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Optimum Arrangement of Traffic Counts for Traffic Demand Estimation by Observed Link-Flow

by

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Abstract

There have been several studies on the estimation of trip distributions using observed link-flow. The accuracy of the estimation depends largely on the location and the number of the traffic counts in the network.

In this paper, two methods are proposed which will enable us to extract the effective links for the estimation of trip distributions. One is a method for the direct estimation of trip distributions, named the “OD-covering method”, and the other is for the estimation of trip generations, named the “zone-covering method”. These two methods were applied to the trunk road network in the Fukuoka City Region, and the results of the estimation were sufficiently accurate based on the link-flows that were observed on the extracted links in the solution.

Keywords: Traffic counts, Optimum arrangement, Trip distribution, Trip generation, Branch and bound method

1. Introduction

The traffic demand, which is required in urban traffic planning and trunk road network planning, is generally calculated using a four-step travel estimation procedure based on measures such as the Person Trip Survey and the Road Traffic Census.

However, in recent years the importance of road traffic management has increased in areas with narrower roads, where data based on such large-scale surveys is useless for traffic planning. Thus, the establishment of effective traffic planning techniques for such areas has become necessary. In response to this need, a technique by which the trip distribution is

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estimated from the observed traffic volumes has been attempted as a realistic solution to this problem, and some estimation models have been developed. Moreover, it has been pointed out that the selection of appropriate observation links is an important problem\(^1\).

Regarding the above-mentioned viewpoint, the current study is a consideration of the method of arranging the traffic observation points to estimate the traffic demand using link flow.

2. Previous Research on Traffic Demand Estimation by Link-Flow

Previous studies have proposed a number of models, all of which can be used to estimate the traffic demand in a district by using the link-flow on the road sections.

Classifying these past models according to the scale of the target network, they can be divided roughly into two types. One type is intended for local areas, while the other is intended for wider areas. The former type attempts to analyze the traffic behavior in a narrow area based on the observed traffic volumes in three directions (through, left turn, and light turn) at each intersection in order to use the results for traffic management and planning, etc. The latter type tries to estimate the traffic demand of a wide-ranging network such as that of an entire city region. Moreover, this type can be further classified into type A, which uses existing trip distribution data, and type B, which uses OD-patterns from the gravity model. The model of type A is useful for wide-range applications because it doesn't require the existing trip distribution data. However, the model's estimation accuracy is not very high due to its dependency on the reliability of the gravity model. On the other hand, the estimation accuracy of the model of type B is relatively high because it uses the data of OD surveys. The model of type B was developed chiefly to correct the OD table, and there are early models in which the screen-line traffic volumes are used as the restriction condition.

The entropy maximization model\(^1\) and the error square minimization model\(^3\) are comparatively practical models among type B models. The former model calculates the trip distribution, which the appearance probability of the OD trip makes the highest under the condition that the calculated traffic volume for each link is equal to the observed volume. The latter model minimizes the square error of the calculated and observed traffic volumes under various restrictions. This model is classified under both the road section model and the generation traffic model due to its combination of objective functions and its restrictions. These methods for traffic demand estimation by link-flow are based on applications of the optimization technique by which the difference between the observed and estimated values is minimized under the restrictions concerning traffic volume.

However, in spite of a large amount of research there are a lot of problems regarding various factors such as 1) changes in road networks, 2) errors and changes of OD data, and 3) the number of observation points and their arrangement.

Although research by Iida and Takayama has dealt with factors 1 and 2, and research by Inoue has dealt with factor 2, there is virtually no research that has examined factor 3. As a general idea, the method of arrangement of observation points known as “OD covering standard” and the method of traffic volume estimation at non-observed road sections have thus far only been proposed by Toi\(^1,5\), with the exception of research by Iida and Takayama\(^3\).

3. Estimation Method of Trip Distribution and Necessary Conditions

In this study, a simple method of OD volume estimation is considered in cases when past
OD survey data exists. The traffic flow of link \( m \) is assumed to be \( t_m \), and the ratio of traffic between zone \( i \) and \( j \) in \( t_m \) is assumed to be \( v_{ijm} \). OD volume between zones \( i \) and \( j \) is assumed to be \( p_{mij} \). Then, the next equation, (1), is introduced because the trip distribution between \( i \) and \( j \) passing through link \( m \) is \( t_m \cdot v_{ijm} = x_{ij} \cdot p_{mij} \).

\[
x_{ij} = \frac{v_{ijm}}{p_{mij}} \cdot t_m
\] (1)

If \( v_{ijm} \) and \( p_{mij} \) are obtained by traffic assignment, then errors are caused between the true value and the estimation. If observed traffic volume \( q_m \) is used instead of \( t_m \), then the observation error is included in \( q_m \), and the time difference is caused between the observation and the calculation of \( v_{ijm} \) and \( p_{mij} \). Therefore, equation (1) does not hold strictly to all combinations of \( (i, j, m) \) that satisfy \( v_{ijm} = 0 \) and \( p_{mij} = 0 \).

\( H_{ij} \) is defined by the next equation, (2), to estimate \( x_{ij} \), minimizing the total error square of equation (1).

\[
H_{ij} = \sum_m (x_{ij} - \frac{v_{ijm}}{p_{mij}} \cdot q_m)^2
\] (2)

Moreover, if it is assumed that \( u_{ijm} = \frac{v_{ijm}}{p_{mij}} \cdot (p_{mij} \neq 0) \), \( \delta_{ijm} = 1 \) (traffic volume is observed at link \( m \)), \( 0 \) (non-observed at link \( m \)), then equation (2) can be expressed as follows.

\[
H_{ij} = \sum_m \xi_m (\delta_{ijm} \cdot x_{ij} - u_{ijm} \cdot q_m)^2
\] (2')

When equation (2') is partially differentiated with \( x_{ij} \), and is assumed to be \( 0 \),

\[
\frac{\partial H_{ij}}{\partial x_{ij}} = 2 \sum_m \xi_m \delta_{ijm} (\delta_{ijm} \cdot x_{ij} - u_{ijm} \cdot q_m) = 0
\]

because \( \delta_{ijm}^2 = \delta_{ijm} \), \( \delta_{ijm} u_{ijm} = u_{ijm} \), and

\[
(\sum_m \xi_m \delta_{ijm}) x_{ij} = \sum_m \xi_m u_{ijm} \cdot q_m
\] (3)

Therefore, the condition that \( x_{ij} \) can be estimated is

\[
\sum_m \xi_m \delta_{ijm} = 1
\] (4)

At this time, \( x_{ij} \), the estimation value of \( x_{ij} \), is given by the next equation, (5).

\[
x_{ij}' = \frac{\sum_m \xi_m u_{ijm} \cdot q_m}{\sum_m \xi_m \delta_{ijm}}
\] (5)

Here, because \( \delta_{ijm} \) is a constant that shows trip distribution between \( i \) and \( j \) passing through link \( m \), \( \xi_m \delta_{ijm} = 1 \) means that traffic volume is observed at the link where the traffic flows between \( i \) and \( j \). Therefore, equation (4) is an example of the “OD covering standard” which has already been proposed. That is, the “optimum arrangement of the traffic observation points by the OD covering standard” means to produce the observation which enables one to estimate \( x_{ij} \) when assuming \( t_m = q_m \) as in equation (1).

4. Traffic Counts Arrangement to Estimate Trip Distribution

The method of arranging observation points is devised using the “OD covering standard” to estimate trip distribution from the reason previously discussed. And it is typical to adopt the observation cost as a target function and to achieve its minimization. The total
observation cost is thought to rise proportionally with the number of traffic counts. This can be formulated by the following optimization problem.

\[
\text{Minimize} \quad Z = \sum_{m} \xi_{m} \\
\text{s.t} \quad \sum_{m} \xi_{m} \delta_{im} \geq 1 \quad (\text{for all } i, j)
\] (6)

To solve this problem, first an initial possible solution is obtained by “Balass’s additional algorithm”, and then a better solution is searched for using the “Branch and Bound Method”. The flow chart of this method is shown in Fig. 1. \(M\) and \(P\) in the figure are sets of links and

![Flow chart of the solution by OD covering standard.](image)
OD pairs, respectively. \(K\) and \(Q\) are sets of indispensable links (when a certain OD pair flows only in link \(k\), link \(k\) is called an indispensable link in this paper), and sets of OD pairs included in an indispensable link, respectively. \(N(I)\) is the sets of links which compose possible solution \((I)\), \(Z\) is the number of the links, and \(\Delta N(I)\) is the sets of the unexamined links. This solution composes an indispensable condition to obtain the estimation value of \(x_{ij}'\) for all of the combinations of \(i\) and \(j\). Next, the problem of traffic count arrangement considering estimation errors can be analyzed as follows.

The errors of estimation are included in \(\nu_{ijm}, p_{mij}\), and \(t_m(q_m)\). The causes are considered to be the following.

1. Errors of OD survey data.
2. Errors which occur in when \(\nu_{ijm}, p_{mij}\), and \(t_m\) are calculated (it is an assignment error in general).
3. Errors caused by changes in trip distribution with the passage of time and changes in the network.
4. Errors of observed link-flows etc.

Errors \(1, 2, \) and \(3\) are expressed as \(\varepsilon_{ijm}\) collectively because they exist in traffic \(t_{ijm}\) between \(i\) and \(j\) on link \(m\). Error \(4\) can be expressed as observed error \(\Delta q_m\). Thus, OD volume \(t_{ijm}'\) between \(i\) and \(j\) on link \(m\) can be expressed as equation (7).

\[
t_{ijm}' = t_{ijm} + \varepsilon_{ijm}
\]

And

\[
\nu_{ijm}' = \frac{t_{ijm} + \varepsilon_{ijm}}{t_m + \varepsilon_m} \quad p_{mij}' = \frac{t_{ijm} + \varepsilon_{ijm}}{x_{ij}} \quad u_{ijm}' = \frac{\nu_{ijm}}{p_{mij}'} = \frac{x_{ij}}{t_m + \varepsilon_m}
\]

where \(e_m = \sum_{i,j} e_{ijm}\).

Substituting these results for equation (5), with approximations of \((1 + e_m/t_m) - 1 \approx 1 - e_m/t_m (e_m/t_m \ll 1)\),

\[
x_{ij} + \Delta x_{ij} = \frac{\sum \xi_m x_{ij}(t_m + \Delta q_m)/(t_m + \varepsilon_m)}{\sum \xi_m \delta_{ijm}}
\]

\[
\approx \left\{ \frac{\sum \xi_m x_{ij} t_m - \sum \xi_m \Delta q_m}{\sum \xi_m \delta_{ijm}} + \frac{\sum \xi_m \varepsilon_{ijm} \Delta q_m}{\sum \xi_m \delta_{ijm}} \right\}
\]

Clause 3, inside the \{\} symbols, is a product of the error element and is extremely small, so it may be omitted. Therefore,

\[
\Delta x_{ij} \approx \sum_m \xi_m \Delta q_m/(1 - e_m) \quad \left( 1 + \frac{e_m}{t_m} \ll 1 \right)
\]

If \(e_m\) is assumed to be a probability variable, then \(\Delta x_{ij}\) is also a probability variable. \(\Delta q_m\) is an observation error at each traffic count, and it has nothing to do with the network. Only \(e_m\) is treated as a probability variable here, and it is assumed that \(\Delta q_m = 0\). When \(e_m^*\) and \(\sigma_m^2\) are assumed as the expectation and the variance of \(e_m\), respectively, and \(\text{Cov}(e_m, e_k) = 0 (m \neq k)\) is assumed, the expectation and variance of \(\Delta x_{ij}\) are expressed as follows, respectively.

\[
\Delta x_{ij}^* = - \sum_m \xi_m u_{ijm} e_m^* \quad \sigma_{\Delta x_{ij}}^2 = \left( \sum_m \xi_m u_{ijm} \right)^2 \sigma_m^2 / \left( \sum_m \xi_m \delta_{ijm}^2 \right)
\]
When the following function $S_\delta$ is defined, clause 1, inside the { } symbols, is superior to clause 2, for:

- $\Delta x_{ij}^* > 0$, otherwise clause 2 is superior, for $\Delta x_{ij}^* < 0$, and $S_\delta$ is then considered to be an index which shows the difference of $\Delta x_{ij}$ around $\Delta x_{ij}^*$.

$$S_\delta^2 = 1/2 \{(\Delta x_{ij}^* + \sigma_{xij})^2 + (\Delta x_{ij}^* - \sigma_{xij})^2\} = \Delta x_{ij}^* + \sigma_{xij}^2$$  \hspace{1cm} (10)

If $S_\delta^2$ is reduced by changing $\xi_m$, the combination of $\xi_m$ gives a small estimation error of $\Delta x_{ij}$. The solution is obtained by limiting $S_\delta^2$ under a constant value $\theta_\delta^2$. That is,

$$S_\delta^2 = \Delta x_{ij}^* + \sigma_{xij}^2 \leq \theta_\delta^2$$  \hspace{1cm} (11)

When $\theta_\delta^2$, the permissible level of $S_\delta^2$, is defined for all $i$ and $j$, the optimum problem which gives the minimum number of traffic counts is formulated under the restrictions of equation (12).

$$\text{Minimize } Z = \sum_m \xi_m$$  
$$\text{s.t. } \Delta x_{ij}^* + \sigma_{xij}^2 \leq \theta_\delta^2 \text{ (for all } i, j)$$  \hspace{1cm} (12)

This is a nonlinear planning problem of 0-1 solution type, and the solution number is limited. Therefore, the optimum solution can be obtained theoretically by examining all the solutions. However, this method is not practical, and the solution can be obtained efficiently using the branch and bound method, shown in chapter 6 of this study.

5. Estimation Method of Trip Generation and the Arrangement of Traffic Counts

This chapter examines the technique used to obtain the trip generation and the essential condition of the arrangement of traffic counts. The probability that traffic generated from zone $i$ flows in link $m$ is assumed to be $r_{im}$. Traffic related to zone $i$ which flows in link $m$ is assumed to be $t_{mi}$, and its ratio to $t_m$ is assumed to be $w_m$. The trip generation of zone $i$ is assumed to be $y_i$. $t_{mi}$ is shown with $t_{mi} = y_i \cdot r_{im}$, and $y_i$ is obtained by equation (13).

$$y_i = (w_m / r_{im})t_m \text{ (for } r_{im} \neq 0)$$  \hspace{1cm} (13)

If the observed value $q_m$ is used instead of $t_m$, equation (13) does not strictly hold due to the various errors described in Chapter 3. $y_i$ is then estimated by minimizing the total error square $G_i$ of equation (13).

$$G_i = \sum_m (\beta_{im} y_i - \gamma_{im} q_m)^2$$  \hspace{1cm} (14)

where

- $\gamma_{im} = w_m / r_{im} \text{ (for } r_{im} \neq 0)$,
- $\beta_{im} = 1 \text{ (traffic related to zone } i \text{ flows in link } m)$,
- $0 \text{ (traffic related to zone } i \text{ doesn't flow in link } m)$.

When equation (14) is partially differentiated with $y_i$, and it is assumed to be 0,

$$\partial G_i / \partial y_i = 2 \sum_m \xi_m \beta_{im} (\beta_{im} y_i - \gamma_{im} q_m) = 0$$

because $\beta_{im}^2 = \beta_{im}$, $\beta_{im} \gamma_{im} = \gamma_{im}$,

$$\sum_m \xi_m \beta_{im} y_i = \sum_m \xi_m \gamma_{im} q_m$$  \hspace{1cm} (15)
Therefore, the condition to obtain $y_i$ can be expressed by equation (16).

$$\sum_m \xi_m \beta_{im} \equiv 1 \quad (16)$$

Then, $y'_i$, the estimation value of $y_i$, is given by equation (17).

$$y'_i = \sum_m \xi_m \gamma_{im} q_m / \sum_m \xi_m \beta_{im} \quad (17)$$

Equation (16) is simply “zone covering standard”. The arrangement based on this standard is obtained by the solution of the next optimal problem.

$$\text{Minimize } Z = \sum_m \xi_m$$

$$\text{s.t. } \sum \xi_m \beta_{im} \equiv 1 \quad \text{for all } i \quad (18)$$

The method used to solve this problem is the same as the one described in Chapter 4. Next, the arrangement of traffic counts to control $\Delta y_i$, the estimation error of $y_i$, is examined through an analysis similar to that of Chapter 4.

$$\gamma_{im}' = w_{im}' / \gamma_{im} = y_i' / (\gamma + e_m) \quad (19)$$

The estimation error of $y_i$ can be shown as equation (20).

$$\Delta y_i = \sum_m \xi_m \gamma_{im} (\Delta q_m - e_m) / \sum_m \xi_m \beta_{im} \quad (20)$$

where $\Delta q_m = 0$ is assumed. When $\text{cov}(e_m, e_k) = 0 (m \neq k)$ is assumed, the expectation and variance of $\Delta y_i$ can be shown as follows, respectively.

$$\Delta y_i = -\sum_m \xi_m \gamma_{im} e_m / \sum_m \xi_m \beta_{im}$$

$$\sigma_{y_i} = \left(\sum_m \xi_m \gamma_{im}^2 \sigma_m^2 / \left(\sum_m \xi_m \beta_{im}\right)^2\right) \quad (21)$$

Moreover, $T_i$ is defined as an index of the $\Delta y_i$ difference.

$$T_i = \Delta y_i^2 + \sigma_{y_i}^2 \quad (22)$$

If permissible value $\tau^2$ is introduced, the following optimization problems can be assumed.

$$\text{Minimize } Z = \sum_m \xi_m$$

$$\text{s.t. } \Delta y_i^2 + \sigma_{y_i}^2 \leq \tau^2 \quad \text{for all } i \quad (23)$$

6. Application to a Real Network

The theory described above was applied to a real network, and the estimation accuracy of the trip distribution and trip generation was analyzed in this chapter. Moreover, the accuracy of the estimation and the relationship between accuracy and the selection of traffic counts was analyzed.

(1) Road network and error occurrence

The outline of the network used in this study is shown in Fig. 2, which includes Fukuoka City and the surrounding area. Sign $\bigcirc$ in Fig. 2 shows the centroids. The roads consist of national highways and main local roads, and they were surveyed in the Road Traffic Census.
Fig. 2 The outline of the network of this research.

The Fukuoka city expressway was excluded because data in 1985 was used. 20 nodes inside and 5 nodes outside of this network were selected from car OD data in 1985. Then the \( t_m \) and \( t_{ijm} \) of each link in the network were calculated by the traffic assignment.

It is thought that there is a main cause of the error in \( w_{mi} \) and \( r_{im} \).

Error \( e_{ijm} \) of equation (7) was made from standard normal random number(s).

\[
e_{ijm} = \frac{t_{ijm} \cdot s}{3}
\]  

(24)

The normal random numbers for which \( 3\sigma \) corresponds to \( t_{ijm} \) were generated, and the numbers that satisfy the following two conditions were adopted from them.

\[
\begin{align*}
|e_{ijm}| & \leq t_{ijm} \\
\sum_{k \in R_i} e_{ijk} &= 0
\end{align*}
\]  

(25)

where \( e_{ijk} \) is the assignment error to route \( k \) of the traffic between \( i \) and \( j \), \( R_i \) is the set of alternative routes for the traffic between \( i \) and \( j \).

The expectation value \( e_m^* \) and variance \( \sigma_m^2 \) in this example were calculated 300 times using the Monte Carlo simulation.

(2) Extraction of essential links by the OD covering standard

(a) The OD covering method

The 77 links shown with thick lines in Fig. 2 are not the subset mutually concerning OD pairs composing the link flow. Therefore, the links that meet the OD covering standard have to be extracted from among sets of these links. In this analysis, the trip distributions of 175 OD pairs for which traffic was not 0 were extracted from among 190 OD pairs.

It should be noted here that the denominator of equation (5) should be positive in order to obtain all the trip distributions in the network. The solution that meets these requirements and has minimum observation points is obtained by solving the problem of equation (6).
Then, the optimization problem of equation (6) is solved for link sets, which are not mutually the subset, and 16 solutions with 20 composition links are extracted. The distribution of the links which compose those solutions are shown in Fig. 3. The links marked by a small ○ sign in Fig. 3 are common links to all of the 16 solutions. If any of these links are missed, the perfect trip distribution cannot be estimated by equation (5). Moreover, sets of OD pairs, which are not covered by the common links, can be covered by combining four links out of the eight shown in Fig. 3 by large ● signs.

(b) Method of controlling estimation errors

The solution for minimizing the observation points is obtained by solving equation (12), under the restriction of suppressing the estimation errors of the trip distribution below a constant value. In this study, the solutions of three cases \( \theta_{ij}/x_{ij}=0.10, 0.15, \) and \( 0.20 \) were obtained. It is necessary to satisfy the restrictions of equation (12) concerning all OD pairs with this method. Moreover, if the OD covering standard of the solution is not met, the left side of equation (12) becomes infinite because the denominator is \(( \sum \xi_n \delta_{om} )^2\) at the left of the restriction. Therefore the solution that consisted of 20 links is necessary. However, in this case the degree of freedom of the solution decreases and the number of solutions decreases because the restrictions concerning the estimation error are added to the OD covering standard.

The solutions obtained incorporating the OD covering method and the error control method are shown in Table 1. The solution by the error control method corresponds to 1-8 of the solutions by the OD covering method in cases of \( \theta_{ij}/x_{ij}=0.10, 0.15, \) and \( 0.20 \). Five cases \( \theta_{ij}/x_{ij}=0.09, 0.08, 0.07, 0.06, \) and \( 0.05 \) were calculated, and only one of the possible solutions was not obtained.
(3) Estimation accuracy of trip distribution

The estimation accuracy of the trip distribution differs according to the solutions. When the accuracy was examined using the estimation error rate (standard deviation of the estimation errors / distribution volume \(\times 100\)), solution 1 showed the highest accuracy and solution 16 showed the lowest. The frequency and the accumulated frequency of the estimation error rate based on the two solutions are shown in Fig. 4. From Fig. 4 it is clear that the error rates are mostly within 6\%, though there are some differences in the estimation error between solutions 1 and 16.

Next, using the Monte Carlo simulation based on solution 1 (Fig. 5), trip distribution \(x_{ij}'\) was calculated by equation (5). The relationships between the given trip distribution \(x_{ij}\) and average value \(x_{ij}'\) are shown in Fig. 6. Both show an extremely high correlation. Moreover, the relationship between trip distribution and standard deviation of estimation error is shown in Fig. 7. Thus, it is understandable that the error rate has little correlation with the scale of OD volume. 

![Fig. 4](image)

**Fig. 4** The frequency of estimation error rate based on solution 1 and 16.

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### Table 1 Solutions based on OD covering method and Error control method.

<table>
<thead>
<tr>
<th>Solution number</th>
<th>OD covering method</th>
<th>Error control method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of links composing solutions</td>
<td>20%</td>
</tr>
<tr>
<td>(1)</td>
<td>4 5 6 25 26 55 56 60 63 9 17 33 35 45 62</td>
<td>○</td>
</tr>
<tr>
<td>(2)</td>
<td>4 5 6 25 26 55 56 60 63 9 17 33 35 45 62</td>
<td>○</td>
</tr>
<tr>
<td>(3)</td>
<td>4 5 6 25 26 55 56 60 63 9 17 33 35 45 62</td>
<td>○</td>
</tr>
<tr>
<td>(4)</td>
<td>4 5 6 25 26 55 56 60 63 9 17 33 35 45 62</td>
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<tr>
<td>(5)</td>
<td>4 5 6 25 26 55 56 60 63 9 17 33 35 45 62</td>
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<td>(6)</td>
<td>4 5 6 25 26 55 56 60 63 9 17 33 35 45 62</td>
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<td>(7)</td>
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</tr>
<tr>
<td>(16)</td>
<td>4 5 6 25 26 55 56 60 63 9 17 33 35 45 62</td>
<td>○</td>
</tr>
</tbody>
</table>
simulation and the calculation value by the second equation of (9) is shown in Fig. 8. Both show a high correlation even though the calculation value by the approximation formula is slightly lower than the simulation value. It can thus be said that the approximation formula of equation (9) is sufficiently practicable. Optimization is therefore possible using \( e_m^* \) and \( \sigma_m^2 \) obtained from one simulation as constants when traffic counts are suppressed to a minimum number given the size of the estimation error such as in equation (12).

From the second equation of (9), the larger \( u \lambda_m \sigma_m^2 \) is, the larger the variance of estimation error becomes.

It is clear that there are differences in the estimation errors of trip distribution according to the arrangement of traffic counts. It has already been noted that \( \sigma_m^2 \) decreases along with the increase of link-flow\(^5\). Therefore, it is thought that the estimation error of trip distribu-

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**Fig. 5** Links composing solution 1 based on the OD covering method.

**Fig. 6** Average of given OD volumes and estimated ones.
Fig. 7 OD volume and standard deviation of estimation errors.

Fig. 8 Simulation value and calculation value on the standard variation of estimation error.

Fig. 9

(4) Extraction of essential links by zone covering standard

When the traffic counts were arranged under the zone covering standard, essential links were extracted from the sets of links of Fig. 2 as well as the OD covering standard. The denominator of equation (17) should be positive in order to obtain the trip generation of all zones in the network. The solution that meets this requirement and has minimum traffic counts was obtained by solving the problem of equation (18). Then, the optimization problem of equation (18) was solved for the sets of links of Fig. 2, and ten solutions with only one composition link were extracted.

The distribution of 10 links is shown in Fig. 9. The link number and the estimation...
accuracy of the obtained solution are shown in Table 2. There is a restriction of equation (16) between the zone covering method and the error control method regarding the trip generation estimation. And the minimum number of links necessary for estimating the trip generation is given without any relation to the size of the error from the restrictions of equation (16).

Because there is only one composition link in this example, the estimation error rate \( T_i/y_i \) of the trip generation in each solution is controlled by error \( (e_n, e_m) \) included in the link-flow. Only when the number of composition links is one, as in this example, will the relationship between the solution and accuracy by the error control method not depend on a special analysis and therefore be able to be read directly from Table 2. For instance, if \( T_i/y_i = 0.05 \) is restricted, the obtained solutions are nine, or (1)-(9) in Table 2.

![Fig. 9 Distribution of the links composing solutions based on zone covering method.](image)

<table>
<thead>
<tr>
<th>Solution number</th>
<th>Link number</th>
<th>Estimation error rate ( T_i/y_i ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>27</td>
<td>0.887</td>
</tr>
<tr>
<td>(2)</td>
<td>3</td>
<td>1.781</td>
</tr>
<tr>
<td>(3)</td>
<td>4</td>
<td>1.797</td>
</tr>
<tr>
<td>(4)</td>
<td>75</td>
<td>2.312</td>
</tr>
<tr>
<td>(5)</td>
<td>62</td>
<td>2.478</td>
</tr>
<tr>
<td>(6)</td>
<td>29</td>
<td>2.52</td>
</tr>
<tr>
<td>(7)</td>
<td>6</td>
<td>2.941</td>
</tr>
<tr>
<td>(8)</td>
<td>30</td>
<td>3.176</td>
</tr>
<tr>
<td>(9)</td>
<td>31</td>
<td>4.526</td>
</tr>
<tr>
<td>(10)</td>
<td>76</td>
<td>6.441</td>
</tr>
</tbody>
</table>

Table 2 Link number and estimation accuracy of the solution.
(5) Estimation accuracy of trip generation

In this section the estimation accuracy of trip generation and the approximation formula of the estimation error are analyzed. Trip generation \( y_i \) was obtained using equation (17) and Monte Carlo simulation where link 3 (represented by the \( \odot \) sign in Fig. 9) was assumed to be an observed link. The relation between \( y_i' \) (average value of \( y_i \)) and trip generation \( \eta_i \) is shown in Fig. 10. Both show an extremely high correlation. Moreover, the correlation between the variance of \( (y_i' - \eta_i) \) and the calculation value by the second equation (21) is shown in Fig. 11. Both show a high correlation, and the approximation formula (21) is sufficiently practicable.

Thus, when the number of traffic counts is suppressed to the minimum given the size of the estimation error, such as in equation (23), traffic counts can be optimized using \( e_m^* \) and \( \sigma_m^2 \) as constants obtained from the simulation. Figure 12 shows the relationship between the trip generation and the standard deviation of estimation error as well as the change caused by the selection of observed links. Standard deviation of the estimation error increases as

![Fig. 10 Traffic generation and the average of estimation value.](image)

![Fig. 11 Standard deviation of the estimation value of traffic generation.](image)
the trip generation increases. On the other hand, the relationships among these values show inclinations for great differences according to the selection of observed links. The inclination of the straight line shows \( \frac{\sigma_{yi}}{y_i} \) as the standard deviation ratio of the estimation error of trip generation, and it varies between a maximum of 6.82% and a minimum of 0.88%. Thus, the range of the estimation error changes through the selection of observation links, and the cause is in \( \sigma_m^2 \). Next, the relationship between \( t_m \) of the observed links and ratio \( \frac{\sigma_{yi}}{y_i} \) was examined in order to analyze the relationship between the size of \( \sigma_m^2 \) and the link characteristics. This is shown in Fig. 13. The value of \( \frac{\sigma_{yi}}{y_i} \) decreases rapidly with the increase of \( t_m \), and it is understood that the minimum value of \( \frac{\sigma_{yi}}{y_i} \) appears in the link where the maximum flow is achieved.

(6) Arrangement of traffic counts and estimation error

It would seem to be more effective to estimate trip generation than trip distribution directly from the viewpoint of estimation accuracy and the observation points. To confirm this idea, the estimation accuracies at the generation and the distribution were compared. Figure 14 shows the comparison of the directly estimated trip generation with the trip
generation that was calculated using the estimation value of trip distribution in the simulation. As trip generation increases, the estimation error of the former increases in a constant ratio, but the absolute value is small. On the other hand, the absolute value of the latter estimation error is large, and it is particularly large in cases of large trip generation.

Figure 15 shows the comparison of the direct estimation of trip distribution with the estimation of trip distribution derived from trip generation using the Fraters method. It is understood that the error of the former is smaller overall and that it is more accurate. Indeed, in the examples of this study, the error of the former was half or less of the error of the latter.

It is said that the accuracy of the direct estimation method is higher in both trip generation and trip distribution. Moreover, the arrangement of traffic counts for the estimation of trip generation is excellent with respect to economical surveys. However, when the estimation of the trip distribution is targeted, the arrangement of traffic counts to estimate the trip distribution is excellent with respect to estimation accuracy.

![Fig. 14](image1.png)  
**Fig. 14** Comparison of estimation method on traffic generation.

![Fig. 15](image2.png)  
**Fig. 15** Comparison of estimation method on OD volume.
7. Conclusions

In this paper a method was proposed for the estimation of traffic demand from the observation value of the link flow taking into consideration the arrangement problem of the traffic counts.

The contents are as follows.

1. A simple estimation equation (5) of the trip distribution was introduced, the condition that enables the estimation was shown, and it was clarified that this condition was the “OD covering standard” which had been previously proposed. Next, the approximation formula for the variance of estimation error $\Delta x_{ij}$ was introduced, and the optimization problem was formulated based on the idea of the arrangement of traffic counts through which the estimation error could be controlled.

2. The simple estimation formula of the trip generation was shown by equation (17), and the necessary condition (zone covering standard) was introduced. Next, the estimation error of the trip generation and the estimation formula of the variance were formulated, and the problem of traffic counts arrangement was formulated, through which the estimation error was controlled.

3. The optimization problem for trip distribution estimation was solved on the real network of the Fukuoka City Region, and the estimation accuracy was examined for 16 solutions consisting of 20 observation links. As a result, it became clear that these methods are effective in obtaining an average estimation of extremely high accuracy, and that the approximation equation of standard deviation of the estimation error is sufficiently accurate. Next, the influence of the observation points was analyzed with regard to the estimation accuracy of the trip distribution, and it became clear that the estimation accuracy of the trip distribution improved when the observation points were put on links of heavy traffic.

4. Ten solutions with one observation link were obtained and the estimation accuracy was examined by solving the optimization problem for the estimation of trip generation. Results clarified that this method gave an average estimation value of extremely high accuracy and that the equation of standard deviation of the estimation error was very accurate. Next, the selection of observation points that influenced the estimation accuracy was analyzed, and it became clear that that estimation accuracy was greatly affected according to the selection of observation points, and that links with heavy traffic improved the estimation accuracy of trip generation when they were adopted as observation points.

5. As for the arranging methods of traffic counts for the estimation of trip distribution and trip generation directly, it was clarified that the former is excellent regarding estimation accuracy and that the latter is excellent with regard to economical concerns.

References

