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THE HEAT SEMIGROUP AND KERNEL ASSOCIATED WITH CERTAIN NON-COMMUTATIVE HARMONIC OSCILLATORS

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Abstract. A probabilistic construction of the heat semigroup and kernel associated with certain non-commutative harmonic oscillators is given. As an application, the unitarily equivalence of the non-commutative harmonic oscillators with the ordinary ones is shown.

Keywords and Phrases: heat semigroup; heat kernel; non-commutative harmonic oscillator; matrix SDE

2000 Mathematics subject classification: Primary 60H30, 60H20

1 Introduction and statement of result

Let $C_{\uparrow}^{\infty}(\mathbb{R}; \mathbb{C}^2)$ be the space of all C^{∞} functions $f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} : \mathbb{R} \rightarrow \mathbb{C}^2$, which and whose derivatives of all orders are of at most polynomial growth. The non-commutative harmonic oscillator for $(\alpha, \beta) \in \mathbb{R}^2$ with $\alpha, \beta > 0$ is by definition the differential operator $Q_{(\alpha, \beta)}$ of $C_{\uparrow}^{\infty}(\mathbb{R}; \mathbb{C}^2)$ to itself defined by

$$Q_{(\alpha, \beta)}f(x) = -\frac{1}{2} \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \{f''(x) - x^2 f(x)\} + J \left\{ x f'(x) + \frac{1}{2} f(x) \right\}$$

for $x \in \mathbb{R}, f \in C_{\uparrow}^{\infty}(\mathbb{R}; \mathbb{C}^2)$,

where $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, and f' and f'' denote the first and the second derivatives of f . Its spectrum was studied in [12, 13, 14] by employing the representation-theoretic method, and extensively studied by many authors [2, 3, 5, 6, 7, 8, 9, 10, 11]. In the present paper, we investigate the case when $\alpha = \beta$ from the point of view of stochastic analysis: we construct probabilistically the heat semigroup and kernel associated with

$$Q_{\alpha} = -Q_{(\alpha, \alpha)} = \frac{\alpha}{2} \left\{ \left(\frac{d}{dx} \right)^2 - x^2 \right\} I - \left\{ x \frac{d}{dx} + \frac{1}{2} \right\} J,$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and, using the semigroup, show the unitarily equivalence of Q_α with the ordinary harmonic oscillator. It should be mentioned that our Q_α has the opposite sign to that in [12, 13, 14].

We state our result more precisely. Assume that

$$\alpha \geq 1.$$

Denote by (\mathcal{W}, μ) the classical 1-dimensional Wiener space: \mathcal{W} is the space of all continuous functions w on $[0, \infty) \rightarrow \mathbb{R}$ with $w(0) = 0$, and μ is the Wiener measure on \mathcal{W} . For $x \in \mathbb{R}$ and $t \geq 0$, set $x_\alpha = x/\sqrt{\alpha}$ and define the $\mathbb{R}^{2 \times 2}$ -valued random variable $M_x^\alpha(t)$ on \mathcal{W} , $\mathbb{R}^{2 \times 2}$ being the space of all real 2×2 matrices, by

$$M_x^\alpha(t, w) = \text{Exp} \left(- \left\{ \frac{1}{2} w(t)^2 + x_\alpha w(t) \right\} J - \left\{ \frac{\alpha^2 - 1}{2} \int_0^t (w(s) + x_\alpha)^2 ds \right\} I \right), \quad (1)$$

where $\text{Exp}(A)$ stands for the exponential matrix of A .

Let U be the unitary matrix given by

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & 1 \\ 1 & -i \end{pmatrix},$$

where $i^2 = -1$. Then, for any $a, b \in \mathbb{R}$, it holds that

$$U \text{Exp}(aJ + bI) U^* = \begin{pmatrix} e^{b+ia} & 0 \\ 0 & e^{b-ia} \end{pmatrix}, \quad (2)$$

and

$$\|\text{Exp}(aJ + bI)\| = e^b, \quad (3)$$

where $\|A\|$ denotes the operator norm of the matrix A . This implies that

$$\|M_x^\alpha(t)\| \leq 1. \quad (4)$$

In particular, $M_x^\alpha(t)$ is p th integrable with respect to μ for any $p \in (1, \infty)$. Moreover, $M_x^\alpha(t) \in \mathbb{D}^{\infty, \infty-}(\mathbb{R}^{2 \times 2})$, where $\mathbb{D}^{\infty, \infty-}(E)$ denotes the E -valued infinitely differentiable functions in the sense of the Malliavin calculus, which and whose Malliavin derivatives of all orders are p th integrable with respect to μ for any $p \in (1, \infty)$. Namely, it is an easy exercise of the Malliavin calculus to see that

$$\exp \left(\pm i \left\{ \frac{1}{2} w(t)^2 + x_\alpha w(t) \right\} \right), \exp \left(- \left\{ \frac{\alpha^2 - 1}{2} \int_0^t (w(s) + x_\alpha)^2 ds \right\} \right) \in \mathbb{D}^{\infty, \infty-}(\mathbb{C}).$$

Then, by (2), we obtain that $M_x^\alpha(t) \in \mathbb{D}^{\infty, \infty-}(\mathbb{R}^{2 \times 2})$.

We can then define

$$\begin{aligned} T_t^\alpha f(x) &= \mathbb{E}[M_x^\alpha(t) f(\sqrt{\alpha} w(t) + x)] \in \mathbb{C}^2 \quad \text{and} \\ K^\alpha(t, x, y) &= \mathbb{E}[M_x^\alpha(t) \delta_y(\sqrt{\alpha} w(t) + x)] \in \mathbb{R}^{2 \times 2} \end{aligned}$$

for $t \geq 0$, $f \in C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2)$, and $x, y \in \mathbb{R}$, where $\mathbb{E}[\cdots]$ stands for the expectation with respect to μ , δ_y is the Dirac measure concentrated at $y \in \mathbb{R}$, $\delta_y(\sqrt{\alpha} w(t) + x)$ denotes Watanabe's pull-back of δ_y through $\sqrt{\alpha} w(t) + x$, and $K^\alpha(0, x, y) = \delta_y(x)$.

The first result of this paper is

Theorem 1. *Let T_t^α and $K^\alpha(t, x, y)$ be as above. Then the followings hold.*

(i) T_t^α maps $C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2)$ into itself.

(ii) For $t, s \geq 0$ and $f \in C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2)$, it holds that

$$T_t^\alpha(T_s^\alpha f) = T_{t+s}^\alpha f \quad \text{and} \quad \frac{d}{dt} T_t^\alpha f = Q_\alpha T_t^\alpha f.$$

(iii) It holds that

$$T_t^\alpha f(x) = \int_{\mathbb{R}} K^\alpha(t, x, y) f(y) dy, \quad f \in C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2).$$

The first and second assertions say that $\{T_t^\alpha\}_{t \geq 0}$ is the heat semigroup generated by Q_α , and the third one does that $K^\alpha(t, x, y)$ is the associated heat kernel.

Let $L^2(\mathbb{R}; \mathbb{C}^2)$ be the space of \mathbb{C}^2 -valued square integrable functions on \mathbb{R} with respect to the Lebesgue measure. Define the differential operator H_α from $C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2)$ to itself by

$$H_\alpha = \left\{ \frac{1}{2} \left(\frac{d}{dx} \right)^2 - \frac{\alpha^2 - 1}{2} x^2 \right\} I.$$

As an application of Theorem 1, we have the following unitarily equivalence between Q_α and H_α in $L^2(\mathbb{R}; \mathbb{C}^2)$, originally shown in [13].

Theorem 2. *Define the unitary operator V_α of $L^2(\mathbb{R}; \mathbb{C}^2)$ to itself by*

$$V_\alpha f(x) = \alpha^{1/4} \begin{pmatrix} e^{-ix^2/2} & 0 \\ 0 & e^{ix^2/2} \end{pmatrix} Uf(\sqrt{\alpha}x), \quad f \in L^2(\mathbb{R}; \mathbb{C}^2), x \in \mathbb{R}.$$

Then

$$Q_\alpha = V_\alpha^{-1} \circ H_\alpha \circ V_\alpha \quad \text{on } C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2).$$

Obviously this implies the closures of Q_α and H_α in $L^2(\mathbb{R}; \mathbb{C}^2)$ are unitarily equivalent.

As will be seen in Remark 1, even if $\alpha \in (0, 1)$, T_t^α can be defined for sufficiently small t . Using this T_t^α , we obtain that the unitarily equivalence continues to hold when $\alpha \in (0, 1)$. Hence, as for the unitarily equivalence, our semigroup method yields the same result as was achieved in [13]. It should be mentioned that if $\alpha \in (0, 1)$ then $T_t f$ may be no longer in $C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2)$ even if $f \in C_\uparrow^\infty(\mathbb{R}; \mathbb{C}^2)$.

The proofs of Theorems are given in the next section.

2 Proofs

Proof of Theorem 1. To prove Theorem 1, we employ the heuristic argument leading us to the expression of $M_x^\alpha(t)$ as described in (1).

We first construct the semigroup $\{T_t^\alpha\}_{t \geq 0}$. Notice that Q_α looks like the Laplacian acting on differential forms on a Riemannian manifold. Inspired by the probabilistic realization of the heat semigroup of differential forms and taking

care of the coefficient $\alpha/2$ in front of f'' in the definition of $Q_\alpha f$, we expect that the semigroup $\{T_t^\alpha\}_{t \geq 0}$ is of the form that

$$T_t^\alpha f(x) = \mathbb{E}[M_x^\alpha(t)f(\sqrt{\alpha}w(t) + x)]$$

for some $\mathbb{C}^{2 \times 2}$ -valued stochastic process $\{M_x^\alpha(t)\}_{t \geq 0}$, which is specified in the sequel.

By the Itô formula, we have that

$$d[f(\sqrt{\alpha}w(t) + x)] = \sqrt{\alpha} f'(\sqrt{\alpha}w(t) + x)dw(t) + \frac{\alpha}{2} f''(\sqrt{\alpha}w(t) + x)dt. \quad (5)$$

A close look at this identity with the Itô calculus in hand leads us to the following stochastic differential equation that $M_x^\alpha(t)$ should obey;

$$dM_x^\alpha(t) = -M_x^\alpha(t)J\{w(t) + x_\alpha\}dw(t) - \frac{1}{2}M_x^\alpha(t)(\alpha^2\{w(t) + x_\alpha\}^2I + J)dt \quad (6)$$

with the initial condition that $M_x^\alpha(0) = I$. Namely, from (5) and (6) it follows that

$$\begin{aligned} d[M_x^\alpha(t)f(\sqrt{\alpha}w(t) + x)] &= M_x^\alpha(t)\sqrt{\alpha}f'(\sqrt{\alpha}w(t) + x)dw(t) \\ &\quad - M_x^\alpha(t)\{w(t) + x_\alpha\}Jf(\sqrt{\alpha}w(t) + x)dw(t) \\ &\quad + M_x^\alpha(t)Q_\alpha f(\sqrt{\alpha}w(t) + x)dt. \end{aligned}$$

If we establish the estimation (4), then this identity implies that

$$\frac{d}{dt}T_t^\alpha f = T_t^\alpha(Q_\alpha f), \quad (7)$$

which is a source to the heat semigroup property as described in the assertion (ii).

As was seen in Section 1, the estimation (4) follows from the expression (1), which is obtained by solving the stochastic differential equation (6). To solve (6), recall that, for semi-martingales $\{X(t)\}_{t \geq 0}$, $\{Y(t)\}_{t \geq 0}$, and $\{Z(t)\}_{t \geq 0}$, the Stratonovich integral $X \circ (Y \circ dZ)$ is represented in terms of Itô integrals as

$$X \circ (Y \circ dZ) = XYdZ + \frac{1}{2}XdY \cdot dZ + \frac{1}{2}dX \cdot YdZ,$$

where $dY \cdot dZ$ denotes the quadratic variation process corresponding to the product of the martingale parts of Y and Z . Hence, it follows from (6) that

$$dM_x^\alpha(t) = M_x^\alpha(t) \circ \left[-J\{w(t) + x_\alpha\} \circ dw(t) - \frac{\alpha^2 - 1}{2}\{w(t) + x_\alpha\}^2 Idt \right].$$

Since $(aJ + bI)(cJ + dI) = (cJ + dI)(aJ + bI)$ for any $a, b, c, d \in \mathbb{C}$ and the Stratonovich integral satisfies the usual differential rule, this implies the expression (1).

Due to the expression (1) of $M_x^\alpha(t)$, the definition of $T_t^\alpha f$, and the estimation (4), it is easily seen that $T_t^\alpha f \in C_1^\infty(\mathbb{R}; \mathbb{C}^2)$ if so is f . Thus the assertion (i) has been verified.

By the definition of $M_x^\alpha(t)$ and the independent increments property of the Brownian motion, we obtain that

$$T_t^\alpha(T_s^\alpha f) = T_{t+s}^\alpha f.$$

Thus the first identity in the assertion (ii) holds. The identity can be rewritten as

$$T_s^\alpha(T_{t-s}^\alpha f) = T_t^\alpha f.$$

Differentiating this in s , plugging (7) into the resulting identity, and then substituting $s = 0$, we get to the the second identity in the assertion (ii).

The assertion (iii) is a straightforward application of the Malliavin calculus. For example, see [4]. \square

Proof of Theorem 2. Let $f \in C_1^\infty(\mathbb{R}; \mathbb{C}^2)$. For $t \geq 0$, define $S_t^\alpha f \in C_1^\infty(\mathbb{R}; \mathbb{C}^2)$ by

$$S_t^\alpha f(x) = \mathbb{E} \left[\exp \left(-\frac{\alpha^2 - 1}{2} \int_0^t \{w(s) + x\}^2 ds \right) f(w(t) + x) \right], \quad x \in \mathbb{R}.$$

By the Feynman-Kac formula, it holds that

$$\frac{d}{dt} S_t^\alpha f = S_t^\alpha (H_\alpha f). \quad (8)$$

Plugging (2) into the definition of T_t^α , we see that

$$\begin{aligned} T_t^\alpha f(x) = U^* & \begin{pmatrix} e^{ix_\alpha^2/2} & 0 \\ 0 & e^{-ix_\alpha^2/2} \end{pmatrix} \mathbb{E} \left[\exp \left(-\frac{\alpha^2 - 1}{2} \int_0^t \{w(s) + x_\alpha\}^2 ds \right) \right. \\ & \times \begin{pmatrix} e^{-i\{w(t)+x_\alpha\}^2/2} & 0 \\ 0 & e^{i\{w(t)+x_\alpha\}^2/2} \end{pmatrix} Uf(\sqrt{\alpha}\{w(t) + x_\alpha\}) \Big]. \end{aligned}$$

Then, by a straightforward computation, we obtain that

$$V_\alpha(T_t^\alpha f) = S_t^\alpha(V_\alpha f).$$

Differentiating the both sides in t and substituting $t = 0$, by virtue of (7) and (8), we have that

$$V_\alpha(Q_\alpha f) = H_\alpha(V_\alpha f).$$

Thus the desired unitarily equivalence has been shown. \square

Remark 1. Let $\alpha \in (0, 1)$. As is well known, $\exp(\sup_{s \leq t} |w(s)|)$ is p th integrable with respect to μ for any $p \in (1, \infty)$, and

$$\mathbb{E} \left[\exp \left(\frac{a^2}{2} \int_0^t w(s)^2 ds \right) \right] = \{\cos(at)\}^{-1/2}$$

for any $a \in [0, \pi/(2t))$. For example, see [1, 4]. By virtue of the estimation (3), we can define T_t^α and S_t^α for $t \in [0, \pi/\{2(1 - \alpha^2)\})$. Then the Itô formula implies (7) and (8) for such t . Hence the previous proof of Theorem 2 still works in this case, and we see that the assertion of Theorem 2 also holds for $\alpha \in (0, 1)$.

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