Modeling and Control of Diffusion Processes by using the CNN Approaches based on the Approximation by the Genetic Programming

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Modeling and Control of Diffusion Processes by using the CNN Approaches based on the Approximation by the Genetic Programming

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1 Introduction

In the last few years, there have been many advances in the study of pattern formation and wave phenomena in the fields such as physics and chemistry[1][2]. These studies have been carried out mostly experimentally and by simulation of nonlinear partial differential equations. An alternative approach is the direct analysis of nonlinear dynamics. For example, it is also showed that interconnections of a sufficiently large number of simple dynamic units such as cellular neural networks (CNNs) can exhibit extremely complex and self-organizing behavior [1]-[5].

The study of chaos has provided new conceptual and theoretical tools enabling us to understand complex behavior and control them [5][6]. Even though the chaotic behavior seems to be universal, and shows up the deterministic model in electrical circuits, lasers, chemical reactors and many other systems, but the applicability means that we learn about chaotic behavior by studying simple mathematical models.

The goal of modeling and control of chaotic behavior is to estimate the dynamics of systems and to predict the future value from a set of known past data. To reconstruct a dynamic model by measuring the time series is a kind of inverse problem existing as a counter part of ordinary approach in numerical analysis where we postulate physical dynamical systems. Sometime, we can only use the immediate past behavior of the time series to reconstruct the current state of systems.

In previous works, we demonstrated that the Genetic Programming (GP) provides the flexibility to evolve the structure and function of chaotic dynamics in their entirety based on observed time series, and is also applicable to the control of chaos [9]-[11]. The estimation method enable us to model complicated variations with simple deterministic equations, especially for the cases where the number of available observation is restricted. We also showed the method is applicable to modeling and control of chaotic behaviors observed in the autowaves in CNNs[8].

However, there still remain problems, since previous works using the GP focused on known system dynamics and artificially generated chaotic time series, and it is not shown whether the method is applicable to real world data. In addition, the conditions for the propagation failure of autowaves are given only for the one-dimensional systems, and are not extensible for higher-dimensional cases [8][17].

We investigate in the paper on modeling and control of the diffusion processes by using the CNN approaches based on the approximation by the GP for real world data. The condition for the propagation failure of the autowave is also discussed based on the estimated equations by using the eigenvalues of coefficients of linearlized equations at the equilibrium point. The conditions are used to estimate the possibility to discuss the propagation failure on the basis of structural changes of system equations.

Our interest is to demonstrate a control scheme of traveling wave on CNN plane without changing the dynamics. We seek taming and controlling chaos by providing a weak control signal to induce the system follow a stability (fixed points and limit cycle).

The chief genetic operator used in the GP is subtree crossover. System equations are usually represented by parse trees (called individuals). One parse tree corresponds to a system of dynamic equations. The performance of each individual (called as fitness) is defined by comparing the output generated by the system equation corresponds to the individual with the observed data to be approximated. Subtree crossover randomly selects two individuals from the pool possessing relatively higher fitness and swaps them at the crosspoint in the reproduction phase.

In our control scheme, we assume the equations governing the autowaves is approximated and estimated by the GP. We only need to impose the small input to the system to achieve the desired propagation failure in autowaves. We assume that the dynamic system $f(x(t))$ with input $s(t) = 0$ is estimated by using the GP, and is denoted as $\hat{f}(x(t))$. Then, the control method is derived straightforward by using the approximation. We impose the input $s(t)$ so that the solution of $dx(t)/dt = \hat{f}(x(t)) + s(t)$ moves to the fixed point.

Simulation studies for approximating known dynamics by using the observed time series show good estimation of the systems equations. As an application, modeling and control method in the paper is applied to the control of autowaves for real world data observed in the propagation of harmful insects and epidemic.

In the following, in Sect.2, we show the fundamentals of the CNN treated in the paper related to the propagation failure of autowaves. In Sect.3, the approximation of the dynamics of the CNN is shown based on the GP. Sect 4 shows simulation studies for modeling and control of known dynamics by the method of the paper. In Sect.5, we show the estimation and control of propagation of autowaves for real world data.

2 Fundamentals of autonomous CNN

In the computational tasks such as image processing, Chua and Roska proposed the CNN as a alternative with a clear mathematical description by partial derivatives and analogue VLSI implementation [1]-[3].

We have several types of CNN, namely autonomous CNN and non-autonomous CNN. In the following, we restrict ourselves to the autonomous CNN but having diffusion terms. As is pointed out, these CNN can simulate very wide class of partial differential equations [1].

In the following, it is assumed that the system equations are shown by using differential equations including the diffusion terms. For eample, if the system is described by a three dimensional systems, then we have

$$du_{ij}(x,t)/dt = f_u(x,t) + D_u \nabla^2 u_{ij}(x,t) \tag{1}$$

$$dv_{ij}(x,t)/dt = f_v(x,t) + D_v \nabla^2 v_{ij}(x,t) \tag{2}$$

$$dw_{ij}(x,t)/dt = f_w(x,t) + D_w \nabla^2 w_{ij}(x,t) \tag{3}$$

where $x = [u_{ij}(x,t), v_{ij}(x,t), w_{ij}(x,t)]$ is three dimensional vector at the location (i,j), and $D_u \nabla^2 u_{ij}(x,t), D_v \nabla^2 v_{ij}(x,t) D_w \nabla^2 w_{ij}(x,t)$ are the diffusion term with diffusion coefficient D_u, D_v, D_w.

The state vector x stands for the location (i, j) on a Cartesian coordinate system of CNN plane. In one dimensional case x means the location i on a line.

The diffusion terms included in equations are discretized in one-dimensional CNN such as

$$\nabla^2 u_i(x,t), = u_{i+1}(t) + u_{i-1}(t) - 2u_i(t) \tag{4}$$

In the same way, at each cell c_{ij} at location (i, j) in a Cartetian coordinate, we have

$$\begin{aligned}\nabla^2 u_{ij}(x,t) = x_{i+1,j}(t) + x_{i-1,j}(t) + x_{i,j+1}(t) + x_{i,j-1}(t) \\ -4x_{i,j}(t)\end{aligned} \tag{5}$$

2.1 Conditions for propagation failure

We begin our study of propagation with the model problem shown in equations (1)-(3). We examine conditions for the exsitence of unique stable traveling solution of equations, or otherwise propagation failure of autowaves so that we can use the control of autowave based on the structural change of system equations. We assume that the dynamics of all cells on CNN is governed by a same differential equation, then the suffix ij for state variables in equations (1)-(3) are omitted if not necessary.

The goal of this section is to prove that equations (1)-(3) have propagation failure for sufficiently small coupling. We assume that the equations in (1)-(3) have the solution in the following form.

$$u(x,t) = u(x-ct), v(x,t) = v(x-ct), w(x,t) = w(x-ct) \tag{6}$$

$$z = x - ct \tag{7}$$

where the number c is independent of diuffusion coefficients D_u, D_v, D_w. By substituting the solution into equations(1)-(3), and we introduce the following intermediate variables.

$$U = \partial u/\partial z, V = \partial v/\partial z, W = \partial w/\partial z \tag{8}$$

Then, the equations (1)-(3) are represented as

$$u' = U, v' = V, w' = W \tag{9}$$

$$\begin{aligned}-cu' = D_u U' + fu(.), -cv' = D_v U' + f_v(.) \\ -cw' = D_w w' + f_w(.)\end{aligned} \tag{10}$$

Then, we assume that the system reaches a stable and equilibrium point $u_0, v_0, w_0, U_0, V_0, W_0$ for variables u, v, w, U, V, W in a sufficient large time. By applying the linearlization and approximation of equations at the equilibrium point, and representing the linearlized equations by using newly introduced variables $u_s, v_s, w_s, U_s, V_s, W_s$ such that

$$u = u_0 + u_s, v = v_0 + v_s, ..., W = W_0 + W_s \tag{11}$$

Then, we have a system of first-order differential equations described in $u_s, v_s, ..., W_s$. We denote the matrix A of coefficients for variables $u_s, v_s, w_s, U_s, V_s, W_s$ included in the system of first-order differential equations. We know that the solution of the first order differential equations is described as a linear combination of $\exp(\lambda_i)$ where $\lambda_i, i = 1, 2, ..., 6$ are the eigenvalues of the matrix A.

In case the autowaves are hindered to propagate on the CNN plane, the system of differential equations should have the solution which do not include the terms $\exp(\lambda_i)$ with negative λ_i.

3 Approximation of system dynamics by the GP

3.1 Applying the GP

The GP tree is interpreted as a coding of functional form of dynamics in CNN showing the growth from a initial structure. We assume that the dynamics $f_u(.), f_v(.), f_w(.)$ in equations(1)-(3) are not known, and should be estimated by using the observed time series for variables u, v, w. Then, we approximate the equations by using the GP procedure.

The GP is an extension of Genetic Algorithm (GA), but its elements consist of arithmetic expression and variables[12]-[16]. A tree structure corresponds to a system dynamics and a set of trees structure (population of individuals) consist of search space for approximation.

We use the prefix representation to describe the tree structure of functions, for example,

$$(6.43 \times y_1 - y_2) \times (y_3 - 3.54) \rightarrow \times - \times 6.43 y_1 y_2 - y_3 3.54 \tag{12}$$

The equation represented by using the prefix are interpreted based upon the stack operation. We begin to scan the prefix representation, and if we meet a set of operator and two terminals (operands) then we perform the calculation and push down the result into the stack again.

To ensure that the underlying GP trees always valid, the so-called stack count (denoted as *StackCount* in the paper) is useful [16]. The *StackCount* is the number of arguments it places on minus the number of arguments it takes off from the stack. The cumulative *StackCount* never becomes positive until we reach the end at which point the overall sum still needs to be 1.

By using the *StackCount* we can see which loci on the prefix expression is available to cut off the tree for the crossover operation, and we can validate whether the mutation operation is allowed. If final count is 1, then the prefix representation (tree) corresponds properly to a system equation. Otherwise, the tree structure is not relevant to represent the equation.

Usually, we calculate the root mean square error ($rmse$) between $x(t)$ and $\tilde{x}(t)$ where $\tilde{x}(t)$ is the prediction of $x(t)$ obtained by the individual, and use it as the fitness. By selecting a pair of individuals having higher fitness, the crossover operation is applied to generate new individuals.

Crossover operations

The chief genetic operator used in the GP is crossover. Contrary to the operation in GA, the crossover operation in GP is applied to restricted cases. Then, we can not choose arbitrary loci in the string of individuals and replace the parts of two tree structures.

To keep the crossover operation always producing syntactically and semantically valid programs, we look for the nodes which can be a subtree in the crossover operation and check for no violation. By using the *StackCount* already mentioned, we know the subtrees which are the candidate for the crossover operation. The basic rule is that any two loci on the two parents genomes can serve as crossover points as long as the ongoing *StackCount* just before those points is the same. The crossover operation creates new offsprings by exchanging sub-trees between two parents.

Mutation

The goal of the mutation operation is the reintroduction of some diversity in an population. Two types of mutation operation in GP is utilized to replace a part of the tree by another element.

(Global mutation :G-mutation)

Generate a individual I_s, and select a subtree which satisfies the consistency of prefix representation. Then, select at random a terminal in the individual, and replace the terminal by the subtree of the individual I_s.

(Local mutation:L-mutation)

Select at random a locus in a parse tree to which the mutation is applied, we replace the place by another value (a primitive function or a variable).

3.2 Optimizing the constants

Even though the structure of the equations describing the system dynamics is improved by the GP, but the constants included in the prefix representation are usually only swapped from one individual to another individual, and never changed from initial value. Besides the mutation operations, there is no way to optimize the constants. In previous works, we utilized the Genetic Algorithm (GA) as one procedure in the GP to optimize the constants in the individuals, but it is time consuming [9][10]. An alternative to optimizing constants is to try to dynamically adjust the value during the run.

In the paper, we optimize the constants by using conventional steepest descent algorithm to simplify the procedure [7][11]. However, the steepest descent algorithm for optimizing the constants is applied only once for each GP iteration. Because the individuals having higher fitness will remain in the pool for a long time, and it is expected that sufficient times of incremental change of the constants are applied to these individuals iteratively. On the other hand, it is not useful to optimize the constants in the individuals with lower fitness which are ultimately removed from the pool.

Define the difference between the observation of the time series $x(t)$ and the prediction $y(t, a)$ obtained by interpreting a certain individual as follows where the difference is accumulated for $t = 1, 2, ..., T$.

$$H(a) = \sum_{t=1}^{T} [y(t, a) - x(t)]^2 \tag{13}$$

where $a = (a_1, a_2, ..., a_m)$ are the constants included in the individual. The incremental value Δa to optimize the constants a are given by

$$\Delta a_i = -\alpha \partial H(a)/\partial a_i \tag{14}$$

where α is used to accelerate the convergence.

The partial derivative is obtained by interpreting the prefix representation. Then, in each GP operation, each individual is interpreted three times to evaluate $y(t, a)$, and Δa_i , and another $y(t, a)$ after the incremental change of a.

From many experimental data we can recognize that the problem arises from the convergence to the local minimum can be avoided in the tasks treated in the paper.

3.3 Algorithm of the GP

We iteratively perform the following steps until the termination criterion has been satisfied.

(Step 1)

Generate an initial population of random composition of possible functions and terminals for the problem at hand. The random tree must be syntactically correct program.

(Step 2)

Execute each individual (evaluation of system equation) in population by applying the optimization of the constants included in the individual. Then, assign it a fitness value giving partial credit for getting close to the correct output. Then, sort the individuals according to the fitness S_i.

(Step 3)

Select a pair of individuals chosen with a probability p_i based on the fitness. The probability p_i is defined for ith individual as follows.

$$p_i = (S_i - S_{min}) / \sum^{N} (S_i - S_{min}) \tag{15}$$

where S_{min} is the minimum value of S_i, and N is the population size.

(Step 4)

Then, create new individuals (offsprings) from the selected pair by genetically recombining randomly chosen parts of two existing individuals using the crossover operation applied at a randomly chosen crossover point. Iterate the procedure several times to replace individuals with lower fitness.

(Step 5)

If the result designation is obtained by the GP (the maximum value of the fitness become larger than the prescribed value), then terminate the algorithm, otherwise go to Step 2.

We can evaluate the approximation of chaotic dynamics generated by known system equations. As a result, the approximation error of system equations for the artificial chaotic time series generated by known 1-D and 2-D dynamics such as (Logistic map, Henon map, and Ushiki map) are about 1.0e-7 and are very small [9][10].

3.4 Control of propagation of autowaves

Our interest in the paper is to demonstrate a control scheme of traveling wave on CNN plane without changing the dynamics. In thinking about the chaotic behavior in CNN such as the autowaves and spatio-temporal chaos, conventional ideas are taming and controlling chaos by providing a weak control signal to induce the system follow a stability (fixed points and limit cycle). Our method is simple and easy to apply compared to conventional methods such as the OGY method [17].

The OGY method is well known control scheme by using the linearlization of a trajectory $x(t+1) = f(x(t), s(t))$ where $s(t)$ is a perturbation input for control [18]. Then, we have a linearlization at a certain fixed point x_f as

$$x(t+1) - x_f = A(x(t) - x_f) + bs(t) \tag{16}$$

where $A = D_x f(x_f, 0), b = D_s f(x_f, 0)$ and are able to be experimentally determined. We then change $s(t)$ slightly from zero to some value (determined by the eigenvalues and eigenvectors of the matrix A) so that the state moves to a stable manifold.

In previous works, we demonstrated that by comparing the result of control by the OGY method with our control scheme, it is revealed that the OGY method still effective if the noise is relatively small, but in the region of with higher level of noise the control fails almost always, while our method still provide in some extent an effective control. Furthermore, the time to complete the control is relatively large in the OGY control compared to the case using the control of our method.

In our control scheme, we assume the equations governing the autowaves is approximated and estimated by the GP. We only need to impose the small input to the system to achieve the desired propagation failure in autowaves.

Consider a non-linear dynamical system

$$dx(t)/dt = f(x(t)) + s(t) \tag{17}$$

where we assume that $x(t)$ is the multi-dimensional state vector, and $s(t) = (s_u(t), s_v(t), s_w(t))$ is the only available control (multi-dimensional) parameter which we allow to vary in a range.

We assume further that we are not far apart from the neighborhood of some steady state x_f (fixed point), which we want to stabilize by choosing an appropriate sequence of admissible input.

Off course, if we need to control the state to a limit cycle, then we replace the fixed point x_f by the trajectory of the limit cycle. We assume that the dynamical system $f(x(t))$ with $s(t) = 0$ is estimated by using the GP, and is denoted as $\hat{f}(x(t))$. Then, the control method is derived straightforward by using the approximation. Since we can obtain the estimated value $\hat{x}(t)$ for the next state, we impose the input $s(t)$ so that we lead the solution of equation

$$dx(t)/dt = \hat{f}(x(t)) + s(t) \tag{18}$$

to the fixed point x_f.

The method is similar to conventional feedback control for chaotic dynamics [6][19], but examined in descretized time points [7].

Since the system is assumed to be nonlinear, the application of linear input will succeed only in a neighborhood of around x_f. Due to the bound, we have to correct the parameters in the next iterations according to the same scheme where we hope to need one iteration of the new set less than the step before.

Since we see a dependency between the maximum parameter perturbation and the expected time to achieve control, there is a trade-off between the maximum allowed parameter changes and the expected time to achieve the target. Since the input $s(t)$ may not be too large, there exist some lower limit of expected time which may still be large.

4 Simulation Studies for Known dynamics

4.1 Approximation of equations

Here we presents examples of estimation and control of CNN under six model of maps listed in Table 1 by using the observation of autowaves. We give these examples since they may be applied to modeling the particular diuffusion processes, and the overview of equations are shown in Table 1 where the coefficients are represented by symbols.

Case 1 denotes the generalized Fisher equation ($D_u = 1$), and has the steady state $u = 1$ or $u = 0$. The equation for Case 2 is well known Fisher equation ($D_u = 1$), and has a constant value for the variable u in a steady state. Case 3 is the FitzHugh-Nagumo equation ($D_u = 1, D_v = 0$), and has a autowave in a pulsive form. Case 4 is a basic model treated in Reference [3] ($D_u = D_v = 0, D_w = 1$), and converges to a stable pattern in a steady state. Case 5 ($D_u = D_v = 0, D_w = 1$) and Case 6 ($D_u = D_v = D_w = 1$) are the CNN generating specific patterns such as the spiral wave [4][5].

Table 1-Examples of CNN systems (a_i mean constants)

name	$f_u(.) =$	$f_v(.) =$	$f_w(.) =$
Case 1	$-u(u-1)(u-a_1)$		
Case 2	$-u(u-1)$		
Case 3	$-(u^3/3-u)-v$	$a_1(u-a_2v)$	
Case 4	$a_1u^2/v-a_2u$	$a_3u^2-a_4v)$	
Case 5	$1/v^2-u$	$w/u-v$	$u-w$
Case 6	$u+v-a_1u^2-uv$	$-v+a_2w-uv$	$u-w$

The parameters for the GP in simulation studies are given as follows.
operators:$||, +, -, \times$

population size=1000
maximum length of array for $u, v, w = 90$
data length of time series=20

The solution for each individual is obtained by the Runge-Kutta method having following parameters.
step size:$h = 0.01$
maximum number of steps:100

Table 2 shows the final value of $n - rmse$ (defined as the $rmse$ divided by the standard deviation of the time series) for the evaluation approximation of system equations.

$$n - rmse = [\sum(x(t) - \tilde{x}(t))^2]^{1/2}/N_s\sigma \tag{19}$$

Table 2 also shows the number of iterations denoted as N_p at which the point we have almost the same functional form of equations as the ultimately obtainable results.

Concerning the iteration of algorithm to terminate the approximation, about after 200 generations of GP we have good approximation (estimation) for CNN dynamics.

Table 2-Approximation error

name	n-rmse	N_p	name	n-rmse	N_p
Case 1	0.015	134	Case 2	0.021	153
Case 3	0.018	184	Case 4	0.021	180
Case 5	0.028	201	Case 6	0.025	230

4.2 Applications of control

Then we show several examples for the control of autowaves for CNN where the dynamics of the original CNN is known but these system equations are estimated by the GP.

We again use examples in Table 1. However, since we need to show the effectiveness of the control for the chaotic dynamics of CNN, we select only two example (Case 3 and 4) from Table 1, and add another two examples of CNN including the Chua's circuits. These two examples are denoted as CNN-1 and CNN-3 , and have following forms of dynamics [4][6].

$$du/dt = a_1u + a_2v + a_3(|u+1| - |u-1|) + a_4 \tag{20}$$

$$dv/dt = u - v + w, dw/dt = a_5v \tag{21}$$

$a_1 = 9, a_2 = -18/7, a_3 = 27/14, a_4 = 9/14, a_5 = -30D_u = 0.6, D_v, D_w = 0$ for CNN-1
$a_1 = 9, a_2 = 15.426, a_3 = 8.356, a_4 = 0, a_5 = -19D_u = 0.5, D_v = 6.5, D_w = 0$ for CNN-3

Fig.1 and 2 show examples of control of CNN-1 and CNN-3. In the CNN-1, the initial condition of all cells are set to one of the stable states, and then controlled to another stable state by imposing the input. In the CNN-3, the initial states of cells are given by using the uniformly distributed random number, and controlled to a stable limit cycle. The left diagram of Fig.1 and 2 shows the transition of the state $u(t)$ in a certain cell. In the following, the solid line denotes the controlled state, and the dashed line means the original (uncontrolled) state. The right side of Fig.1 denotes the corresponding input signal $s(t)$ to control the chaotic behavior of the CNN-1. As is seen from Fig.1 , the state of CNN-1 is moved to a fixed point after imposing a input $s_u(t)$ at a time point.

In a similar manner, the process of control of the CNN-3 is depicted in Fig.2, where the left diagram shows the controlled state of $u(t)$ in a certain cell, and the right signal means the corresponding input. The state $u(t)$ is moved to a limit cycle, and we need only a small input at a time point to complete the control.

Table 3. summarizes the mean value of the absolute value of the input signal denoted such as In_u, In_v, In_w for variables u, v, w, and the time steps N_s necessary for controlling the state to the steady state.

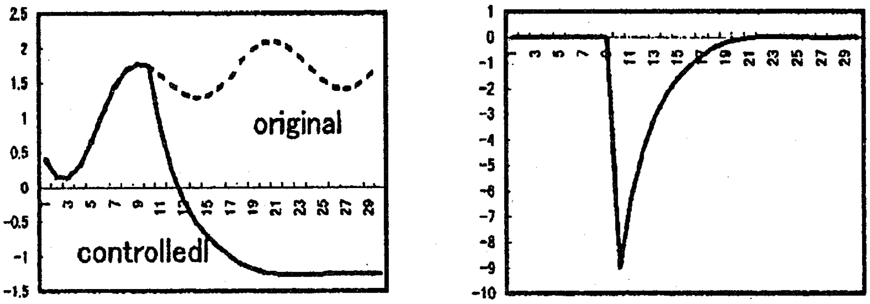

Fig.1-Control of CNN-1 (left: u,right: s_u)

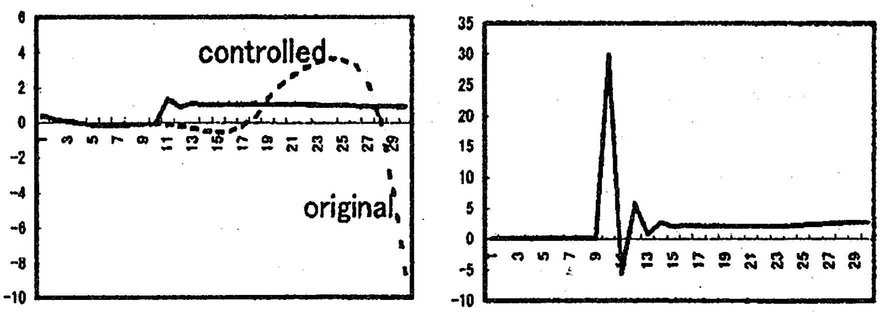

Fig.2-Control of CNN-3 (left:u,right:s_u)

Table 3-Result for control of chaos

name	N_s	In_u	In_v	In_w
Case 3	5	0.66	0.01	
Case 4	5	0.52	0.02	
Case 7	5	1.23	0.01	0.01
Case 8	5	2.15	0.01	0.02

5 Applications for real world data

5.1 Estimated equations

To illustrate the applicability of the method of the paper for modeling and control of diffusion processes in real world data, we treat following three examples of diffusion processes available through published researches on the diffusion of insect and epidemic [20]. For convenience, only the sketch of examples are summarized as follows.

(Case A) one-dimensional data

The data shows the diffusion process of harmful insects in Japan. At the beginning of the diffusion the number of insects denoted as u grows rapidly, but in a elaps of time the number reaches a level which seems to be a saturation level acceptable in the environment.

(Case B) two-dimensional data

The data shows the increase of patent corrupted by a kind of epidemic (denoted as the variable u), and the number of unaffected people(denoted as the variable v) . The variable u behaves like a sinusoidal curve, namely, it increases form zero to a large value at the beginning, but after a elaps of time, it decreases to a small level close to zero. At the same time, the variable v which is constant at the beginning decreases gradually, and reaches to a level corresponding to the cease of epedemic.

(Case C) three-dimensional data

In these case, another variable w is added to Case B representing the number of epidemic carrier who is not yet ill but having the otential to become ill.

For these examples, the details of the data are omitted here for the sake of limited pages.

Table 4 shows the result of estimated equations of dynamics for the examples. In the table, a part of the coefficients are represented by symbols which are used later for the discussion of the structural changes of system equations, and their initial value are one. Table 5 shows the result of approximation (estimation) error of equations using the approximation error denoted by $n - rmse$.

As is seen from the result, the estimated equations are not so complicated, and usable to analyze the mechanism of the diffusion processes. The time until we get attainable level of approximation is relatively longer than the cases for simulation studies where the time series is generated by known dynamics (deterministic function). The main reason for relatively poor approximation compared to the cases for known dynamics is that for real world data it is not insured that the system does not necessarily follows a deterministic process, and sometime behaves like a stochastic process.

Table 4-Estimated system equations

name	equations
Case A	$f_u(.) = c_1 \times 2u - c_2 u^2 + c_3 \lvert c_4 \times 2u \rvert, D_u = 1.2$
Case B	$f_u(.) = c_1 u(c_5 u - c_2 \times 2 \lvert u \rvert u + c_3 v) - c_4 u + c_6$
	$f_v(.) = c_7(-2uv(c_9 u + 2v)) + c_8$
	$D_u = 1.3, D_v = 0.9$
Case C	$f_u(.) = c_1 \lvert 2u - c_2 \times 0.97 + (-1.42) - c_3 v - w \rvert$
	$-c_5 u - c_4 w$
	$f_v(.) = 4.01 - c_6 \times 0.5 \lvert c_7 w + \lvert v + \lvert u + v \rvert + w \rvert$
	$+ \lvert u + v \rvert + \lvert u + 1.85 \rvert + v \rvert + 0.023 \rvert + c_8$
	$f_w(.) = c_9 \lvert u \rvert - 2w + v$
	$D_u = 1.1, D_v = 1.1, D_w = 0.9$

Table 5-Approximation error

name	n-rmse	N_p
Case A	0.08	234
Case B	0.12	273
Case C	0.15	344

5.2 Control of autowaves

The goal of this section is to present a control (propagation failure) of autowaves of CNN whose dynamics are approximated in previous section. Since we are sure of the estimation for f_u, f_v, f_w

in equations (8)-(9), we can achieve the control by imposing appropriate small input.

To emphasize the effect of control based on the GP, we assume that the control input is imposed only in a certain period of time allowed for control. In discretized time points, we begin with the control at around $t = 3$, and then stop imposing the input at around $t = 8$.

Figures 3, 4 and 5 show the result of control by depicting the controlled state variables and related input. As described in previous examples, the solid lines mean the controlled states, and dashed lines stand for the uncontrolled (original) states. Table 6 show the summary of the result of control. In Table 4, N_s mean the length of time during the control, and In_u, In_v, In_w mean the average of absolute value of input for the variables u, v, w.

As is seen from the result, the autowaves in CNN are almost successfully hindered to propagate by the control method The fact lead us the possibility that by estimating the dynamics of autowaves modeled by the CNN, the diffusion of harmful insects or the epidemic could be prevented to propagate if we begin to impose appropriate input in early stage.

Table 6-Result for control of chaos

name	N_s	In_u	In_v	In_w
Case A	6	0.30		
Case B	6	0.013	0.05	
Case C	6	0.05	0.02	0.02

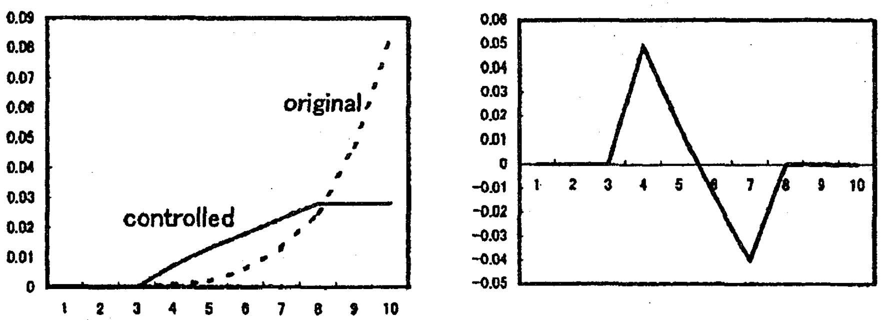

Fig-3. Control of Case A (left:u, right:u_s)

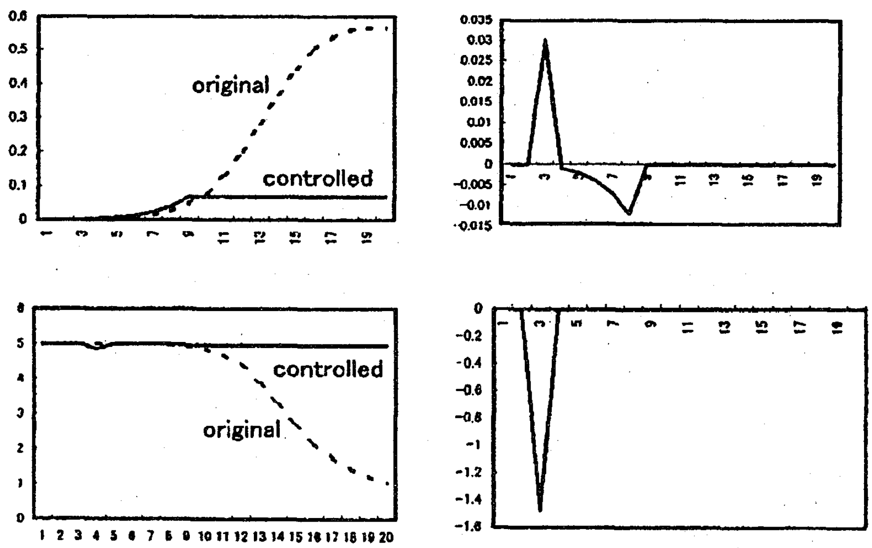

Fig-4.Control of Case B (top left:u, top right:u_s, down left:v, down right:v_s)

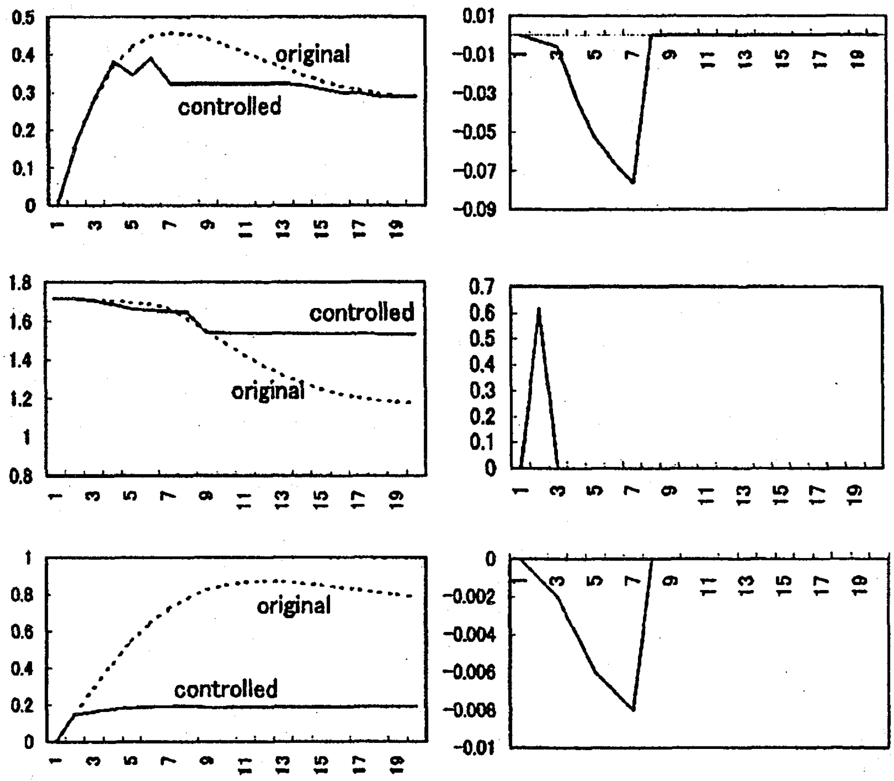

Fig-5.Control of Case C (top left:u, top right:s_u, middle left:v,middle right:s_v,down left:w, down right:s_w)

5.3 Control by structural changes of dynamics

Propagation failure of autowaves is realizable also by selecting sufficiently small diffusion coefficients. We show an alternative to control autowaves by estimating the smallest value of diffusion coefficients D_u, D_v, D_w based on the eigenvalues of coefficient for variables obtained for the linearlized equations of dynamics at the equilibrium points.

In a similar manner, no traveling wave solution is obtained if the dynamics of CNN has a particular form of equations. Actually, we have the estimation for the dynamics of CNN as in Table 4, we can choose an alternative to hinder autowaves by changing the newly introduces coefficient in equations such as $c_1, c_2, \ldots$.

Table 7 show the smallest value of diffusion coefficients to hinder autowaves in Case A, B, C. In the table, "independent" means the coefficient mas no effect on the diffusion. Also, "condition" in the table means the restriction on two coefficients to represent simply the condition.

Table 8 depicts the range of coefficients in Table 4 under which the condition autowaves are hindered to propagate on CNN (only effective changes are shown). The result is useful to discuss the possibility of structural changes of dynamics on CNN if the impose of control input is not available.

Table 7-Smallest value of diffusion coefficients

name	D_u	D_v	D_w	condition
Case A	0.07	-	-	-
Case B	0.15	independent	-	
Case C	0.5	0.5	independent	$D_u + D_v \leq 0.85$

Table 8-Range of constants for control

name	
Case A	$c_1 = 2.5, c_2 = 2.5c_3 = 2.5, c_4 = 2.5$
Case B	$c_3 = 4.5$
Case C	$c_2 = 9.0$

6 Conclusion

This paper showed modeling and control of diffusion processes by using the CNN approaches based on the approximation by the GP. Previous works were extended to treat the real world data, and the structural changes of system equations are discussed . The condition for the propagation failure of the autowave was discussed based on the estimated equations by using the eigenvalues of coefficients of linearlized equations at the equilibrium point. Then, we used the control method to stabilize the chaotic dynamics in the CNN. Since the system equations are estimated, we only need to change the input so that the system moves to the stable region. Simulation studies showed system dynamics of known CNNs were estimated by using the observation of state variables, and the prediction error was discussed. As an application, the method in the paper was applied to the control of autowaves for real world data observed in the propagation of harmful insects and epidemic. As a result, examples showed effectiveness of the method to control diffusion processes by imposing the input so that the system moves to stable region.

The problems remained to be solved include the extension of the method to various real time series, and further works will be continuously done by the authors.

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