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Finite Element Schemes for Two-Fluids Flow Problems

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A weak formulation suitable for two-fluids flow problems is shown. Stability of the velocity is proved for a wide class of finite element spaces under a mild assumption on the density. Full stability is proved for a mass-conservative upwind scheme without any condition.

Keywords: two-fluids flow, density-dependent Navier-Stokes equations, finite element method, mass-conservative upwind scheme

AMS Subject classification: Primary 65M12, 65M60; Secondary 76M10

1. Introduction

Two-fluids flow problems describe many interesting phenomena such as Rayleigh-Taylor instability and sloshing of fluid, and various computations have been done by many authors, e.g., [5], [4]. The problems include unknown interfaces, that is, the domains where the problems are solved are also unknown. To the best of our knowledge there are no numerical schemes whose solutions are proved to converge to the exact one. Recently we have developed a finite element scheme for density-dependent Navier-Stokes equations and proved the convergence of the finite element solutions [3]. In this paper we consider two-fluids flow problems in the framework of density-dependent Navier-Stokes problems. We present a weak formulation suitable for free-interface problems. In this formulation a wide class of finite element methods is stable with respect to the velocity under the non-negativity assumption on the density. The assumption is satisfied by a mass-conservative upwind finite element approximation developed by Baba-Tabata [1]. The finite element scheme is proved unconditionally stable for all variables. The study to show the convergence of the finite element solutions is ongoing.

2. Two-layers flow problems

Let Ω be a bounded domain in $\mathbf{R}^d$, $d = 2, 3$, with piecewise smooth boundary Γ, and T be a positive number. At the initial time $t = 0$ the domain Ω is occupied by two immiscible incompressible viscous fluids; each domain is denoted by Ω_k^0, $k = 1, 2$, whose interface $\partial\Omega_1^0 \cap \partial\Omega_2^0$ is denoted by Γ_{12}^0. At $t \in (0, T)$ the two fluids occupy domains $\Omega_k(t)$, $k = 1, 2$, and the interface $\partial\Omega_1(t) \cap \partial\Omega_2(t)$ is denoted by $\Gamma_{12}(t)$.

Let ρ_k and μ_k, $k = 1, 2$, be the densities and the viscosities of the two fluids. Let

$$u : \Omega \times (0, T) \to \mathbf{R}^d, \quad p : \Omega \times (0, T) \to \mathbf{R}$$

be the velocity and the pressure to be found. The Navier-Stokes equations are satisfied in each domain $\Omega_k(t)$, $k = 1, 2$, $t \in (0, T)$,

$$\rho_k \left\{ \frac{\partial u}{\partial t} + (u \cdot \nabla) u \right\} - \nabla (2\mu_k D(u)) + \nabla p = \rho_k f \tag{1a}$$

$$\nabla \cdot u = 0 \tag{1b}$$

where $f : \Omega \times (0, T) \to \mathbf{R}^d$ is a given function, $D(u)$ is the strain tensor defined by

$$D_{ij}(u) = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

and

$$[\nabla (2\mu_k D(u))]_i = \sum_{j=1}^{d} \frac{\partial}{\partial x_j} (2\mu_k D_{ij}(u)).$$

On the boundary Γ, $t \in (0, T)$, the non-slip conditions

$$u = 0 \tag{2}$$

are imposed. On the inteface $\Gamma_{12}(t)$, $t \in (0, T)$, the velocity and the stress vector should be continuous,

$$[u] = 0 \tag{3a}$$

$$[\sigma(\mu, u, p) n_{12}] = 0 \tag{3b}$$

where $[\cdot]$ means the difference of the values approaching from the domain Ω_2 and the domain Ω_1, n_{12} is the unit outer normal to $\Gamma_{12}(t)$ from $\Omega_1(t)$ to $\Omega_2(t)$, and σ is the stress tensor defined by

$$\sigma(\mu, u, p) = -pI + 2\mu D(u).$$

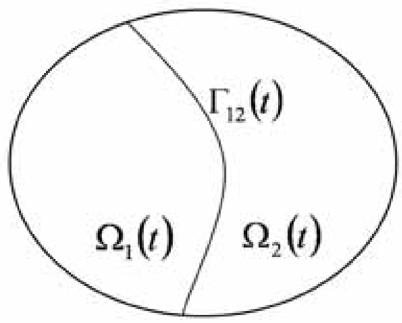

Figure 1. Domains $\Omega_k(t)$,$k = 1, 2$, and the interface $\Gamma_{12}(t)$

Let $\rho(x, t)$ be the solution of the convection equation

$$\frac{\partial \rho}{\partial t} + u \cdot \nabla \rho = 0, \qquad (x, t) \in \Omega \times (0, T) \tag{4a}$$

$$\rho(x,0)=\rho^0(x), \qquad x\in\Omega \tag{4b}$$

where u is the unknown velocity and

$$\rho^0(x)=\begin{cases}\rho_1 & (x\in\Omega_1^0)\\ \rho_2 & (x\in\Omega_2^0)\end{cases} \tag{5}$$

Note that no boundary condition is required as the velocity u vanishes on the boundary. Then, $\Omega_k(t)$ can be written as

$$\Omega_k(t)=\{x\in\Omega;\ \rho(x,t)=\rho_k\}, \qquad k=1,2.$$

Using $\rho(x,t)$, we can express the viscosity $\mu(x,t)$ as the function of ρ,

$$\mu(\rho)=\mu_1\frac{\rho_2-\rho}{\rho_2-\rho_1}+\mu_2\frac{\rho-\rho_1}{\rho_2-\rho_1}. \tag{6}$$

3. A weak formulation

The function ρ^0 is discontinuous and does not belong to $H^1(\Omega)$. We, however, assume that $\rho(\cdot,t)\in H^1(\Omega)$, $t\in(0,T)$, for the time being. Deriving a weak formulation under the assumption, we devise a finite element scheme for the original problem. Apart from (5) and (6) we set a more general setting on $\mu(\rho)$,

$$\mu:\mathbf{R}\to[\mu_1,\mu_2] \tag{7}$$

where μ_i, $i=1,2$, are positive constants and μ is continuously differentiable.

Let $X=H^1(\Omega)$, $Y=X^d$, and V and Q be function spaces defined by

$$V=H_0^1(\Omega)^d, \quad Q=\{q\in L^2(\Omega);\ \int_\Omega q\ dx=0\}.$$

We consider the varitational problem to find the functions

$$\rho:(0,T)\to X, \quad u:(0,T)\to V, \quad p:(0,T)\to Q \tag{8}$$

satisfying

$$\left(\frac{\partial\rho}{\partial t},\phi\right)+c_1(u,\rho,\phi)=0, \quad \forall\phi\in X \tag{9a}$$

$$\left(\rho\frac{\partial u}{\partial t}+\frac{1}{2}u\frac{\partial\rho}{\partial t},v\right)+a_1(\rho,u,u,v)+a_0(\rho,u,v)$$
$$+b(v,p)=(\rho f,v)\,, \quad \forall v\in V \tag{9b}$$
$$b(u,q)=0, \quad \forall q\in Q \tag{9c}$$

subject to the initial conditions

$$\rho(0)=\rho^0, \quad u(0)=u^0 \tag{10}$$

where

$$\rho^0:\Omega\to\mathbf{R}, \quad u^0:\Omega\to\mathbf{R}^d$$

are given functions, $(\cdot,\cdot)$ shows the inner product in $L^2(\Omega)$ or $L^2(\Omega)^d$, and

$$\begin{aligned}
c_1(u,\rho,\phi) &= \int_\Omega (\nabla\cdot(u\rho))\,\phi\ dx \\
a_1(\rho,w,u,v) &= \int_\Omega \left(\frac{1}{2}(w\cdot\nabla\rho)u + \frac{1}{2}\rho(\nabla\cdot w)u + \rho(w\cdot\nabla)u\right)\cdot v\ dx \\
a_0(\rho,u,v) &= \int_\Omega 2\mu(\rho)D(u) : D(v)\ dx \\
b(v,q) &= -\int_\Omega (\nabla\cdot v)q\ dx.
\end{aligned}$$

The Sobolev theorem implies the following.

Proposition 1. (i) c_1 is a continuous tri-linear form on $Y\times X\times X$.

(ii) a_1 is a continuous tetra-linear form on $X\times Y\times Y\times Y$.

(iii) a_0 is a continuous tri-linear form on $L^\infty(\Omega)\times Y\times Y$.

(iv) b is a continuous bi-linear form on $Y\times X$.

We now assume that the space-time domain $Q(T)\equiv\Omega\times(0,T)$ is divided into two domains $Q_1(T)$ and $Q_2(T)$ such that the common surface $\Sigma\equiv\bar{Q}_1(T)\cap\bar{Q}_2(T)$ is a smooth surface in $\mathbf{R}^{d+1}$, and that $\Sigma\cap\{(x,t);\ x\in\Omega, t=s\}$ is not empty for all $s\in[0,T]$.

Theorem 2. Suppose that (ρ,u,p) is smooth in $Q_1(T)$ and $Q_2(T)$, and continuous in $Q(T)$. If (ρ,u,p) is the solution of the variational problem (8)-(10), then (ρ,u,p) satisfies the equation

$$\rho\left\{\frac{\partial u}{\partial t} + (u\cdot\nabla)u\right\} - \nabla\left(2\mu(\rho)D(u)\right) - \nabla p = \rho f$$

and (1b) in each domain $Q_k(T)$, $k=1,2$, and the interface conditions (3) on Σ.

We omit the proof.

Remark 3. In Theorem 2 we assumed that ρ is continuous in $Q(T)$. In (9a), however, the required differentiability of ρ is concerned only with the material differential

$$\frac{D}{Dt}\equiv\frac{\partial}{\partial t}+u\cdot\nabla.$$

Hence, the continuity to the direction $(u,1)$ in $\mathbf{R}^{d+1}$ is required, but the continuity to the transverse direction is not necessary. If the normal to Σ is orthogonal to $(u,1)$, ρ can be discontinuous at Σ. It is the case of the two-fluids flow problems.

4. Finite element schemes

Now we consider the finite element approximation to the variational problem (8)-(10). Let $\Phi_h\subset X$, $V_h\subset V$, and $Q_h\subset Q$ be finite element spaces for the density, the velocity, and the pressure. These spaces are equipped with the norms $L^2(\Omega)$, $H^1(\Omega)^d$, and $L^2(\Omega)$, respectively. L^2-norm and H^1-norm are denoted simply by $||\cdot||_0$ and $||\cdot||_1$. We assume the inf-sup condition on V_h and Q_h

$$\inf_{q_h \in Q_h} \sup_{v_h \in V_h} \frac{b(v_h, q_h)}{||q_h||_0 ||v_h||_1} \geq \beta \tag{11}$$

where β is a positive constant independent of h, the representative mesh size. As the problem (8)-(10) includes the Navier-Stokes equations, the inf-sup condition is necessary when we discuss the convergence of the finite element solutions, though we do not do it in this paper. Let Δt be a time increment and $N_T = \lfloor T/\Delta t \rfloor$. We denote by (ρ_h^n, u_h^n, p_h^n) the value at time $n\Delta t$, and by $\bar{D}_{\Delta t}$ the backward difference operator, e.g.,

$$\bar{D}_{\Delta t} u_h^n = \frac{u_h^n - u_h^{n-1}}{\Delta t}.$$

We find $\{(\rho_h^n, u_h^n, p_h^n) \in \Phi_h \times V_h \times Q_h;\ n = 1, \cdots, N_T\}$ satisfying

$$(\bar{D}_{\Delta t} \rho_h^n, \phi_h) + c_1(u_h^{n-1}, \rho_h^n, \phi_h) = 0, \quad \forall \phi_h \in \Phi_h \tag{12a}$$

$$\left(\rho_h^{n-1} \bar{D}_{\Delta t} u_h^n + \frac{1}{2} u_h^n \bar{D}_{\Delta t} \rho_h^n, v_h\right) + a_1(\rho_h^n, u_h^{n-1}, u_h^n, v_h) + a_0(\rho_h^n, u_h^n, v_h)$$
$$+ b(v_h, p_h^n) = (\rho_h^n \Pi_h f^n, v_h), \quad \forall v_h \in V_h \tag{12b}$$

$$b(u_h^n, q_h) = 0, \quad \forall q_h \in Q_h \tag{12c}$$

subject to the initial conditions

$$\rho_h^0 = \Pi_h \rho^0, \quad u_h^0 = \Pi_h u^0 \tag{13}$$

where Π_h is the interpolation operator to the corresponding finite element spaces.

For a series of functions $\phi_h = \{\phi_h^n\}_{n=0}^{N_T}$ in a Banach space W we prepare the norms defined by

$$||\phi_h||_{\ell^\infty(W)} \equiv \max\{||\phi_h^n||_W;\ 0 \leq n \leq N_T\},$$
$$||\phi_h||_{\ell^2(W)} \equiv \left\{\Delta t \sum_{n=0}^{N_T} ||\phi_h^n||_W^2\right\}^{1/2}.$$

Lemma 4. Suppose that $(\rho_h^n, u_h^n, p_h^n) \in \Phi_h \times V_h \times Q_h$, $n = 0, \cdots, N_T$, satisfy (12b) and (12c) and that $\rho_h \geq 0$. Then we have

$$||\sqrt{\rho_h} u_h||_{\ell^\infty(L^2)},\ ||\sqrt{\mu_1} u_h||_{\ell^2(H^1)} \leq c\{||\sqrt{\rho_h^0} u_h^0||_0 + ||\sqrt{\rho_h}\ \Pi_h f||_{\ell^2(L^2)}\} \tag{14}$$

where c is a positive constant independent of h and Δt.

Proof. We substitute $v_h = u_h^n$ in (12b). The first term is equal to

$$\left(\rho_h^{n-1} \bar{D}_{\Delta t} u_h^n + \frac{1}{2} u_h^n \bar{D}_{\Delta t} \rho_h^n, u_h^n\right)$$
$$= \bar{D}_{\Delta t}\left(\frac{1}{2}||\sqrt{\rho_h^n} u_h^n||_0^2\right) + \frac{1}{2}||\sqrt{\Delta t}\sqrt{\rho_h^{n-1}} \bar{D}_{\Delta t} u_h^n||_0^2$$

By using the Gauss-Green theorem and the boundary condition, the second term becomes

$$a_1(\rho_h^n, u_h^{n-1}, u_h^n, u_h^n) = \int_\Gamma \rho_h^n (u_h^{n-1} \cdot n)|u_h^n|^2\ ds = 0.$$

The third term is estimated as

$$a_0(\rho_h^n, u_h^n, u_h^n) \geq ||\sqrt{2\mu_1}D(u_h^n)||_0^2 \geq c_0||\sqrt{2\mu_1}u_h^n||_1^2$$

where c_0 is a positive constant in the Korn inequality. The fourth term vanishes from (11c). The right-hand side is evaluated as

$$|\,(\rho_h^n \Pi_h f^n, u_h^n)\,| \leq \epsilon||\sqrt{\rho_h^n}u_h^n||_0^2 + \frac{1}{4\epsilon}||\sqrt{\rho_h^n}\Pi_h f^n||_0^2$$

where ϵ is any positive constant. Combining these estimates and applying the discrete Gronwall inequality, we get (14). □

Remark 5. If we replace the tri-linear form $c_1(u, \rho, \phi)$ by a skew symmetric form

$$\hat{c}_1(u, \rho, \phi) = \frac{1}{2}\int_\Omega ((u \cdot \nabla\rho)\phi - (u \cdot \nabla\phi)\rho)\, dx,$$

we can easily get the estimate

$$||\rho_h||_{\ell^\infty(L^2)} \leq ||\rho_h^0||_0$$

by substituting $\phi_h = \rho_h^n$ in (12a). When ρ is continuous, $\hat{c}_1(u, \rho, \phi) = c_1(u, \rho, \phi)$. If not so, $\hat{c}_1$ includes another condition $[\rho u \cdot n_{12}] = 0$ on the interface.

5. A mass-conservative upwind finite element scheme

We now replace the tri-linear form $c_1(\rho, u, p)$ by a mass-conservative upwind approximation $c_{1h}(u_h, \rho_h, \phi_h)$ [1]. Using the lumping technique, we can show the stability of ρ_h, u_h, and p_h without any assumption. We describe the scheme only in the case $d = 2$. At first we make the dual decomposition

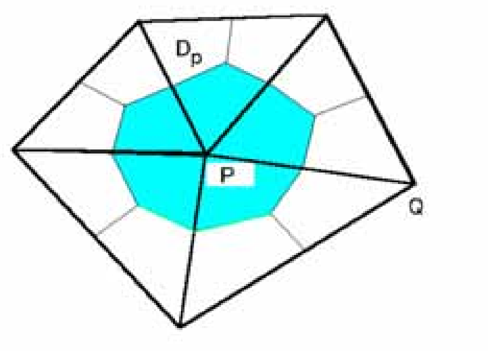

Figure 2. A node P and the barycentric domain D_P

$\{D_P\}$, where D_P is the barycentric domain of the node P, see Fig. 2. The D_P is made by connecting the midpoints of the sides and the centroids of the triangles around $P \in \Omega$. When the node P is on Γ, some parts of the (approximate) boundary are used. Let Φ_h be the P1-finite element space. We define the lumping operator $^-$ from Φ_h to $L^2(\Omega)$ by

$$\phi_h \longmapsto \bar{\phi}_h(x) \equiv \sum_P \phi_h(P)\chi_P(x)$$

where χ_P is the characteristic function of D_P. We denote by Λ_P the set of nodes adjacent to P. Let $u_h \in V_h$ and $\rho_h, \phi_h \in \Phi_h$. The mass-conservative upwind approximation c_{1h} is defined by

$$c_{1h}(u_h, \rho_h, \phi_h) = \sum_P \phi_h(P) \sum_{Q \in \Lambda_P} \left\{ \beta^+_{PQ}(u_h)\rho_h(P) - \beta^-_{PQ}(u_h)\rho_h(Q) \right\}$$

where

$$\beta_{PQ}(u_h) = \int_{\Gamma_{PQ}} u_h \cdot n_{PQ} \; ds, \quad \Gamma_{PQ} = \partial D_P \cap \partial D_Q,$$

n_{PQ} is the unit outer normal to Γ_{PQ} from D_P to D_Q,

$$\beta^+ = \max(\beta, 0), \quad \beta^- = \max(-\beta, 0).$$

We replace (12a) by

$$(\bar{D}_{\Delta t}\bar{\rho}^n_h, \bar{\phi}_h) + c_{1h}(u^{n-1}_h, \rho^n_h, \phi_h) = 0, \quad \forall \phi_h \in \Phi_h. \tag{15}$$

Our variational problem is to find $\{(\rho^n_h, u^n_h, p^n_h) \in \Phi_h \times V_h \times Q_h;\ n = 1, \cdots, N_T\}$ satisfying (15), (12b), (12c), and (13).

Theorem 6. Suppose that Φ_h is the P1-finite element space and $V_h \subset V$ and $Q_h \subset Q$ satisfy the inf-sup condition (11).

(i) For any functions $\rho^0 \geq 0$, u^0, and f there exists the unique solution (ρ_h, u_h, p_h) of (15), (12b), (12c), and (13).

(ii) The solution ρ_h is non-negative and it satisfies

$$||\bar{\rho}^n_h||_{L^1(\Omega)} = ||\bar{\rho}^0_h||_{L^1(\Omega)}, \quad \forall n = 0, \cdots, N_T.$$

(iii) The solution satisfies the estimate (14).

Remark 7. Examples of the choice of V_h and Q_h are P2/P1 elements, and P1+/P1 elements [2].

6. Concluding remarks

We have shown finite element schemes for two-fluids flow problems. For a wide class of finite element spaces we have shown the stability of the velocity under a mild assumption on the density. For a mass-conservative upwind scheme we have shown the full stability of the density, the velocity, and the pressure. Numerical results and the discussion on the convergence will be given in a forthcoming paper.

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