The heat semigroup and kernel associated with certain non-commutative harmonic oscillators

Taniguchi, Setsuo
Faculty of Mathematics, Kyushu University

出版情報：Kyushu Journal of Mathematics. 62 (1), pp.63-68, 2008-03. Faculty of Mathematics, Kyushu University
バージョン：
権利関係：

MHF Preprint Series

Kyushu University
21st Century COE Program
Development of Dynamic Mathematics with High Functionality

The heat semigroup and kernel associated with certain non-commutative harmonic oscillators

S. Taniguchi

MHF 2006-29

(Received November 1, 2006)

Faculty of Mathematics
Kyushu University
Fukuoka, JAPAN

THE HEAT SEMIGROUP AND KERNEL ASSOCIATED WITH CERTAIN NON-COMMUTATIVE HARMONIC OSCILLATORS

Setsuo Taniguchi

November 1, 2006

Abstract. A probabilistic construction of the heat semigroup and kernel associated with certain non-commutative harmonic oscillators is given. As an application, the unitarily equivalence of the non-commutative harmonic oscillators with the ordinary ones is shown.

Keywords and Phrases: heat semigroup; heat kernel; non-commutative harmonic oscillator; matrix SDE

2000 Mathematics subject classification: Primary 60H30, 60H20

1 Introduction and statement of result

Let $C^\infty_\uparrow(\mathbb{R};\mathbb{C}^2)$ be the space of all C^∞ functions $f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} : \mathbb{R} \to \mathbb{C}^2$, which and whose derivatives of all orders are of at most polynomial growth. The non-commutative harmonic oscillator for $(\alpha,\beta) \in \mathbb{R}^2$ with $\alpha,\beta > 0$ is by definition the differential operator $Q_{(\alpha,\beta)}$ of $C^\infty_\uparrow(\mathbb{R};\mathbb{C}^2)$ to itself defined by

$$Q_{(\alpha,\beta)}f(x) = -\frac{1}{2}\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}\{f''(x) - x^2 f(x)\} + J\Big\{xf'(x) + \frac{1}{2}f(x)\Big\}$$
$$\text{for } x \in \mathbb{R}, f \in C^\infty_\uparrow(\mathbb{R};\mathbb{C}^2),$$

where $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, and f' and f'' denote the first and the second derivatives of f. Its spectrum was studied in [12, 13, 14] by employing the representation-theoretic method, and extensively studied by many authors [2, 3, 5, 6, 7, 8, 9, 10, 11]. In the present paper, we investigate the case when $\alpha = \beta$ from the point of view of stochastic analysis: we construct probabilistically the heat semigroup and kernel associated with

$$Q_\alpha = -Q_{(\alpha,\alpha)} = \frac{\alpha}{2}\Big\{\Big(\frac{d}{dx}\Big)^2 - x^2\Big\}I - \Big\{x\frac{d}{dx} + \frac{1}{2}\Big\}J,$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and, using the semigroup, show the unitarily equivalence of Q_α with the ordinary harmonic oscillator. It should be mentioned that our Q_α has the opposite sign to that in [12, 13, 14].

We state our result more precisely. Assume that

$$\alpha \geq 1.$$

Denote by $(\mathcal{W}, \mu)$ the classical 1-dimensional Wiener space: $\mathcal{W}$ is the space of all continuous functions w on $[0, \infty) \to \mathbb{R}$ with $w(0) = 0$, and μ is the Wiener measure on $\mathcal{W}$. For $x \in \mathbb{R}$ and $t \geq 0$, set $x_\alpha = x/\sqrt{\alpha}$ and define the $\mathbb{R}^{2\times 2}$-valued random variable $M_x^\alpha(t)$ on $\mathcal{W}$, $\mathbb{R}^{2\times 2}$ being the space of all real 2×2 matrices, by

$$M_x^\alpha(t, w) = \mathrm{Exp}\Big(-\Big\{\frac{1}{2}w(t)^2 + x_\alpha w(t)\Big\}J - \Big\{\frac{\alpha^2 - 1}{2}\int_0^t (w(s) + x_\alpha)^2 ds\Big\}I\Big), \quad (1)$$

where $\mathrm{Exp}(A)$ stands for the exponential matrix of A.

Let U be the unitary matrix given by

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & 1 \\ 1 & -i \end{pmatrix},$$

where $i^2 = -1$. Then, for any $a, b \in \mathbb{R}$, it holds that

$$U \mathrm{Exp}(aJ + bI) U^* = \begin{pmatrix} e^{b+ia} & 0 \\ 0 & e^{b-ia} \end{pmatrix}, \quad (2)$$

and

$$\|\mathrm{Exp}(aJ + bI)\| = e^b, \quad (3)$$

where $\|A\|$ denotes the operator norm of the matrix A. This implies that

$$\|M_x^\alpha(t)\| \leq 1. \quad (4)$$

In particular, $M_x^\alpha(t)$ is pth integrable with respect to μ for any $p \in (1, \infty)$. Moreover, $M_x^\alpha(t) \in \mathbb{D}^{\infty,\infty-}(\mathbb{R}^{2\times 2})$, where $\mathbb{D}^{\infty,\infty-}(E)$ denotes the E-valued infinitely differentiable functions in the sense of the Malliavin calculus, which and whose Malliavin derivatives of all orders are pth integrable with respect to μ for any $p \in (1, \infty)$. Namely, it is an easy exercise of the Malliavin calculus to see that

$$\exp\Big(\pm i\Big\{\frac{1}{2}w(t)^2 + x_\alpha w(t)\Big\}\Big), \exp\Big(-\Big\{\frac{\alpha^2 - 1}{2}\int_0^t (w(s) + x_\alpha)^2 ds\Big\}\Big) \in \mathbb{D}^{\infty,\infty-}(\mathbb{C}).$$

Then, by (2), we obtain that $M_x^\alpha(t) \in \mathbb{D}^{\infty,\infty-}(\mathbb{R}^{2\times 2})$.

We can then define

$$T_t^\alpha f(x) = \mathbb{E}[M_x^\alpha(t) f(\sqrt{\alpha}w(t) + x)] \in \mathbb{C}^2 \quad \text{and}$$
$$K^\alpha(t, x, y) = \mathbb{E}[M_x^\alpha(t)\delta_y(\sqrt{\alpha}w(t) + x)] \in \mathbb{R}^{2\times 2}$$

for $t \geq 0$, $f \in C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2)$, and $x, y \in \mathbb{R}$, where $\mathbb{E}[\cdots]$ stands for the expectation with respect to μ, δ_y is the Dirac measure concentrated at $y \in \mathbb{R}$, $\delta_y(\sqrt{\alpha}w(t) + x)$ denotes Watanabe's pull-back of δ_y through $\sqrt{\alpha}w(t) + x$, and $K^\alpha(0, x, y) = \delta_y(x)$.

The first result of this paper is

Theorem 1. *Let T_t^α and $K^\alpha(t,x,y)$ be as above. Then the followings hold.*
(i) T_t^α maps $C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2)$ into itself.
(ii) For $t,s \geq 0$ and $f \in C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2)$, it holds that

$$T_t^\alpha(T_s^\alpha f) = T_{t+s}^\alpha f \quad \textit{and} \quad \frac{d}{dt}T_t^\alpha f = Q_\alpha T_t^\alpha f.$$

(iii) It holds that

$$T_t^\alpha f(x) = \int_{\mathbb{R}} K^\alpha(t,x,y)f(y)dy, \quad f \in C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2).$$

The first and second assertions say that $\{T_t^\alpha\}_{t\geq 0}$ is the heat semigroup generated by Q_α, and the third one does that $K^\alpha(t,x,y)$ is the associated heat kernel.

Let $L^2(\mathbb{R};\mathbb{C}^2)$ be the space of $\mathbb{C}^2$-valued square integrable functions on $\mathbb{R}$ with respect to the Lebesgue measure. Define the differential operator H_α from $C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2)$ to itself by

$$H_\alpha = \left\{\frac{1}{2}\left(\frac{d}{dx}\right)^2 - \frac{\alpha^2-1}{2}x^2\right\}I.$$

As an application of Theorem 1, we have the following unitarily equivalence between Q_α and H_α in $L^2(\mathbb{R};\mathbb{C}^2)$, originally shown in [13].

Theorem 2. *Define the unitary operator V_α of $L^2(\mathbb{R};\mathbb{C}^2)$ to itself by*

$$V_\alpha f(x) = \alpha^{1/4}\begin{pmatrix} e^{-ix^2/2} & 0 \\ 0 & e^{ix^2/2}\end{pmatrix} Uf(\sqrt{\alpha}x), \quad f \in L^2(\mathbb{R};\mathbb{C}^2), x \in \mathbb{R}.$$

Then

$$Q_\alpha = V_\alpha^{-1} \circ H_\alpha \circ V_\alpha \quad \textit{on } C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2).$$

Obviously this implies the closures of Q_α and H_α in $L^2(\mathbb{R};\mathbb{C}^2)$ are unitarily equivalent.

As will be seen in Remark 1, even if $\alpha \in (0,1)$, T_t^α can be defined for sufficiently small t. Using this T_t^α, we obtain that the unitarily equivalence continues to hold when $\alpha \in (0,1)$. Hence, as for the unitarily equivalence, our semigroup method yields the same result as was achieved in [13]. It should be mentioned that if $\alpha \in (0,1)$ then $T_t f$ may be no longer in $C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2)$ even if $f \in C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2)$.

The proofs of Theorems are given in the next section.

2 Proofs

Proof of Theorem 1. To prove Theorem 1, we employ the heuristic argument leading us to the expression of $M_x^\alpha(t)$ as described in (1).

We first construct the semigroup $\{T_t^\alpha\}_{t\geq 0}$. Notice that Q_α looks like the Laplacian acting on differential forms on a Riemannian manifold. Inspired by the probabilistic realization of the heat semigroup of differential forms and taking care of the coefficient $\alpha/2$ in front of f'' in the definition of $Q_\alpha f$, we expect that the semigroup $\{T_t^\alpha\}_{t\geq 0}$ is of the form that

$$T_t^\alpha f(x) = \mathbb{E}[M_x^\alpha(t) f(\sqrt{\alpha} w(t) + x)]$$

for some $\mathbb{C}^{2\times 2}$-valued stochastic process $\{M_x^\alpha(t)\}_{t\geq 0}$, which is specified in the sequel.

By the Itô formula, we have that

$$d[f(\sqrt{\alpha} w(t) + x)] = \sqrt{\alpha}\, f'(\sqrt{\alpha} w(t) + x) dw(t) + \frac{\alpha}{2} f''(\sqrt{\alpha} w(t) + x) dt. \tag{5}$$

A close look at this identity with the Itô calculus in hand leads us to the following stochastic differential equation that $M_x^\alpha(t)$ should obey;

$$dM_x^\alpha(t) = -M_x^\alpha(t) J\{w(t) + x_\alpha\} dw(t) - \frac{1}{2} M_x^\alpha(t)\big(\alpha^2\{w(t) + x_\alpha\}^2 I + J\big) dt \tag{6}$$

with the initial condition that $M_x^\alpha(0) = I$. Namely, from (5) and (6) it follows that

$$\begin{aligned} d[M_x^\alpha(t) f(\sqrt{\alpha} w(t) + x)] =& M_x^\alpha(t)\sqrt{\alpha} f'(\sqrt{\alpha} w(t) + x) dw(t) \\ &- M_x^\alpha(t)\{w(t) + x_\alpha\} J f(\sqrt{\alpha} w(t) + x) dw(t) \\ &+ M_x^\alpha(t) Q_\alpha f(\sqrt{\alpha} w(t) + x) dt. \end{aligned}$$

If we establish the estimation (4), then this identity implies that

$$\frac{d}{dt} T_t^\alpha f = T_t^\alpha (Q_\alpha f), \tag{7}$$

which is a source to the heat semigroup property as described in the assertion (ii).

As was seen in Section 1, the estimation (4) follows from the expression (1), which is obtained by solving the stochastic differential equation (6). To solve (6), recall that, for semi-martingales $\{X(t)\}_{t\geq 0}$, $\{Y(t)\}_{t\geq 0}$, and $\{Z(t)\}_{t\geq 0}$, the Stratonovich integral $X \circ (Y \circ dZ)$ is represented in terms of Itô integrals as

$$X \circ (Y \circ dZ) = XY dZ + \frac{1}{2} X dY \cdot dZ + \frac{1}{2} dX \cdot Y dZ,$$

where $dY \cdot dZ$ denotes the quadratic variation process corresponding to the product of the martingale parts of Y and Z. Hence, it follows from (6) that

$$dM_x^\alpha(t) = M_x^\alpha(t) \circ \Big[-J\{w(t) + x_\alpha\} \circ dw(t) - \frac{\alpha^2 - 1}{2}\{w(t) + x_\alpha\}^2 I dt\Big].$$

Since $(aJ + bI)(cJ + dI) = (cJ + dI)(aJ + bI)$ for any $a, b, c, d \in \mathbb{C}$ and the Stratonovich integral satisfies the usual differential rule, this implies the expression (1).

Due to the expression (1) of $M_x^\alpha(t)$, the definition of $T_t^\alpha f$, and the estimation (4), it is easily seen that $T_t^\alpha f \in C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2)$ if so is f. Thus the assertion (i) has been verified.

By the definition of $M_x^\alpha(t)$ and the independent increments property of the Brownian motion, we obtain that

$$T_t^\alpha(T_s^\alpha f) = T_{t+s}^\alpha f.$$

Thus the first identity in the assertion (ii) holds. The identity can be rewritten as

$$T_s^\alpha(T_{t-s}^\alpha f) = T_t^\alpha f.$$

Differentiating this in s, plugging (7) into the resulting identity, and then substituting $s = 0$, we get to the the second identity in the assertion (ii).

The assertion (iii) is a straightforward application of the Malliavin calculus. For example, see [4]. $\square$

Proof of Theorem 2. Let $f \in C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2)$. For $t \geq 0$, define $S_t^\alpha f \in C_\uparrow^\infty(\mathbb{R};\mathbb{C}^2)$ by

$$S_t^\alpha f(x) = \mathbb{E}\Big[\exp\Big(-\frac{\alpha^2-1}{2}\int_0^t \{w(s)+x\}^2 ds\Big) f(w(t)+x)\Big], \quad x \in \mathbb{R}.$$

By the Feynman-Kac formula, it holds that

$$\frac{d}{dt} S_t^\alpha f = S_t^\alpha (H_\alpha f). \tag{8}$$

Plugging (2) into the definition of T_t^α, we see that

$$\begin{aligned} T_t^\alpha f(x) = U^* \begin{pmatrix} e^{ix_\alpha^2/2} & 0 \\ 0 & e^{-ix_\alpha^2/2} \end{pmatrix} \mathbb{E}\Big[\exp\Big(-\frac{\alpha^2-1}{2}\int_0^t \{w(s)+x_\alpha\}^2 ds\Big) \\ \times \begin{pmatrix} e^{-i\{w(t)+x_\alpha\}^2/2} & 0 \\ 0 & e^{i\{w(t)+x_\alpha\}^2/2} \end{pmatrix} Uf(\sqrt{\alpha}\{w(t)+x_\alpha\})\Big]. \end{aligned}$$

Then, by a straightforward computation, we obtain that

$$V_\alpha(T_t^\alpha f) = S_t^\alpha(V_\alpha f).$$

Differentiating the both sides in t and substituting $t = 0$, by virtue of (7) and (8), we have that

$$V_\alpha(Q_\alpha f) = H_\alpha(V_\alpha f).$$

Thus the desired unitarily equivalence has been shown. $\square$

Remark 1. Let $\alpha \in (0,1)$. As is well known, $\exp(\sup_{s\leq t}|w(s)|)$ is pth integrable with respect to μ for any $p \in (1,\infty)$, and

$$\mathbb{E}\Big[\exp\Big(\frac{a^2}{2}\int_0^t w(s)^2 ds\Big)\Big] = \big\{\cos(at)\big\}^{-1/2}$$

for any $a \in [0, \pi/(2t))$. For example, see [1, 4]. By virtue of the estimation (3), we can define T_t^α and S_t^α for $t \in [0, \pi/\{2(1-\alpha^2)\})$. Then the Itô formula implies (7) and (8) for such t. Hence the previous proof of Theorem 2 still works in this case, and we see that the assertion of Theorem 2 also holds for $\alpha \in (0,1)$.

Acknowledgement. The Research is supported in part by Grant-in-Aid for Scientific Research (B) 18340038.

References

[1] R. H. Cameron and W. T. Martin. The Wiener measure of Hilbert neighborhoods in the space of real continuous functions. Jour. Math. Phys. M.I.T. **23** (1944), 195-209.

[2] T. Ichinose and M. Wakayama. Zeta functions for the spectrum of the non-commutative harmonic oscillators. Commun. Math. Phys. **258** (2005), 697-739.

[3] T. Ichinose and M. Wakayama. Special values of the spectral zeta functions of the non-commutative harmonic oscillators and confluent Heun equations. Kyushu J. Math. **59** (2005), 39-100.

[4] N. Ikeda and S. Watanabe. Stochastic differential equations and diffusion processes. 2nd ed. North-Holland/Kodansha, Amsterdam/Tokyo, 1989.

[5] K. Kimoto and M. Wakayama. Apéry-like numbers arising from spectral values of spectral zeta functions for non-commutative harmonic oscillators. Kyushu J. Math. **60** (2006), 383-404.

[6] K. Nagatou, M. Nakao, and M. Wakayama. Verified numerical computations of eigenvalue problems for non-commutative harmonic oscillators. Numer. Funct. Analy. Optim. **23** (2002), 633-650.

[7] H. Ochiai. Non-commutative harmonic oscillators and Fuchsian ordinary differential operators. Commun. Math. Phys. **217** (2001), 357-373.

[8] H. Ochiai. Non-commutative harmonic oscillators and the connection problems for the Heun differential equation. Lett. Math. Phys. **70** (2004), 133-139.

[9] A. Parmeggiani. On the spectrum and the lowest eigenvalue of certain non-commutative harmonic oscillators. Kyushu J. Math. **58** (2004), 277-322.

[10] A. Parmeggiani. On the spectrum of certain noncommutative harmonic oscillators. to apper in "AROUND HYPERBOLIC SYSTEMS, Ferrara, June 15-17, 2005, in memory of Stefano Benvenuti", Annali Universita' di Ferrara. Sec. 7, Sci. Math. **52** (2006).

[11] A. Parmeggiani. On the spectrum of certain noncommutative harmonic oscillators and semiclassical analysis. Preprint, 2006.

[12] A. Parmeggiani and M. Wakayama. Oscillator representations and systems of ordinary differential equations. Proc. Nat. Acad. Sci. USA **98** (2001), 26-30.

[13] A. Parmeggiani and M. Wakayama. Non-commutative harmonic oscillators – I. Forum Math. **14** (2002), 539-604.

[14] A. Parmeggiani and M. Wakayama. Non-commutative harmonic oscillators – II. Forum Math. **14** (2002), 669-690.

Setsuo Taniguchi
Faculty of Mathematics
Kyushu University
Fukuoka 812-8581, Japan
(E-mail: taniguch@math.kyushu-u.ac.jp)

List of MHF Preprint Series, Kyushu University

21st Century COE Program
Development of Dynamic Mathematics with High Functionality

MHF2005-1 Hideki KOSAKI
Matrix trace inequalities related to uncertainty principle

MHF2005-2 Masahisa TABATA
Discrepancy between theory and real computation on the stability of some finite element schemes

MHF2005-3 Yuko ARAKI & Sadanori KONISHI
Functional regression modeling via regularized basis expansions and model selection

MHF2005-4 Yuko ARAKI & Sadanori KONISHI
Functional discriminant analysis via regularized basis expansions

MHF2005-5 Kenji KAJIWARA, Tetsu MASUDA, Masatoshi NOUMI, Yasuhiro OHTA & Yasuhiko YAMADA
Point configurations, Cremona transformations and the elliptic difference Painlevé equations

MHF2005-6 Kenji KAJIWARA, Tetsu MASUDA, Masatoshi NOUMI, Yasuhiro OHTA & Yasuhiko YAMADA
Construction of hypergeometric solutions to the q - Painlevé equations

MHF2005-7 Hiroki MASUDA
Simple estimators for non-linear Markovian trend from sampled data:
I. ergodic cases

MHF2005-8 Hiroki MASUDA & Nakahiro YOSHIDA
Edgeworth expansion for a class of Ornstein-Uhlenbeck-based models

MHF2005-9 Masayuki UCHIDA
Approximate martingale estimating functions under small perturbations of dynamical systems

MHF2005-10 Ryo MATSUZAKI & Masayuki UCHIDA
One-step estimators for diffusion processes with small dispersion parameters from discrete observations

MHF2005-11 Junichi MATSUKUBO, Ryo MATSUZAKI & Masayuki UCHIDA
Estimation for a discretely observed small diffusion process with a linear drift

MHF2005-12 Masayuki UCHIDA & Nakahiro YOSHIDA
AIC for ergodic diffusion processes from discrete observations

MHF2005-13 Hiromichi GOTO & Kenji KAJIWARA
Generating function related to the Okamoto polynomials for the Painlevé IV equation

MHF2005-14 Masato KIMURA & Shin-ichi NAGATA
Precise asymptotic behaviour of the first eigenvalue of Sturm-Liouville problems with large drift

MHF2005-15 Daisuke TAGAMI & Masahisa TABATA
Numerical computations of a melting glass convection in the furnace

MHF2005-16 Raimundas VIDŪNAS
Normalized Leonard pairs and Askey-Wilson relations

MHF2005-17 Raimundas VIDŪNAS
Askey-Wilson relations and Leonard pairs

MHF2005-18 Kenji KAJIWARA & Atsushi MUKAIHIRA
Soliton solutions for the non-autonomous discrete-time Toda lattice equation

MHF2005-19 Yuu HARIYA
Construction of Gibbs measures for 1-dimensional continuum fields

MHF2005-20 Yuu HARIYA
Integration by parts formulae for the Wiener measure restricted to subsets in $\mathbb{R}^d$

MHF2005-21 Yuu HARIYA
A time-change approach to Kotani's extension of Yor's formula

MHF2005-22 Tadahisa FUNAKI, Yuu HARIYA & Mark YOR
Wiener integrals for centered powers of Bessel processes, I

MHF2005-23 Masahisa TABATA & Satoshi KAIZU
Finite element schemes for two-fluids flow problems

MHF2005-24 Ken-ichi MARUNO & Yasuhiro OHTA
Determinant form of dark soliton solutions of the discrete nonlinear Schrödinger equation

MHF2005-25 Alexander V. KITAEV & Raimundas VIDŪNAS
Quadratic transformations of the sixth Painlevé equation

MHF2005-26 Toru FUJII & Sadanori KONISHI
Nonlinear regression modeling via regularized wavelets and smoothing parameter selection

MHF2005-27 Shuichi INOKUCHI, Kazumasa HONDA, Hyen Yeal LEE, Tatsuro SATO, Yoshihiro MIZOGUCHI & Yasuo KAWAHARA
On reversible cellular automata with finite cell array

MHF2005-28 Toru KOMATSU
Cyclic cubic field with explicit Artin symbols

MHF2005-29 Mitsuhiro T. NAKAO, Kouji HASHIMOTO & Kaori NAGATOU
A computational approach to constructive a priori and a posteriori error estimates for finite element approximations of bi-harmonic problems

MHF2005-30 Kaori NAGATOU, Kouji HASHIMOTO & Mitsuhiro T. NAKAO
Numerical verification of stationary solutions for Navier-Stokes problems

MHF2005-31 Hidefumi KAWASAKI
A duality theorem for a three-phase partition problem

MHF2005-32 Hidefumi KAWASAKI
A duality theorem based on triangles separating three convex sets

MHF2005-33 Takeaki FUCHIKAMI & Hidefumi KAWASAKI
An explicit formula of the Shapley value for a cooperative game induced from the conjugate point

MHF2005-34 Hideki MURAKAWA
A regularization of a reaction-diffusion system approximation to the two-phase Stefan problem

MHF2006-1 Masahisa TABATA
Numerical simulation of Rayleigh-Taylor problems by an energy-stable finite element scheme

MHF2006-2 Ken-ichi MARUNO & G R W QUISPEL
Construction of integrals of higher-order mappings

MHF2006-3 Setsuo TANIGUCHI
On the Jacobi field approach to stochastic oscillatory integrals with quadratic phase function

MHF2006-4 Kouji HASHIMOTO, Kaori NAGATOU & Mitsuhiro T. NAKAO
A computational approach to constructive a priori error estimate for finite element approximations of bi-harmonic problems in nonconvex polygonal domains

MHF2006-5 Hidefumi KAWASAKI
A duality theory based on triangular cylinders separating three convex sets in R^n

MHF2006-6 Raimundas VIDŪNAS
Uniform convergence of hypergeometric series

MHF2006-7 Yuji KODAMA & Ken-ichi MARUNO
N-Soliton solutions to the DKP equation and Weyl group actions

MHF2006-8 Toru KOMATSU
Potentially generic polynomial

MHF2006-9 Toru KOMATSU
Generic sextic polynomial related to the subfield problem of a cubic polynomial

MHF2006-10 Shu TEZUKA & Anargyros PAPAGEORGIOU
Exact cubature for a class of functions of maximum effective dimension

MHF2006-11 Shu TEZUKA
On high-discrepancy sequences

MHF2006-12 Raimundas VIDŪNAS
Detecting persistent regimes in the North Atlantic Oscillation time series

MHF2006-13 Toru KOMATSU
Tamely Eisenstein field with prime power discriminant

MHF2006-14 Nalini JOSHI, Kenji KAJIWARA & Marta MAZZOCCO
Generating function associated with the Hankel determinant formula for the solutions of the Painlevé IV equation

MHF2006-15 Raimundas VIDŪNAS
Darboux evaluations of algebraic Gauss hypergeometric functions

MHF2006-16 Masato KIMURA & Isao WAKANO
New mathematical approach to the energy release rate in crack extension

MHF2006-17 Toru KOMATSU
Arithmetic of the splitting field of Alexander polynomial

MHF2006-18 Hiroki MASUDA
Likelihood estimation of stable Lévy processes from discrete data

MHF2006-19 Hiroshi KAWABI & Michael RÖCKNER
Essential self-adjointness of Dirichlet operators on a path space with Gibbs measures via an SPDE approach

MHF2006-20 Masahisa TABATA
Energy stable finite element schemes and their applications to two-fluid flow problems

MHF2006-21 Yuzuru INAHAMA & Hiroshi KAWABI
Asymptotic expansions for the Laplace approximations for Itô functionals of Brownian rough paths

MHF2006-22 Yoshiyuki KAGEI
Resolvent estimates for the linearized compressible Navier-Stokes equation in an infinite layer

MHF2006-23 Yoshiyuki KAGEI
Asymptotic behavior of the semigroup associated with the linearized compressible Navier-Stokes equation in an infinite layer

MHF2006-24 Akihiro MIKODA, Shuichi INOKUCHI, Yoshihiro MIZOGUCHI & Mitsuhiko FUJIO
The number of orbits of box-ball systems

MHF2006-25 Toru FUJII & Sadanori KONISHI
Multi-class Logistic Discrimination via Wavelet-based Functionalization and Model Selection Criteria

MHF2006-26 Taro HAMAMOTO, Kenji KAJIWARA & Nicholas S. WITTE
Hypergeometric Solutions to the q-Painlevé Equation of Type $(A_1 + A_1')^{(1)}$

MHF2006-27 Hiroshi KAWABI & Tomohiro MIYOKAWA
The Littlewood-Paley-Stein inequality for diffusion processes on general metric spaces

MHF2006-28 Hiroki MASUDA
Notes on estimating inverse-Gaussian and gamma subordinators under high-frequency sampling

MHF2006-29 Setsuo TANIGUCHI
The heat semigroup and kernel associated with certain non-commutative harmonic oscillators