A DUALITY THEORY BASED ON TRIANGULAR CYLINDERS SEPARATING THREE CONVEX SETS IN $ R^N $

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A duality theory based on triangular cylinders separating three convex sets in R^n

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A DUALITY THEORY BASED ON TRIANGULAR CYLINDERS SEPARATING THREE CONVEX SETS IN R^N *

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Abstract. Separation theorems play the central role in the duality theory. Recently, we proposed a duality theorem for a three-phase partition problem in [7]. It is based on triangles separating three convex sets in R^2. The aim of this paper is to extend the duality theorem to R^n.

Key words. duality theorem, triangular cylinder, partition problem, separation theorem

AMS subject classifications. 49K10, 90C30, 26B99

1. Introduction. The three-phase partition problem is to divide a given domain $\Omega \subset R^2$ into three subdomains with a triple junction having least interfacial area (Fig.1.1).

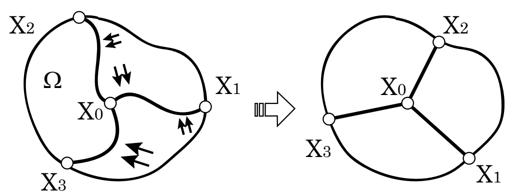

Fig. 1.1. *Three-phase partition problem*

Sternberg and Zeimer [10] and Ikota and Yanagida [4] formulated this problem as variational problems and discussed stability of stationary solutions. However, since the shortest curve joining two points X_0 and X_i is the line segment X_0X_i, they can be formulated as extremal problems in a Euclidean space. From this point of view, we discussed stability and studied its game-theoretic aspect in [5][6]. Further, we gave a duality theorem for an extremal problem (P_0) below induced from the three-phase partition problem in [7].

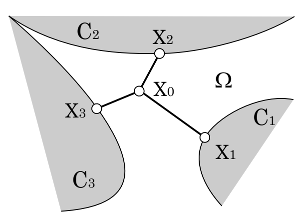

Fig. 1.2. *Primal problem* (P_0)

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$$(P_0) \qquad \begin{array}{ll} \text{Minimize} & f(X_0, \ldots, X_3) := \displaystyle\sum_{i=1}^{3} ||X_i - X_0|| \\ \text{subject to} & X_0 \in \Omega,\ X_i \in C_i\ (i = 1, 2, 3), \end{array}$$

where $||\cdot||$ denotes the Euclidean norm and C_i $(i = 1, 2, 3)$ are closed convex sets with non-empty interior in R^2 such that $\Omega := \mathrm{cl}(\cap_{i=1}^{3} C_i^c)$ is not empty (Fig. 1.2). The main aim of this paper is to extend the duality theorem (Theorem 2.3 below) to R^n.

This paper is organized as follows. In Section 2, we briefly review the first-order optimality condition for the primal problem (P_0) and the duality theorem given in [7]. In Section 3, we give a first-order optimality condition for the primal problem in R^n. In Section 4, we introduce the notion of separating three convex sets by a triangular cylinder in R^n to define the dual problem, and show the strong duality.

We close this section with our notations. For any closed convex sets C_1 and C_2, we define $d(C_1, C_2) := \inf\{||X_1 - X_2|| \mid X_i \in C_i\ (i = 1, 2)\}$. We denote by $N(X_i; C_i)$ the normal cone of C_i at X_i, that is, $N(X_i; C_i) := \{Y \in R^n \mid Y^T(X - X_i) \le 0\ \forall X \in C_i\}$.

2. Preliminaries. As is easily seen from Fig. 1.2, Ω is not always a convex set. So the primal problem (P_0) is not a convex programming problem. We modify it so that it becomes a convex programming problem.

$$(P) \qquad \begin{array}{ll} \text{Minimize} & \displaystyle\sum_{i=1}^{3} ||X_i - X_0|| \\ \text{subject to} & X_0 \in R^2,\ X_i \in C_i\ (i = 1, 2, 3). \end{array}$$

The only difference is that Ω is replaced by the whole space R^2. We say a feasible solution $(X_0, \ldots, X_3)$ for (P_0) (or (P)) non-degenerate if $X_i \ne X_j$ for any $i \ne j$. The following is a straightforward consequence of Torricelli's Theorem and the projection theorem on the convex set C_i.

THEOREM 2.1. *Let $(X_0, \ldots, X_3)$ be a non-degenerate minimal solution for (P_0) (or (P)). Then it satisfies*

$$\angle X_i X_0 X_j = 2\pi/3 \text{ for any } i \ne j \tag{2.1}$$

and

$$X_0 - X_i \in N(X_i; C_i) \quad (i = 1, 2, 3). \tag{2.2}$$

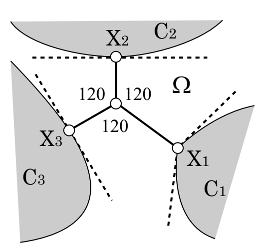

FIG. 2.1. *First-order optimality conditions*

Lemma 2.2. *([7]) When Ω is a triangle in R^2, it holds that*

$$\min(P) = \min(P_0) = \text{the smallest height of } \Omega.$$

We say that a triangle $\Delta \subset \Omega$ separates $\{C_i\}_{i=1}^3$ if there are three closed half spaces $\{H_i^-\}_{i=1}^3$ such that $C_i \subset H_i^-$ for every i and $\Delta = \cap_{i=1}^3 H_i^+$, where H_i^+ denotes the closed half space opposite to H_i^- (Fig. 2.2).

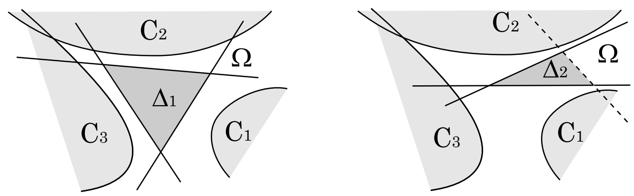

Fig. 2.2. Δ_1 *separates* $\{C_i\}_{i=1}^3$, *and* Δ_2 *does not separate* $\{C_i\}_{i=1}^3$.

Then the dual problem is defined as follows.

(D) Maximize the smallest height of a triangle that separates $\{C_i\}_{i=1}^3$.

Theorem 2.3. *([7]) If (P_0) (or (P)) has a non-degenerate minimal solution, then*

$$\max(D) = \min(P) = \min(P_0). \tag{2.3}$$

3. The primal problem in R^n. In this section, we extend Theorem 2.1 to R^n. Let C_i $(i = 1, 2, 3)$ be closed convex sets with non-empty interior in R^n such that $\Omega = \mathrm{cl}(\cap_{i=1}^3 C_i^c)$ is not empty. Then (P_0) and (P) are defined as well as in Sections 1 and 2, respectively, where the base space is R^n.

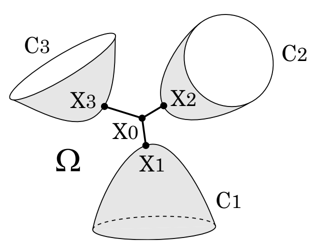

Fig. 3.1. *The primal problem* (P) *in* R^3.

Theorem 3.1. *Let $(X_0, \ldots, X_3)$ be a non-degenerate minimal solution for (P_0). Then X_i's are on a two-dimensional affine set and $(X_0, \ldots, X_3)$ satisfies (2.1) and (2.2). Further, it is a minimum solution for (P).*

Proof. According to Kuhn-Tucker's theorem, see e.g. Rockafellar (Ref. 6, Section 28), there exist multipliers $\lambda_i \geq 0$ $(i = 1, 2, 3)$ such that $0 \in R^{4n}$ belongs to the subdifferential of the Lagrange function $L(X_0, \ldots, X_3) := \sum_{i=1}^3 ||X_i - X_0|| + \sum_{i=1}^3 \lambda_i \delta(X_i|C_i)$, where $\delta(X_i|C_i)$ denotes the characteristic function of C_i. Picking up X_0-component of the subdifferential ∂L, we have

$$n_1 + n_2 + n_3 = 0 \in R^n, \tag{3.1}$$

where $n_i := (X_0 - X_i)/||X_i - X_0||$, which implies the first assertion. From (3.1), $||n_k||^2 = ||n_i||^2 + ||n_j||^2 + 2n_i^T n_j$ for any $\{i,j,k\} = \{1,2,3\}$. Thus $n_i^T n_j = -1/2$, which implies (2.1). Picking up X_i-component $(i = 1,2,3)$ of ∂L, we have $0 \in -n_i + \lambda_i N(X_i; C_i)$, which implies (2.2). Next, there exists an open convex neighborhood C_0 of X_0 such that $(X_0, \dots, X_3)$ is a minimum point of f on $C := C_0 \times C_1 \times C_2 \times C_3$. Since f and C are convex, $(X_0, \dots, X_3)$ is a minimum point of f on $R^n \times C_1 \times C_2 \times C_3$. Hence it is a minimum solution for (P). □

4. Duality theorem. In this section, we first introduce the notion of separation of three convex sets by a triangular cylinder. Next, we define the dual problem and show strong duality.

Definition 4.1. *Let n_i $(i = 1,2,3)$ be nonzero (unit) vectors in R^n satisfying (3.1), and α_i $(i = 1,2,3)$ negative real numbers. Define*

$$H_i^+ := \{\xi \in R^n \mid n_i^T \xi \geq \alpha_i\}, \tag{4.1}$$

$H_i^- := \{\xi \in R^n \mid n_i^T \xi \leq \alpha_i\}$, and $H_i := H_i^+ \cap H_i^-$ for any $i = 1,2,3$. Then we call a shifted figure of $H_1^+ \cap H_2^+ \cap H_3^+$ a (regular) triangular cylinder, see Fig. 4.1. (Here we remark that the origin of R^n belongs to $H_1^+ \cap H_2^+ \cap H_3^+$.) Further, we say a triangular cylinder $H_1^+ \cap H_2^+ \cap H_3^+ + X$ separates three convex sets $\{C_i\}_{i=1}^3$ if $C_i \subset H_i^- + X$ for any $i = 1,2,3$. Let $M := \cap_{i=1}^3 \{\xi \in R^n \mid n_i^T \xi = 0\}$ and

$$N := M^\top = \{X \in R^n \mid \xi^T X = 0 \quad \forall \xi \in M\}. \tag{4.2}$$

Then, by (3.1), N is a 2-dimensional subspace, and

$$\Delta := N \cap (H_1^+ \cap H_2^+ \cap H_3^+) \tag{4.3}$$

is a (regular) triangle. We call the smallest height of Δ the width of the triangular cylinder $H_1^+ \cap H_2^+ \cap H_3^+ + X$.

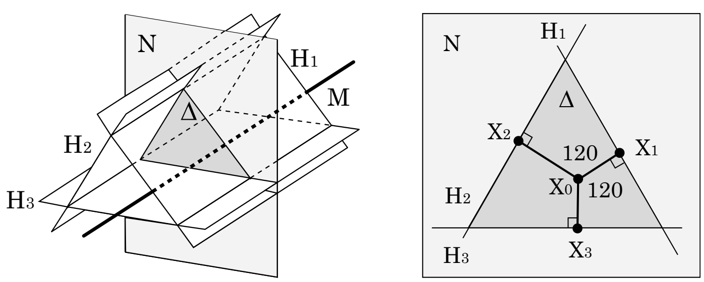

Fig. 4.1. *Regular triangular cylinder in R^3*

Our dual problem is defined as follows.

(D) Maximize the width of a triangular cylinder that separates $\{C_i\}_{i=1}^3$.

Theorem 4.2. *(Weak duality)*

$$\sup(D) \leq \inf(P) \leq \inf(P_0). \tag{4.4}$$

Proof. Second inequality is trivial. Let $(Y_0, \dots, Y_3)$ be a feasible solution for (P), and $H_1^+ \cap H_2^+ \cap H_3^+ + X$ a feasible solution for (D). Then $Y_i \in C_i \subset H_i^- + X$ for any $i = 1, 2, 3$. Hence

$$||Y_0 - Y_i|| \geq d(Y_0, H_i^- + X) = d(Y_0 - X, H_i^-) = d(P_N(Y_0 - X), H_i^-),$$

where P_N denotes the projection to N defined by (4.2). Hence

$$\begin{aligned}\sum_{i=1}^{3} ||Y_0 - Y_i|| &\geq \sum_{i=1}^{3} d(P_N(Y_0 - X), H_i^-) \\ &= \min \left\{ \sum_{i=1}^{3} ||P_N(Y_0 - X) - X_i|| \,\middle|\, X_i \in H_i^- \ (i = 1, 2, 3) \right\} \\ &\geq \min \left\{ \sum_{i=1}^{3} ||X_0 - X_i|| \,\middle|\, X_i \in H_i^- \ (i = 1, 2, 3),\ X_0 \in N \right\} \\ &\geq \text{the smallest height of the triangle } \Delta \text{ defined by (4.3)} \\ &= \text{the width of } H_1^+ \cap H_2^+ \cap H_3^+ + X,\end{aligned}$$

where the last inequality follows from Lemma 2.2. Hence we get $\sup(D) \leq \inf(P)$. □

THEOREM 4.3. *(Strong duality) If (P_0) (or (P)) has a non-degenerate minimal solution, then it holds that*

$$\max(D) = \min(P) = \min(P_0). \tag{4.5}$$

Proof. Second equality follows from Theorem 3.1. Let $(X_0, \dots, X_3)$ be a non-degenerate minimal solution for (P_0). Then, the regular triangular cylinder $H_1^+ \cap H_2^+ \cap H_3^+ + X_0$ determined by $n_i := (X_0 - X_i)/||X_i - X_0||$ and $\alpha_i := -||X_i - X_0||$ separates $\{C_i\}_{i=1}^3$. Indeed, it follows from (2.2) that $n_i^T X \leq n_i^T X_i$ for any $X \in C_i$, so that

$$n_i^T (X - X_0) \leq n_i^T (X_i - X_0) = -||X_i - X_0|| = \alpha_i.$$

Namely, $C_i - X_0$ is contained in H_i^-. Moreover, since X_i $(i = 0, 1, 2, 3)$ belong to the regular triangle $\Delta + X_0$, it follows from Lemma 2.2 that $\sum_{i=1}^3 ||X_0 - X_i||$ is equal to the width of the regular triangular cylinder $H_1^+ \cap H_2^+ \cap H_3^+ + X_0$. Therefore we get (4.5). □

REMARK 4.1. *We can replace triangular cylinders by regular triangular cylinders in the dual problem (D), because the maximum value is attained by a regular triangular cylinder. However, it is clear that regular triangular cylinders are not enough when Ω is a non-regular triangular cylinder. That's why we defined the dual problem with (general) triangular cylinders.*

5. Extension to the weighted objective function. In [4], the objective function was weighted. It is not hard to extend the present results to the weighted objective function

$$\sum_{i=1}^{3} \sigma_i ||X_i - X_0||,$$

where $\sigma_i > 0$ $(i = 1, 2, 3)$ can be interpreted as interface tension, see Fig. 5.1. We impose an assumption that $\sigma_i < \sigma_j + \sigma_k$ for any $\{i, j, k\} = \{1, 2, 3\}$. If this assumption is violated, the primal problem does not have any non-degenerate minimal solution, see (5.2) below. We denote (P_0) and (P) with weighted objective function by (P_0^σ) and (P^σ), respectively.

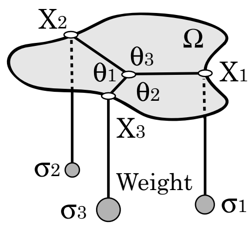

FIG. 5.1. $\sigma_i > 0$ $(i = 1, 2, 3)$ *can be regarded as interface tensions.*

THEOREM 5.1. *Let* $(X_0, \ldots, X_3)$ *be a non-degenerate minimal solution for* (P_0^σ). *Then it is a minimum solution for* (P^σ), *and satisfies (2.2) and*

$$\frac{\sin\theta_1}{\sigma_1} = \frac{\sin\theta_2}{\sigma_2} = \frac{\sin\theta_3}{\sigma_3}, \tag{5.1}$$

where θ_i $(i = 1, 2, 3)$ *are defined as in Fig. 5.1.*

Proof. The first assertion and (2.2) are proved in the same way with Theorem 3.1. As well as (3.1), we get $\sigma_1 n_1 + \sigma_2 n_2 + \sigma_3 n_3 = 0 \in R^n$. Hence we have

$$\frac{\sigma_k^2 - \sigma_i^2 - \sigma_j^2}{2\sigma_i\sigma_j} = n_i^T n_j = \cos\theta_k \tag{5.2}$$

for any $\{i, j, k\} = \{1, 2, 3\}$. So we get

$$\frac{\sin^2\theta_k}{\sigma_k^2} = \frac{2(\sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2) - \sigma_1^4 - \sigma_2^4 - \sigma_3^4}{4\sigma_1^2\sigma_2^2\sigma_3^2}.$$

Since the right-hand side does not depend on k, we get (5.1). ∎

LEMMA 5.2. *When* Ω *is a triangle in* R^2, *it holds that*

$$\min(P^\sigma) = \min(P_0^\sigma) = \min_{i=1,2,3} \sigma_i h_i,$$

where h_i *is the height of the triangle with base length* a_i.

Proof. It's enough to consider KKT condition for the following extremal problem.

$$\begin{array}{ll} \text{Minimize} & \sigma_1 l_1 + \sigma_2 l_2 + \sigma_3 l_3 \\ \text{subject to} & a_1 l_1 + a_2 l_2 + a_3 l_3 = 2S \\ & l_i \geq 0 \quad (i = 1, 2, 3). \end{array}$$

∎

For a triangular cylinder, let Δ be a triangle defined by (4.3) and σ_i $(i = 1, 2, 3)$ positive real numbers. We call the smallest weighted height of Δ the weighted width of the triangular cylinder. Then the dual problem (D^σ) is defined as follows.

Maximize the weighted width of a triangular cylinder that separates $\{C_i\}_{i=1}^3$.

Theorem 5.3. *If* (P_0^σ) *(or* (P^σ)*) has a non-degenerate minimal solution, then it holds that*

$$\min(P_0^\sigma) = \min(P^\sigma) = \max(D^\sigma). \tag{5.3}$$

Proof. Since the proof of this theorem is the same as that of Theorem 4.3 except obvious modifications, we omit it. □

6. Concluding remark. Gale and Klee [3] gave separation theorems for an infinite number of convex sets with empty intersection in R^n. For example, if a family of open convex proper subsets $\{C_\nu\}$ of R^n has empty intersection, then there are open halfspaces $\{H_\nu^-\}$ with empty intersection such that $C_\nu \subset H_\nu^-$ for any ν, see also a survey paper Klee [8] and Bair [1]. Since their separation theorems are not quantitative but qualitative, they do not seem to derive us to our duality theorems.

Further, a generalization of Duboviskiĭ and Miljutin's theorem [2] asserts that if a finite number of convex sets $\{C_i\}_{i=1}^k$ in R^n has empty intersection, then there exist non-trivial vectors $\xi_i \in R^n$ $(i = 1, \ldots, k)$ such that

$$\xi_1 + \cdots + \xi_k = 0 \tag{6.1}$$

and

$$\delta^*(\xi_1|C_1) + \cdots + \delta^*(\xi_k|C_k) \leq 0, \tag{6.2}$$

where $\delta^*(\xi_i|C_i)$ denotes the support function $\sup\{\xi_i^T X_i \mid X_i \in C_i\}$. If we directly apply this separation theorem to the linear case $C_i = \{X_i \mid \xi_i^T X_i \leq \alpha_i\}$, (6.1) and (6.2) reduce to $\sum_{i=1}^3 \lambda_i \xi_i = 0$ and $\sum_{i=1}^3 \lambda_i \alpha_i \leq 0$ for some non-trivial $\lambda_i \geq 0$ $(i = 1, 2, 3)$, respectively. They do not seem enough to derive our duality theorems, either.

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