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出版情報：MHF Preprint Series. 2006-30, 2006-11-17. 九州大学大学院数理学研究院
バージョン：
権利関係：

MHF Preprint Series

Kyushu University
21st Century COE Program
Development of Dynamic Mathematics with High Functionality

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MHF 2006-30

(Received November 17, 2006)

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Contemporary Mathematics

Stochastic analysis and the KdV equation

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Dedicated to Professor Yoichiro Takahashi on the occasion of his 60th birthday

Abstract. N. Ikeda and the author established a mapping from a class of Gaussian measures parameterized by linear combinations of Dirac measures on $\mathbb{R}$ to that of reflectionless potentials. The bijectivity of the mapping was shown by the author, and was extended to a class of more general Gaussian measures.

In this paper, a brief review on the bijection and its application to the KdV equation is given first. Next another application to the stochastic KdV equation is discussed. Finally presented is an alternative approach to the bijection via the linear filtering theory.

1. Introduction

Applications of stochastic analysis to the theory of partial differential equations (PDE in short) have their source in stochastic representations of solutions; let $\mathcal{L}^V$ be a second order differential operator on $\mathbb{R}^n$ of the form

$$\mathcal{L}^V = \frac{1}{2}\sum_{i,j=1}^{n} a^{ij}\frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b^i \frac{\partial}{\partial x_i} + V,$$

where a^{ij}, b^i, $1 \le i, j \le n$, and V are appropriately smooth functions from $\mathbb{R}^n$ to $\mathbb{R}$. The solution $u = u(x,t)$ of the Cauchy problem of the PDE

$$\frac{\partial u}{\partial t} = \mathcal{L}^V u, \quad u(\cdot, 0) = f$$

is represented as

$$u(x,t) = \mathbb{E}\left[f(X(t,x))\exp\left(\int_0^t V(X(s,x))ds\right)\right],$$

where $\{X(t,x)\}_{t\ge 0}$ is the diffusion process stating from x at time 0, which is generated by $\mathcal{L}^0$, and $\mathbb{E}$ stands for the expectation with respect to the underling probability measure. This kind of expression goes back to the studies in 1940's made

2000 *Mathematics Subject Classification.* Primary 60H30; 60B10; Secondary 34L25.

Key words and phrases. Wiener integral, Reflectionless potential, KdV equation, Linear filtering theory.

Research supported in part by Grant-in-Aid for Scientific Research (A) 14204010.

by K. Itô [**It**], R. Cameron-W. Martin [**CM**], and M. Kac [**Ka**]. In this paper, we shall first give a review on such expressions for classical and generalized reflectionless potentials, and n-soliton solutions of the Korteweg-de Vries (KdV in short) equation

$$\frac{\partial v}{\partial t} = \frac{3}{2}\, v\, \frac{\partial v}{\partial x} + \frac{1}{4}\, \frac{\partial^3 v}{\partial x^3}. \tag{1.1}$$

Secondly, we shall apply our probabilistic expression to the stochastic KdV equation observed in [**W**]. Finally we shall give an alternative proof of the probabilistic expressions of reflectionless potentials by using the filtering theory.

The author is grateful to Professors Gui-Qiang Chen, Elton Hsu, and Mark Pinsky for the stimulating conference at the Northwestern University in June, 2005, and their hospitality.

2. Reflectionless potentials

Let

$$\mathcal{S} = \Big\{ \{\eta_j, m_j\}_{1\le j\le n} \,\Big|\, n \in \mathbb{N},\ \eta_j, m_j > 0,\ \eta_i \ne \eta_j \text{ if } i \ne j \Big\}.$$

A classical reflectionless potential $u_{\mathbf{s}}$ with scattering data $\mathbf{s} = \{\eta_j, m_j\}_{1\le j\le n} \in \mathcal{S}$ is by definition the function

$$u_{\mathbf{s}}(x) = -2\frac{d^2}{dx^2} \log\det(I + G_{\mathbf{s}}(x))$$

where

$$G_{\mathbf{s}}(x) = \left(\frac{\sqrt{m_i m_j}\, e^{-(\eta_i+\eta_j)x}}{\eta_i + \eta_j} \right)_{1\le i,j\le n}.$$

Applying the inverse scattering theory to the Schrödinger operator $-(d/dx)^2 + u_{\mathbf{s}}$, one can spell out the scattering data $\mathbf{s}$ from $u_{\mathbf{s}}$. Thus we can identify the space

$$\Xi_0 = \{u_{\mathbf{s}} \,|\, \mathbf{s} \in S\}$$

of all classical reflectionless potentials with $\mathcal{S}$. We say that u is a generalized reflectionless potential if there exist $\mu > 0$ and $\{u_n\}_{n=1}^{\infty} \subset \Xi_0$ such that u_n converges to u uniformly on compacts and

$$\mathrm{Spec}\left(-\frac{d^2}{dx^2} + u_n \right) \subset [-\mu, \infty), \quad n = 1, 2, \dots,$$

where $\mathrm{Spec}(-(d/dx)^2 + u_n)$ stands for the spectrum of $-(d/dx)^2 + u_n$. Let Ξ be the space of all generalized reflectionless potentials. The space Ξ was used by D. Lundina [**L**] and V. Marchenko [**Ma2**] to study the Cauchy problem of the KdV equation, and by S. Kotani [**Ko1, Ko2**] to construct the KdV-flow.

Let $\mathcal{W}$ be the space of all continuous functions $w : [0, \infty) \to \mathbb{R}$ with $w(0) = 0$. We denote by $\{X(x)\}_{x\ge 0}$ the coordinate mapping on $\mathcal{W}$; $X(x) : \mathcal{W} \ni w \mapsto X(x, w) = w(x) \in \mathbb{R}$. Let Σ be the space of all finite measures on $\mathbb{R}$ with compact support. For $\sigma \in \Sigma$, P^σ denotes the probability measure on $\mathcal{W}$ under which $\{X(x)\}_{x\ge 0}$ is a centered Gaussian process with covariance function

$$\int_{\mathcal{W}} X(x)X(y)dP^\sigma = \int_{\mathbb{R}} \frac{e^{\zeta(x+y)} - e^{\zeta|x-y|}}{2\zeta} \sigma(d\zeta).$$

Set

$$\mathcal{G} = \{P^\sigma \,|\, \sigma \in \Sigma\}.$$

Since

$$\frac{d}{dx}\int_{\mathcal{W}} X(x)^2 dP^\sigma = \int_{\mathbb{R}} e^{2\zeta x}\sigma(d\zeta),$$

we may and will identify Σ with $\mathcal{G}$.

For $P^\sigma \in \mathcal{G}$, we define $\Phi_\sigma : [0,\infty) \to (0,\infty)$ by

$$\Phi_\sigma(x) = \int_{\mathcal{W}} \exp\Big(-\frac{1}{2}\int_0^x X(y)^2 dy\Big) dP^\sigma, \quad x \geq 0.$$

Then $\Phi_\sigma \in C^\infty([0,\infty))$ (see [**T2**]), and hence we can define $\psi : \mathcal{G} \to C([0,\infty))$ by

$$\psi(P^\sigma)(x) = 4\frac{d^2}{dx^2}\log \Phi_\sigma(x), \quad x \geq 0.$$

Put

$$\Sigma_0 = \Big\{\sigma = \sum_{j=1}^n c_j^2 \delta_{p_j} \,\Big|\, n \in \mathbb{N}, c_j > 0, p_j \in \mathbb{R},\ j = 1,\dots,n, p_j \neq p_i\ (i \neq j)\Big\}.$$

For $\sigma = \sum_{j=1}^n c_j^2\delta_{p_j} \in \Sigma_0$, we can construct P^σ by taking advantage of stochastic integrals. Namely, take an n-dimensional Brownian motion $\{b(x)\}_{x\geq 0}$ on the probability space $(\Omega, \mathcal{F}, P)$. Representing elements of $\mathbb{R}^n$ as column vectors, we define the $\mathbb{R}^n$-valued Ornstein-Uhlenbeck process $\{\xi_\sigma(x)\}_{x\geq 0}$ and the $\mathbb{R}$-valued centered Gaussian process $\{X_\sigma(x)\}_{x\geq 0}$ by

$$\xi_\sigma(x) = e^{xD_\sigma}\int_0^x e^{-yD_\sigma} db(y) \quad \text{and} \quad X_\sigma(x) = \langle \mathbf{c}, \xi_\sigma(x)\rangle,$$

respectively, where $D_\sigma = \mathrm{diag}[p_j]$ (the diagonal matrix with $p_1,\dots,p_n$ being diagonal entries), $e^A = \sum_{k=0}^\infty A^k/k!$ for $n\times n$-matrix A, $\langle\cdot,\cdot\rangle$ is the standard inner product in $\mathbb{R}^n$, and $\mathbf{c} = {}^t(c_1,\dots,c_n)$, the transposed vector of $(c_1,\dots,c_n)$. It is easily seen that P^σ coincides with the induced measure $P\circ X_\sigma^{-1}$ on $\mathcal{W}$ through $X_\sigma : \Omega \to \mathcal{W}$.

For $\sigma = \sum_{j=1}^n c_j^2\delta_{p_j} \in \Sigma_0$, without loss of generality, we may and will assume that there exist $m < n$ and $1 \leq j(1) < \cdots < j(m) \leq n$ such that

$$|p_j| \leq |p_{j+1}|, \quad p_{j(\ell)} > 0, \quad p_{j(\ell)+1} = -p_{j(\ell)}, \quad \#\{|p_1|,\dots,|p_n|\} = n-m.$$

Let $0 < r_1 < \cdots < r_{n-m}$ be the roots of the algebraic equation

$$\sum_{j=1}^n \frac{c_j^2}{p_j^2 - r} = -1. \tag{2.1}$$

We define the mapping $\overline{\psi} : \Sigma_0 \ni \sigma \mapsto \overline{\psi}(\sigma) = \{\eta_j, m_j\} \in \mathcal{S}$ by

$$\{\eta_1 < \cdots < \eta_n\} = \big\{p_{j(1)},\dots,p_{j(m)}, \sqrt{r_1},\dots,\sqrt{r_{n-m}}\big\}$$

and

$$m_i = \begin{cases} 2\eta_i \dfrac{c_{j(\ell)+1}^2}{c_{j(\ell)}^2}\displaystyle\prod_{k\neq i}\frac{\eta_k+\eta_i}{\eta_k-\eta_i}\prod_{k\neq j(\ell), j(\ell)+1}\frac{p_k+\eta_i}{p_k-\eta_i}, & \text{if } i = j(\ell), \\ -2\eta_i\displaystyle\prod_{k\neq i}\frac{\eta_k+\eta_i}{\eta_k-\eta_i}\prod_{k=1}^n\frac{p_k+\eta_i}{p_k-\eta_i}, & \text{otherwise.}\end{cases} \tag{2.2}$$

THEOREM 2.1 ([**IkT, T2**]). *Let $P^\sigma \in \mathcal{G}_0$ and $\overline{\psi}(\sigma) = \{\eta_j, m_j\}_{1\le j\le n}$.*
(i) *It holds that*

$$4\log\Phi_\sigma(x) = -2\log\det\Big(I + G_{\overline{\psi}(\sigma)}(x)\Big) + 2\log\det\Big(I + G_{\overline{\psi}(\sigma)}(0)\Big) - 2x\sum_{i=1}^n(p_i+\eta_i), \quad x \ge 0. \tag{2.3}$$

(ii) *$\psi(\mathcal{G}_0) \subset \Xi_0$ and $\psi(P^\sigma) = u_{\overline{\psi}(\sigma)}$.*
(iii) *$\psi : \mathcal{G}_0 \to \Xi_0$ is bijective.*

The second assertion should be understood as follows. Due to the first assertion, one has that $\psi(P^\sigma) = u_{\overline{\psi}(\sigma)}$ on $[0,\infty)$. Being real analytic on $\mathbb{R}$, $u_{\overline{\psi}(\sigma)}$ is determined completely by $\psi(P^\sigma)$. In this sense, one thinks of $\psi(P^\sigma)$ as an element in Ξ_0.

In the proof of (2.3) given in [**IkT**], the change of variable formula on $\mathcal{W}$, so called the Cameron-Martin formula, played a key role. In the next section, we shall give a new proof of the identity (2.3) by using the filtering theory.

To see the bijectivity described in the assertion (iii), it suffices to construct the inverse mapping of $\overline{\psi}$. For this purpose, let $u = u_{\mathbf{s}} \in \Xi_0$ ($\mathbf{s} \in \mathcal{S}$) and $e^+(x;\zeta)$ be the right Jost solution of the Schrödinger operator $\mathcal{L} = -(d/dx)^2 + u_{\mathbf{s}}$;

$$\mathcal{L}e^+(*;\zeta) = \zeta^2 e^+(*;\zeta), \quad e^+(x;\zeta) \sim e^{i\zeta x}\ (x\to\infty).$$

Then, it is known ([**L, Ma2**]) that there are $\lambda_j \in C^\infty(\mathbb{R};\mathbb{R})$, $1 \le j \le n$, such that

$$e^+(x;\zeta) = e^{\sqrt{-1}\,\zeta x}\prod_{j=1}^n \frac{\zeta - \sqrt{-1}\,\lambda_j(x)}{\zeta + \sqrt{-1}\,\eta_j} \quad \text{and} \quad \lambda_j' < 0,\ 1 \le j \le n.$$

If we set $\kappa(\mathbf{s}) = \sum_{j=1}^n(-\lambda_j'(0))\delta_{\lambda_j(0)}$, then $\kappa = \overline{\psi}^{-1}$.

On account of the above observation on the Jost solution, one can give stochastic expressions on $(-\infty, 0]$;

THEOREM 2.2 ([**T2**]). *Let $P^\sigma \in \mathcal{G}_0$.*
(i) *If we define $\mu \in \Sigma_0$ by $\mu(A) = \sigma(-A)$, $A \in \mathcal{B}(\mathbb{R})$ ($\equiv$the Borel field on $\mathbb{R}$), then*

$$u_{\overline{\psi}(\sigma)}(x) = \psi(P^\mu)(-x) \quad \text{for } x \in (-\infty, 0].$$

(ii) *For $y \le 0$, let $b(y) = b(-y)$. Define*

$$\xi_\sigma(y) = -e^{yD_\sigma}\int_y^0 e^{-zD_\sigma}db(z), \quad X_\sigma(y) = \langle \mathbf{c}, \xi_\sigma(y)\rangle.$$

Then $u = \psi(P^\sigma)$ is represented as

$$u(x) = 4\frac{d^2}{dx^2}\log\bigg(\int_\Omega \exp\bigg(-\frac{1}{2}\int_{\min\{0,x\}}^{\max\{0,x\}} X_\sigma(y)^2dy\bigg)dP\bigg) \quad \text{for every } x \in \mathbb{R}.$$

The above bijectivity extends to that between $\mathcal{G}$ and Ξ;

THEOREM 2.3 ([**T2**]). (i) *Let $\sigma_n \in \Sigma_0$, $n = 1, 2, \ldots$, and $\sigma \in \Sigma$. Assume that σ_n converges to σ vaguely as $n \to \infty$ and $\bigcup_{n\in\mathbb{N}}\operatorname{supp}\sigma_n \subset [-\beta, \beta]$ for some $\beta > 0$. Then $\psi(P^{\sigma_n})$ tends to $\psi(P^\sigma)$ uniformly on compacts in $[0,\infty)$ as $n \to \infty$. Moreover, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that*

$$\bigcup_{n\ge n_0}\operatorname{Spec}\Big(-\frac{d^2}{dx^2} + \psi(P^{\sigma_n})\Big) \subset [-\beta^2 - \sigma(\mathbb{R}) - \varepsilon, \infty).$$

(ii) *Let $\sigma \in \Sigma$, and define $\mu \in \Sigma$ by $\mu(A) = \sigma(-A)$, $A \in \mathcal{B}(\mathbb{R})$. Set*

$$u(x) = \begin{cases} \psi(P^\sigma)(x), & x \geq 0, \\ \psi(P^\mu)(-x), & x \leq 0. \end{cases}$$

Then $u \in \Xi$. Conversely every $u \in \Xi$ is of the above form.

For $\sigma \in \Sigma$, σ_n's constructed as follows satisfy the conditions described in (i);

$$\sigma_n = \sum_{j=-n}^{n} \Big\{ \sigma\big([j\beta/n, (j+1)\beta/n)\big) + \frac{1}{n} \Big\} \delta_{j\beta/n},$$

where $\beta > 0$ has been chosen so that $\operatorname{supp} \sigma \subset [-\beta, \beta]$.

3. The KdV equation

As is well known ([**MiJD**]), soliton solutions of the KdV equation and the τ-function of the KdV hierarchy are of the same form as classical reflectionless potential. Namely, the τ-function of the KdV hierarchy is given by

$$\tau(x, \mathbf{t}) = \det(I + A(x, \mathbf{t}))$$

where $x \in \mathbb{R}$, $\mathbf{t} = (t_1, t_2, \dots) \in \mathbb{R}^{\mathbb{N}}$ satisfies that $\#\{t_j \neq 0\} < \infty$, $\{\eta_j, m_j\} \in \mathcal{S}$, $\zeta_i(x, \mathbf{t}) = x\eta_i + \sum_{\alpha=1}^{\infty} t_\alpha \eta_i^{2\alpha+1}$, and

$$A(x, \mathbf{t}) = \left(\frac{\sqrt{m_i m_j}}{\eta_i + \eta_j} \, e^{-\{\zeta_i(x,\mathbf{t}) + \zeta_j(x,\mathbf{t})\}} \right)_{1 \leq i,j \leq n}.$$

If $\mathbf{t} = (t, 0, \dots)$, then $v(x, t) = 2(\partial/\partial x)^2 \log \tau(x, \mathbf{t})$ is a soliton solution of the KdV equation (1.1) ([**MiJD**]).

As was seen in [**IkT**], we also have a stochastic expression of the τ-function. To recall this, let $\sigma \in \Sigma_0$ and $\overline{\psi}(\sigma) = \{\eta_j, m_j\} \in \mathcal{S}$. If we denote by R the diagonal matrix $\operatorname{diag}[\eta_j]$ with η_j's being the diagonal entries, then there exists $U \in O(n)$ such that $D_\sigma^2 + \mathbf{c} \otimes \mathbf{c} = U R^2 U^{-1}$, where $\mathbf{c} \otimes \mathbf{c} = \big(c_i c_j\big)_{1 \leq i,j \leq n}$. Set $\zeta(x, \mathbf{t}) = \operatorname{diag}[\zeta_j(x, \mathbf{t})]$, and define the $n \times n$-matrix $\phi(x, \mathbf{t})$ by

$$\phi(x, \mathbf{t}) = U\big\{\cosh(\zeta(x, \mathbf{t})) - \sinh(\zeta(x, \mathbf{t}))R^{-1}U^{-1}D_\sigma U\big\}U^{-1}.$$

Then, for every $(x, \mathbf{t})$, $\det \phi(x, \mathbf{t}) \neq 0$ and one can define the $n \times n$-matrix $\beta_{\mathbf{t}}(x)$ by

$$\beta_{\mathbf{t}}(x) = -\frac{\partial \phi}{\partial x}(x, \mathbf{t})\, \phi^{-1}(x, \mathbf{t}).$$

Set

$$I_\sigma(x, \mathbf{t}) = \int_\Omega \exp\Big(-\frac{1}{2} \int_0^x X_\sigma(y)^2 dy + \frac{1}{2} \langle (\beta_{\mathbf{t}}(0) - D_\sigma)\xi_\sigma(x), \xi_\sigma(x) \rangle \Big) dP.$$

We then have that

THEOREM 3.1 ([**IkT**]). (i) *It holds that*

$$\log I_\sigma(x, \mathbf{t}) = -\frac{1}{2} \log \tau(x, \mathbf{t}) + \frac{1}{2} \log \tau(0, \mathbf{t}) - \frac{x}{2} \sum_{i=1}^{n} (p_i + \eta_i).$$

(ii) *If $\mathbf{t} = (t, 0, \dots)$, then the function*

$$v_\sigma(x, \mathbf{t}) = -4 \left(\frac{\partial}{\partial x} \right)^2 \log I_\sigma(x, \mathbf{t})$$

is an n-soliton solution of the KdV equation (1.1).

We next consider the stochastic KdV equation dealt with in [**W**];

$$d_t v - \left\{\frac{3}{2}\, v\, \frac{\partial v}{\partial x} + \frac{1}{4}\, \frac{\partial^3 v}{\partial x^3}\right\} dt = a\, dW(t), \tag{3.1}$$

where $a \in \mathbb{R}$ and $\{W(t)\}_{t\geq 0}$ is a 1-dimensional Brownian motion which is independent of $\{b(x)\}_{x\geq 0}$. For $t \geq 0$, let $\mathbf{t} = (t, 0, \dots)$ and write $A(x,t)$ and $\beta_t(x)$ for $A(x, \mathbf{t})$ and $\beta_{\mathbf{t}}(x)$. Define

$$I_\sigma(x,t;W) = \int_\Omega \exp\biggl(-\frac{1}{2}\int_0^x X_\sigma(y)^2 dy + \frac{1}{2}\biggl\langle\biggl\{\beta_t\biggl(-\frac{3a}{2}\int_0^t W(u)du\biggr) - D_\sigma\biggr\}\xi_\sigma(x), \xi_\sigma(x)\biggr\rangle\biggr) dP.$$

Then we have that

THEOREM 3.2. *The function given by*

$$v(x,t;W) = aW(t) - 4\biggl(\frac{\partial}{\partial x}\biggr)^2 \log I_\sigma(x,t;W)$$

is a solution of the stochastic KdV equation (3.1).

PROOF. In repetition of the argument in [**IkT**], we have that

$$\begin{aligned}\log\biggl(\int_\Omega \exp\biggl(-\frac{1}{2}\int_0^x X_\sigma(y)^2 dy + \frac{1}{2}\langle\{\beta_t(x_0) - D_\sigma\}\xi_\sigma(x), \xi_\sigma(x)\rangle\biggr) dP\biggr) \\ = -\frac{1}{2}\log\det(I + A(x+x_0,t)) + \frac{1}{2}\log\det(I + A(x_0,t)) - \frac{x}{2}\sum_{i=1}^n (p_i + \eta_i)\end{aligned} \tag{3.2}$$

for any $x_0 \in \mathbb{R}$. While the stochastic KdV equation dealt with by Wadati [**W**] was of the form

$$d_t v - \left\{6v\frac{\partial v}{\partial x} - \frac{\partial^3 v}{\partial x^3}\right\} dt = dW(t),$$

after the standard change of variables, his result is rewritten in our setting as follows; the function

$$v(x,t;W) = aW(t) + 2\biggl(\frac{\partial}{\partial x}\biggr)^2 \log\biggl(\det\biggl[I + A\biggl(x - \frac{3a}{2}\int_0^t W(u)du, t\biggr)\biggr]\biggr)$$

solves the stochastic KdV equation (3.1). Combining this with (3.2), we arrive at the desired assertion. □

4. The filtering theory

The results in the previous sections, which connects the Gaussian measures and reflectionless potentials and the KdV equation, start from the identity (2.3). In [**IkT**], the identity was shown by using the Cameron-Martin formula on the Wiener space. In this section, we first give an alternative proof of (2.3) with the help of the linear filtering theory. Secondly we investigate Φ_σ from the Gaussian filtering theoretical point of view to revisit the Marchenko formula.

4.1. Proof of (2.3) via the linear filtering theory. Let $\sigma = \sum_{j=1}^n c_j^2 \delta_{p_j} \in \Sigma_0$. Throughout this subsection, we assume that

$$|p_1| < \cdots < |p_n|.$$

The general case follows from this one in the standard limiting procedure as was seen in [**IkT**]. Let $\{b(x)\}_{x\geq 0}$ be an n-dimensional Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, P, \{\mathcal{F}_x\}_{x\geq 0})$, and define $\{\xi_\sigma(x) = {}^t(\xi_\sigma^1(x), \dots, \xi_\sigma^n(x))\}_{x\geq 0}$ and $\{X_\sigma(x)\}_{x\geq 0}$ as in Section 2.

Take a 1-dimensional Brownian motion $\{N(x)\}_{x\geq 0}$ with $N(0) = 0$, which is independent of $\{b(x)\}_{x\geq 0}$, and set

$$Z_\sigma(x) = \int_0^x X_\sigma(y)dy + N(x),$$

$\mathcal{Z}_x = \sigma[Z_\sigma(y); y \leq x]$, $\widehat{X}_\sigma(x) = \mathbb{E}[X_\sigma(x)|\mathcal{Z}_x]$, the conditional expectation of $X_\sigma(x)$ given $\mathcal{Z}_x$, and

$$S_\sigma(x) = \int_\Omega \big(X_\sigma(x) - \widehat{X}_\sigma(x)\big)^2 dP.$$

It was shown by Kleptsyna-Le Breton [**KlB**] that

$$\int_\Omega \exp\biggl(-\frac{1}{2}\int_0^x X_\sigma(y)^2 dy\biggr)dP = \exp\biggl(-\frac{1}{2}\int_0^x S_\sigma(y)dy\biggr).$$

Combined with the definition of $X_\sigma(x)$, this implies that

$$\log\biggl(\int_\Omega \exp\biggl(-\frac{1}{2}\int_0^x X_\sigma(y)^2 dy\biggr)dP\biggr) = -\frac{1}{2}\int_0^x \biggl(\sum_{i,j=1}^n c_i c_j P_{ij}(y)\biggr)dy, \tag{4.1}$$

where

$$P_{ij}(y) = \int_\Omega (\xi_\sigma^i(y) - \widehat{\xi_\sigma^i}(y))(\xi_\sigma^j(y) - \widehat{\xi_\sigma^j}(y))dP.$$

Since the $n \times n$-matrix valued function $P = (P_{ij})$ is the error matrix of the linear filtering problem

$$\begin{cases} d\xi_\sigma(x) = D_\sigma \xi_\sigma(x)dx + db(x) & \text{(system)}, \\ dZ_\sigma(x) = \langle \mathbf{c}, \xi_\sigma(x)\rangle dx + dN(x) & \text{(observation)}, \end{cases}$$

it obeys the ordinary differential equation

$$\frac{dP}{dx}(x) = D_\sigma P(x) + P(x)D_\sigma - P(x)(\mathbf{c}\otimes\mathbf{c})P(x) + I, \quad P(0) = 0, \tag{4.2}$$

where I denotes the $n \times n$ unit matrix. See [**BJ**]. Thus, what is needed to show (2.3) is a precise expression of $\sum_{i,j=1}^n c_i c_j P_{ij}(x)$.

Put

$$H = \begin{pmatrix} -D_\sigma & \mathbf{c}\otimes\mathbf{c} \\ I & D_\sigma \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} U_0(x) \\ V_0(x) \end{pmatrix} = \exp(xH)\begin{pmatrix} I \\ 0 \end{pmatrix},$$

where 0 denotes the $n \times n$ zero matrix. Since

$$\langle U_0(x)v, V_0(x)v\rangle = \int_0^x \bigl\{|U_0(y)v|^2 + \langle \mathbf{c}, V_0(y)v\rangle^2\bigr\}dy, \quad v \in \mathbb{R}^n,$$

we see that $\det U_0(x) \neq 0$ for any $x \geq 0$. It is then easily checked that

$$P(x) = V_0(x)U_0(x)^{-1}. \tag{4.3}$$

Let $0 < r_1 < \cdots < r_n$ be the roots of (2.1), and set $\eta_j = \sqrt{r_j}$. Define $n \times n$-matrices A, B, C and $(2n) \times (2n)$-matrix S by

$$A = \big((D_\sigma + \eta_j)^{-1}\mathbf{c}\big)_{1\le j\le n}, \quad B = \big((D_\sigma - \eta_j)^{-1}\mathbf{c}\big)_{1\le j\le n},$$
$$C = \big(-(D_\sigma^2 - r_j)^{-1}\mathbf{c}\big)_{1\le j\le n}, \quad S = \begin{pmatrix} A & B \\ C & C \end{pmatrix}.$$

Then $\det S \neq 0$ and it holds that

$$H = S\begin{pmatrix} R & 0 \\ 0 & -R \end{pmatrix} S^{-1}.$$

Since

$$S = \begin{pmatrix} A-B & B \\ 0 & C \end{pmatrix}\begin{pmatrix} I & 0 \\ I & I \end{pmatrix},$$

$\det(A-B) \neq 0$ and $\det C \neq 0$. We then have that

$$\exp(xH) = \begin{pmatrix} Ae^{xR} & Be^{-xR} \\ Ce^{xR} & Ce^{-xR} \end{pmatrix}\begin{pmatrix} (A-B)^{-1} & -(A-B)^{-1}BC^{-1} \\ -(A-B)^{-1} & (A-B)^{-1}BC^{-1} + C^{-1} \end{pmatrix}.$$

Due to (4.3), this implies that

$$P(x) = C(e^{xR} - e^{-xR})\big\{Ae^{xR} - Be^{-xR}\big\}^{-1}.$$

Define $n \times n$-matrices X, Y, Z by

$$X = \left(\frac{1}{p_i + \eta_j}\right)_{1\le i,j\le n}, \quad Y = \left(\frac{1}{p_i - \eta_j}\right)_{1\le i,j\le n}, \quad Z = \left(\frac{1}{p_i^2 - r_j}\right)_{1\le i,j\le n}.$$

Since $A = \mathrm{diag}[c_j]X$, $B = \mathrm{diag}[c_j]Y$, $C = -\mathrm{diag}[c_j]Z$, we obtain that

$$P(x) = -\mathrm{diag}[c_j]Z\big(e^{xR} - e^{-xR}\big)\big\{Xe^{xR} - Ye^{-xR}\big\}^{-1}\mathrm{diag}[1/c_j].$$

In conjunction with the definition of r_k's, this yields that

$$\sum_{i,j=1}^n c_i c_j P_{ij}(x) = \sum_{i,j=1}^n Q_{ij}(x), \tag{4.4}$$

where

$$Q(x) = \big(I - e^{-2xR}\big)\big\{X - Ye^{-2xR}\big\}^{-1}.$$

In the sequel, we compute $\sum_{i,j=1}^n Q_{ij}(x)$ algebraically, and do not use the dependence of η_j's on p_i's. Hence, in what follows we only assume that $p_i \neq p_j$ and $\eta_i \neq \eta_j$ if $i \neq j$, $\eta_j > 0$, $1 \le j \le n$, and $\{p_1, \ldots, p_n\} \cap \{\eta_1, \ldots \eta_n\} = \emptyset$. Define $m_i > 0$ by (2.2);

$$m_i = -2\eta_i \prod_{k\neq i} \frac{\eta_k + \eta_i}{\eta_k - \eta_i} \prod_{k=1}^n \frac{p_k + \eta_i}{p_k - \eta_i}.$$

If we set $T(\zeta) = X - Y\mathrm{diag}[\zeta_j^2]$, $\zeta = {}^t(\zeta_1, \ldots, \zeta_n) \in \mathbb{R}^n$, then

$$T({}^t(e^{-x\eta_1}, \ldots, e^{-x\eta_n})) = X - Ye^{-2xR}$$

and

$$\det T(\zeta) = \det X + \sum_{p=1}^n \sum_{1\le j_1<\cdots<j_p\le n} (\det X_{j_1\ldots j_p})\zeta_{j_1}^2 \ldots \zeta_{j_p}^2,$$

where $X_{j_1\dots j_p}$ is the matrix obtained by replacing all j_k-th column, $1\le k\le p$, of X by ${}^t\big(-1/(p_i-\eta_{j_k})\big)_{1\le i\le n}$, $1\le k\le p$, respectively. Due to Cauchy's identity

$$\det\left[\left(\frac{1}{\alpha_i+\beta_j}\right)_{1\le i,j\le n}\right]=\frac{\prod_{1\le i<j\le n}(\alpha_i-\alpha_j)(\beta_i-\beta_j)}{\prod_{i,j=1}^n(\alpha_i+\beta_j)},$$

we obtain that

$$\det X_{j_1\dots j_p}=\prod_{k=1}^p\frac{m_{j_k}}{2\eta_{j_k}}\prod_{1\le k<\ell<p}\left(\frac{\eta_{j_k}-\eta_{j_\ell}}{\eta_{j_k}+\eta_{j_\ell}}\right)^2\times\det X. \tag{4.5}$$

Since

$$\begin{aligned}&\det(I+G_{\mathbf{s}}(x))\\&\quad=1+\sum_{p=1}^n\sum_{1\le i_1<\cdots<i_p\le n}\prod_{j=1}^p\frac{m_{i_j}}{2\eta_{i_j}}\prod_{1\le j<k\le p}\left(\frac{\eta_{i_j}-\eta_{i_k}}{\eta_{i_j}+\eta_{i_k}}\right)^2\exp\Big(-2x\sum_{j=1}^p\eta_{i_j}\Big),\end{aligned} \tag{4.6}$$

we have that

$$\det(X-Ye^{-2xR})=\det X\cdot\det(I+G_{\mathbf{s}}(x)). \tag{4.7}$$

Denote by $\widetilde{T}(\zeta)$ the cofactor matrix of $T(\zeta)$, and set $\widehat{Q}(\zeta)=(I-\mathrm{diag}[\zeta_j^2])\widetilde{T}(\zeta)$. It then holds that

$$Q(x)=\frac{1}{\det T(\zeta)}\widehat{Q}(\zeta)\Big|_{\zeta={}^t(e^{-x\eta_1},\dots,e^{-x\eta_n})}. \tag{4.8}$$

If we use $T_{<i>}(\zeta)$ to indicate the matrix obtained by replacing the i-th line of $T(\zeta)$ by $(1-\zeta_1^2,\dots,1-\zeta_n^2)$, then it holds that

$$\sum_{k,i=1}^n\widehat{Q}_{ki}(\zeta)=\sum_{k,i=1}^n(1-\zeta_k^2)\widetilde{T}_{ki}(\zeta)=\sum_{i=1}^n\det T_{<i>}(\zeta).$$

Since $\lim_{p_i\to\infty}p_iT_{ik}(\zeta)=1-\zeta_k^2$, $1\le k\le n$, we observe that

$$\det T_{<i>}(\zeta)=\lim_{p_i\to\infty}p_i\det T(\zeta)=\sum_{p=0}^n\sum_{1\le j_1<\cdots<j_p\le n}\big(\det X_{j_1\dots j_p;i}\big)\zeta_{j_1}^2\dots\zeta_{j_p}^2,$$

where $X_{j_1\dots j_p;i}=\big((X_{j_1\dots j_p;i})_{k\ell}\big)_{1\le k,\ell\le n}$ is given by

$$\big(X_{j_1\dots j_p;i}\big)_{k\ell}=\begin{cases}1/(p_k+\eta_\ell), & k\ne i,\ \ell\notin\{j_1,\dots,j_p\},\\ -1/(p_k-\eta_\ell), & k\ne i,\ \ell\in\{j_1,\dots,j_p\},\\ 1, & k=i,\ \ell\notin\{j_1,\dots,j_p\},\\ -1, & k=i,\ \ell\in\{j_1,\dots,j_p\}.\end{cases}$$

In the above expression, for $p=0$, $\zeta_{j_1}^2\dots\zeta_{j_p}^2=1$ and the summation $\sum_{1\le j_1<\cdots<j_p\le n}$ is just one term determined as $\{j_1,\dots,j_p\}=\emptyset$. Thus we obtain that

$$\sum_{i,j=1}^n\widehat{Q}_{ij}(\zeta)=\sum_{p=0}^n\sum_{1\le j_1<\cdots<j_p\le n}\Big(\sum_{i=1}^n\det X_{j_1\dots j_p;i}\Big)\zeta_{j_1}^2\dots\zeta_{j_p}^2. \tag{4.9}$$

For

$$K=\left(\frac{1}{\alpha_i+\beta_j}\right)_{1\le i,j\le n}$$

and its cofactor matrix $\widetilde{K}$, it holds that

$$\sum_{i,j=1}^{n} \widetilde{K}_{ij} = \det K \sum_{i=1}^{n} (\alpha_i + \beta_i).$$

Hence if we set

$$\left(X'_{j_1 \dots j_p}\right)_{k\ell} = \begin{cases} \dfrac{1}{p_k + \eta_\ell}, & \ell \notin \{j_1, \dots, j_p\} \\ \\ \dfrac{1}{p_k - \eta_\ell}, & \ell \in \{j_1, \dots, j_p\} \end{cases}$$

and denote by $\widetilde{X'_{j_1 \dots j_p}}$ the cofactor matrix of $X'_{j_1 \dots j_p}$, then, by (4.5), we have that

$$\begin{aligned} \sum_{i=1}^{n} \det X_{j_1 \dots j_p; i} &= (-1)^p \sum_{k,\ell=1}^{n} \left(\widetilde{X'_{j_1 \dots j_p}}\right)_{k\ell} \\ &= (-1)^p \left\{ A - 2 \sum_{k=1}^{p} \eta_{j_k} \right\} \det X'_{j_1 \dots j_p} = \left\{ A - 2 \sum_{k=1}^{p} \eta_{j_k} \right\} \det X_{j_1 \dots j_p} \\ &= \left\{ A - 2 \sum_{k=1}^{p} \eta_{j_k} \right\} \det X \prod_{k=1}^{p} \frac{m_{j_k}}{2\eta_{j_k}} \prod_{1 \le k < \ell \le p} \left(\frac{\eta_{j_k} - \eta_{j_\ell}}{\eta_{j_k} + \eta_{j_\ell}} \right)^2, \end{aligned}$$

where $A = \sum_{i=1}^{n} (p_i + \eta_i)$. Plugging this into (4.9), we obtain that

$$\begin{aligned} \sum_{i,j=1}^{n} \widehat{Q}_{ij}(\zeta) = \det X \Bigg\{ A + \sum_{p=1}^{n} \sum_{1 \le j_1 < \dots < j_p \le n} \Big\{ A - 2 \sum_{k=1}^{p} \eta_{j_k} \Big\} \times \\ \times \prod_{k=1}^{p} \frac{m_{j_k}}{2\eta_{j_k}} \prod_{1 \le k < \ell \le p} \left(\frac{\eta_{j_k} - \eta_{j_\ell}}{\eta_{j_k} + \eta_{j_\ell}} \right)^2 \zeta_{j_1}^2 \dots \zeta_{j_p}^2 \Bigg\}. \end{aligned}$$

Combining this with (4.6), (4.7) and (4.8), we arrive at

$$\sum_{i,j=1}^{n} Q_{ij}(x) = \sum_{i=1}^{n} (p_i + \eta_i) + \frac{d}{dx} \log \det(I + G_{\mathbf{s}}(x)).$$

From this, (4.1) and (4.4), the identity (2.3) follows.

4.2. The Gaussian filtering theory. In this subsection, we give a probabilistic interpretation of the Marchenko formula ([**Ma1**]) in the inverse scattering theory through the Gaussian filtering theory.

We start with recalling the Gaussian filtering theory. Let $u \in \Xi$ and, as an application of Theorem 2.3, take $P^\sigma \in \mathcal{G}$ so that $u = \psi(P^\sigma)$ on $[0, \infty)$. Let $\Omega = \mathcal{W} \times \mathcal{W}$, and define the probability measure $P = P^\sigma \times P^{\delta_0}$ on Ω. Notice that P^{δ_0} is the Wiener measure on $\mathcal{W}$. Denote by $\{(X(x), N(x))\}_{x \ge 0}$ the coordinate mapping of Ω; $X(x, w) = w^1(x)$, $N(x, w) = w^2(x)$ for $w = (w^1, w^2) \in \Omega$. Define

$$Z(x) = \int_0^x X(y) dy + N(x), \quad x \ge 0.$$

Set $\mathcal{Z}_x = \sigma[Z(y) : y \le x]$, $\widehat{X}(x) = \mathbb{E}[X(x) | \mathcal{Z}_x]$, and

$$S(x) = \int_\Omega (X(x) - \widehat{X}(x))^2 dP.$$

Let $K(x,y)$ be the unique Volterra kernel solving the Wiener-Hopf type equation

$$K(x,y)+\int_0^x K(x,z)R_\sigma(z,y)dz=R_\sigma(x,y),$$

where $R_\sigma(x,y)=\int_{\mathcal{W}}X(x)X(y)dP^\sigma$, and $L(x,y)$ be the unique Volterra kernel to the resolvent equation

$$L(x,y)-K(x,y)-\int_y^x K(x,z)L(z,y)dz=0.$$

Then it holds that

$$L(x,y)=\begin{cases}\displaystyle\int_\Omega X(x)(X(y)-\widehat{X}(y))dP, & x\geq y,\\ 0, & x<y,\end{cases}$$

and

$$S(x)=L(x,x)=K(x,x).$$

It was shown by Kleptsyna-Le Breton [**KlB**] that

$$\int_\Omega\exp\left(-\frac{1}{2}\int_0^x X(y)^2dy\right)dP=\exp\left(-\frac{1}{2}\int_0^x S(y)dy\right).$$

Hence we have that

$$u(x)=\psi(P^\sigma)(x)=2\frac{d}{dx}(-K(x,x)),\quad x\geq 0.$$

This identity is exactly the Marchenko formula in the inverse scattering theory ([**Ma1**]). Thus we have revisited the Marchenko formula via the stochastic calculus.

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