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ON THE JACOBI FIELD APPROACH TO STOCHASTIC OSCILLATORY INTEGRALS WITH QUADRATIC PHASE FUNCTION

Setsuo Taniguchi *

Abstract. The approach taking advantage of Jacobi fields to represent explicitly stochastic oscillatory integrals with quadratic phase function, which approach was introduced by N. Ikeda, S. Kusuoka, and S. Manabe, is completed in the general scheme and is testified in several examples.

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1 Introduction and Statement of results

Let $d \in \mathbf{N}$, $T > 0$, $\mathcal{W}$ be the space of all continuous functions $w : [0, T] \to \mathbf{R}^d$ with $w(0) = 0$, and μ the Wiener measure on $\mathcal{W}$. Denote by H the Cameron-Martin subspace of $\mathcal{W}$, the space of all $h \in \mathcal{W}$ which is absolutely continuous on $[0, T]$ and has the derivative h' square integrable with respect to the Lebesgue measure. A Wiener functional q, which is smooth in the sense of the Malliavin calculus, is said to be quadratic if it is of the form $q = Q_A = (\nabla^*)^2 A$, where $A : H \to H$ is a symmetric Hilbert-Schmidt operator and ∇^* is the adjoint operator of the Malliavin gradient ∇. Such q and A are in one to one correspondence; $A = \nabla^2 q/2$. For Q_A's, there are lots of investigations to give explicit expression of stochastic oscillatory integral

$$I(Q_A; \zeta) = \int_{\mathcal{W}} e^{\zeta Q_A/2} \delta_x(\boldsymbol{\eta}) d\mu,$$

where $\zeta \in \mathbf{C}$ and $\delta_x(\boldsymbol{\eta})$ denotes Watanabe's pull back of the Dirac measure δ_x on $\mathbf{R}^N$ concentrated at x through the non-degenerate and smooth Wiener functional $\boldsymbol{\eta} : \mathcal{W} \to \mathbf{R}^N$. In [3, 4], N. Ikeda, S. Kusuoka, and S. Manabe introduced the Jacobi field for Q_A and used it to evaluate $I(Q_A; \zeta)$ in two cases; one is when Q_A is the quadratic Wiener functional related to harmonic oscillator in uniform magnetic field, and the other is when Q_A is the generalized stochastic area for Gaussian process. In this paper, we extend their formula to more general quadratic Wiener functionals and $\boldsymbol{\eta}$'s to complete the Jacobi field approach to stochastic oscillatory integrals. Moreover, all exact expressions of stochastic

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oscillatory integrals under various $\delta_x(\boldsymbol{\eta})d\mu$'s will be unified in terms of Grassmannians.

We shall state our result precisely. Let $A : H \to H$ be a symmetric Hilbert-Schmidt operator. Throughout the paper, we assume that

(A1) there exist a Volterra operator $A_V : H \to H$, a bounded operator $A_F : H \to H$, and linearly independent $\eta_1, \dots, \eta_M \in H$ such that

(i) $A = A_V + A_F$, and
(ii) the subspace of H spanned by $\eta_1, \dots, \eta_M$, say $\mathcal{R}$, includes the range $\mathcal{R}(A_F)$ of A_F.

Let $\zeta \in \mathbf{C}$ and $p = (p_1, \dots, p_M)^\dagger \in \mathbf{C}^M$, where we have thought of elements of $\mathbf{C}^n$, $n \in \mathbf{N}$, as column vectors, and $(p_1, \dots, p_M)^\dagger$ is the transposed vector of the row vector $(p_1, \dots, p_M)$. Define $\widetilde{J}_\zeta(\cdot; p) \in H \otimes \mathbf{C}$ ($\equiv$ the standard complexification of H) by

$$\widetilde{J}_\zeta(\cdot; p) = (I - \zeta A_V)^{-1}\left(\sum_{j=1}^{M} p_j \eta_j\right),$$

where $I - \zeta A_V$ is extended to a complex operator on $H \otimes \mathbf{C}$ in the usual manner. Let $0 \le N \le M$ and L be the subspace of $\mathcal{R}$ spanned by $\eta_1, \dots, \eta_N$, where $L = \{0\}$ if $N = 0$. Denote by $\pi_L : H \to H$ the orthogonal projection onto L, and set $A^\natural = (I - \pi_L)A$ and $A^\# = (I - \pi_{\mathcal{R}})A$. Define the linear mapping $\widetilde{J}_{\zeta,N} : \mathbf{C}^M \to \mathbf{C}^M$ by

$$\widetilde{J}_{\zeta,N}\, p = \Big(\big\langle \widetilde{J}_\zeta(\cdot; p), \{I - \zeta(A^\natural - A^\#)^*\}\eta_j \big\rangle_H\Big)^\dagger_{1 \le j \le M},$$

where $(A^\natural - A^\#)^*$ is the adjoint operator of $A^\natural - A^\#$. Denote by $\boldsymbol{\eta}^{(N)}$ the $\mathbf{R}^N$-valued Wiener functional $(\nabla^*\eta_1, \dots, \nabla^*\eta_N)^\dagger$. Note that $\boldsymbol{\eta}^{(N)}$ is smooth and non-degenerate in the sense of the Malliavin calculus.

We are ready to state our results.

Theorem 1. *There exists* $\varepsilon > 0$ *such that for every* $\zeta \in \mathbf{C}$ *with* $|\zeta| < \varepsilon$, $\det \widetilde{J}_{\zeta,N} \neq 0$ *and it holds that*

$$\int_{\mathcal{W}} e^{\zeta Q_A/2} \delta_0(\boldsymbol{\eta}^{(N)}) d\mu = \left\{ \frac{\det C(\boldsymbol{\eta}^{(M)})}{(2\pi)^N \det C(\boldsymbol{\eta}^{(N)}) \det \widetilde{J}_{\zeta,N}} \right\}^{1/2} e^{-(\zeta/2)\mathrm{tr}\, A_F}, \qquad (1)$$

where $C(\boldsymbol{\eta}^{(k)}) = \big(\langle \eta_i, \eta_j \rangle_H\big)_{1 \le i,j \le k}$, *and* $\delta_0(\boldsymbol{\eta}^{(0)})d\mu = d\mu$ *and* $C(\boldsymbol{\eta}^{(0)}) = 1$.

Once the identity (1) has been obtained, it is routine to extend it holomorphically to much wider domain in $\mathbf{C}$. Moreover, we have the following assertion with $\delta_x(\boldsymbol{\eta}^{(N)})$ instead of $\delta_0(\boldsymbol{\eta}^{(N)})$.

Theorem 2. *Let* $N \ge 1$ *and* $x = (x_1, \dots, x_N)^\dagger \in \mathbf{R}^N$.
(i) Suppose that there exists $h \in H \otimes \mathbf{C}$ *such that* $\langle h, \eta_i \rangle_H = x_i$, $1 \le i \le N$, *and*

$\langle (I-\zeta A)h, k\rangle_H = 0$ for any $k \in L^\perp$ ($\equiv$ the orthogonal complement of L in H). Then it holds that

$$\int_{\mathcal{W}} e^{\zeta Q_A/2}\delta_x(\boldsymbol{\eta}^{(N)})d\mu = e^{-\langle (I-\zeta A)h,h\rangle_H/2}\int_{\mathcal{W}} e^{\zeta Q_A/2}\delta_0(\boldsymbol{\eta}^{(N)})d\mu. \tag{2}$$

(ii) If there exists $y = (y_1, \dots, y_{M-N})^\dagger \in \mathbf{C}^{M-N}$ such that

$$\langle (I-\zeta A)\widetilde{J}_\zeta(\cdot; \widetilde{J}_{\zeta,M}^{-1}p_{x,y}), k\rangle_H = 0 \quad \text{for every } k \in L^\perp,$$

where $p_{x,y} = (x_1, \dots, x_N, y_1, \dots, y_{M-N})^\dagger$, then $h = \widetilde{J}_\zeta(\cdot; \widetilde{J}_{\zeta,M}^{-1}p_{x,y})$ satisfies the condition in (i).
(iii) If $N = M$, then $h = \widetilde{J}_\zeta(\cdot; \widetilde{J}_{\zeta,M}^{-1}x)$ fulfills the condition in (i).

Under suitable assumptions, $\widetilde{J}_\zeta(\cdot; p)$ can be characterized only by A and η_j's;

Theorem 3. *Suppose that*

(A2) *there exists $m \in \mathbf{N}$ with $md \geq M$ such that*

(a) $A_V h(t)$ is differentiable around $t = 0$ for any $h \in H$,
(b) $A_V k(t)$ is $(i+1)$-times differentiable around $t = 0$ if $1 \leq i < m$ and $k \in H$ is i-times differentiable around $t = 0$,
(c) the j-th derivative $(A_V g)^{(j)}(0)$ of $A_V g$ at $t = 0$ vanishes for every $1 \leq j \leq m$ provided that $g \in H$ is m-times differentiable around $t = 0$, and
(d) each η_j is m-times differentiable around $t = 0$ and the matrix

$$D = \begin{pmatrix} \eta_1^{(1)}(0) & \cdots & \eta_M^{(1)}(0) \\ \vdots & \ddots & \vdots \\ \eta_1^{(m)}(0) & \cdots & \eta_M^{(m)}(0) \end{pmatrix} \in \mathbf{R}^{md \times M}$$

has the rank M.

Then, for each $p \in \mathbf{C}^M$, $\widetilde{J}_\zeta(\cdot; t)$ is the unique element $J \in H \otimes \mathbf{C}$ which is differentiable around $t = 0$ and satisfies that

$$\langle (I-\zeta A)J, h\rangle_H = 0 \ \text{ for any } h \in \mathcal{R}^\perp, \text{ and } \begin{pmatrix} J^{(1)}(0) \\ \vdots \\ J^{(m)}(0) \end{pmatrix} = Dp. \tag{3}$$

It should be mentioned that the assumption (b) guarantees the m-times differentiability of $A_V g$ around $t = 0$ in (c).

In [4], the case where $N = M$ was observed for the special two types of quadratic Wiener functionals as mentioned above. Remembering that the equation (3) was called the Jacobi equation associated with Q_A in [3, 4], we call Theorems 1, 2, and 3 the Jacobi field approach to stochastic oscillatory integrals.

From Theorems 1 and 2, we see that $\widetilde{J}(\cdot;p)$ governs the all stochastic oscillatory integrals $I(Q_A;\zeta)$. In terms of Grassmannians, we can give another unified interpretation for this phenomenon. Namely, let

$$\Phi_\zeta = \begin{pmatrix} \widetilde{J}_{\zeta,M} \\ \widetilde{J}_{\zeta,0} \end{pmatrix} \in \mathbf{C}^{2M\times M}.$$

Since $\det \widetilde{J}_{\zeta,N} \neq 0$ for any $0 \le N \le M$, Φ_ζ determines an M-frame in a $2M$-dimensional vector space $V(2M)$ over $\mathbf{C}$, and hence a point, say W_ζ, in the Grassmannian $GM(M, V(2M))$, the set of all M-dimensional vector subspaces of $V(2M)$. Without loss of generality, we may and will assume that $\langle \eta_i, \eta_j\rangle_H = 0$ if $i \neq j$. Then, denoting by $\Phi_\zeta^{(i)}$ the i-th row vector of Φ_ζ, by the very definition of $\widetilde{J}_{\zeta,N}$, we have that

$$\widetilde{J}_{\zeta,N} = \begin{pmatrix} \Phi_\zeta^{(1)} \\ \vdots \\ \Phi_\zeta^{(N)} \\ \Phi_\zeta^{(M+N+1)} \\ \vdots \\ \Phi_\zeta^{(2M)} \end{pmatrix}.$$

Hence $\det \widetilde{J}_{\zeta,N}$ is the $(1,\dots,N,M+N+1,\dots,2M)$-th Plücker coordinate of W_ζ. In this manner, through the Plücker coordinates, the point W_ζ governs the all stochastic oscillatory integrals $I(Q_A;\zeta)$. This kind of representation using the Plücker coordinates was first investigated by K. Hara and N. Ikeda [1] for the classical and generalized stochastic areas.

In Section 2, the proofs of the theorems will be given. Some examples, to which the theorems are applicable, will be discussed in Section 3.

2 Proofs of Theorems 1, 2 and 3

In this section, we always assume that the assumption (A1) is satisfied.

We shall start this section by recalling the general expression of stochastic oscillatory integral in terms of A_V and A_F, which was found out in [2]. Let $A_F^\natural = -\pi_L A_V + (I-\pi_L)A_F$.

Proposition 1. *If $|\Re\zeta| < 1/\|A\|_{\mathrm{op}}$, where $\|A\|_{\mathrm{op}}$ is the operator norm of A, then the following two identities hold.*

$$\int_{\mathcal{W}} e^{\zeta Q_A/2} d\mu = \big\{\det\big(I - \zeta A_F(I-\zeta A_V)^{-1}\big)\big\}^{-1/2} e^{-(\zeta/2)\mathrm{tr}\, A_F}, \tag{4}$$

$$\int_{\mathcal{W}} e^{\zeta Q_A/2}\delta_0(\boldsymbol{\eta}^{(N)})\, d\mu \tag{5}$$

$$= \frac{1}{\sqrt{(2\pi)^N \det C(\boldsymbol{\eta}^{(N)})}} \{\det(I - \zeta A_F^\natural (I-\zeta A_V)^{-1})\}^{-1/2} e^{-(\zeta/2)\mathrm{tr}\, A_F}.$$

Note that (5) includes (4). Namely, if $N=0$, then $A_F^\natural = A_F$. Hence what to do is just replacing $\delta_x(\boldsymbol{\eta})d\mu$ by $d\mu$ and substituting $C(\boldsymbol{\eta}^{(0)})=1$, as was stated in Theorem 1.

Proof. While the original proof can be found in [2], for the sake of completeness and preciseness, we give the proof.

If $S: H \to H$ is of trace class and $T: H \to H$ is a Hilbert-Schmidt operator, then we have that

$$\det{}_2(I+S)(I+T) = \det(I+S)\det{}_2(I+T)e^{-\operatorname{tr} S(I+T)}. \tag{6}$$

If $|\Re\zeta| < 1/\|A\|_{\mathrm{op}}$, then we have that

$$\int_{\mathcal{W}} e^{\zeta Q_A/2} dP = \big\{\det{}_2(I-\zeta A)\big\}^{-1/2}, \tag{7}$$

$$\int_{\mathcal{W}} e^{\zeta Q_A/2}\delta_0(\boldsymbol{\eta}^{(N)})d\mu = \frac{1}{\sqrt{(2\pi)^N \det C(\boldsymbol{\eta}^{(N)})}}\{\det{}_2(I-\zeta A_0)\}^{-1/2}e^{-(\zeta/2)\operatorname{tr} A_1}, \tag{8}$$

where $A_0 = (I-\pi_L)A(I-\pi_L)$ and $A_1 = \pi_L A \pi_L$. For example, see [5, 7]. Substituting (6) with $S = -\zeta A_F(I-\zeta A_V)^{-1}$ and $T = -\zeta A_V$ into (7), we obtain (4). Since $\operatorname{tr} A_F^\natural + \operatorname{tr} A_1 = \operatorname{tr} A_F$ and $\det_2(I-\zeta(I-\pi_L)A) = \det_2(I-\zeta A_0)$, plugging (6) with $S = -\zeta A_F^\natural(I-\zeta A_V)^{-1}$ and $T = -\zeta A_V$ into (8), we obtain (5). $\square$

Proof of Theorem 1. Let $|\zeta| < 1/\|A\|_{\mathrm{op}}$. Define $Q_L(\zeta): H\otimes \mathbf{C} \to H \otimes \mathbf{C}$ and the matrix $q_L(\zeta) = \big(q_L^{ij}(\zeta)\big)_{1\le i,j\le M} \in \mathbf{C}^{M\times M}$ by

$$Q_L(\zeta) = \pi_{\mathcal{R}}\big(I - \zeta A_F^\natural (I-\zeta A_V)^{-1}\big)\pi_{\mathcal{R}}, \quad Q_L(\zeta)\eta_i = \sum_{j=1}^M q_L^{ij}(\zeta)\eta_j, \quad 1 \le i \le M.$$

Then

$$\det q_L(\zeta) = \det\big(I - \zeta A_F^\natural(I-\zeta A_V)^{-1}\big), \tag{9}$$

which, in conjunction with Proposition 1, implies that

$$\det q_L(\zeta) \neq 0.$$

Since $\|A^\natural\|_{\mathrm{op}} \le \|A\|_{\mathrm{op}}$, for $p = (p_1, \dots, p_M)^\dagger \in \mathbf{C}^M$, one can define

$$J_L(\cdot; p) = (I - \zeta A^\natural)^{-1}\bigg(\sum_{j=1}^M p_j \eta_j\bigg).$$

Since $\eta_i \in \mathcal{R}$ and $\mathcal{R}(A_F^\natural) \subset \mathcal{R}$, observe that

$$(I - \zeta A_F^\natural (I-\zeta A_V)^{-1})\eta_i = Q_L(\zeta)\eta_i, \quad i = 1, \dots, M.$$

Noting that $A^\natural = A_V + A_F^\natural$, we have that

$$I - \zeta A_F^\natural(I-\zeta A_V)^{-1} = (I-\zeta A^\natural)(I-\zeta A_V)^{-1}.$$

Hence it holds that

$$(I-\zeta A_V)^{-1}\eta_i = (I-\zeta A^\natural)^{-1}Q_L(\zeta)\eta_i = (I-\zeta A^\natural)^{-1}\left(\sum_{j=1}^M q_L^{ij}(\zeta)\eta_j\right),$$

which implies that

$$\widetilde{J}_\zeta(\cdot;p) = J_L(\cdot;q_L(\zeta)^\dagger p). \tag{10}$$

By the very definition of $A^\#$, for every $p' = (p'_1,\dots,p'_M)^\dagger \in \mathbf{C}^M$, it holds that

$$\begin{aligned}\langle J_L(\cdot;p'),\eta_j\rangle_H &= \langle (I-\zeta A^\#)J_L(\cdot;p'),\eta_j\rangle_H \\ &= \langle (I-\zeta A^\natural)J_L(\cdot;p'),\eta_j\rangle_H + \zeta\langle (A^\natural - A^\#)J_L(\cdot;p'),\eta_j\rangle_H \\ &= \sum_{i=1}^M p'_i\langle \eta_i,\eta_j\rangle_H + \zeta\langle (A^\natural - A^\#)J_L(\cdot;p'),\eta_j\rangle_H, \quad j=1,\dots,M.\end{aligned}$$

Hence we have that

$$\Big(\big\langle J_L(\cdot;p'),\eta_j - \zeta(A^\natural - A^\#)^*\eta_j\big\rangle_H\Big)_{1\le j\le M} = C(\boldsymbol{\eta}^{(M)})p'.$$

By (10), we obtain that

$$\widetilde{J}_{\zeta,N} = C(\boldsymbol{\eta}^{(M)})q_L(\zeta)^\dagger,$$

which yields that

$$q_L(\zeta)^\dagger = C(\boldsymbol{\eta}^{(M)})^{-1}\widetilde{J}_{\zeta,N}.$$

From this, Proposition 1, and (9), the assertion of Theorem 1 follows immediately. □

Proof of Theorem 2. Let h be as described in (i). It is easily seen that

$$Q_A(\cdot+h) = Q_A + 2\nabla^*(Ah) + \langle Ah,h\rangle_H, \quad \boldsymbol{\eta}^{(N)}(\cdot - h) = \boldsymbol{\eta}^{(N)} - x.$$

Due to the Cameron-Martin theorem, we have that

$$\begin{aligned}\int_{\mathcal{W}} e^{\zeta Q_A/2}\delta_x(\boldsymbol{\eta}^{(N)})d\mu &= \int_{\mathcal{W}} e^{\zeta Q_A/2}\delta_0(\boldsymbol{\eta}^{(N)} - x)d\mu \\ &= \int_{\mathcal{W}} e^{(\zeta/2)Q_A(\cdot+h)}e^{-\nabla^*h-(\|h\|_H^2/2)}\delta_0(\boldsymbol{\eta}^{(N)})d\mu \\ &= e^{-\langle (I-\zeta A)h,h\rangle_H/2}\int_{\mathcal{W}} e^{(\zeta Q_A/2)-\nabla^*[(I-\zeta A)h]}\delta_0(\boldsymbol{\eta}^{(N)})d\mu.\end{aligned}$$

Since $(I-\zeta A)h = \sum_{j=1}^N c_j\eta_j$ for some $c_1,\dots,c_N \in \mathbf{C}$, $\nabla^*[(I-\zeta A)h] = 0$ $\delta_0(\boldsymbol{\eta}^{(N)})d\mu$-a.e. Thus we obtain (2).

If $N=M$, then $A^\natural = A^\#$ and hence

$$\widetilde{J}_{\zeta,M}p = \big(\langle \widetilde{J}_\zeta(\cdot;p),\eta_j\rangle_H\big)^\dagger_{1\le j\le M}.$$

This implies the second and the third assertions. □

Proof of Theorem 3. For any $h \in \mathcal{R}^\perp$, we have that

$$\langle (I-\zeta A)\widetilde{J}_\zeta(\cdot;p), h\rangle_H = \langle (I-\zeta A_V)\widetilde{J}_\zeta(\cdot;p), h\rangle_H = \sum_{j=1}^M p_j \langle \eta_j, h\rangle_H = 0.$$

By virtue of the assumption (A2) and the expression that

$$\widetilde{J}_\zeta(\cdot;p) = \zeta A_V \widetilde{J}_\zeta(\cdot;p) + \sum_{j=1}^M p_j \eta_j,$$

$\widetilde{J}_\zeta(\cdot;p)$ is m-times differentiable around $t=0$ and satisfies that

$$[\widetilde{J}_\zeta(\cdot;p)]^{(i)}(0) = \sum_{j=1}^M p_j \eta_j^{(i)}(0) = \big((Dp)_k\big)^\dagger_{di+1\le k\le di+d}, \quad 1 \le i \le m.$$

Thus $\widetilde{J}_\zeta(\cdot;p)$ solves (3).

If both $J_1, J_2 \in H \otimes \mathbf{C}$ are m-times differentiable around $t=0$ and satisfy (3), then, for any $h \in \mathcal{R}^\perp$,

$$\langle (I-\zeta A_V)(J_1-J_2), h\rangle_H = \langle (I-\zeta A)(J_1-J_2), h\rangle_H = 0.$$

Hence there exists $q=(q_1,\dots,q_M)^\dagger \in \mathbf{C}^M$ so that

$$(I-\zeta A_V)(J_1-J_2) = \sum_{j=1}^M q_j\eta_j \text{ and hence } J_1 - J_2 = \zeta A_V(J_1-J_2) + \sum_{j=1}^M q_j\eta_j.$$

Since $J_1^{(i)}(0) = J_2^{(i)}(0)$, $1 \le i \le m$, this implies that $Dq=0$. By the assumption (d), this implies that $q=0$, and hence $(I-\zeta A_V)(J_1-J_2)=0$. We then see that $J_1 - J_2 = 0$. □

3 Examples

In this section, we shall give several examples to which Theorems 1, 2, and 3 are applicable.

Example 1. In this example, we consider the quadratic Wiener functional related with harmonic oscillator; let $d=1$ and set

$$\mathfrak{h}_T(w) = \int_0^T w(t)^2 dt, \quad w \in \mathcal{W}.$$

If we define the symmetric Hilbert-Schmidt operator $A: H \to H$ by

$$Ah(t) = \int_0^t \int_s^T h(u)duds, \quad t \in [0,T],\ h \in H,$$

then $\mathfrak{h}_T = Q_A + (T^2/2)$. See [6]. Put

$$(A_V h)(t) = -\int_0^t \int_0^s h(u)duds, \quad (A_F h)(t) = \left(\int_0^T h(s)ds\right)t,$$

and $\eta_1(t) = t$ for $t \in [0,T]$. Then A_V is a Volterra operator, $A = A_V + A_F$, $\mathcal{R}(A_F) = \{c\eta_1 \,|\, c \in \mathbf{R}\}$, and $D = 1$. Thus the assumptions (A1) and (A2) are fulfilled with $m = M = 1$ and this η_1.

The condition that $\langle (I - \zeta A)h, g\rangle_H = 0$ for any $g \in \mathcal{R}^\perp$ can be rewritten as

$$h(t) - \zeta \int_0^t \int_s^T h(u)duds = B(h) \times t, \quad t \in [0,T], \tag{11}$$

where $B(h) \in \mathbf{C}$ is a constant depending on only h. This is equivalent to that

$$h'' + \zeta h = 0, \quad h(0) = 0.$$

Solving this with the additional initial condition that $h'(0) = p$, we obtain

$$\widetilde{J}_\zeta(t;p) = \frac{\sin(\sqrt{\zeta}\,t)}{\sqrt{\zeta}} p.$$

Setting $b(p) = B(\widetilde{J}_\zeta(\cdot;p))$, we conclude from this and (11) that

$$b(p) = [(I - \zeta A)\widetilde{J}_\zeta(\cdot;p)]'(T) = \cos(\sqrt{\zeta}\,T)p.$$

Furthermore we have that

$$\widetilde{J}_{\zeta,1} = \frac{\sin(\sqrt{\zeta}\,T)}{\sqrt{\zeta}}.$$

First let $N = 1$, i.e. $L = \mathcal{R}$. For $x \in \mathbf{R}$, set $p_x = (\sqrt{\zeta}/\sin(\sqrt{\zeta}\,T))x$ and $h_x = \widetilde{J}_\zeta(\cdot;p_x)$. From (11) it follows that

$$\langle (I - \zeta A)h_x, h_x\rangle_H = \langle b(p_x)\eta_1, h_x\rangle_H = \frac{\sqrt{\zeta}}{\tan(\sqrt{\zeta}\,T)} x^2.$$

Since $C(\boldsymbol{\eta}^{(1)}) = T$ and $\operatorname{tr} A_F = T^2/2$, by Theorems 1 and 2,

$$\int_{\mathcal{W}} e^{\zeta \mathfrak{h}_T/2} \delta_x(w(T))d\mu = \frac{1}{\sqrt{2\pi T}} \sqrt{\frac{\sqrt{\zeta}\,T}{\sin(\sqrt{\zeta}\,T)}} \exp\left(-\frac{1}{2}\frac{\sqrt{\zeta}}{\tan(\sqrt{\zeta}\,T)} x^2\right).$$

Next let $N = 0$, i.e., $L = \{0\}$. Then $A^\natural = A$ and $\langle A^\# h, \eta_1\rangle_H = 0$ for any $h \in H$. Due to (11), we obtain that

$$\begin{aligned}\langle \widetilde{J}_\zeta(\cdot;p), \eta_1 - \zeta(A^\natural - A^\#)^*\eta_1\rangle_H &= \langle (I - \zeta A)\widetilde{J}_\zeta(\cdot;p), \eta_1\rangle_H \\ &= [(I - \zeta A)\widetilde{J}_\zeta(\cdot;p)](T) = b(p)T = T\cos(\sqrt{\zeta}\,T)p.\end{aligned}$$

Hence

$$\widetilde{J}_{\zeta,0} = T\cos(\sqrt{\zeta} T).$$

By Theorem 1, it holds that

$$\int_{\mathcal{W}} e^{\zeta \mathfrak{h}_T/2} d\mu = \sqrt{\frac{1}{\cos(\sqrt{\zeta}\,T)}}\ .$$

Example 2. In this example, we deal with the quadratic Wiener functional corresponding to the classical stochastic area; let $d = 2$ and put

$$\mathfrak{s}_T(w) = \frac{1}{2}\int_0^T \{w^1(t)dw^2(t) - w^2(t)dw^1(t)\},$$

where $w(t) = (w^1(t), w^2(t))$ denotes the position of w at time t and $dw^i(t)$ stands for the Itô integral with respect to $w^i(t)$, $i = 1, 2$. If we define the symmetric Hilbert-Schmidt operator $A : H \to H$ by

$$Ah(t) = \int_0^t J\Big(h(s) - \frac{1}{2}h(T)\Big)ds, \quad t \in [0, T],\ h \in H, \tag{12}$$

where $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, then $\mathfrak{s}_T = Q_A/2$. See [6]. Set

$$(A_V h)(t) = \int_0^t Jh(s)ds, \quad (A_F h)(t) = -\frac{t}{2}Jh(T), \quad t \in [0, T].$$

Define $\eta_i \in H$ by $\eta_i(t) = te_i$, $i = 1, 2$, where $e_1 = (1, 0)^\dagger$, $e_2 = (0, 1)^\dagger \in \mathbf{R}^2$. Then A_V is a Volterra operator, $A = A_V + A_F$, $\mathcal{R}(A_F) = \{c_1\eta_1 + c_2\eta_2 \,|\, c_1, c_2 \in \mathbf{R}\}$, and D is the 2×2 unit matrix. Thus the assumptions (A1) and (A2) are satisfied with $m = 1$, $M = 2$, and these η_1 and η_2. Moreover, we see that

$$\operatorname{tr} A_F = \frac{1}{T}\sum_{i=1}^2 \langle \eta_i, A_F\eta_i\rangle_H = -\frac{1}{2}\sum_{i=1}^2 \langle e_i, Je_i\rangle_{\mathbf{R}^2} = 0.$$

The condition that $\langle (I - \zeta A)h, g\rangle = 0$ for any $g \in R^\perp$ can be rewritten as

$$h(t) - \zeta\Big(\int_0^t Jh(s)ds - \frac{1}{2}Jh(T)t\Big) = t\,B(h), \quad t \in [0, T], \tag{13}$$

where $B(h) \in \mathbf{C}^2$ is a constant vector depending only on h. This is equivalent to that

$$h'' - \zeta Jh' = 0, \quad h(0) = 0.$$

Solving this with the additional condition that $h'(0) = p$, we obtain that

$$\widetilde{J}_\zeta(t; p) = \frac{\sin(\zeta t/2)}{\zeta/2}\begin{pmatrix} \cos(\zeta t/2) & -\sin(\zeta t/2) \\ \sin(\zeta t/2) & \cos(\zeta t/2) \end{pmatrix} p.$$

If we set $b(p) = B(\widetilde{J}_\zeta(\cdot; p))$, then it follows from this expression and (13) that

$$\begin{aligned} b(p) &= [(I - \zeta A)\widetilde{J}_\zeta(\cdot; p)]'(0) = \Big\{p + \frac{\zeta}{2}J\widetilde{J}_\zeta(T; p)\Big\} \\ &= \cos(\zeta T/2)\begin{pmatrix} \cos(\zeta T/2) & -\sin(\zeta T/2) \\ \sin(\zeta T/2) & \cos(\zeta T/2) \end{pmatrix} p. \end{aligned}$$

Furthermore we have that

$$\widetilde{J}_{\zeta,2} = \frac{\sin(\zeta T/2)}{\zeta/2} \begin{pmatrix} \cos(\zeta T/2) & -\sin(\zeta T/2) \\ \sin(\zeta T/2) & \cos(\zeta T/2) \end{pmatrix}.$$

First let $N = 2$, i.e., $L = \mathcal{R}$. For $x = (x_1, x_2)^\dagger \in \mathbf{R}^2$, set

$$p_x = \frac{\zeta/2}{\sin(\zeta T/2)} \begin{pmatrix} \cos(\zeta T/2) & \sin(\zeta t/2) \\ -\sin(\zeta T/2) & \cos(\zeta T/2) \end{pmatrix} x$$

and $h_x = \widetilde{J}_\zeta(\cdot; p_x)$. By virtue of (13), we see that

$$\langle (I - \zeta A)h_x, h_x \rangle_H = \langle b(p_x), h_x(T) \rangle_{\mathbf{R}^2} = \frac{\zeta/2}{\tan(\zeta T/2)} |x|^2.$$

Since $\det C(\boldsymbol{\eta}^{(2)}) = T^2$, by Theorems 1 and 2,

$$\int_{\mathcal{W}} e^{\zeta \mathfrak{s}_T} \delta_x(w(T)) d\mu = \frac{1}{2\pi T} \frac{\zeta T/2}{\sin(\zeta T/2)} \exp\biggl(-\frac{1}{2} \frac{\zeta/2}{\tan(\zeta T/2)} |x|^2 \biggr).$$

Next let $N = 0$, i.e., $L = \{0\}$. Since $A^\natural = A$ and $\langle A^\# h, \eta_i \rangle_H = 0$ for any $h \in H$, we have that

$$\begin{aligned} &\langle \widetilde{J}_\zeta(\cdot; p), \eta_j - \zeta(A^\natural - A^\#)^* \eta_j \rangle_H = \langle (I - \zeta A)\widetilde{J}_\zeta(\cdot; p), \eta_j \rangle_H \\ &\quad = \langle [(I - \zeta A)\widetilde{J}_\zeta(\cdot; p)](T), e_j \rangle_{\mathbf{R}^2} = T \langle b(p), e_j \rangle_{\mathbf{R}^2}. \end{aligned}$$

Hence

$$\widetilde{J}_{\zeta,0} = T \cos(\sqrt{\zeta} T) \begin{pmatrix} \cos(\sqrt{\zeta} T) & -\sin(\sqrt{\zeta} T) \\ \sin(\sqrt{\zeta} T) & \cos(\sqrt{\zeta} T) \end{pmatrix}.$$

This implies that

$$\int_{\mathcal{W}} e^{\zeta \mathfrak{s}_T} d\mu = \frac{1}{\cos(\zeta T/2)}.$$

Finally let $N = 1$ and $L = \{c\eta_1 \,|\, c \in \mathbf{R}\}$. Since $\langle \eta_1, \eta_2 \rangle_H = 0$, due to the observation made in Section 1, we have that

$$\widetilde{J}_{\zeta,1} = \begin{pmatrix} \sin(\zeta T/2)\cos(\zeta T/2)/(\zeta/2) & -\sin^2(\zeta T/2)/(\zeta/2) \\ T \sin(\zeta T/2)\cos(\zeta T/2) & T\cos^2(\zeta T/2) \end{pmatrix}.$$

If we set $h_a = \widetilde{J}_\zeta(\cdot; p_{(a,0)^\dagger})$ for $a \in \mathbf{R}$, then by (13), it holds that

$$(I - \zeta A)h_a = \frac{a\zeta/2}{\tan(\zeta T/2)} \eta_1, \quad \langle (I - \zeta A)h_a, h_a \rangle_H = \frac{\zeta/2}{\tan(\zeta T/2)} a^2.$$

Thus

$$\int_{\mathcal{W}} e^{\zeta \mathfrak{s}_T} \delta_a(w^1(T)) d\mu = \frac{1}{\sqrt{2\pi T}} \sqrt{\frac{\zeta T}{\sin(\zeta T)}} \exp\biggl(-\frac{1}{2} \frac{\zeta/2}{\tan(\zeta T/2)} a^2 \biggr).$$

Example 3. As a combination of Examples 1 and 2, we can apply our results to the quadratic Wiener functional associated with harmonic oscillator in uniform magnetic field; let $d = 2$, $\alpha, \beta_1, \beta_2 \in \mathbf{R}$, $\neq 0$, and put

$$\mathfrak{q}(w) = \alpha \mathfrak{s}_T(w) + \frac{1}{2}\int_0^T \langle Bw(t), w(t)\rangle_{\mathbf{R}^2} dt,$$

where $B = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix}$. Define $A, A_V, A_F : H \to H$ by

$$\begin{aligned} A_V h(t) &= \int_0^t \left\{ \alpha J h(s) - \int_0^s Bh(u)du \right\} ds, \\ A_F h(t) &= \left\{ -\frac{\alpha}{2} Jh(T) + \int_0^T Bh(u)du \right\} t, \quad t \in [0, T], \\ A &= A_V + A_F. \end{aligned}$$

Due to the observations in Examples 1 and 2, we have that

$$q = \frac{1}{2} Q_A + \frac{(\beta_1 + \beta_2)T^2}{4}.$$

Moreover, A_V is a Volterra operator, $\mathcal{R}(A_F) = \{c_1\eta_1 + c_2\eta_2 \,|\, c_1, c_2 \in \mathbf{R}\}$, and D is the 2×2 unit matrix, where $\eta_1, \eta_2 \in H$ are defined as in Example 2. Thus the assumptions (A1) and (A2) are fulfilled with $m = 1$, $M = 2$, and these η_1 and η_2. Moreover, $\operatorname{tr} A_F = \{\beta_1 + \beta_2\} T^2/4$.

The equation (3) to determine $\widetilde{J}_\zeta(t; p)$ now reads as

$$h'' - \alpha\zeta J h' + \zeta B h = 0, \quad h(0) = 0, \quad h'(0) = p,$$

which is nothing but the Jacobi equation associated with the Lagrange function for harmonic oscillator in uniform magnetic field. The Jacobi equation leads us to the expression of $\widetilde{J}_\zeta(t; p)$ as follows. Let $\pm\lambda_1, \pm\lambda_2$ be the roots of the equation

$$(\lambda^2 + \zeta\beta_1)(\lambda^2 + \zeta\beta_2) + \zeta^2\alpha^2\lambda^2 = 0.$$

In the sequel, we assume that ζ satisfies that $\zeta\{\alpha^4\zeta^2 + 2\alpha^2(\beta_1+\beta_2)\zeta + (\beta_1-\beta_2)^2\} \neq 0$, which implies that $\pm\lambda_j$'s are different from each other. Put

$$\begin{aligned} a_\zeta^{11}(t) &= \frac{\lambda_1(\lambda_2^2 + \zeta\beta_1)}{(\lambda_1^2 - \lambda_2^2)\zeta\beta_1}\sinh(\lambda_1 t) - \frac{\lambda_2(\lambda_1^2 + \zeta\beta_1)}{(\lambda_1^2 - \lambda_2^2)\zeta\beta_1}\sinh(\lambda_2 t), \\ a_\zeta^{12}(t) &= -a_\zeta^{21}(t) = -\frac{\alpha\zeta}{\lambda_1^2 - \lambda_2^2}\cosh(\lambda_1 t) + \frac{\alpha\zeta}{\lambda_1^2 - \lambda_2^2}\cosh(\lambda_2 t), \\ a_\zeta^{22}(t) &= \frac{\lambda_1^2 + \zeta\beta_1}{\lambda_1(\lambda_1^2 - \lambda_2^2)}\sinh(\lambda_1 t) - \frac{\lambda_2^2 + \zeta\beta_1}{\lambda_2(\lambda_1^2 - \lambda_2^2)}\sinh(\lambda_2 t), \end{aligned}$$

and define

$$A_\zeta(t) = \begin{pmatrix} a_\zeta^{11}(t) & a_\zeta^{12}(t) \\ a_\zeta^{21}(t) & a_\zeta^{22}(t) \end{pmatrix}.$$

Then we have that

$$\widetilde{J}_\zeta(t;p) = A_\zeta(t)p. \tag{14}$$

It should be noticed that $a_\zeta^{ij}(t)$'s are all symmetric functions of λ_1^2 and λ_2^2, and hence neither the choice of the sign of λ_i's nor the ordering of λ_1 and λ_2 matters.

From (14), we can conclude that

$$\widetilde{J}_{\zeta,2} = A_\zeta(T),$$
$$\widetilde{J}_{\zeta,0} = \Big(I - \frac{\zeta\alpha T}{2}J\Big)A_\zeta(T) - \zeta\alpha J\int_0^T A_\zeta(s)ds - \zeta B\int_0^T sA_\zeta(s)ds.$$

In particular, we obtain that

$$\det \widetilde{J}_{\zeta,2} = \frac{2\zeta^2\alpha^2}{(\lambda_1^2-\lambda_2^2)^2}\{1-\cosh(\lambda_1 T)\cosh(\lambda_2 T)\} + \frac{\zeta^2\alpha^2(\zeta\beta_1+\zeta\beta_2)+(\zeta\beta_1-\zeta\beta_2)^2}{\lambda_1\lambda_2(\lambda_1^2-\lambda_2)^2}\sinh(\lambda_1 T)\sinh(\lambda_2 T),$$

which brings us the expression that

$$\int_{\mathcal{W}} e^{\zeta\mathfrak{q}}\delta_0(w(T))d\mu = (2\pi)^{-1}(\det \widetilde{J}_{\zeta,2})^{-1/2}.$$

In the remaining of this section, we shall give two examples where the number m corresponding to the differentiability in the assumptions (A2) is greater than 1.

Example 4. In this example, we consider the quadratic Wiener function obtained as the norm of the Malliavin derivative of $\mathfrak{h}_T$; let $d=1$ and set

$$\mathfrak{g}_T(w) = \int_0^T\Big(\int_t^T w(s)ds\Big)^2 dt, \quad w\in\mathcal{W}.$$

It is then easily checked that

$$\mathfrak{g}_T(w) = \frac{1}{4}\|\nabla\mathfrak{h}_T(w)\|_H^2.$$

Set

$$Ah(t) = \int_0^t ds\int_s^T du\int_0^u dv\int_v^T dah(a),$$
$$A_V h(t) = \int_0^t ds\int_0^s du\int_0^u dv\int_0^v dah(a), \quad t\in[0,T],$$
$$A_F = A - A_V.$$

Define $\eta_1,\eta_2\in H$ by $\eta_1(t)=t$ and $\eta_2(t)=t^3-T^2t$, $t\in[0,T]$. Then $q = Q_A + (T^4/6)$, $\operatorname{tr} A_F = T^4/6$, and $\mathcal{R}(A_F) = \{c_1\eta_1 + c_2\eta_2 \,|\, c_1,c_2\in\mathbf{R}\}$. See [8]. Moreover, it holds that

$$D = \begin{pmatrix} 1 & -T^2 \\ 0 & 0 \\ 0 & 6 \end{pmatrix}.$$

Thus the assumptions (A1) and (A2) are fulfilled with $m = 3$, $M = 2$, and these η_1 and η_2.

The equation (3) is equivalent to the ordinary differential equation

$$h^{(4)} - \zeta h = 0, \qquad \begin{pmatrix} h(0) \\ h^{(1)}(0) \\ h^{(2)}(0) \\ h^{(3)}(0) \end{pmatrix} = \begin{pmatrix} 0 \\ p_1 - T^2 p_2 \\ 0 \\ 6p_2 \end{pmatrix},$$

where $p = (p_1, p_2)^\dagger$ and $h^{(j)}$ denotes the jth derivative of h. Solving this, we obtain that

$$\widetilde{J}_\zeta(t; p) = \langle x_\zeta(t), p \rangle_{\mathbf{R}^2},$$

where

$$x_\zeta(t) = \frac{1}{2\zeta^{1/4}} \begin{pmatrix} \sinh(\zeta^{1/4}t) + \sin(\zeta^{1/4}t) \\ -(T^2 - 6\zeta^{-1/2}) \sinh(\zeta^{1/4}t) - (T^2 + 6\zeta^{-1/2}) \sin(\zeta^{1/4}t) \end{pmatrix}.$$

We first consider the case where $N = 0$. Setting $\alpha_\zeta = \cosh(\zeta^{1/4}/T)$, $\beta_\zeta = \cos(\zeta^{1/4}/T)$, and defining $a_\zeta, b_\zeta \in \mathbf{R}^2$ by

$$b_\zeta = \frac{\zeta^{1/2}}{12} \begin{pmatrix} \alpha_\zeta - \beta_\zeta \\ -(T^2 - 6\zeta^{-1/2})\alpha_\zeta + (T^2 + 6\zeta^{-1/2})\beta_\zeta \end{pmatrix}, \quad a_\zeta = x_\zeta^{(1)}(T) - 2T^2 b_\zeta,$$

we observe that

$$x_\zeta^{(1)}(T) = a_\zeta + 2T^2 b_\zeta, \quad x_\zeta^{(3)}(T) = 6b_\zeta,$$

and hence that

$$(I - \zeta A)\widetilde{J}_\zeta(\cdot; p) = \langle a_\zeta, p \rangle_{\mathbf{R}^2} \eta_1 + \langle b_\zeta, p \rangle_{\mathbf{R}^2} \eta_2.$$

Since $\langle \eta_1, \eta_2 \rangle_H = 0$, we then have that

$$\widetilde{J}_{\zeta,0} = \begin{pmatrix} \|\eta_1\|_H^2 a_\zeta^\dagger \\ \|\eta_2\|_H^2 b_\zeta^\dagger \end{pmatrix}.$$

This implies that

$$\det \widetilde{J}_{\zeta,0} = \det C(\boldsymbol{\eta}^{(2)}) \alpha_\zeta \beta_\zeta.$$

Thus we obtain that

$$\int_{\mathcal{W}} e^{\zeta \mathfrak{g}_T/2} d\mu = \left\{ \cosh(\zeta^{1/4}T) \cos(\zeta^{1/4}T) \right\}^{-1/2}.$$

We next consider the case where $N = 1$. If we set $\gamma_\zeta = \sinh(\zeta^{1/4}T)$ and $\delta_\zeta = \sin(\zeta^{1/4}T)$ and define $c_\zeta \in \mathbf{R}^2$ by

$$c_\zeta = \frac{1}{2\zeta^{1/4}} \begin{pmatrix} \gamma_\zeta + \delta_\zeta \\ -(T^2 - 6\zeta^{-1/2})\gamma_\zeta - (T^2 + 6\zeta^{-1/2})\delta_\zeta \end{pmatrix},$$

then it holds that

$$\langle \widetilde{J}_\zeta(\cdot; p), \eta_1 \rangle_H = \langle c_\zeta, p \rangle_{\mathbf{R}^2}.$$

Since η_1 and η_2 are perpendicular to each other, due to the observation made in Section 1, we have that

$$\widetilde{J}_{\zeta,1} = \begin{pmatrix} c_\zeta^\dagger \\ \|\eta_2\|_H^2 b_\zeta^\dagger \end{pmatrix} \quad \text{and} \quad \det \widetilde{J}_{\zeta,1} = \frac{\|\eta_2\|_H^2}{2\zeta^{1/4}}\{\alpha_\zeta\delta_\zeta + \beta_\zeta\gamma_\zeta\}.$$

Thus we obtain that

$$\int_{\mathcal{W}} e^{\zeta \mathfrak{g}_T/2}\delta_0(w(T))d\mu = \frac{\zeta^{1/8}}{\sqrt{\pi\{\cosh(\zeta^{1/4}T)\sin(\zeta^{1/4}T) + \cos(\zeta^{1/4}T)\sinh(\zeta^{1/4}T)\}}}.$$

We finally consider the case where $N = 2$. Define $d_\zeta = (d_\zeta^1, d_\zeta^2)^\dagger \in \mathbf{R}^2$ by

$$\begin{aligned} d_\zeta^1 &= \frac{3}{2\zeta^{1/4}}\{-2\zeta^{-1/4}T\alpha_\zeta + 2\zeta^{-1/4}T\beta_\zeta + (T^2 + 2\zeta^{-1/2})\gamma_\zeta + (T^2 - 2\zeta^{-1/2})\delta_\zeta\} \\ d_\zeta^2 &= \frac{3}{2\zeta^{1/4}}\{2\zeta^{-1/4}T(T^2 - 6\zeta^{-1/2})\alpha_\zeta - 2\zeta^{-1/4}T(T^2 + 6\zeta^{-1/2})\beta_\zeta \\ &\qquad - (T^2 + 2\zeta^{-1/2})(T^2 - 6\zeta^{-1/2})\gamma_\zeta - (T^2 - 2\zeta^{-1/2})(T^2 + 6\zeta^{-1/2})\delta_\zeta\}. \end{aligned}$$

Then we have that

$$\langle \widetilde{J}_\zeta(\cdot; p), \eta_2\rangle_H = \langle d_\zeta - T^2 c_\zeta, p\rangle_{\mathbf{R}^2}.$$

This implies that

$$\widetilde{J}_{\zeta,2} = \begin{pmatrix} c_\zeta^\dagger \\ (d_\zeta - T^2 c_\zeta)^\dagger \end{pmatrix}$$

and

$$\det \widetilde{J}_{\zeta,2} = 18\zeta^{-5/4}\{2\zeta^{-1/4}\gamma_\zeta\delta_\zeta - T(\alpha_\zeta\delta_\zeta + \beta_\zeta\gamma_\zeta)\}.$$

Thus we obtain that

$$\int_{\mathcal{W}} e^{\zeta \mathfrak{g}_T/2}\delta_0(\boldsymbol{\eta}^{(2)})d\mu = \frac{\zeta^{5/8}}{\sqrt{72\pi^2\{2\zeta^{-1/4}\gamma_\zeta\delta_\zeta - T(\alpha_\zeta\delta_\zeta + \beta_\zeta\gamma_\zeta)\}}}.$$

Example 5. In this example, we consider the generalized stochastic area investigated by K. Hara, N. Ikeda, S. Kusuoka and S. Manabe [1, 2, 3, 4]. For this purpose, let $d = 2$ and $n \in \mathbf{N}$. Define $\mathcal{I} : \mathcal{W} \ni w \mapsto \mathcal{I}w \in \mathcal{W}$ by

$$\mathcal{I}w(t) = \int_0^t w(s)ds, \quad t \in [0, T].$$

Put

$$\mathfrak{a}(w) = \frac{1}{2}\int_0^T \langle J\mathcal{I}^n w(t), \mathcal{I}^{n-1}w(t)\rangle dt,$$

where $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \in \mathbf{R}^{2\times 2}$ as in Example 2, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathbf{R}^2$. If we set

$$\begin{aligned}
A_V h &= (-1)^n \mathcal{I}^{2n+1} J h, \\
(A_F h)'(t) &= \frac{1}{2}\sum_{j=0}^{n-1}(-1)^j \left\{ \mathcal{I}^{n+j+1} Jh(T)\,\frac{(T-t)^{n-j-1}}{(n-j-1)!} - \mathcal{I}^{n+j} Jh(T)\,\frac{(T-t)^{n-j}}{(n-j)!} \right\} \\
&\quad - \frac{(-1)^n}{2}\mathcal{I}^{2n} Jh(T), \\
A &= A_V + A_F,
\end{aligned}$$

then A_V is a Volterra operator and $q = Q_A/2$. Putting

$$\eta_{2j-1}(t) = \frac{t^j}{j}\begin{pmatrix}1\\0\end{pmatrix}, \quad \eta_{2j}(t) = \frac{t^j}{j}\begin{pmatrix}0\\1\end{pmatrix}, \qquad 1 \le j \le n+1,$$

we see that $\mathcal{R}(A_F) \subset \left\{\sum_{j=1}^{2n+2} c_j \eta_j \,\middle|\, c_1, \dots, c_{2n+2} \in \mathbf{R}\right\}$ and

$$D = \begin{pmatrix}
I & 0 & \cdots & \cdots & 0 \\
0 & I & \ddots & & \vdots \\
\vdots & \ddots & 2I & \ddots & \vdots \\
\vdots & & \ddots & \ddots & 0 \\
0 & \cdots & \cdots & 0 & n!I \\
0 & \cdots & \cdots & \cdots & 0 \\
\vdots & & & & \vdots \\
0 & \cdots & \cdots & \cdots & 0
\end{pmatrix} \in \mathbf{R}^{2(2n+1)\times(2n+2)},$$

where I denotes the 2×2 unit matrix. Hence the assumptions (A1) and (A2) are satisfied with $m = 2n+1$, $M = 2n+2$, and these η_j's. Thus our results are applicable to the generalized stochastic area $\mathfrak{a}$ studied in [1, 2, 3, 4].

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