Numerical Simulation of Rayleigh-Taylor Problems by an Energy-Stable Finite Element Scheme

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Abstract

Numerical simulation of some Rayleigh-Taylor problems is performed by an energy-stable finite element scheme. The problems are solved in the frame work of the density-dependent Navier-Stokes equations. Mesh dependency of numerical results is discussed for the Rayleigh-Taylor instability problem and perturbed stable problems.

1 Introduction

The Rayleigh-Taylor problem describes the movement of two fluids subject to an initial state, where a heavy fluid is located in the upper part of a container and a light fluid is in the lower part. It is one of two-fluid flow problems with unknown interfaces. In this paper we simulate such problems by an energy-stable P1/P2/P1 finite element scheme developed recently by ourselves [6], where the density, the velocity, and the pressure are approximated by linear, quadratic, and linear finite element, respectively.

Many numerical results have been obtained for two-fluid flow problems, e.g. [5], [8], but, to the best of our knowledge, there are no numerical schemes

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whose solutions are proved to converge to the exact one. Recently we have developed a P0/P1N/P0 finite element scheme for density-dependent Navier-Stokes equations and proved the convergence of the finite element solutions [4]. Here P0 and P1N stand for constant and non-conforming linear element, respectively. After reformulating the original two-fluid flow problem by a density-dependent Navier-Stokes problem, we can apply the P0/P1N/P0 scheme to two-fluid flow problems. Some numerical results for the Rayleigh-Taylor problem by this scheme have been reported in [7].

The P1/P2/P1 finite element scheme is applied to Rayleigh-Taylor problems also in the framework of a density-dependent Navier-Stokes problem, where the density is treated as a field function defined in the whole domain. This scheme has the advantage that the stress interface condition can be treated more easily and the approximation ability is higher than the P0/P1N/P0 scheme. We solve some Rayleigh-Taylor problems subject to an unstable equilibrium and perturbed non-equilibrium initial states and discuss the effect caused by the mesh subdivision.

2 Two-fluid flow problems

We begin by describing the general two-fluid flow problem. Let Ω be a bounded domain in $\mathbf{R}^d$, $d = 2, 3$, with piecewise smooth boundary Γ, and T be a positive number. At the initial time $t = 0$ the domain Ω is occupied by two immiscible incompressible viscous fluids; each domain is denoted by Ω_k^0, $k = 1, 2$, whose interface $\partial\Omega_1^0 \cap \partial\Omega_2^0$ is denoted by Γ_{12}^0. At $t \in (0, T)$ the two fluids occupy domains $\Omega_k(t)$, $k = 1, 2$, and the interface $\partial\Omega_1(t) \cap \partial\Omega_2(t)$ is denoted by $\Gamma_{12}(t)$. Let ρ_k and μ_k, $k = 1, 2$, be the densities and the viscosities of the two fluids. Let

$$u : \Omega \times (0, T) \to \mathbf{R}^d, \quad p : \Omega \times (0, T) \to \mathbf{R}$$

be the velocity and the pressure to be found. The Navier-Stokes equations are satisfied in each domain $\Omega_k(t)$, $k = 1, 2$, $t \in (0, T)$,

$$\rho_k \left\{ \frac{\partial u}{\partial t} + (u \cdot \nabla) u \right\} - \nabla \left(2\mu_k D(u) \right) - \nabla p = \rho_k f \quad \text{(1a)}$$

$$\nabla \cdot u = 0, \quad \text{(1b)}$$

where $f : \Omega \times (0, T) \to \mathbf{R}^d$ is a given function, $D(u)$ is the strain tensor defined by

$$D_{ij}(u) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

and

$$\left[\nabla\left(2\mu_k D(u)\right)\right]_i = \sum_{j=1}^{d} \frac{\partial}{\partial x_j}\left(2\mu_k D_{ij}(u)\right).$$

On the boundary Γ, $t \in (0, T)$, the slip conditions

$$u \cdot n = 0, \quad \sigma(\mu, u, p)n \times n = 0 \tag{2}$$

are imposed, where n is the unit outer normal to Γ and σ is the stress tensor defined by

$$\sigma(\mu, u, p) = -pI + 2\mu D(u).$$

On the interface $\Gamma_{12}(t)$, $t \in (0, T)$, the velocity and the stress vector should be continuous,

$$[u] = 0, \quad [\sigma(\mu, u, p)n_{12}] = 0, \tag{3}$$

where $[\cdot]$ means the difference of the values approaching from the domain Ω_2 and the domain Ω_1, n_{12} is the unit outer normal to $\Gamma_{12}(t)$ from $\Omega_1(t)$ to $\Omega_2(t)$. The initial condition at $t = 0$ for the velocity

$$u = u^0 \tag{4}$$

is imposed.

We consider the case where a heavy fluid is located in the upper part and a light fluid is in the lower part as shown in Fig. 1. Given such an initial state, we consider the transitional movement of the fluids.

3 An energy-stable finite element scheme

We have developed a class of energy-stable finite element schemes for problem (1)–(4) in [6]. The main idea is to convert the original problem to a density-dependent Navier-Stokes problem, where the density is treated as a field function of x and t. The viscosity μ is also supposed to be a function of ρ. In the case of two-fluid flow problems μ is defined by

$$\mu(\rho) = \mu_1 \frac{\rho_2 - \rho}{\rho_2 - \rho_1} + \mu_2 \frac{\rho - \rho_1}{\rho_2 - \rho_1}. \tag{5}$$

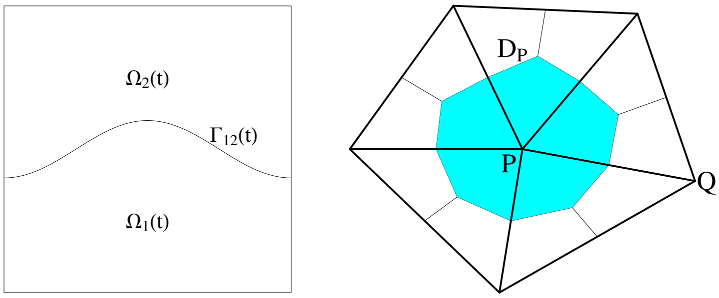

Figure 1: The interface $\Gamma_{12}(t)$ (left) and the barycentric domain D_P (right)

Here we only describe a P1/P2/P1 scheme, that is, the density, the velocity, and the pressure are approximated by P1-, P2-, and P1-element, respectively. We consider the 2-dimensional case $d = 2$. For the extension to the 3-dimensional case and the derivation of the scheme we refer to [6].

Let Φ_h, V_h, and Q_h be P1-, P2-, and P1-finite element space for the density, the velocity, and the pressure. The essential boundary condition is imposed on V_h,

$$V_h = \{v_h;\ (v_h \cdot n)(P) = 0 \text{ (node } P \in \Gamma)\},$$

but is not on Φ_h and Q_h. Let Δt be a time increment and $N_T = \lfloor T/\Delta t \rfloor$. We denote by (ρ_h^n, u_h^n, p_h^n) the value at time $n\Delta t$, and by $\bar{D}_{\Delta t}$ the backward difference operator, e.g.,

$$\bar{D}_{\Delta t} u_h^n = \frac{u_h^n - u_h^{n-1}}{\Delta t}.$$

We find $\{(\rho_h^n, u_h^n, p_h^n) \in \Phi_h \times V_h \times Q_h;\ n = 1, \cdots, N_T\}$ satisfying

$$\begin{aligned}
\left(\bar{D}_{\Delta t}\bar{\rho}_h^n, \bar{\phi}_h\right) + c_{1h}(u_h^{n-1}, \rho_h^n, \phi_h) &= 0, \quad \forall \phi_h \in \Phi_h && \text{(6a)}\\
\left(\rho_h^{n-1}\bar{D}_{\Delta t}u_h^n + \frac{1}{2}u_h^n\bar{D}_{\Delta t}\rho_h^n, v_h\right) &+ a_1(\rho_h^n, u_h^{n-1}, u_h^n, v_h) + a_0(\rho_h^n, u_h^n, v_h) \\
+b(v_h, p_h^n) &= \left(\rho_h^n \Pi_h f^n, v_h\right), \quad \forall v_h \in V_h && \text{(6b)}\\
b(u_h^n, q_h) &= 0, \quad \forall q_h \in Q_h && \text{(6c)}
\end{aligned}$$

subject to the initial conditions

$$\rho_h^0 = \Pi_h \rho^0, \quad u_h^0 = \Pi_h u^0. \tag{7}$$

Here ρ^0 is a function defined by

$$\rho^0(x) = \begin{cases} \rho_1 & (x \in \Omega_1^0) \\ \rho_2 & (x \in \Omega_2^0), \end{cases}$$

Π_h is the interpolation operator to the corresponding finite element spaces, $(\cdot,\cdot)$ is the inner product in $L^2(\Omega)$ or $L^2(\Omega)^2$, and linear forms a_1, a_0, and b are defined by

$$\begin{aligned} a_1(\rho, w, u, v) &= \int_\Omega \left(\frac{1}{2}(w \cdot \nabla\rho)u + \frac{1}{2}\rho(\nabla \cdot w)u + \rho(w \cdot \nabla)u \right) \cdot v \ dx \\ a_0(\rho, u, v) &= \int_\Omega \mu(\rho) D(u) : D(v) \ dx, \quad b(v,q) = -\int_\Omega (\nabla \cdot v) q \ dx. \end{aligned}$$

We now explain (6a), which is based on an upwind approximation developed in [1]. At first we make the dual decomposition $\{D_P\}$ of the domain Ω, where D_P is the barycentric domain of the node P, see Fig. 1. The D_P is made by connecting the midpoints of the sides and the centroids of the triangles around $P \in \Omega$. When the node P is on Γ, some parts of the (approximate) boundary are used. We define the lumping operator $^-$ from Φ_h to $L^2(\Omega)$ by

$$\phi_h \longmapsto \bar{\phi}_h(x) \equiv \sum_P \phi_h(P)\chi_P(x),$$

where χ_P is the characteristic function of D_P. We denote by Λ_P the set of nodes adjacent to P. The form c_{1h} is defined by

$$c_{1h}(u_h, \rho_h, \phi_h) = \sum_P \phi_h(P) \sum_{Q \in \Lambda_P} \beta_{PQ}^-(u_h)(\rho_h(P) - \rho_h(Q)), \tag{8}$$

where

$$\beta_{PQ}^-(u_h) = \max(-\int_{\Gamma_{PQ}} u_h \cdot n_{PQ} \ ds, \ 0), \quad \Gamma_{PQ} = \partial D_P \cap \partial D_Q,$$

and n_{PQ} is the unit outer normal to Γ_{PQ} from D_P to D_Q. (8) is an upwind approximation of the trilinear form

$$c_1(u, \rho, \phi) = \int_\Omega (u \cdot \nabla\rho)\phi \ dx,$$

and we refer to (4.13) of [1] for the derivation of (8). Without any restriction on Δt it can be proved that the solution ρ_h of (6) satisfies the maximum principle and that scheme (6) is energy-stable, i.e.,

$$\int_\Omega \rho_h^n |u_h^n|^2 dx$$

is uniformly bounded for $n = 0, \cdots, N_T$, see [1] and [6].

4 Numerical results

Let $\Omega \equiv (0,1) \times (0,1)$ be a unit square in $\mathbf{R}^2$. Letting $\eta : [0,1] \to \mathbf{R}$ be a given continuous function, we define

$$\Omega_2^0 \equiv \{x \in \Omega; x_2 \geq \eta(x_1)\}, \quad \Omega_1^0 \equiv \{x \in \Omega; x_2 < \eta(x_1)\}.$$

We take the following values,

$$(\rho_1, \mu_1) = (10, 1), \quad (\rho_2, \mu_2) = (100, 2)$$

and the initial velocity and the outer force,

$$u^0 = (0,0)^T, \quad f = (0,-1)^T.$$

We divide the domain Ω into a union of triangles. We consider three kinds of meshes, Union-Jack type (UJ), Friedricks-Keller type (FK), and FreeFEM type (Free) shown in Fig. 2. The last mesh is made by FreeFEM [2]. Each side of the square is divided into 32 equal segments. We take $\Delta t = \frac{1}{2}$. The function η is chosen to satisfy

$$\int_0^1 \eta(x_1) dx_1 = \frac{1}{2}.$$

In the following figures we show computed interface curves approximating $\Gamma_{12}(t)$. They are determined so that the areas of the two fluids are equal to each other. Driven by the outer force f, the heavy fluid goes down. After a period of time elapses, the heavy fluid finally occupies the lower half part. We observe the transient movement for the period $[0, 10]$.

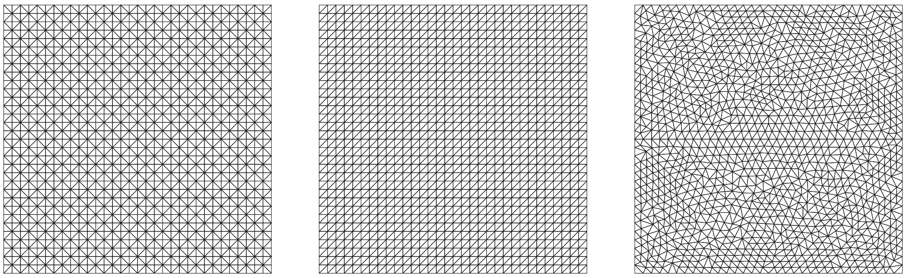

Figure 2: Meshes of type UJ, FK, and Free

4.1 The Rayleigh-Taylor instability problem

The function η is defined by

$$\eta(x_1) = \frac{1}{2}. \tag{9}$$

The initial state is in equilibrium but unstable as the solution of partial differential equations. Figs. 3, 4, and 5 show the interface curves at $t = 0, 2, 4, 6, 8, 10$ in meshes UJ, FK, and Free, respectively. Mesh Free is not uniform. Mesh FK is not symmetric with respect to $x_1 = 1/2$, which induces the movement but it differs from the one in mesh Free. The heavy fluid goes down from left. In mesh UJ the interface almost keeps the original position in this period, though a very small change can be observed. Later it develops and finally the heavy fluid goes down. Three movements are completely different from one another and are heavily dependent on the meshes. Such numerical solutions are often called "ghost solutions" because they never appear in the partial differential equation but only in discrete approximations. Applying numerical methods to unstable problems, we may have ghost solutions. In the next subsection, however, our scheme is shown to work well for well-posed problems.

4.2 A perturbed Rayleigh-Taylor problem

The function η is defined by

$$\eta(x_1) = \frac{1}{2} - a \cos 2\pi x_1, \quad a = 0.01. \tag{10}$$

Figure 3: Interfaces at $t = 0, 2, 4, 6, 8, 10$. η:(9), mesh UJ

The initial state is a little perturbed, that is, the heavy fluid intrudes a little from both sides near the boundary. Figs. 6, 7, and 8 show the interface curves in meshes UJ, FK, and Free, respectively. These three figures are much similar to one another, and the mesh dependency is small. The differences decrease when the mesh size tends to zero.

4.3 The other problems

In mesh UJ we solve problems with other initial states.

Fig. 9 shows the case where the function η is defined by

$$\eta(x_1) = \frac{1}{2} - a \cos 2\pi x_1, \quad a = -0.01. \tag{11}$$

The initial state is a little perturbed, that is, the heavy fluid intrudes a little from the center. The small intrusion ignites the fall of the heavy fluid from the center.

Fig. 10 shows the case where the function η is defined by

$$\eta(x_1) = \frac{1}{2} - a \cos 4\pi x_1, \quad a = 0.01. \tag{12}$$

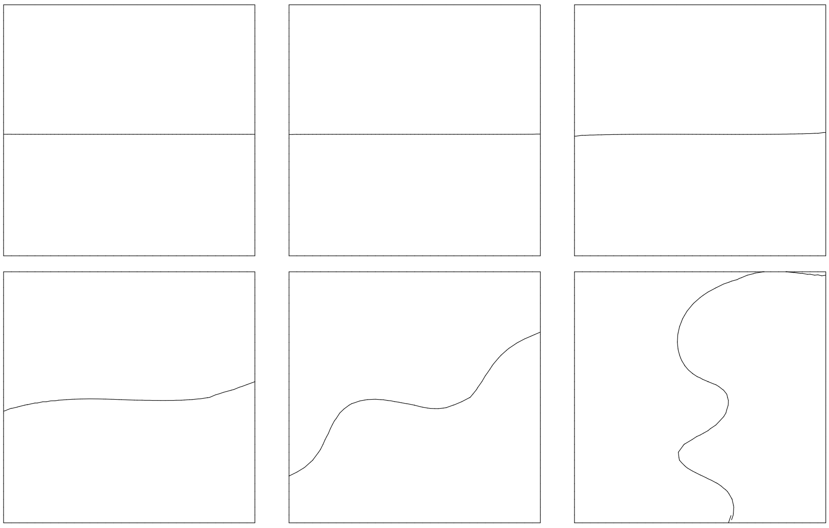

Figure 4: Interfaces at $t = 0, 2, 4, 6, 8, 10$. η:(9), mesh FK

The small intrusion of the heavy fluid from both sides and the center ignites the fall of the fluid from these parts.

5 Concluding remarks

We have simulated numerically Rayleigh-Taylor problems by an energy-stable P1/P2/P1 finite element scheme. For Rayleigh-Taylor problems subject to non-equilibrium states numerical interface curves are almost mesh-independent. For the Rayleigh-Taylor instability problem subject to an unstable equilibrium state ghost solutions are obtained. Such phenomenon is often observed in solving numerically unstable problems.

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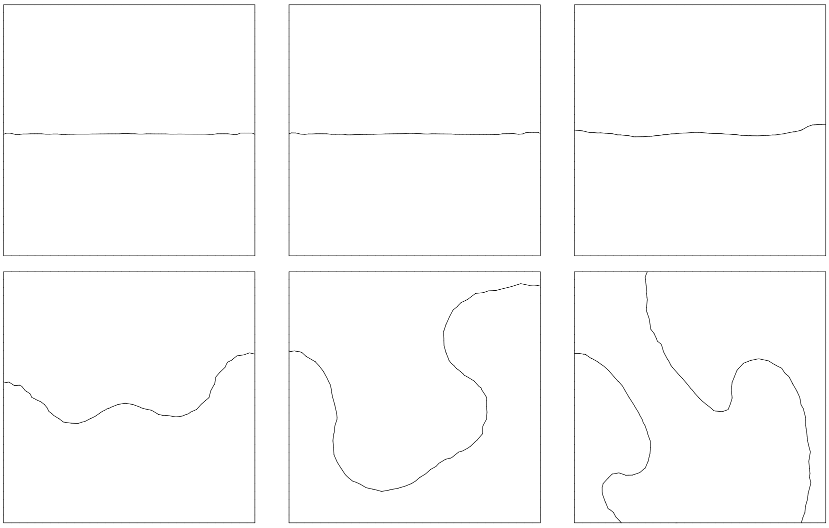

Figure 5: Interfacesat $t = 0, 2, 4, 6, 8, 10$. η:(9), mesh Free

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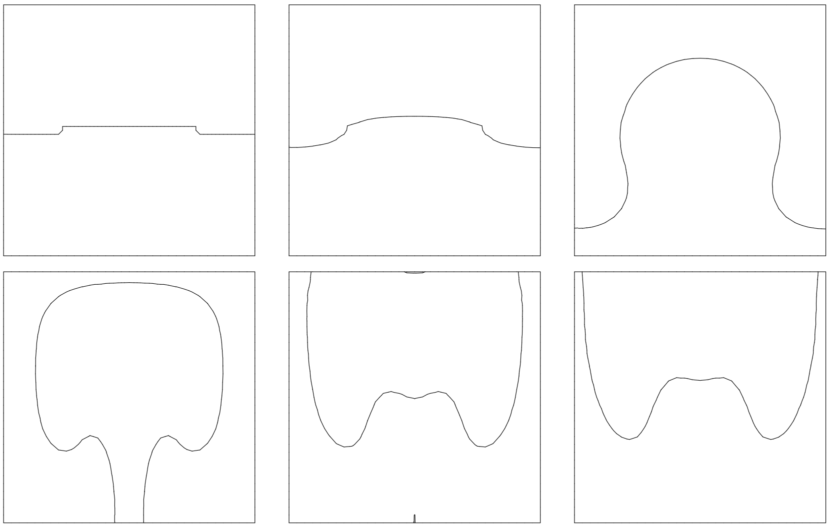

Figure 6: Interfaces at $t = 0, 2, 4, 6, 8, 10$. η:(10), mesh UJ

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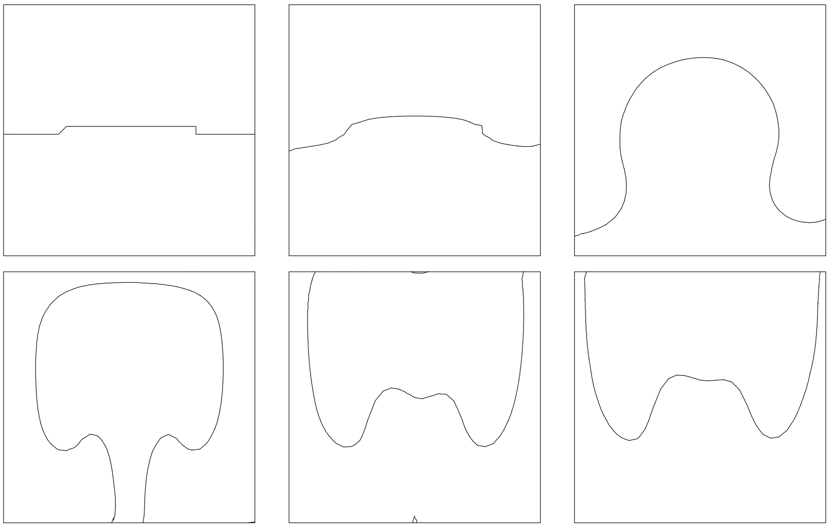

Figure 7: Interfaces at $t = 0, 2, 4, 6, 8, 10$. η:(10). mesh FK

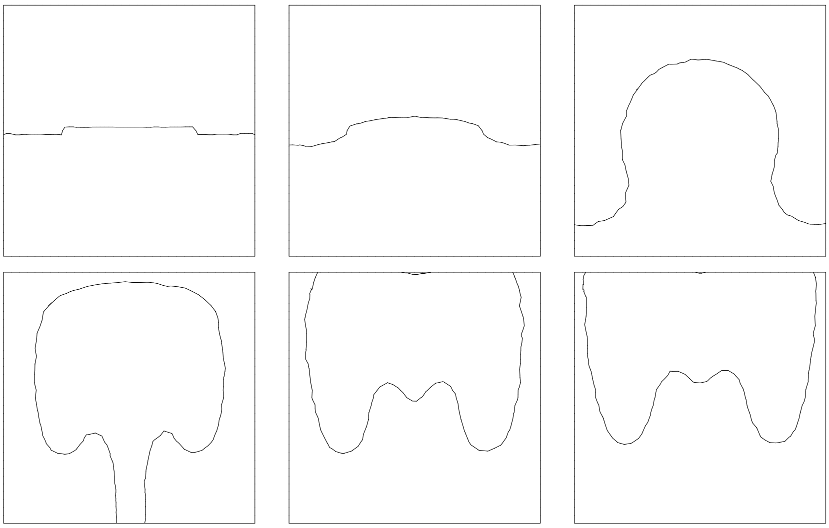

Figure 8: Interfaces at $t = 0, 2, 4, 6, 8, 10$. η:(10), mesh Free

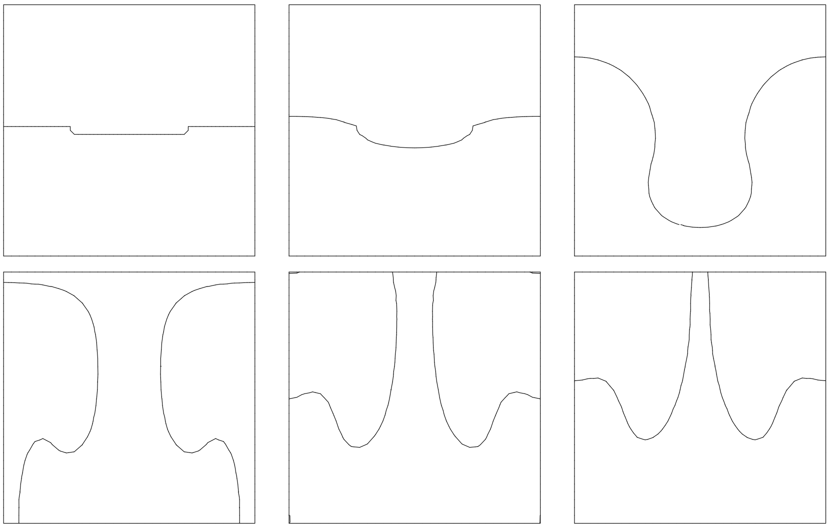

Figure 9: Interfaces at $t = 0, 2, 4, 6, 8, 10$. η:(11), mesh UJ

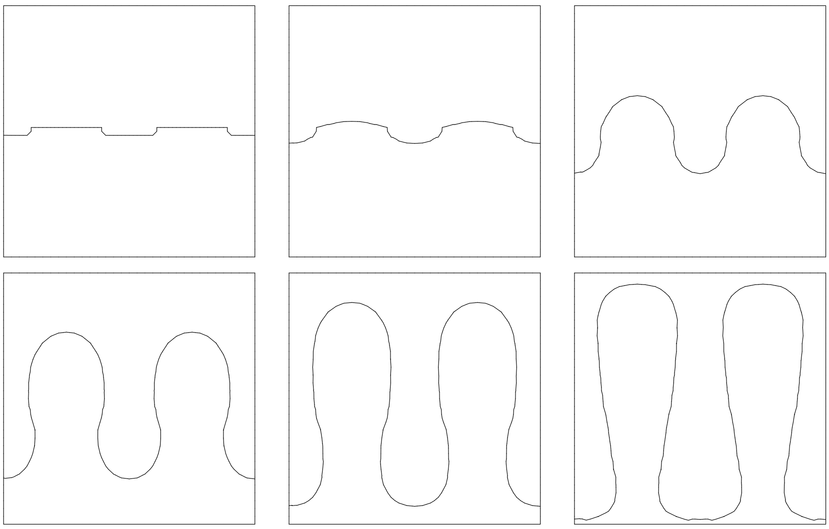

Figure 10: Interfaces at $t = 0, 2, 4, 6, 8, 10$. η:(12), mesh UJ

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