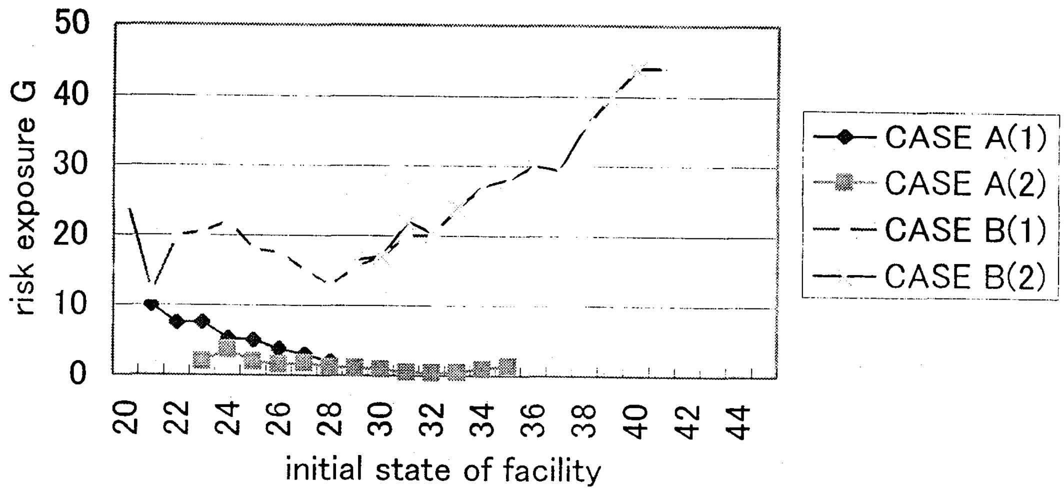

Fig.6-Credit risk with different demand variability

4 Risk aversion with traded asset

4.1 The Merton investment problem

Next, we describe the model for risk hedging in the investment where agents can not exit from the project and simultaneously they buy or sell tradable asset . The classical Merton wealth problem involves an agent investing in a risky share with price P, growth rate μ and volatility sigma, and a risk less bank account with constant interest rate r .

For simplicity, we assume $r = 0$. The agent chooses to invest the cash amount θ_t in the risky share and starting with the initial wealth x. Then, the wealth of agent evolves as Where B is a standard Brownian motion. The agent's aim is to maximize expected utility of terminal wealth of utility function $U(y)$, namely , $U(X_T)$. The utility function $U(y)$ has following form with constant risk aversion

$$U(y) = y^{1-R}/(1-R) \tag{12}$$

where $R(R > 0, R \neq 1)$ is a constant representing risk aversion parameter, Let the value function is defined as

$$V(t,x) = \frac{x^{1-R}}{1-R}\exp[\frac{\mu^2(1-R)}{2\sigma^2 R}(T-t)] \tag{13}$$

Then, applying the Ito's formula to $V(t, X_t)$, we find

$$dV = [dX/X - 0.5R(dX)^2/X^2 - \mu^2/(\sigma^2 R)dt]X^{1-R}e^{m(1-R)(T-t)} \tag{14}$$

where $m = \mu^2/(2\sigma^2 R)$, and V is a supermartingale for any strategy θ and a martingale for the optimal strategy $\theta = (\mu/\sigma^2 R)X_t$. Then, V given in equation (13) is the value function for the utility maximization problem. Not that, if $\pi_t = \theta_t/X_t$ and is the proportion of wealth invested in the risky asset, then the optimal strategy for agent is $\pi_t = \mu/(\sigma^2 R)$ which is constant, so the called "Merton's proportion" [11].

In particular, in a complete market an agent with constant relative risk aversion has a simple optimal strategy.

Then, we introduce another risky asset with price S on which no trading is allowed. Assume S follows

$$dS/S = \nu dt + \eta dW^+ \tag{15}$$

where W^+ is a Brownian motion, and ν, η are constants. We assume W^+ is correlated to the Brownian motion B , the process P, with correlation coefficient γ. It is convenient to think of W^+ as a linear combination of two independent Brownian motions B and W. Thus,

$$W^+ = \gamma B_t + \sqrt{1-\rho^2}W_t \tag{16}$$

If two processes B and W^+ are the same process ($\gamma = 1$), and the conditions that P and S are martingale is assumed, then the optimal value of investing for the asset is written as

$$\theta = \mu X_t/(\sigma^2 R) + \lambda S_t[\mu/(\sigma^2 R) - \eta/\sigma] \tag{17}$$

Then, we can interpret the formula in a simplified manner. If we have a single asset, then the risk aversion is attained by using the first term in equation (17), but for hedging the bearing risk for non-traded asset, we must also need the second term in equation (17).

By the way, the general case for $\gamma < 1$, it is not possible to obtain a closed form solution, and then in Refernce [10] a kind of approximation to get the partial differential equations for time t and $Z_t = S_t/X_t$, and the power series expansion is utilized to obtain the option value.

However, the general case for $\gamma < 1$ is very hard to analyze, and the main purpose of the paper is to describe the basic idea for risk hedging, then we focus only on the case where $\gamma = 1$.

Moreover, the numerical results included in Reference [10] show that in the evaluation for the case $\gamma < 1$ P is a linear combination of γ, and the result for $\gamma = 1$ is still useful to discuss the general case. Then, we combine the evaluation method of project based on the DP and the risk hedging using the tradable asset to remove the risk incurred in non-traded asset (investment). But, we must introduce several kind of restriction (conditions) for the simulation studies.

5 Simulation studies

5.1 Approximation for simulation

In the following, we will discuss the simulation studies for evaluation of real option for project finance based on the risk aversion using the traded assets. At first, we assume several conditions necessary to combine the investment flexibility based on the DP and the risk aversion process.

Basically, the model treated in Reference supposes no exogenous input from the outside, then the capacity (status) of the project subjects to a kind of random process such as the Brownian motion, and the replication of the process by ordinary process correlated to the change of status is not difficult. However, in the investment flexibility in which we utilize the exogenous input such as the improvement of investment or shrinking of investment, the process of status of capacity may diverge from ideal Brownian motion. We must note that a closed form solution for the case is very difficult in the form resemble to the result discussed above. At the same way, in the risk aversion process, the initial investment is modified to suffice the additional investment, and sometime increased by the abandonment of investment. Then, the theoretical result for the risk aversion using the initial or single shot of investment treated in Section 4.1 is not applicable in our cases.

Therefore, the following simulation studies show an approximation for complicated situations by using the result of ideal cases. Then, we must note that in the simulation studies we assume following conditions for the approximation.

(1) Investment is composed of flexible investment besides initial payment

In the original model, agent pays an initial payment at time 0, and in subsequent time, he adjust and determine the optimal amount of cash invested in P denoted as θ^*, which is not affected by another factors. But, in our model, the total amount of cash which agent has will change depending on the investment flexibility. Then, we regard this adjusted amount of cash including investment flexibility as the cash the agent possesses. The procedure of calculating the additional investment inherited from the Investment flexibility is shown in Fig.7. The amount of investment is found by tracing the pass in the DP from the final stage to the initial stage (determined backward and traced forward).

(2) time series of status change is approximated by a Brownian motion

Originally, it is supposed that the price (time series) of underlying asset S and tradable asset P follow the Brownian motion. But, these processes deviate from the ideal Brownian motion. To apply the closed form result for the risk aversion, we assume that two processes approximately follow Brownian motion.

Based on two assumptions, we can proceed to the simulation studies to find the real option using the traded asset. It must be noticed that the price change of underlying asset (non-traded asset) corresponds to the capacity of facility plays an important role.

(1) find the optimal pass to realize targeted capacity

At first, we find the optimal pass to realize the final status of facility (capacity) by using the DP , for example, as shown in Fig.3. In the pass, the change of the capacity corresponds to the price of asset S is found, and is regarded as the price change of asset S.

(2) find optimal risk aversion using traded asset

Secondly, by trading the tradable asset with price P which is correlated with the price S, we adjust the position for the tradable asset by using the result for optimal theta. The time series for price P is found by tracing the DP pass forward as in Fig.7.

(3) evaluate the effect of risk aversion

In the Monte Carlo simulation, we try sufficient number of runs for the Step (1) and (2), we aggregate the result of real option by weighting the result with the probability the underlying pass has. The definition of real option used here is the difference between the amount of two kinds of investment, namely, the investment with risk aversion utilizing tradable asset and without risk aversion. Off course, in two investment in this case the flexible investment is assumed. In the evaluation, the expected accumulated cash flow obtained by operating the facility after construction (investment) is calculated by using the diagram such as Fig.5. By multiplying the coefficient λ, we obtain the the profit form the investment to the project. .

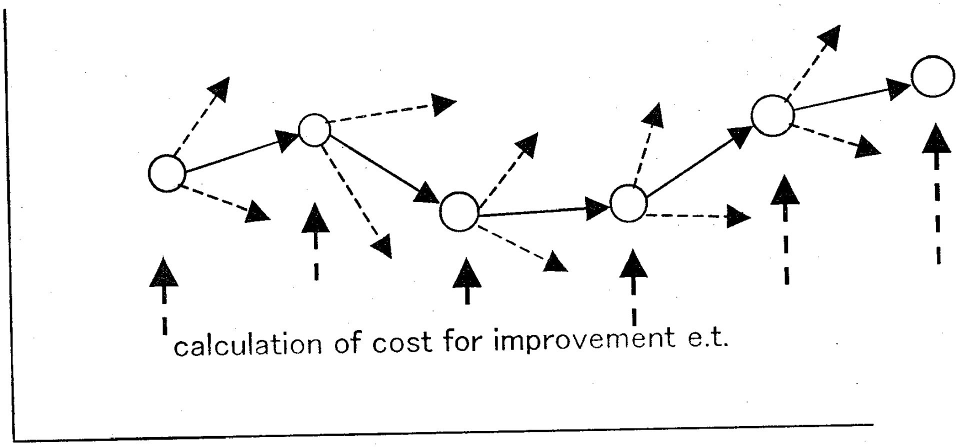

Fig.7 Calculation of various cost incurred in investment

By the way, there exists a kind of comparison in cash flow (risk exposure) in cases with and without risk aversion, but the similar simulation is available, and we skip the discussion here.

5.2 Result of simulation studies

In the following, we summarize the comparison of investment (value of investment) using the risk aversion based on the simulation studies. There are three kinds of strategies to realize the profit from the investment to project.

Case I: without investment flexibility

Case II: with investment flexibility but without risk aversion

Case III: with flexible investment using risk aversion with traded asset

The first Case I is evaluated by using ordinary NPV. The next two strategies are evaluated by the procedure discussed so far. Following two conditions are assumed for simulation studies.

(1) single risk factor

Usually, the evaluation of underlying asset is described by several risk factors such as the interest rate. But, we simplify the problem by assuming that the evaluation of the asset is represented by the price of asset itself, similarly to the stock holding where the value of asset is the stock price. Then, the price of asset is changed depending on the investment options described in equations (1), (2) and (3). The figures like Fig.2 and Fig.4 are used to follow determine price.

(2) generating time series of risk factor

The time series for the price of tradable asset is assumed to be identical to the time series of project (capacity of facility) discussed above. Then, the correlation coefficients for two prices S and P becomes one.

The only parameter to be varied is the rate λ of investment (commitment) to the construction of facility. If λ is set to be large, then the obtainable profit from the facility becomes also large, but the amount to hedge the risk also becomes large.

At first, Fig.8 shows the cash flow obtained by the investment under the flexible investment and risk aversion using traded asset. In Fig.8, the vertical axis denotes the obtainable cash flow, and horizontal axis means the initial state of facility. As is seen from Fig.8, the maximum value of cash flow is located in the middle of the axis of initial state of facility, but it becomes to be flat compared to Fig.5. The fact means that the risk aversion using tradable asset adjust the difference caused from the selection of initial state of facility.

Fig.9 depicts the result of simulation by showing the total amount of obtainable return of investment. In Fig.9, the vertical axis represents the obtainable return, and the horizontal axis is the initial state of facility. Two cases of initial amount of cash for investment are depicted in upper and lower figures in Fig.9.

As is seen from Fig.9, the maximum value of return is found in the middle of horizontal axis (initial state of facility), but the shape of the curve is slightly different depending on the initial cash of investment. However, it is also found that the difference of return depending on the selection of initial state of facility is very small compared to Fig.5, and the fact implies that the risk aversion using the tradable asset mitigate the gap of investment. .

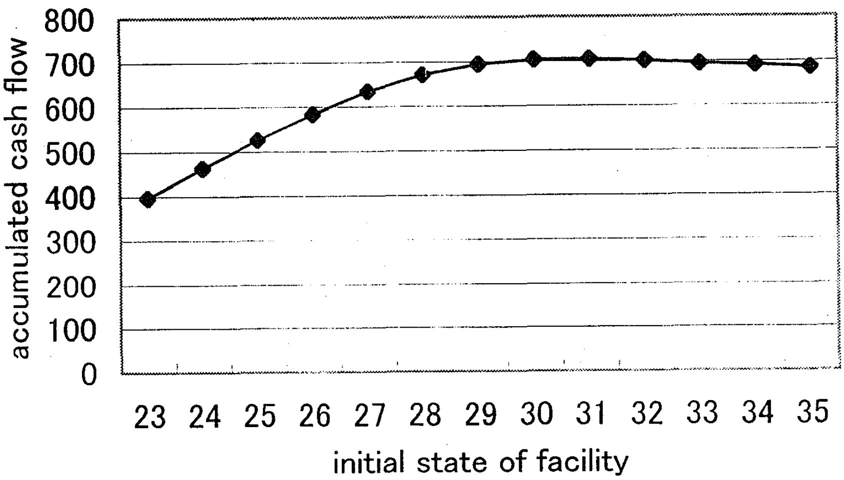

Fig.8 Accumulated cash flow versus initial state of facility

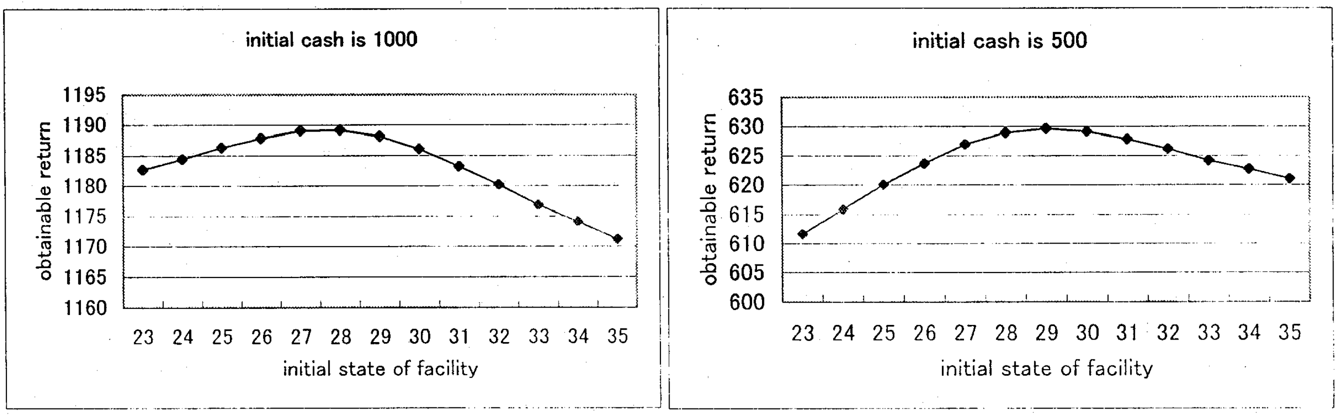

Fig.9 Obtainable return versus initial state of facility

6 Conclusion

In this paper, we showed the evaluation of real option for the project finance on which no trading is allowed and risk hedging is realized by using another tradable asset. We took the real option of investment flexibility into consideration in the context of project finance. Specifically, we divide the project management into two phases consisting of the investment phase and the repayment phase from the viewpoints of finance, although usually it is divided into construction phase and implementation phase. The evaluation schema of a project with investment flexibility was developed in the investment phase by applying the approach of DP. Then, we utilized the option formula on the non-traded asset with a multiple λ units of the share by the predefined position of traded asset as a risk hedging. The simulation studies showed the effectiveness of risk aversion using tradable asset, and the increase of obtainable return.

The problem remained to be solved are the real applications for various project management such as the Built Operate and Transfer systems, and our research will be still continued.

References

[1] R.L.McDonald and D.R.Siegel,"Investment and the valuation of firms when there is an option to shut down", International Economic Review, vol.26(June), pp.331-349, 1985.

[2] R.L.McDonald and D.R.Siegel,"The value of waiting to invest", Quarterly Journal of Economics, vol.101, pp.707-727, 1986.

[3] R.S. Pindyck,"Irreversible investment, capacity choice, and the value of the firm," American Economic Review, 78,5, pp.23-34, 1988.

[4] R.S.Pindyck,"Irreversibility, uncertainty and investment", Journal of Economic Literature, Sept., pp.1110-1148, 1991.

[5] A. Dixit,"Hysteresis, import penetration, and exchange rate pass-through," Quarterly Journal of Economics, 104, pp.205-228 ,1989.

[6] A. Dixit,"Entry and exit decisions under uncertainty," Journal of Polotical Economy, 97, pp.620-638 ,1989.

[7] A.Dixit and R.S.Pindyck:"Investment under uncertainty", Princeton University Press, 1991.

[8] S.A.Johnson and Y.S.Tian,"The value and incentive effects of non-traditional executive stock option plans", Journal of Financial Economics, vol.57, pp.3-34, 2000.

[9] M.H.A.Davis,"ption pricing in incomplete markets", in Mathematics of Derivative Securities,M.A.H (eds) and S.R.Pliska, Cambridge University Press, 1997.

[10] V.Henderson and D.G.Hobson,"Real options with constant relative risk aversion",Journal of Economic Dynamics & Control, vol.27,pp.329-355 , 2002.

[11] R.C.Merton,"Lifetime portfolio selection under uncertainty:the continuous time case", Review of Economics and Statistics, vol.51 , pp.247-257, 1996.

[12] X.Chen and S.Tokinaga,"Analysis of project finance based on the dynamic programming and its applications (in Japanese)" ,Proc of 2001 Spring National Conference of ORSJ,pp.106-107 ,2001.

[13] X.Chen,Analysis of Dynamical Systems based on Nonlinear Modeling and its Applications to Optimization,Dissertation of Graduate School of Economics, Kyushu University,pp.94-132,2002.

[14] A.Huchzermeier and C.H.Loch,"Project management under risk:Using the real options approach to evaluate flexibility in R&D", Management Science, vol.47,no.1, pp.100-113,1996.

[15] B.Kogut and N.Kulatilaka,"Operational flexibility, global manufacturing and the option value of a multinational networks", Management Science, vol.40, no.1 ,pp.123-139 ,1994.

[16] N.Takagi and S.Tokinaga," Prediction of chaotic time series by using the multi-stage fuzzy inference systems and its applications to the analysis of operational flexibility, Journal of Operations Research Society of Japan, vol.23, no.3,pp.243-259,2002.

[17] J.D.Finnerty,"Credit derivatives, infrastructure finance and emerging market risk", The Financier, Feb.1996, pp.64-78, 1996. bibitemb18 Peter K. Nevitt,Project financing, Euromoney publication,1989.

[18] A.Saunders,Credit Risk Measurement, John Wiley & Sons Inc. , 1999.

[19] G.D.Vinter,Project finance,Lonod Sweet and Maxwell, 1994.

Xiaorong Chen
(Antai School of Management, Shanghai Jiaotong University)
Shozo Tokinaga
(Graduate School of Economics, Kyushu University)