Convective cell dynamics in universal drift wave turbulence

斉藤, 剛
九州大学大学院総合理工学府

伊藤, 早苗
九州大学応用力学研究所

矢木, 雅敏
九州大学応用力学研究所

https://doi.org/10.15017/3549


バージョン:

権利関係:
Convective cell dynamics in universal drift wave turbulence

Tsuyoshi SAITO *1, Sanae-I. ITOH *2, Masatoshi YAGI*2;
E-mail of corresponding author: tsuyoshi@riam.kyushu-u.ac.jp
(Received July 30, 2004)

Abstract

The interaction between the universal drift wave and the convective cell in the shearless slab geometry is investigated using a kinetic-fluid model based on nonlinear gyro-kinetic equations. It is found that the convective cell is nonlinearly excited in a way, which is similar to the zonal flow generation ofITG turbulence. On the other hand, if we introduce the large ion-ion collision effect, the zonal flow is damped and instead the streamer and finite ky modes are excited in the convective cell mode. As the result, the flux is larger than the one without collision. The forward cascade is observed in kx space and the forward/inverse cascade is observed in ky space in the time evolution of energy spectrum.

Key words: Universal drift wave, Convective cell, Zonal flow, Streamer, Nonlinear simulation, The effect of ion-ion collision, Kinetic-fluid model

1. Introduction

Research on anomalous transport in high temperature plasmas is important issue in thermo-nuclear fusion research. It is considered that the drift wave contributes to anomalous transport. The ion temperature gradient (ITG) driven turbulence is caused by the drift wave destabilized by the ion temperature gradient. Recently, there has been a considerable progress in the gyro-kinetic particle-in-cell simulation of ITG driven turbulence 1, 2). These simulations have demonstrated that zonal flows 3) play a crucial role in regulating drift wave turbulence and the level of the anomalous ion transport. The theoretical explanation for the generation of zonal flow has been proposed based on a modulational instability 4, 5). Later, a four-mode model of zonal flow growth in toroidal geometry has been developed 6). While works 4, 5, 6) have provided a firm theoretical foundation on this subject, the direct comparisons to the gyro-kinetic simulation results have been only possible in certain idealized cases. Simplified assumptions made for analytic theory are not always satisfied in simulations of turbulence. It is worth performing the simulations in a simplified geometry and comparing the results with those from the theory.

In this thesis, a kinetic-fluid model 7) in a shearless slab geometry is used to study the problem. This model is an extension of the model proposed by Smolyakov 8) and Hinton 9), which include the nonlinear interaction between z-axisymmetric potentials in zonal flows and z-nonaxisymmetric potentials in drift waves 9). The adiabatic electron is assumed, therefore, no unstable drift wave is included. They assume the marginally stable drift wave with \( \omega_i = 0 \). Our model can describe the universal drift wave turbulence and nonlinear excitation of convective cells (CC) 11), in which zonal flows and streamers by drift waves are included. The schematic diagram of zonal flow and streamer is described in Fig.1. The zonal flow and the streamer are one of Fourier components which constitute the convective cell. In chapter 2, we review simulation and theory of drift wave turbulence and convective cell dynamics in later 70's, and the recent theory of ITG and zonal flow turbulence dynamics. In chapter 3, we explain our model equations. In chapter 4, simulation results are discussed. Finally, discussion and summary are given in chapter 5.

2. Review

2.1 Convective Cell

2.1.1 Numerical Simulation of Drift Wave Turbulence

In later 70's, three dimensional particles simulations were performed on the collisionless drift instabilities in a cylindrical geometry 11). The simulation
Fig. 1 Schematic diagram of zonal flow and streamer. Red lines express maximum amplitude, blue lines express minimum amplitude of the wave

...model used was a straight cylinder in an uniform external magnetic field \( B_0 \) in the \( z \) direction with the length \( L_z/\rho_i = 640, \rho_i = (T_i/m_i)^{1/2}/\Omega_i \), being the ion gyroradius, where \( m_i \) is the ion mass, \( T_i \) is the ion temperature and \( \Omega_i \) is the ion cyclotron frequency. For numerical computation with its physical length \( L_z/\rho_i = 32, \) a \( 64 \times 64 (L_z^2) \) spatial grid was used. The plasma was assumed to be periodic in \( z \) direction. Initial conditions of ions and electrons had Maxwellian velocity distributions with \( T_e/T_i = 4, \) where \( T_e \) is the electron temperature.

Initial plasma density profile was taken to be \( n_e(r) = n_i(r) = n_0 \exp(-4r^2/a^2) \) with the average density given by \( \Omega_e/\Omega_{pe} = 5, m_i/m_e = 100, a = L_z/2, \) where \( \Omega_e \) is the electron cyclotron frequency, \( \omega_{pe} \) is the electron plasma frequency and \( a \) is a half width of density profile; and there was no initial temperature gradient. Seven Fourier modes \( n = 0, \pm 1, \pm 2, \pm 3, \) where \( n \) is the Fourier mode number in \( z \) direction and the mode with \( n = 0 (\omega = 0) \) is defined as convective-cell mode, were kept in \( z \) direction with the wave number \( k_z = 2\pi n/L_z. \)

Linear analysis predicts that the collisionless drift instability (universal mode) is strongly unstable against \( n = \pm 1 \) perturbations, which becomes stable with increasing \( |n| \) due to the onset of ion Landau damping.

Fig. 2 shows the particle diffusion, heat transfer, electron velocity distribution, and radial-mode structure for the \( E_0(k_z) \) associated with the instability of the electrostatic potential \( \phi_m, \) where \( m \) is the azimuthal mode number with \( k_0 = m/r. \) While ions and electrons diffuse more or less together in the early stage, large charge separation built up at a later stage. This is because of the difference of radial \( cE_0/B \) drifts of ions and electrons, where \( E_0 \) is the fluctuating azimuthal electric field associated with the drift wave. The observed anomalous particle diffusion was mainly due to drift instabilities at the early stage. At the later stage, it was due to nonlinearly excited convective cells, which enhance the diffusion even when the drift instability is quenched.

The electron parallel temperature \( T_{ei} \) rapidly decreases for \( r/a > 0.5 \) because of the inverse Landau damping. It increases for \( r/a < 0.5 \) because of the absorption of wave energy. The electron parallel velocity distribution is steepened because of inverse Landau damping. Ion perpendicular temperature \( T_{ii} \) increases due to the convective cells. Ion parallel temperature \( T_{ii} \) changes little because the contribution from the ion Landau damping is weak. A typical drift-wave radial mode-structure of \( [\phi_m(r)/T_s] \) with \( m/n = 3/1 \) shows a peak near \( r/a = 0.5 \) at the early stage. After reaching the maximum amplitude of 10% at \( \omega_{pe}t = 1040, \) it begins to decrease as a result of nonlinear excitation of convective cells (\( n = 0, \omega = 0) \).

Fig. 3 shows more detailed diagnostic of the instability: spectral distributions \( |E^2(m)| \) and \( |E^2(n)|, \) and power spectrum \( P_{mn}(\omega, r) \) where \( |E^2(m)| = \sum_n |\phi_m(r)/T_s|^2, |E^2(n)| = \sum_m |\phi_m(r)/T_s|^2 \) and \( P_{mn}(\omega, r) = |\phi_m(r)/T_s|^2 \) and \( E^2(m) \) indicates typical spectral profile for drift turbulence as seen in the experiments; it peaks around \( k_0 \rho_i = 0.3 \sim 0.6, \) corresponding to the \( m = 2, 3 \) and 4 modes for this case. The amplitude decreases rapidly for \( k_0 \rho_i \geq 1 (m \geq 8) \). The shape of the spectral distribution little changes from early stage. The shift of the peak of the spectrum toward low \( m \) numbers was seen. The shift of peak was due to the nonlinear excitation of convective cells which tend to coalesce into large cells. \( |E^2(n)|, \) on the other hand, shows a drastic change when the instability develops into a nonlinear regime. We see that at earlier times the spectrum was peaked at \( n = 1 (k_z = 2\pi/L_z) \) and decreases rapidly with increasing \( n. \) The \( n = 0 \) mode, which may be called 'convective-cell mode' \( (k_z = 0, \omega = 0) \), also grows as the drift instability grows; and eventually they dominate over the drift instability. Note that this state is not the ambipolar field \( (m = n = 0) \) (it is called zonal flow in the recent terminology). The excitation of convective cells is clearly seen in the power-spectrum plots \( P_{mn}(\omega) \) at \( r/a = 0.5, \) where we see both drift modes \( (n \neq 0, \omega \neq 0) \) and convective cells \( (n = 0, \omega = 0) \).

For the case of \( m = 3 \) mode, the power spectrum indicates that the amplitude of drift mode \( (n = 1) \) was much larger than that of the convective cells \( (n = 0) \), while for \( m = 1 \) case, convective cell components are much stronger than the drift mode ones.

2.1.2 Nonlinear Mechanism for Excitation of Convective Cell

In this section, the theoretical work for the excitation of CC is reviewed. It is based on the decay instability.

A system of equations describing the nonlinear interaction of the drift waves with the convective mode are
Fig. 2 Time variation of (a) ion and electron density profiles $n_i(r)$ and $n_e(r)$ in terms of number of particles summed over a given volume; (b) electron parallel temperature $T_{ee}(r)$, ion parallel temperature $T_{ii}(r)$, and ion perpendicular temperature $T_{li}(r)$ in terms of thermal electron energy $m_e v_e^2/2 = 6.2$; (c) electron velocity distribution; and (d) radial-mode structures for the $(m,n) = (3,1)$ mode.

Fig. 3 Time variation of (a) spectral distribution $|E^z(m)|$ and spectral distribution $|E^y(n)|$. Power spectra for (c) the $(m,n) = (1,1)$ and $(1,0)$ modes indicate two peaks near $\omega \approx 0$ and $\omega^*$. written by [13]

\[
\frac{\partial n}{\partial t} + \frac{C}{H} \left( \frac{\partial \phi}{\partial x} \frac{\partial n}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial n}{\partial x} \right) - \frac{C}{H} \frac{n_0}{\omega_{HI}} \left( \frac{\partial \phi}{\partial t} - \mu \Delta \right) U
\]

\[
- \frac{C^2}{H^2} \frac{1}{\omega_{HI}} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{\partial \phi}{\partial y} \left( \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial x} \right) \frac{\partial U}{\partial x}
\]

\[
+ \frac{\partial \phi}{\partial y} \left( \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial x} \right) \frac{\partial U}{\partial y} = 0
\] (1)

Fig. 4 Comparison of electron density diffusion with and without convective cells ($k_l = 0$). Note that at $\omega_{pe,l} = 1960$, convective cells have larger amplitudes than the drift waves when the convective cells are not detected.

\[
\frac{\partial n}{\partial t} + \frac{C}{H} \left( \frac{\partial \phi}{\partial x} \frac{\partial n}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial n}{\partial x} \right) + n_0 \frac{\partial U_{IC}^c}{\partial x} = 0
\] (2)

Here $\mu = \frac{3}{16} \frac{T_e}{n_e m_e}$ is the ion viscosity, $U = \phi + (T_i/e) \ln n$ is the generalized potential ($\phi$ is the electrostatic potential), and $\Delta = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$. The last term in eq.(2) describes the density perturbation due to the longitudinal electron motion and is important only for the drift mode, where $H (B = \mu H)$ is the magnetic intensity and $\omega_{HI}$ is the cyclotron frequency.

All quantities corresponding to the collective particle motions were separated into two parts, describing the drift and convective cell modes, respectively as:

\[
\phi = \phi^c(t,x,y) + \phi^d(t,x)
\]

\[
\delta n = \delta n^c(t,x,y) + \delta n^d(t,x)
\] (3)

where superfix c and d denote the convective cell and drift wave, respectively. For the existence of the drift wave, the condition $|\omega^*| \ll k_B T_e$ holds, so the Boltzmann distribution was always established because of the fast electron motion in the longitudinal direction;

\[
\phi^d = \frac{T_e}{c} \ln \left[ 1 + \frac{\delta n^d(x)}{n_0(x)} + \delta n^c \right] (4)
\]

Using eq.(4) and assuming that $\delta n^c \ll n_0$ and $|\frac{1}{2} \frac{d}{dx} \ln n_0| \ll 1$, the equations for the density and the potential variation in the drift wave are obtained from eq.(1). For the slow mode of plasma motions which are homogeneous along the magnetic field, we obtain the following system of equations from eqs.(1) and (2);[13]

\[
\frac{\partial \delta n^c}{\partial t} - \frac{C}{H} \frac{\partial \phi^c}{\partial x} \frac{\partial \delta n^c}{\partial y} = \frac{C}{H} \left( \frac{\partial \phi^d}{\partial y} \frac{\partial \delta n^c}{\partial x} - \frac{\partial \phi^d}{\partial x} \frac{\partial \delta n^c}{\partial y} \right)
\] (5)
where \( U^e = \phi^e + (T_i/e)(\delta n^e/n_0) \) and \( D_B \) is the Bohm diffusion coefficient. The angle brackets on the right hand side of eq.(6) denote an averaging over a spatial interval \( \sim 1/k_x \). The nonlinear terms retained on the left hand side of eq.(6) are responsible for the inherent nonlinearity of the \( k_x = 0 \) mode and describe the energy pumping between different \( k_z \) scales. The terms in the right hand side of eq.(6) are responsible for the excitation of \( k_x = 0 \) mode in the nonlinear interaction of drift waves.

Assuming that the contribution to anomalous plasma transport is due to slower mode of the convective cells, the analysis is restricted below to this mode, and assumes \( \omega^d < \omega, \delta n^d \rightarrow 0 \). Using the latter condition and omitting those terms which are responsible for the inherent nonlinearity of the drift waves and convective mode, the following system of equations, which describes the interaction of the drift waves and convective cells, is finally derived [13].

\[
\frac{\partial \delta n^d}{\partial t} - D_B \frac{d \ln n_0}{d x} \frac{\partial \delta n^d}{\partial y} = - D_B \left( \frac{\partial \delta n^d}{\partial y} \frac{\partial \delta n^e}{\partial x} - \frac{\partial \delta n^d}{\partial x} \frac{\partial \delta n^e}{\partial y} \right) \Delta \delta n^d
\]

(7)

\[
\frac{\partial \delta n^e}{\partial t} - \mu \Delta \delta n^e + D_B \frac{d \ln n_0}{d x} \frac{\partial \delta n^e}{\partial y} - \frac{\partial^2 \delta n^e}{\partial y^2} \Delta \delta n^d
\]

(8)

The dispersion relation of the drift waves in eq.(7)

\[
\omega^d = -k_y D_B \frac{d \ln n_0}{d x} \frac{1}{1 + k_y^2 \rho_i^2}
\]

(9)

follows from the general equation for the drift frequency [12]:

\[
\omega^d = -k_y D_B \frac{d \ln n_0}{d x} \frac{\Gamma(\xi)}{1 + \frac{k_y^2}{\rho_i^2} (1 - \Gamma(\xi))}
\]

(10)

in the limit of \( T_i \ll T_e, k_x \rho_i (T_e/T_i)^{1/2} \ll 1 \), where \( \Gamma(\xi) = e^{-\xi} I_0(\xi) \) and \( \xi = k_y^2 \rho_i^2 T_e/T_i \).

In the short-wave length part of the spectrum, \( k_y \rho_i \sim 1 \), we can always distinguish the waves with identical values of \( \omega \) and \( k_z \) (and different values of \( k_x \) and \( k_y \)), which interact and give rise to convective cells (Fig.5). This is described in the right side of eq.(8) since an averaging over the spatial interval \( \sim k_x^{-1} \) and over the first time \( (\omega^d)^{-1} \) is taken.

A simplest mechanism for the nonlinear generation of convective motions is known to be the decay instability of a monochromatic drift wave. Let us examine the decay instability of a mode which is localized along the \( x \) direction

\[
\delta n^d = \delta n_0(x) \exp(ik_y y + ik_z z - \omega_0^d t) + \delta n_1(x) \exp(ik_y y + ik_z z - \omega_1^d t - i\omega t) + c.c. 
\]

(11)

\[
\delta n^e = \delta n_2(x) \exp(ik_y y - i\omega t) + c.c.
\]

(12)

where the subscript "0" refers to the fundamental wave, "1" refers to the test drift wave, and \( k_2 = k_1 - k_0 \). Substituting this equation into eqs.(7) and (8) the equation for \( \delta n_2 \) is obtained as

\[
\frac{d^2 \delta n_2}{d x^2} + \left[ \frac{\omega^d + i\omega k_y^2}{D_B} \frac{T_e}{T_i} \frac{n_0^2}{\delta n_0^2} (k_y^2 - k_z^2) \right] \delta n_2
\]

\[
+ \frac{k_y^2 - k_z^2}{k_y^2 - k_z^2} \frac{1}{\delta n_0^2} \left( \frac{d \delta n_0}{d x} \right)^2 \frac{k_y^2 - k_z^2}{k_y^2 - k_z^2} \ln \delta n_0 \delta n_2 = 0
\]

(13)

Multiplying eq.(13) by \( \delta n_2 \) and integrating it over the plasma slab, \(-a \leq x \leq a \) and assuming that the spatial region to which the convective mode is confined is much smaller than the drift wave region, \( \Delta^e \ll \Delta^d \), we obtain the following approximate equation for the instability growth rate

\[
\gamma^d = D_B k_y \delta n_0 \frac{\Gamma(\xi)}{1 + \frac{k_y^2}{\rho_i^2} (1 - \Gamma(\xi))}
\]

(14)

where \( k_x^2 = \pi/\Delta^e \). It is found that the characteristic time of the convective mode growth in numerical simulations agrees with that for the decay instability of drift waves.

The mechanism for the nonlinear generation of convective cells studied here is extremely important to magnetic plasma confinement, since it is obvious that the slow plasma motions in these cells can substantially raise the diffusion confinement, as demonstrated in the numerical simulations [11].

Fig. 5 Dispersion of drift waves in a nonisothermal \( (T_e > T_i) \) plasma. \( \omega_0 = -D_B \frac{d \ln n_0}{d x} / 2\rho_i, \omega_2 = \sqrt{\frac{2 \omega_1}{2 \omega_1^2}} \)

---

16  Tsuyoshi Saito, et al. : Convective cell dynamics in universal drift wave turbulence

The dispersion relation of the drift waves in eq.(7),

\[
\omega^d = -k_y D_B \frac{d \ln n_0}{d x} \frac{1}{1 + k_y^2 \rho_i^2}
\]

(9)

follows from the general equation for the drift frequency [12]:

\[
\omega^d = -k_y D_B \frac{d \ln n_0}{d x} \frac{\Gamma(\xi)}{1 + \frac{k_y^2}{\rho_i^2} (1 - \Gamma(\xi))}
\]

(10)

in the limit of \( T_i \ll T_e, k_x \rho_i (T_e/T_i)^{1/2} \ll 1 \), where \( \Gamma(\xi) = e^{-\xi} I_0(\xi) \) and \( \xi = k_y^2 \rho_i^2 T_e/T_i \).

In the short-wave length part of the spectrum, \( k_y \rho_i \sim 1 \), we can always distinguish the waves with identical values of \( \omega \) and \( k_z \) (and different values of \( k_x \) and \( k_y \)), which interact and give rise to convective cells (Fig.5). This is described in the right side of eq.(8) since an averaging over the spatial interval \( \sim k_x^{-1} \) and over the first time \( (\omega^d)^{-1} \) is taken.

A simplest mechanism for the nonlinear generation of convective motions is known to be the decay instability of a monochromatic drift wave. Let us examine the decay instability of a mode which is localized along the \( x \) direction

\[
\delta n^d = \delta n_0(x) \exp(ik_y y + ik_z z - \omega_0^d t) + \delta n_1(x) \exp(ik_y y + ik_z z - \omega_1^d t - i\omega t) + c.c. 
\]

(11)

\[
\delta n^e = \delta n_2(x) \exp(ik_y y - i\omega t) + c.c.
\]

(12)

where the subscript "0" refers to the fundamental wave, "1" refers to the test drift wave, and \( k_2 = k_1 - k_0 \). Substituting this equation into eqs.(7) and (8) the equation for \( \delta n_2 \) is obtained as

\[
\frac{d^2 \delta n_2}{d x^2} + \left[ \frac{\omega(\omega + i\omega k_y^2)}{D_B} \frac{T_e}{T_i} \frac{n_0^2}{\delta n_0^2} (k_y^2 - k_z^2) \right] \delta n_2
\]

\[
+ \frac{k_y^2 - k_z^2}{k_y^2 - k_z^2} \frac{1}{\delta n_0^2} \left( \frac{d \delta n_0}{d x} \right)^2 \frac{k_y^2 - k_z^2}{k_y^2 - k_z^2} \ln \delta n_0 \delta n_2 = 0
\]

(13)

Multiplying eq.(13) by \( \delta n_2 \) and integrating it over the plasma slab, \(-a \leq x \leq a \) and assuming that the spatial region to which the convective mode is confined is much smaller than the drift wave region, \( \Delta^e \ll \Delta^d \), we obtain the following approximate equation for the instability growth rate

\[
\gamma^d = D_B k_y \delta n_0 \frac{\Gamma(\xi)}{1 + \frac{k_y^2}{\rho_i^2} (1 - \Gamma(\xi))}
\]

(14)

where \( k_x^2 = \pi/\Delta^e \). It is found that the characteristic time of the convective mode growth in numerical simulations agrees with that for the decay instability of drift waves.

The mechanism for the nonlinear generation of convective cells studied here is extremely important to magnetic plasma confinement, since it is obvious that the slow plasma motions in these cells can substantially raise the diffusion confinement, as demonstrated in the numerical simulations [11].

---

16  Tsuyoshi Saito, et al. : Convective cell dynamics in universal drift wave turbulence
2.2 Dynamics of Zonal Flows and Drift Wave Turbulence

In this section, the recent theoretical works on the zonal flow generation in ITG turbulence is reviewed. Zonal flows\(^{13}\) are poloidally and toroidally symmetric \((k_p = k_z = 0)\), zero-frequency \((\omega = 0)\) vortex modes with finite radial scale \((k_r \text{ finite})\), and thus constitute a limiting case of the more general notion of a "convective cell"\(^{14}\). Since zonal flows are azimuthally symmetric, they are unable to directly tap expansion free energy stored in radial gradients and are thus excited exclusively via nonlinear process, such as "inverse cascade" of drift wave turbulence. Zonal flows are of great significance to confinement physics, since they are, effectively, sheared \(E \times B\) flow layers which strain and distort the drift waves they co-exist with\(^{15}\).

Zonal flow shearing is the principle saturation mechanism operating in drift wave turbulence. Generic drift (or drift-ITG) turbulence may be modeled as a self-regulating, two component system, consisting of:

a) drift waves (with \(k_p \neq 0\)), which cause anomalous transport, and for which \(\tilde{n}/n \sim \dot{\phi}/T\).

b) zonal flows with \(k_p = 0\), for which \(\tilde{n}/n \sim (k_r^2, k_z^2)\dot{\phi}/T\), which share available gradient free energy. The confinement regime quality is thus characterized by the branching ratio of zonal flow and drift wave energy.

2.2.1 Nonlinear Dynamics of Zonal Flows

The equation for the zonal flow intensity \(U = |\dot{\phi}|^2\) is modeled as

\[
\frac{\partial}{\partial t} U + \gamma_d U = |\text{Growth}| U + |\text{Noise}|.
\]

Here \(\gamma_d\) refers to the collisional damping, \(|\text{Growth}| U\) refers to amplification due to drift-wave coupling and \(|\text{Noise}|\) refers to incoherent emission of drift wave energy into the zonal flow. Since the drift wave gas must maintain a divergence-free radial current (composed of polarization and transport-induced currents) and since the zonal shear flows modulate the transport-induced current, it follows that zonal flow stability is determined by:

\[
\rho^2 \frac{\partial}{\partial t} \left( \frac{\partial^2 \phi}{\partial \nu^2} \right) + \frac{\partial}{\partial \nu} \left[ \delta \langle J_r \rangle / \delta \phi \right] = 0
\]

Here \(\phi\) is the zonal flow potential, \(\langle J_r \rangle\) is the transport-induced radial current and \(\rho\) is the polarization screening length. Since the radial current is simply the difference of electron and ion radial flux, it follows that the frequency \(\Omega_q\) of a zonal flow mode with radial wavenumber \(q\) is

\[
\Omega_q = \left( q^2 \rho^2 \right)^{-1} (q \delta \Gamma_- / \delta \phi_q)
\]

Here \(\Gamma_-\) is the relative flux. \(\Gamma_-\) is very detail-sensitive, since it involves the dissipative couplings of the various species. However, the generalized quasi-linear wave energy theorem (Poynting theorem) directly relates \(\Gamma_-\) to radial wave energy density flux at stationarity. Thus, the zonal flow frequency can be written as:

\[
\Omega_q q^2 \rho^2 = -q^2 \frac{\delta}{\delta \phi_q} \left[ \rho \sigma \sum_k \frac{\partial}{\partial \nu} \left( \frac{\delta \phi_k}{\delta \nu} \right) \frac{\delta \rho_k}{\delta \nu} \right] (17)
\]

Here \(k\) and \(\dot{\phi}_k/T\) refer to drift waves \((k_p \neq 0)\) and \(\epsilon\) is the difference of the real part of ion and electron susceptibilities. For simple drift wave models, \(\partial \epsilon/\partial k_r \sim -k_r \rho^2\)
so \(\omega \sim k_p k_r [\dot{\phi}/T]^2 \sim (\dot{V}_r, \dot{V}_\varphi)\), the Reynolds stress. Thus, zonal flows are seen to arise from modulations of the drift wave Reynolds stress.

The appropriate adiabatic invariant for drift wave turbulence in azimuthally symmetric shear flows is

\[
N(k) = (1 + k_z^2, k_z^2) [\dot{\phi}/T]^2
\]

Here \(k\) and \(k_z^2\) are determined by drift waves \((k_p \neq 0)\) and \(\epsilon\) is the difference of the real part of ion and electron susceptibilities. For simple drift wave models, \(\partial \epsilon/\partial k_r \sim -k_r \rho^2\)
so \(\omega \sim k_p k_r [\dot{\phi}/T]^2 \sim (\dot{V}_r, \dot{V}_\varphi)\), the Reynolds stress. Thus, zonal flows are seen to arise from modulations of the drift wave Reynolds stress.
most efficiently done using two dimensional hydrodynamics. The result is:
\[
\frac{\partial}{\partial t} |\psi|^2 \big|_{\text{noise}} = (q^2 c_s^2) \sum_k \rho_k (k_x^2 k_y^2) \left( N k_x k_y \right) R(k, q) \right) |\psi|^2 \quad (23)
\]
Note that the noise resembles the square of Reynolds stress, as it should. The zonal flow spectrum equation thus finally is:
\[
\left( \frac{\partial}{\partial t} + \gamma_d \right) |\psi|^2 = -q^2 c_s^2 \sum_k \rho_k (k_x^2 k_y^2) \left( N k_x k_y \right) R(k, q) + q^2 c_s^2 \sum_k \rho_k (k_x^2 k_y^2) \left( N k_x k_y \right) R(k, q) \quad (24)
\]
where \( f(q_0) = \rho^2 / [1 - q^2 \rho^2 / (1 + k_x^2 \rho^2)] \).
Zonal flows shear and distort the drift wave spectrum which drives them. Thus, the zonal flows constitute a random strain field which randomly refracts drift waves, causing a diffusive increase in \( k_s \) which in turn enhances their coupling to small scale dissipation. The quasi-linear equation is given by:
\[
\frac{\partial |N|^2}{\partial t} - \frac{\partial}{\partial k_x} D \frac{\partial |N|^2}{\partial k_x} = \gamma(k) |N|^2 - \Delta \omega(k) |N|^2 \quad (25)
\]
where
\[
D = \sum_k \frac{k_x^2 k_y^2}{q^2} \left( 1 - \frac{q^2 \rho^2}{1 + k_x^2 \rho^2} \right) (\rho^2 c_s q)^2 R(k, q) |\psi|^2 \quad (26)
\]
\[\rho^2 c_s q = \rho q 2 \left( 1 + k_x^2 \rho^2 \right) \frac{c_s}{L} \frac{\gamma_d}{\sigma} \frac{k_x^2}{q} \quad (27)\]
where \( \sigma \approx \sqrt{\Lambda} \), \( \sigma \approx (\rho^2) c_s / L \), and \( \alpha = (q^2 c_s^2) / (\sigma c_s) / L \). The stable fixed point in these equations is the finite flow state with
\[
\left| \frac{n}{n_i} \right|^2 \sim \frac{k_x^2 k_y^2}{c_s^2} \sqrt{\frac{\gamma_d}{\sigma}} \quad (28)
\]
where \( D_{GB} \) is the gyro-Bohm diffusion coefficient. A striking feature of these results is the explicit proportionality of the fluctuation level and transport coefficient to the collisional flow damping \( \gamma_d \) and \( \sigma \).

### 3. Kinetic-Fluid Model for Drift Wave

To investigate the interaction between universal drift wave and convective cell in shearless slab geometry, kinetic-fluid model \( 11, 12 \) is derived. To derive the model, we assume the distribution function is Maxwellian in \( v \) space and integrate the nonlinear gyro-kinetic equation \( 17 \) over the velocity space. The zonal flow solution is used for the first moment of \( v \) which ensures the exact linear dispersion relation. The ion continuity equation is written by
\[
\frac{\partial}{\partial t} n_k - \omega_s \Gamma \xi Z \phi_k = \Gamma (1 + \xi Z) \frac{\partial}{\partial \xi} \phi_k + \frac{\tau}{2} \sum_{k' = k + k'} \left( b \cdot k' \times b \right) \phi_k' \chi_{\xi} (n_{\xi} \phi' + \xi \phi_k') = 0, \quad (31)
\]
and the electron continuity equation is
\[
\frac{\partial}{\partial t} n_k - \omega_s \xi Z \phi_k - (1 + \xi Z + k^2 \Lambda^2) \frac{\partial}{\partial \xi} \phi_k + \frac{\tau}{2} \sum_{k' = k + k'} \left( b \cdot k' \times b \right) \phi_k' \chi_{\xi} (n_{\xi} \phi' + \xi \phi_k') = 0, \quad (32)
\]
where \( n_k \) is the fluctuating ion density and \( \phi_k \) is the electrostatic potential, \( \omega_s = \tau k_n p_i^3 / (2 L_n) \) is the normalized diamagnetic drift frequency by the ion cyclotron frequency \( \Omega_i \), \( \rho_i \) is the ion gyro-radius, \( \tau = T_e / T_i \) is the ratio of electron and ion temperature, \( \lambda^2 = \sqrt{\omega_p^2 / (2 \omega_n^2)} \), \( \omega_n = \tau k_i p_i^3 / (2 L_n) \) is the plasma frequency, \( \Gamma \) is the ion plasma frequency, \( \Omega_i \) is the 0-th order modified Bessel function and \( J_0 \) is the 0-th order Bessel function. The factor 1/2 in eqs.(31) and (32) appears due to the definition of thermal velocity. The Poisson equation is used to eliminate the fluctuating electron density in the electron continuity equation \( 10 \).

The energy conservation relation is obtained from eqs.(31) and (32) as
\[
\sum_k \left( \text{Re}(s_2) \frac{\partial}{\partial t} |\phi|^2 + 2 \text{Re}(s_1) |\phi|^2 - 2 \text{Im}(s_2) \right) \text{Re}(\phi) - \text{Im}(\phi) \text{Re}(\phi) = 0 \quad (33)
\]
where \( \text{Re} \) shows real part, \( \text{Im} \) shows imaginary part of the argument, \( s_1 = i \omega_e (\xi Z \xi + \Gamma \xi Z \xi) \), \( s_2 = 1 + \xi Z + k^2 \lambda^2 + \xi \phi \frac{\partial}{\partial \xi} \left( 1 + \xi Z \xi \right) \) and \( \phi = \frac{\phi}{c_s} \). In the fluid ion and
near adiabatic electron response $1 \ll \zeta_e, \zeta_i \ll 1$, i.e.,
\[ v_{th i} \ll \frac{e}{k_B T_i} \ll v_{th e}, \]
it is reduced to
\[ \frac{\partial}{\partial t} \left( \sum_{k_x, k_y, k_z \neq 0} \left( 1 + \kappa^2 \lambda^2 + \tau(1 - \Gamma_{ok}) \right) |\phi_k|^2 \right) + \sum_{k_x, k_y, k_z = 0} \left( \kappa^2 \lambda^2 + \tau(1 - \Gamma_{ok}) \right) |\phi_k|^2 = 0. \] (34)

The first term in LHS (left hand side) represents the drift wave energy and the second term represents the convective cell energy. In the same limit, the dispersion relation of universal drift wave is analytically evaluated as
\[ \omega = \frac{\omega_{\Gamma_{ok}}}{1 + \tau(1 - \Gamma_{ok})} + \frac{\sqrt{\pi}}{1 + \tau(1 - \Gamma_{ok})^2} \times \left( 1 - \Gamma_{ok} k_0^2 e^{-k_0^2} - \Gamma_{ok} e^{-k_0^2} \right). \] (35)

The dispersion relation shows that the universal drift wave is destabilized by the inverse Landau damping and is stabilized by the ion Landau damping. Based on this model, the universal drift wave turbulence is investigated numerically and is discussed in the next Chapter.

4. Simulation results

4.1 Numerical Scheme

Eqs. (31) and (32) are solved as follows. Initially the linear eigenvalue $\omega$ and eigenfunction are calculated for given parameters, then $\omega$ is fixed as a constant in plasma dispersion function. Now, eqs. (31) and (32) are formally expressed by the following equation
\[ \frac{\partial}{\partial t} Y_k + f(Y_k, Y_k) + AY_k = 0 \] (36)
where $Y_k = \{\phi_k, n_k\}$, $A$ is a numerical constant including plasma dispersion function. The time step is advanced by the predictor-corrector scheme. Nonlinear Landau damping is not taken into account in this scheme, however, it might be possible if $A = A(\omega)$ is advanced in time numerically.

In present numerical simulations, the following parameters are used: $16 \times 32 \times 8$ modes in $k_x, k_y, k_z$ space, $k_{z0} = k_{\phi0} = 0.15, k_{x0} = 0.02, m_i/m_e = 1836, \Omega_i^2/\omega_{pi}^2 = 10, \rho_i/L_n = 0.2$ (the wave number is normalized by ion gyro-radius). Fig.6 shows $k_0$ dependence of the real frequency, which is normalized by the ion cyclotron frequency $\Omega_i$, and growth rate of universal drift wave in the cases with $k_z = 0.15$ and $2.4$ for $k_x = 0.02$, respectively. It is seen that the universal drift wave is unstable in the short wavelength region (large-$|k_z|$). Larger $|k_z|$ makes the universal drift wave more stable. It should also be mentioned that for larger $k_z$, the universal drift wave is stable.

4.2 1D and 2D Spectra of Energy

To investigate the normal cascade and inverse cascade processes in the wave vector space, we define the following power spectra according to eq. (34). The energy spectrum in $k_z$ space of drift wave is defined by
\[ E_{DW}(k_z) = \sum_{k_x, k_y} \left( 1 + k^2 \lambda^2 + \tau(1 - \Gamma_{ok}) \right) |\phi_k|^2. \] (37)

Then the drift wave energy $E_{DW}$ and the convective cell energy $E_{CC}$ are written by
\[ E_{DW} = \sum_{k_x} E_{DW}(k_z), \] (38)
\[ E_{CC} = \sum_{k_x, k_y, k_z = 0} \left( k^2 \lambda^2 + \tau(1 - \Gamma_{ok}) \right) |\phi_k|^2. \] (39)

In the components of Fourier modes which constitute the convective cell, the zonal flow component corresponds to $k_z \neq 0, k_y = k_x = 0$ and the streamer component corresponds to $k_z = k_x = 0, k_y \neq 0$. Similarly, 1-D(dimensional) and 2-D spectra of the potential and internal energy $E_p$ and $E_n$ are defined by
\[ E_p(k_z) = \sum_{k_x, k_y} |\phi_k|^2, E_n(k_z) = \sum_{k_x, k_y} |n_k|^2, \] (40)
\[ E_p(k_x, k_y) = \sum_{k_z} |\phi_k|^2, E_n(k_x, k_y) = \sum_{k_z} |n_k|^2, \] (41)
respectively.

4.3 Collisionless Simulation of Universal Drift wave

Fig.7 shows the time evolution of the drift wave energy $E_{DW}$ and the convective cell energy $E_{CC}$. The energy of each mode with $k_z = 0.02, 0.04, 0.06$ is also shown. The maximum growth rate of the drift energy comes from the longest wavelength mode with $k_z = \pm 0.02$. It is seen that the convective cell and the drift wave with short wavelength of $k_z$ are also excited at $t \geq 800$. The amplitudes of these modes are seen to reach almost the same level as those with $k_z = \pm 0.02$ at $t \geq 1300$.

Fig.8 shows the time evolution of 1-D spectrum of the potential and internal energy as the function of $k_z$. Time slices of $t = 100, 200, \ldots, 1600$ are shown. It is seen that the convective cell grows after the excitation of $k_z = \pm 0.06$ modes and the cascade occurs in $k_z$ space, simultaneously. As is also seen from Figure 4.3, the spectrum are almost flat in $k_z$ space at $t = 1300$. The normal cascade is observed in addition to the excitation of the convective cell.
The real frequency and growth rate of the universal drift wave in the cases with $k_x = 0.15, 2.4$ for $k_z = 0.02$ is shown.

Fig. 6 $k_y$ dependence of the real frequency and growth rate of the universal drift wave in the cases with $k_x = 0.15, 2.4$ for $k_z = 0.02$.

Fig. 7 Time evolution of the drift wave energy $E_{DW}$ and the convective cell energy $E_{cc}$. The energy for each mode with $k_x = 0.02, 0.04, 0.06$ is also shown.

Fig. 8 Time evolution of 1-D spectrum of the potential and internal energy. Time slices of $t = 100, 200, ..., 1600$ are shown.

is clearly seen that the components of zonal flow with $k_y = \pm 0.15$ are dominant in the convective cell energy of potential. On the other hand, Fourier modes with $k_y = 0.3, k_y = 0.3$ and $k_y = -0.3, k_y = 0.3$ are dominant in the convective cell energy of density.

Fig. 10 shows the 3-D power spectrum of convective cell energy of potential and density in $k_y$ space at $t = 1500$. The excitation of zonal flow with $k_y = 0.15$ grows gradually and becomes dominant at $t \approx 1600$.

Fig. 11 shows the time evolution of the Fourier modes in the convective cell energy of potential. The streamer with $k_y = 0.15$ is dominant in convective cell energy at $t = 1300$ as indicated by the arrow. The zonal flow with $k_y = 0.15$ grows gradually and becomes dominant at $t \approx 1600$.

Fig. 12 shows the time evolution of the convective cell energy spectrum of density for finite $k_x$ modes. This is plotted because the spectrum of $E_{\rho}$ is different from the spectrum of $E_p$. The modes with $k_x = 0.15$ and $k_x = 0.30$ are plotted for $k_y = 0.0, 0.15, 0.30$ and 0.45. It is seen that all Fourier modes in CC start to grow $t \approx 600$. This is a clear identification of the nonlinear growth of CC with finite $k_y$ modes.

Fig. 13 shows the contour plot of the flux $\Gamma(x, y, z = 0)$ at $t = 390$ and $t = 1590$. The flux is calculated by the inverse Fourier transformation (which means the transforming a physical quantity from the wave vector space into the real vector space) of $\Gamma = \sum_{k_x, k_y, k_z} n_{k_x}(-i k_x \phi_{k_y})$. Longitudinal or transverse axis is the $y$ axis or the $x$ axis, respectively. The red part
Fig. 9 Time evolution of 2-D spectrum of the potential and internal energy with $k_x = 0.15$. Time slices of $t = 10, 50, 100, 200, ... , 1600$ are shown.

Fig. 10 3-D power spectrum of the convective cell energy of potential and density in $k_y$ space at $t = 1500$. The cases with $k_x = 0, \pm 0.15, \pm 0.30$ are plotted.

Fig. 11 Time evolution of Fourier modes in CC energy of potential

indicates high level of flux. It is seen that the large size vortex collapses into the small size elongated vortex at $t = 1590$. This collapse means that the irregular contour of flux (including the convective cell mode and the other modes) becomes eventually the similar level. These vortices move to the direction of $E \times B$, which are similar to the motion of the zonal flow.

4.4 Collisional Simulation of Universal Drift Wave

To investigate zonal flow damped by collision $^5)$, the effect of ion-ion collision is investigated. It is included into the model equations by replacing $\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \nu_{ii}$ in the ion continuity equation.

Fig.14 shows the time evolution of the drift wave energy $E_{Dw}$ and the convective cell energy $E_{CC}$ in collisional case. It is found that the convective cell energy is more rapidly growing and is larger than the drift wave energy compared with those in collisionless case.

Fig.15 shows the 1-D spectrum of the potential energy $E_{\phi}$ and internal energy $E_{s}$ as the function of $k_z$. Cascade occurs in $k_z$ space and the convective cell is excited. It is found that the excitation of CC is stronger than that in collisionless case. The growth of linear drift wave is not clearly observed in spectral space.

Fig.16 shows the time evolution of 2-D spectrum of the potential and internal energy as the function of $k_y$ for the case with $k_x = 0.15$. The cascade and inverse cascade occur in $k_y$ space. It is found that the excitation of zonal flow is weak in this case. Other Fourier modes are dominant in CC energy as is shown in the next two figures.

Fig.17 shows the 3-D power spectrum of CC energy at $t = 800$ in the collisional case with $\nu_{ii} = 0.01$. The time $t = 800$ corresponds to the almost final phase of simulation and after that, the simulation breaks down due to the numerical instability. It is found that the zonal flow is damped by collision, however, the finite $k_y$
modes are excited instead, which are large contribution of CC energy.

Fig. 18 shows the time evolution of CC energy spectrum of potential for the case with $\nu_{hi} = 0.01$. It is seen that the component of streamer $k_x = 0.0, k_y = 0.3$ is dominant in CC energy in this case. Zonal flow $k_x = 0.15, k_y = 0.0$ gradually stays in quasi-steady state during $t = 670 \sim 720$, then starts to grow again. The collisional effect makes the zonal damping, however it also contributes to the excitation of other modes. The simple conclusion for saturation mechanism is not drawn in this case comparing to ITG case. The difference
might come from shearless and sheared slab geometry.

Fig.19 shows the time evolution of the CC energy spectrum of density for finite k_x modes in the presence of ion-ion collision (v_{ii} = 0.01). The modes with k_x = 0.15 and k_x = 0.30 are plotted for k_y = 0.0, 0.15, 0.30 and 0.45. For k_x = 0.15 modes, k_y = 0.15 mode starts to grow rapidly at t = 500 and reaches almost the same amplitude as other modes at t \approx 700. For k_x = 0.3 modes, the growth of zonal flow and k_y = 0.15 mode is similar for t \geq 500 and both become dominant in t \approx 650. The amplitude of the CC energy is almost the same as the one of collisionless case (Figure 4.7) when the CC energy starts to grow. Comparing the result in Figure 4.7 and Figure 4.14, we see that the time necessary to reach the same level is different. Namely, the collisional case is almost the half of the collisionless case.

Fig.20 shows the contour plot of the flux \Gamma(x, y, z = 0) at t = 490 and t = 690 for v_{ii} = 0.01, respectively. The similar patterns are observed comparing with collisionless case. However, the vortex size is smaller than that in collisionless case.

Fig.21 shows the time evolution of convective cell energy and flux for the cases with v_{ii} = 0 and v_{ii} = 0.01. In the case with v_{ii} = 0.01, the growth of flux and convective cell energy is rapid compared to the one without collision, since the zonal flow is damped by collision. It is found that the flux and convective cell energy correlate to each other and reach almost the same levels.

5. Summary and Discussion

Kinetic-fluid model in the shearless slab geometry is introduced \(^{10}\) using linear closure relation for the first moment of v_{ij}, which is an extension of the model proposed by Smolyakov \(^{8}\) and Hinton \(^{9}\). This model can simultaneously describe the universal drift wave turbulence and the convective cell dynamics, in which zonal flows and streamers are included.

Nonlinear simulations of universal drift wave turbu-
Fig. 19 Time evolution of Fourier modes in CC energy of density for finite $k_y$ modes with ion-ion collision ($\nu_{ii} = 0.01$). The modes with $k_y = 0.15$ and $k_y = 0.30$ are plotted for $k_z = 0.0, 0.15, 0.30, 0.45$.

Fig. 20 Contour plot of the flux $\Gamma(x, y, z = 0)$ at $t = 490$ and $t = 690$ for $\nu_{ii} = 0.01$ (collisional-case).

Fig. 21 Time evolution of CC energy and flux in cases with $\nu_{ii} = 0.0$ and $\nu_{ii} = 0.01$. linearly self-sustained DW turbulence.

The ion-ion collision effect on zonal flow is also investigated. This effect makes the growth of CC energy stronger. It is concluded that the ion-ion collision damps the zonal flow and breaks down a self-regulating system between drift wave and zonal flow. It also excites the streamer and finite $k_y$ modes, which dominate the amplitude of convective cell instead of the zonal flow. As the result, the flux is larger than the one without collision.

Fig. 22 shows the flow diagram of the universal drift
wave energy and convective cell energy. We have shown that the collision causes the damping of the zonal flow and the rapid excitation of streamer and other finite \(k_y\) modes. However, we don’t know how much energy is transferred from the universal drift wave to each Fourier mode (zonal flow, streamer and other finite \(k_y\) modes) in CC energy. It should be clarified. We should also explore the mechanism of the drift wave in sheared slab case and the zonal flow induced random shearing as possible nonlinear saturation mechanism 5, 16).

Fig. 22 Flow diagram about the universal drift wave energy and the convective cell energy.

References


13) F. H. Busse, CHAOS 4, 123 (1949).


