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# Methods for the Orbit Determination of a Tethered Satellite System by a Single Ground Station

by

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## Abstract

In general, motion of orbital objects is following the Kepler's law, and an orbit determination system in ground stations has the algorithm to analyze the Kepler's motion. A tethered satellite system (TSS) is the representative future space system, and does not follow the conventional space dynamics. Therefore, new methods for the orbit determination of TSS are necessary.

Reviewing the previous researches, the following three problems are considered in this paper. At first, the sensitivity that the length of tether and the librational motion influence observations, which are range, range rate, and direction, is researched. Secondly, the new filtering algorithm is proposed for the orbit determination of a TSS. Thirdly, the initial orbit determination of a TSS is proposed.

After the considerations, it was made clear that observations of range and range rate can reflect the motion of a TSS, on the other hand, observations of direction are not effective for the detection of a TSS motion. Moreover, the new algorithm for the orbit determination of a TSS showed the excellent performance. When the motion of the center-of-mass is already known, methods of the initial determination of the length of tether, librational angle, and angular velocity were introduced, and the performance was proved.

**Keywords:** Tethered satellite system, Orbit determination, State estimation filter, Initial orbit determination

## 1. Introduction

At the present age, space organizations such as AFSPC (Air Force Space Command), U. S. Air Force are observing various space objects like asteroids and spacecrafts. In general, motion of orbital objects is following the Kepler's law, and an orbit determination system in ground stations has the algorithm to analyze the Kepler's motion. In the near future, however, special space systems

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are expected to increase, and they will not follow the conventional space dynamics. The representative system is a tethered satellite system (TSS). TSS is the system to connect two or more satellites, including a space shuttle, by a tether.

The original idea of a space tether is proposed by Tsiolkovskii in 1895. Since then, the many application technologies are proposed by world researchers<sup>1)</sup>: power or propulsion generation by a conductive tether, upper atmosphere observation, artificial gravity generation, orbital transfer, and so on. Since the orbital experiment by Gemini-11 in 1966, experiments of a space tether are continuing, and in the near future, a space tether and TSS will be expected to become a standard space system.

It is widely known that the orbit of the center-of-mass of TSS follows the Kepler's motion<sup>2)</sup>, when neglecting all the perturbations. The motion of sub satellites of a system, however, differs from the Kepler's motion. In general, a ground station is observing an end satellite of TSS. If an observer misunderstands the orbit of the end satellite as the general Kepler's motion, the wrong ballistic coefficient is estimated and the orbit is misunderstood as a re-entry orbit<sup>3)</sup>. Therefore, new methods for the orbit determination of TSS are necessary.

The orbit determination of TSS is demonstrated in TiPS (Tether Physics and Survivability) project<sup>4)-6)</sup> by Naval Research Laboratory (NRL). TiPS is a micro satellite and consists of 43.2-kg mother, 10.2-kg daughter, and 5.4-kg tether. The 4-km tether deployment in orbit was successful, and it survived for more than one year for the first time in space tether systems. The main objective of TiPS is to analyze the librational motion after the tether deployment, and the detail motion was confirmed by ground observation systems. The satellite does not have any sensors for the orbit determination, and the motion was observed from ground lidar systems, laser systems, and optical observations. The review of the TiPS mission shows the past results and future problems of the orbit determination of TSS.

The researches by D. A. Cicci et al<sup>7)-11)</sup> consider the automatic identification algorithm and the filtering process of a TSS, which will be used in a future observation system. At the present age, a TSS is operated as an experimental mission, and the number is very few. Therefore, the observation can be processed as a special case, and the misunderstanding with a general single satellite can be avoided. In the near future, however, the age that many TSS are simultaneously operated in orbit will be expected, therefore a conventional observation system will have to include the ability to identify a TSS and shift to the special filtering process.

Reviewing the previous researches, some problems are defined. In the TiPS mission, it was observed that the large libration amplitude just after the tether deployment was gradually decreasing. This phenomenon was assumed by a numerical simulation, and the actual observation was helpful to verify the simulation results. On the other hand, it was made clear that the state filtering process to estimate the center-of-mass motion and the tether librational motion was difficult in the over six-hour observation. This is because the non-linearity of the motion is large, and the in-plane and out-of-plane librational motions are interfering each other. In the TiPS mission, to avoid this difficulty, the motion of the center-of-mass is firstly estimated by the long-span observation, and then the librational motion is estimated by the short-span observation.

In both papers of TiPS mission and D. A. Cicci et al., similar problem was pointed out: in large librational amplitude motion, the allowable error in the initial state for the state estimation filter is decreased. In previous researches, the methods to calculate the initial estimation except the filtering methods have not been proposed. If an initial estimation method is available, the processing time for the orbit determination will be much improved.

From the mentioned backgrounds, the following three problems are considered in this paper. At first, the sensitivity that the length of tether and the librational motion influence observations, which are range, range rate, and direction, is researched. It is confirmed what observation source is

suitable to the observation of a TSS. Secondly, the new filtering algorithm is proposed for the orbit determination of a TSS. The conventional algorithms are not used in this paper, and the extended method of recursive least-squares estimation is newly proposed. In this algorithm, it is unique that one motion of the center-of-mass and the individual librational motion in an each pass are defined. In the offline estimation process by long-span observation, all the parameters can be simultaneously estimated. Thirdly, the initial orbit determination of a TSS is proposed. Supposing that the motion of the center-of-mass is known, the length of tether, libration angles, and angular velocity can be estimated by at least three observations of range or range rate. In this study, the in-plane motion is only considered as the first step, and the analytical expressions of range and range rate are introduced. Then, the performances of estimation methods are compared by numerical simulations.

The methods proposed in this study will be utilized in the project of the original TSS observation system in Space Systems and Dynamics Laboratory (SSDL), Kyushu University. SSDL is conducting the project to develop the original micro tethered satellite<sup>12)</sup>. In this mission, 50-kg class micro satellite will be launched as a piggyback satellite, and then it will deploy the sub satellite with a tether. The primary objective is to observe the motion of a TSS before and after the tether deployment. SSDL is planning to observe this satellite by self-constructed original ground station. From the cost aspect, it is difficult to attain the similar ability of a general ground station. Therefore, the ground station adopted the one-way range-rate observation style, which observes a Doppler frequency of a signal emitted by a beacon transmitter of a satellite. This low-cost ground station is already constructed and the orbit determination of an amateur radio satellite with a CW beacon transmitter was successful<sup>13)-14)</sup>. The previous researches for the orbit determination of a TSS does not mention the case where range rate only is observed by a single ground station. Therefore, the results of this study are expected to be grate helpful to consider the required facility performance, the attained estimation error. Moreover, the cultivated technique and developed software in this study will be used in the analysis of real observations of a TSS.

## 2. Theory

### 2.1 Model of motion

In this study, the actual observations are not used. Instead of that, dummy observations are generated in numerical simulations. Therefore, the model of the motion of a two-body tethered satellite system is defined here. To save the simulation time, the model with mass-less and constant-length tether is adopted. The center-of-mass of this model follows the Kepler's motion. About the mass-less tether model, the equations by Beletsky and Levin<sup>2)</sup> are famous. In this study, the coordinate frames are defined in **Fig. 1**, and the equations equivalent to ones of Beletsky and Levin are defined as follows. The change rate of length of tether is neglected here.

$$\ddot{\theta} = -\ddot{u} + 2(\dot{u} + \dot{\theta})\dot{\phi} \tan \phi - 3\frac{\mu}{r^3} \cos \theta \sin \theta \quad (1)$$

$$\ddot{\phi} = -\left\{(\dot{u} + \dot{\theta})^2 + 3\frac{\mu}{r^3} \cos^2 \theta\right\} \cos \phi \sin \phi \quad (2)$$

Where,  $\theta$  is the in-plane libration angle,  $\phi$  is the out-of-plane libration angle, and dots show the time derivatives. In addition,  $r$  is the radius of the center-of-mass,  $u$  is the argument of latitude, that is sum of the argument of perigee and the true anomaly, and  $\mu$  is the gravity constant. The mass of the upper satellite is defined by  $m_1$ , and the mass of the lower satellite is defined by  $m_2$ . These masses do not affect the librational motion.

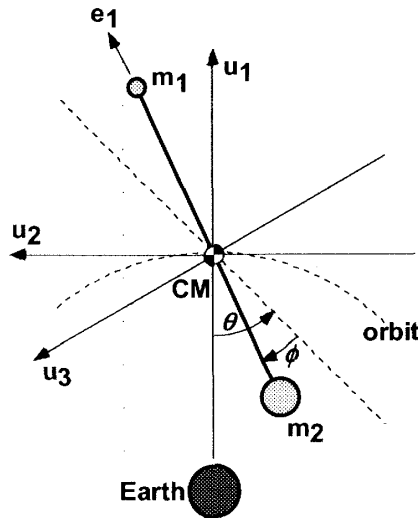


Fig. 1 Model of the motion of a two-body tethered satellite system.

## 2.2 State vectors

In state estimation filters, the parameters to be estimated should be arranged in a state vector. At first, the state vector of the center-of-mass is defined by  $\mathbf{X}_{\text{CM}(I)}$ . The attached ( $I$ ) means inertial coordinates.

$$\mathbf{X}_{\text{CM}(I)} = \{x_c, y_c, z_c, \dot{x}_c, \dot{y}_c, \dot{z}_c\}^T \quad (3)$$

On the other hand, the state vector of a tether motion is defined by  $\mathbf{X}_T$ . The length of tether  $\ell$  is the distance from the center-of-mass to the end mass, here. Supposing the constant-length tether, the change rate of the length of tether is not contained in  $\mathbf{X}_T$ .

$$\mathbf{X}_T = \{\ell, \theta, \dot{\theta}, \phi, \dot{\phi}\}^T \quad (4)$$

When the single motion of the center-of-mass and the single motion of tether are supposed, the following state vector is used in numerical simulations. When several motions of tether are supposed,  $\mathbf{X}_{\text{TS}}$  should have the plural  $\mathbf{X}_T$ .

$$\mathbf{X}_{\text{TS}} = \begin{bmatrix} \mathbf{X}_{\text{CM}(I)} \\ \mathbf{X}_T \end{bmatrix} = \{x_c, y_c, z_c, \dot{x}_c, \dot{y}_c, \dot{z}_c, \ell, \theta, \dot{\theta}, \phi, \dot{\phi}\}^T \quad (5)$$

## 3. Software for numerical simulations

To smoothly conduct numerical simulations, the following software are self developed.

- OPROPS (Orbit PROPagation Simulator)
- OOBS (Orbit OBServation Simulator)
- ODEF (Orbit DETERmination Filter)

In the normal orbit determination simulation, the dummy observations are generated by OOBS, and then they are processed in ODEF, and the estimate state vector and covariance matrix can be obtained. Processing the estimate state vector in OPROPS, the future orbits can be predicted.

In the development of software, object-oriented language such as C++ is adopted. Therefore, the flexible modification of algorithms and models is possible. This software can also process the actual observations of general single satellites as well as a tethered satellite system. The orbit propagation models and the combinations of state vectors can be flexibly selected. The observa-

tions sources include range, range rate, azimuth, and elevation, and the all combinations are available. The flexible choices will expand the possibility of simulations.

#### 4. Sensitivity of the motion of a TSS to the observations

##### 4.1 Observation model

In numerical simulations, general precisions of a modern ground station are used. In this study, a general radar observation system is supposed, and a laser system, Global Positioning System (GPS), and optical observation are not considered. The radar system is the most general observing system, which can observe the range, range rate, and direction of a spacecraft. The general precisions of modern radar system are defined as follows<sup>15)</sup>. About a range and a range rate, root mean square (RMS) of a single observation is defined. About an angle, 0.6-deg half beam width is supposed, and  $\pm 0.3$  deg is defined as uncertainty.

- Range: 15 m (RMS, carrier frequency 100 kHz)
- Range rate: 1 mm/s (RMS, carrier frequency 2 GHz)
- Angle: less than  $\pm 0.3$  deg (carrier frequency 2 GHz, diameter of antenna 15 m)

The altitude, inclination, length of tether, and mass ratio of TSS-1R (1996), which is the famous TSS mission, are adopted as the orbit of the center-of-mass,

$$i = 28.45 \text{ deg}, \quad a = 296 + 6378.137 \text{ km}, \quad e = 0, \quad \ell = 20 \text{ km}, \quad \hat{m} = m_2/m_1 = \infty$$

Where,  $i$  is the inclination,  $a$  is the semi-major axis, and  $e$  is the eccentricity. The  $\hat{m}$  is the mass ratio of two satellites, and the lower satellite is defined as a space shuttle (large mass), and the upper satellite is defined as a small probe (light mass).

In the generation of the dummy observations, the following observing ground station is defined. In this study, the observation by a single ground station is supposed.

$$\lambda_{\text{earth}} = 130.429144 \text{ deg}, \quad \phi_{\text{north}} = 33.621442 \text{ deg}, \quad h = 31.210 \text{ km (Fukuoka, Japan)}$$

Under the mentioned conditions, average observation span per day is 37 min, and average span per single pass is 7.4 min. Because the altitude is low, the observations in more than 20-deg elevation are only 6%.

##### 4.2 Deviations of observations in no librational motion

Compared to the sensitivity that the center-of-mass of a TSS influence the observations, the sensitivity that librational motion influence the observations is much smaller. In order to estimate the length of tether and the librational motion from the observations, the observations must largely reflect the motion compared to the measurement noise.

The observation of the center-of-mass (CM) is defined by  $z_{\text{CM}}$ , which is an actual observation or estimate, and the observation of the end mass (EM) in no libration motion is defined by  $z_{\text{EM0}}$ . The difference of them is defined as the observational deviation  $\delta z_{\text{EM0-CM}}$ , and the expression is as follows.

$$\delta z_{\text{EM0-CM}} = z_{\text{EM0}} - z_{\text{CM}} \tag{6}$$

It is very important to understand the size of the observational deviation before starting a numerical simulation. Therefore, in range, range rate, and angle observations of a radar ground system, the ratio of observational deviation  $\delta z_{\text{EM0-CM}}$  to observation error  $\sigma_z$  is researched.

The histories of observational deviations for observational sources are shown in **Fig. 2**, where the 1-week observations are generated. In these graphs, the observation times are neglected, and all the data are consecutively placed side by side.

From the graphs, RMS with zero mean is 6.949 km in range, 12.022 m/s in range rate, 0.002 deg in azimuth, and 0.872 deg in elevation. To compare the RMS to general observation error, range is 463 times sensitivity to 15-m error, range rate is 12022 times sensitivity to 1-mm/s error. On the other hand, in angle observation, most data are largely affected by noise of 0.3 deg.

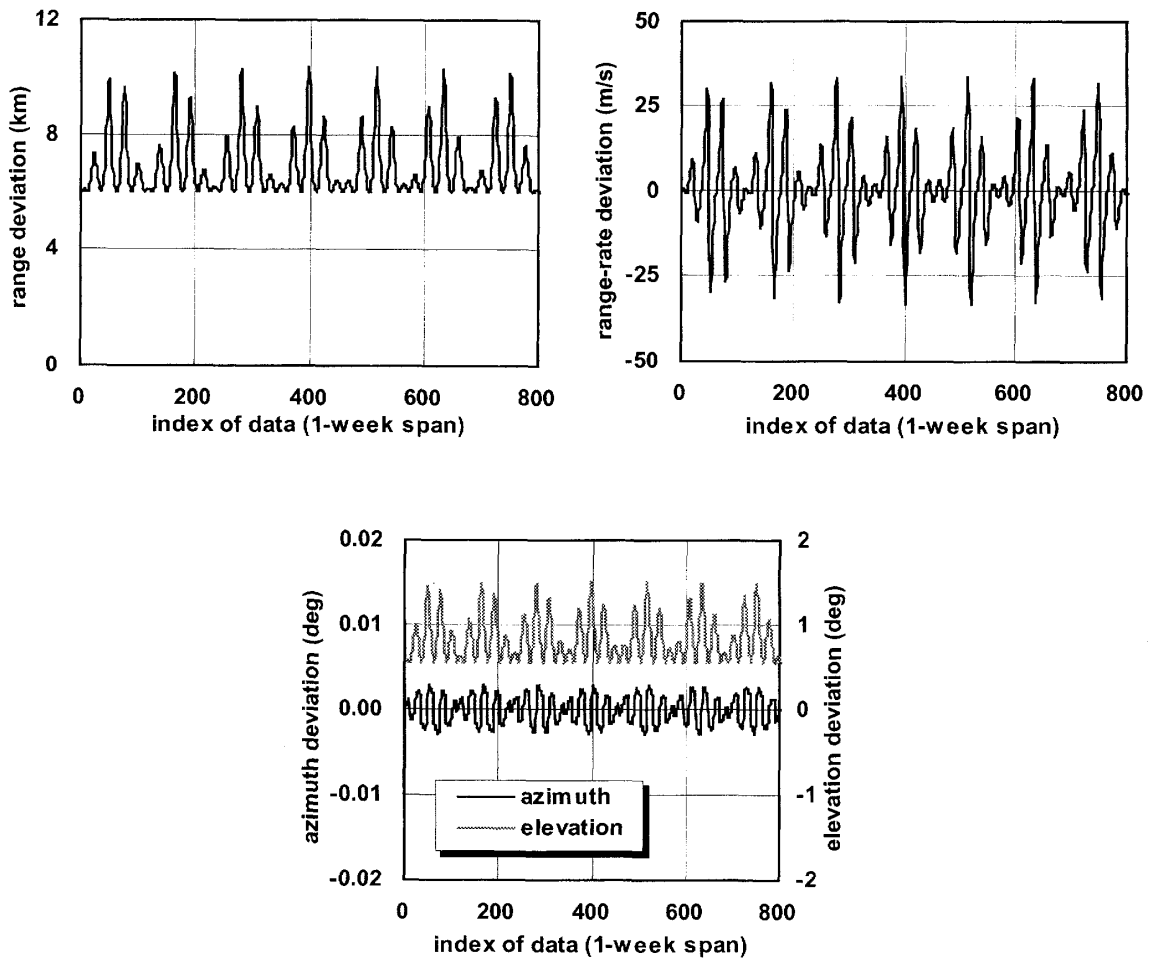
#### 4.3 Deviations of observations in 30-deg amplitude librational motion

The observation of the end mass (EM) in x-deg amplitude libration motion is defined by  $z_{EMx}$ , and the difference  $z_{EMx}$  and  $z_{EM0}$  is defined as the observational deviation  $\delta z_{EMx-EM0}$ , and the expression is as follows.

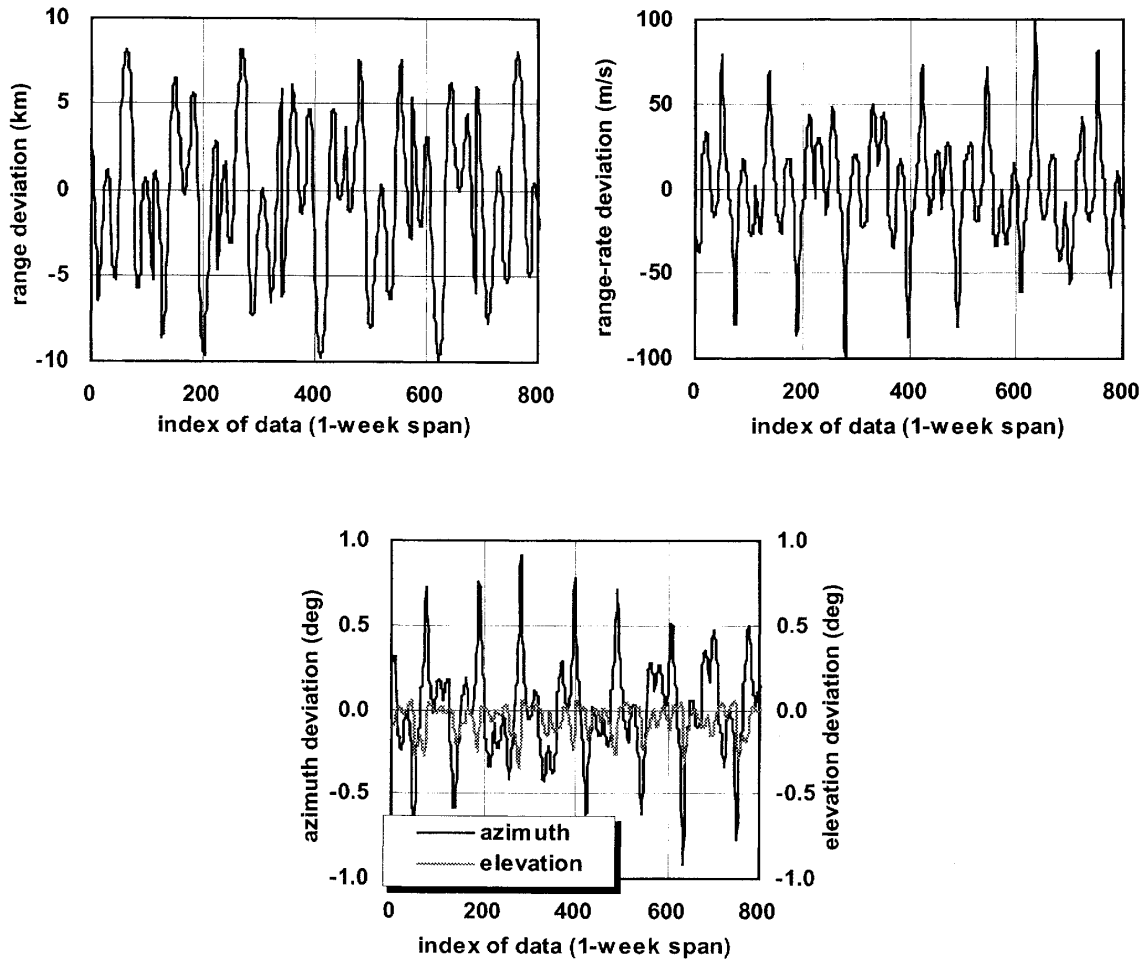
$$\delta z_{EMx-EM0} = z_{EMx} - z_{EM0} \quad (7)$$

Where,  $z_{EMx}$  is the observation of the end mass, which is in-plane moving between -x deg to +x deg. This deviation shows the sensitivity that the librational motion influence observations. The histories of observational deviations  $\delta z_{EM30-EM0}$  for observational sources are shown in **Fig.3**.

From the graphs, RMS with zero mean is 4.458 km in range, 31.043 m/s in range rate, 0.287 deg in azimuth, and 0.102 deg in elevation. To compare the RMS to general observation error, range is 297 times sensitivity to 15-m error, range rate is 31043 times sensitivity to 1-mm/s error. On the other hand, in angle observation, most data are largely affected by noise of 0.3 deg. This



**Fig. 2** History of observational deviations for 1-week span in no librational motion.



**Fig. 3** History of observational deviations for 1-week span in 30-deg amplitude librational motion.

**Table 1** Statistics of observational deviations.

RMS of		EM0-CM	EM5-EM0	EM30-EM0	EM60-EM0
range	km	6.949	0.760	4.458	8.464
range-rate	m/s	12.022	5.337	31.043	56.857
azimuth	deg	0.002	0.049	0.287	0.530
elevation	deg	0.872	0.013	0.102	0.311

tendency is the same as the case in no librational motion.

In this trial, the deviation of range rate is much more superior to one of range compare to the case in zero librational motion. The ratio of the sensitivities of range and range rate is 105 (= 31043/297).

From these results, angle measurement is neglected in following orbit determination simulations. Similar trials are conducted in cases of 5-deg and 60-deg amplitude librational motions, and the statistics of observational deviations are summarized in **Table 1**.



### 5. Simulations of orbit determination

#### 5.1 Estimation of the center-of-mass and the length of tether by no librational model

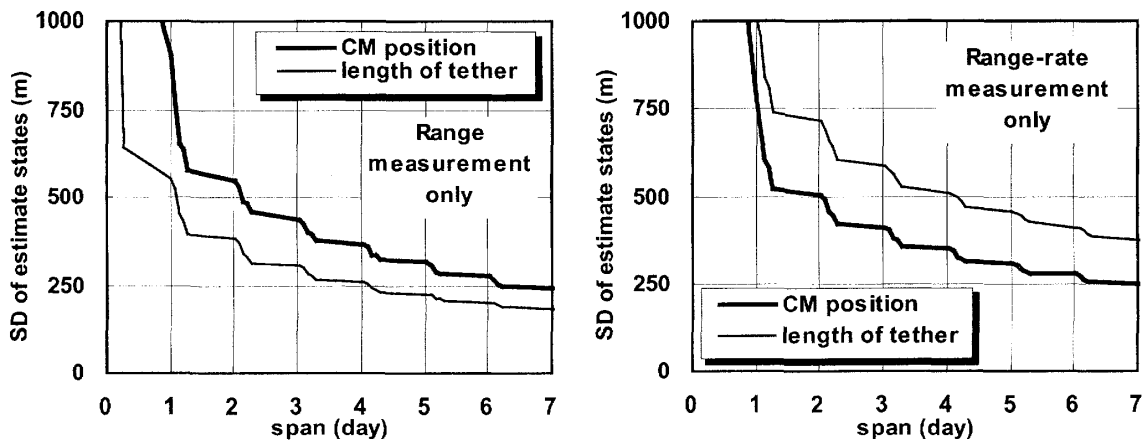
In methods of orbit determination of a TSS, the simplest method is the estimation of the center-of-mass by no libration model. With no a priori initial state, the estimation of all parameters from a short-period observation is quite difficult. To obtain the initial guess of the center-of-mass, this simple method is very useful.

By neglecting the librational motion, the apparent observational error is generated. The RMS of this error is equivalent to the RMS in **Table 1**. That is, in the case of 5-deg amplitude librational motion, the apparent observational error is same as the RMS of  $\delta z_{EM5-EM0}$ . Supposing the general observation error of 15 m and 1 mm/s, the apparent observational error will be 760 m and 5337 mm/s in 5-deg amplitude librational motion. This apparent error is much larger than pure observational error.

In this method, the following state vector  $\mathbf{X}_{TS}$  is defined.

$$\mathbf{X}_{TS} = \{x_c, y_c, z_c, \dot{x}_c, \dot{y}_c, \dot{z}_c, \ell\}^T \tag{8}$$

When assuming the observation RMS of 760 m and 5337 mm/s, the result of covariance analy-



**Fig. 4** Estimate error of the center-of-mass and the length of tether by no librational model (5-deg in-plane amplitude librational motion, 5-s measurement interval).

**Table 2** Results of orbit determination by no librational model (5-deg in-plane amplitude librational motion, 5-s measurement interval).

			Result				
			true	estimate	abs. error (estimate - true)	uncertainty (SD)	error / SD
observation RMS (range-rate)	sigma_drho	mm/s	1.000	5162.000	-	-	-
center-of-mass inertia position	x	km	6674.137	6674.658	0.521	0.155	3.4
	y	km	0.000	0.700	0.700	0.223	3.1
	z	km	0.000	1.155	1.155	0.301	3.8
center-of-mass inertia velocity	dx	m/s	0.000	-1.444	1.444	0.332	4.4
	dy	m/s	6794.780	6794.426	0.353	0.198	1.8
	dz	m/s	3681.591	3680.979	0.612	0.216	2.8
length of tether	lt	km	20.000	19.253	0.747	0.567	1.3

range-rate observation only, 3-day span, 5-sec interval

sis is shown in **Fig. 4**. These graphs show the RMS of the errors in estimate parameters. When the initial guess of center-of-mass is considered, this result is reasonable to some extent.

In the next simulation, the 3-day dummy range-rate observations without measurement error are generated, and orbit determination is conducted. The results are shown in **Table 2**. From this result, the RMS of observations is equivalent to the RMS of  $\delta z_{EM5-EM0}$ , and the difference of estimate and true states are less than 4.4 times of estimate standard deviation (SD).

Comparing the RMS of  $\delta z_{EMx-EM0}$ , in the case of 30-deg in-plane amplitude librational motion, estimate error will be 5.8 times larger than those of **Fig. 4** and **Table 2**, and in the case of 60-deg motion, the error will be 10.7 times larger. That is, the estimate error of the length of tether from the 3-day observation will be 3.3 km (in 30-deg motion), 6.1 km (in 60-deg motion). On the other hand, the estimate error of the CM position will be 400 m (in 5-deg motion), 2300 m (in 30-deg motion), 4300 m (in 60-deg motion).

As a result, the simple estimation method by no librational model will be effective in small librational motion, and the estimate CM will be reasonable as the initial guess in large librational motion. The length of tether, however, is difficult to estimate within reasonable error in the case of 30-deg and 60-deg motion.

## 5.2 State estimation algorithm for the orbit determination of a TSS

The conventional state estimation filters are introduced in many reference books<sup>16)</sup>, and batch least-squares estimation, recursive least-squares estimation, and extended Kalman filter (EKF) are the famous filters.

In the previous study<sup>17)</sup>, when the single motion of the center-of-mass and the single motion of the tether libration are assumed, the estimate error from the long-span observation was considered from the covariance analyses. In the actual observation, however, only one librational motion will be invalid in the over 6-hour observations<sup>5)</sup>. Therefore, when the long-span observations are processed, the span should be divided into small sections, and individual librational motions should be defined in each section.

In this section, new state estimation filter only for the orbit determination of a TSS is introduced. This algorithm is derived from recursive least-squares method. In this filter, libration angles and angular velocities are defined in each pass, or short span. That is, the orbit propagation of the librational angles are limited in the scope of several minutes to several tens of minutes. Therefore, in the actual observation, this estimation filter will work well.

The state vector  $\mathbf{X}$  is defined as follows, where the state vector  $\mathbf{X}_0$  includes the motion of the center-of-mass  $\mathbf{r}_c$ ,  $\dot{\mathbf{r}}_c$ , and the length of tether  $\ell$ . When observation span can be divided into  $p$  passes, the same numbers of  $\mathbf{X}_i$  are defined. The state vector  $\mathbf{X}_i$  includes in-plane and out-of-plane librational angles  $\theta_i$ ,  $\phi_i$  and angular velocities  $\dot{\theta}_i$ ,  $\dot{\phi}_i$ . When the motion of the center-of-mass or the length of tether is changed, all the librational parameters will be changed.

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_0 \\ \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_p \end{bmatrix}, \mathbf{X}_0 = \begin{bmatrix} \mathbf{r}_c \\ \dot{\mathbf{r}}_c \\ \ell \end{bmatrix}, \mathbf{X}_i = \begin{bmatrix} \theta_i \\ \dot{\theta}_i \\ \phi_i \\ \dot{\phi}_i \end{bmatrix} \quad (i = 1 \cdots p) \quad (9)$$

When  $p = 2$ , the state vector  $\mathbf{X}$ , the observation vector residual  $\Delta \mathbf{Z}$ , the inverse matrix of observation covariance matrix  $R^{-1}$ , the sensitivity matrix  $H = \partial \mathbf{Z} / \partial \mathbf{X}$  are defined as follows, where attached  $m \times n$  shows the sizes of row and column. The size of state vector is  $m$ , and the size of observation vector is  $n$ .

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_0 \\ \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix}_{m \times 1}, \quad \Delta \mathbf{Z} = \begin{bmatrix} \Delta \mathbf{Z}_1 \\ \Delta \mathbf{Z}_2 \end{bmatrix}_{n \times 1}, \quad R^{-1} = \begin{bmatrix} R_1^{-1} & 0 \\ 0 & R_2^{-1} \end{bmatrix}_{n \times n} \quad (10)$$

$$H = \begin{bmatrix} H_{10} & H_{11} & 0 \\ H_{20} & 0 & H_{22} \end{bmatrix}_{n \times m}, \quad H^T = \begin{bmatrix} H_{10}^T & H_{20}^T \\ H_{11}^T & 0 \\ 0 & H_{22}^T \end{bmatrix}_{m \times n} \quad (11)$$

Substituting into the general formula of  $P^{-1}$ , this is inverse matrix of estimate covariance matrix,

$$P^{-1} = H^T R^{-1} H = \begin{bmatrix} H_{10}^T & H_{20}^T \\ H_{11}^T & 0 \\ 0 & H_{22}^T \end{bmatrix}_{m \times n} \begin{bmatrix} R_1^{-1} & 0 \\ 0 & R_2^{-1} \end{bmatrix}_{n \times n} \begin{bmatrix} H_{10} & H_{11} & 0 \\ H_{20} & 0 & H_{22} \end{bmatrix}_{n \times m}$$

$$\therefore P_{i=2}^{-1} = \begin{bmatrix} H_{10}^T R_1^{-1} H_{10} + H_{20}^T R_2^{-1} H_{20} & H_{10}^T R_1^{-1} H_{11} & H_{20}^T R_2^{-1} H_{22} \\ H_{11}^T R_1^{-1} H_{10} & H_{11}^T R_1^{-1} H_{11} & 0 \\ H_{22}^T R_2^{-1} H_{20} & 0 & H_{22}^T R_2^{-1} H_{22} \end{bmatrix}_{m \times m} \quad (12)$$

Where,  $P_{i=2}$  is the estimate covariance matrix when the state vector  $\mathbf{X}$  is estimated by the observations  $\mathbf{Z}_1 \cdots \mathbf{Z}_2$ . When  $i = p$ ,  $P^{-1}$  is defined by the following expression.

$$\therefore P^{-1} = \begin{bmatrix} \sum_{i=1}^p H_{i0}^T R_i^{-1} H_{i0} & H_{10}^T R_1^{-1} H_{11} & \cdots & H_{p0}^T R_p^{-1} H_{pp} \\ H_{11}^T R_1^{-1} H_{10} & H_{11}^T R_1^{-1} H_{11} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ H_{pp}^T R_p^{-1} H_{p0} & 0 & 0 & H_{pp}^T R_p^{-1} H_{pp} \end{bmatrix}_{m \times m} \quad (13)$$

$P^{-1}$  can be simplified as follows

$$P^{-1} = \begin{bmatrix} P_0^{-1} & \Phi_{10}^T & \cdots & \Phi_{p0}^T \\ \Phi_{10} & P_{11}^{-1} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \Phi_{p0} & 0 & 0 & P_{pp}^{-1} \end{bmatrix} \quad (14)$$

where

$$H_{i0} = \frac{\partial \mathbf{Z}_i}{\partial \mathbf{X}_0}, \quad H_{ii} = \frac{\partial \mathbf{Z}_i}{\partial \mathbf{X}_i}, \quad P_0^{-1} = \sum_{i=1}^p H_{i0}^T R_i^{-1} H_{i0}, \quad P_{ii}^{-1} = H_{ii}^T R_i^{-1} H_{ii}, \quad \Phi_{i0} = H_{ii}^T R_i^{-1} H_{i0} \quad (15)$$

On the other hand, substituting the vectors of  $p=2$  into the general formula of  $\Delta \mathbf{X}$ ,

$$P^{-1} \Delta \mathbf{X} = H^T R^{-1} \Delta \mathbf{Z} = \begin{bmatrix} H_{10}^T & H_{20}^T \\ H_{11}^T & 0 \\ 0 & H_{22}^T \end{bmatrix}_{m \times n} \begin{bmatrix} R_1^{-1} & 0 \\ 0 & R_2^{-1} \end{bmatrix}_{n \times n} \begin{bmatrix} \Delta \mathbf{Z}_1 \\ \Delta \mathbf{Z}_2 \end{bmatrix}_{n \times 1}$$

$$\therefore P_{i=2}^{-1} \Delta \mathbf{X}_{i=2} = \begin{bmatrix} H_{10}^T R_1^{-1} \Delta \mathbf{Z}_1 + H_{20}^T R_2^{-1} \Delta \mathbf{Z}_2 \\ H_{11}^T R_1^{-1} \Delta \mathbf{Z}_1 \\ H_{22}^T R_2^{-1} \Delta \mathbf{Z}_2 \end{bmatrix}_{m \times 1} \quad (16)$$

That is,

$$\begin{bmatrix} P_0^{-1} & \Phi_{10}^T & \Phi_{20}^T \\ \Phi_{10} & P_{11}^{-1} & 0 \\ \Phi_{20} & 0 & P_{22}^{-1} \end{bmatrix}_{m \times m} \begin{bmatrix} \Delta \mathbf{X}_0 \\ \Delta \mathbf{X}_1 \\ \Delta \mathbf{X}_2 \end{bmatrix}_{m \times 1} = \begin{bmatrix} H_{10}^T R_1^{-1} \Delta \mathbf{Z}_1 + H_{20}^T R_2^{-1} \Delta \mathbf{Z}_2 \\ H_{11}^T R_1^{-1} \Delta \mathbf{Z}_1 \\ H_{22}^T R_2^{-1} \Delta \mathbf{Z}_2 \end{bmatrix}_{m \times 1}$$

$$\therefore P_0^{-1} \Delta \mathbf{X}_0 = H_{10}^T R_1^{-1} \Delta \mathbf{Z}_1 + H_{20}^T R_2^{-1} \Delta \mathbf{Z}_2 - \Phi_{10}^T \Delta \mathbf{X}_1 - \Phi_{20}^T \Delta \mathbf{X}_2 \quad (17)$$

$$\therefore P_{11}^{-1} \Delta \mathbf{X}_1 = H_{11}^T R_1^{-1} \Delta \mathbf{Z}_1 - \Phi_{10} \Delta \mathbf{X}_0 \quad (18)$$

$$\therefore P_{22}^{-1} \Delta \mathbf{X}_2 = H_{22}^T R_2^{-1} \Delta \mathbf{Z}_2 - \Phi_{20} \Delta \mathbf{X}_0 \quad (19)$$

Therefore, the following general expressions about the state correction vector  $\Delta \mathbf{X}_0$  and  $\Delta \mathbf{X}_i$  can be obtained.

$$P_0^{-1} \Delta \mathbf{X}_0 = \sum_{i=1}^p \{ H_{i0}^T R_i^{-1} \Delta \mathbf{Z}_i - \Phi_{i0}^T \Delta \mathbf{X}_i \} \quad (20)$$

$$P_{ii}^{-1} \Delta \mathbf{X}_i = H_{ii}^T R_i^{-1} \Delta \mathbf{Z}_i - \Phi_{i0} \Delta \mathbf{X}_0 \quad (i = 1 \dots p) \quad (21)$$

Substituting (21) into (20),

$$\left( P_0^{-1} - \sum_{i=1}^p \Phi_{i0}^T P_{ii}^{-1} \Phi_{i0} \right) \Delta \mathbf{X}_0 = \sum_{i=1}^p H_{i0}^T R_i^{-1} \Delta \mathbf{Z}_i - \sum_{i=1}^p \Phi_{i0}^T P_{ii}^{-1} H_{ii}^T R_i^{-1} \Delta \mathbf{Z}_i \quad (22)$$

From this formula, the state correction vector  $\Delta \mathbf{X}_0$  is obtained for the observation vector residual  $\Delta \mathbf{Z}$ , and then, the state correction vector  $\Delta \mathbf{X}_i$  in each pass can be obtained from (21).

In this study, long-span dummy observation is prepared, and the state vectors  $\mathbf{X}_0$  and  $\mathbf{X}_i$  are estimated. This kind of orbit determination is called as offline method. The real-time method will be obtained from mentioned formulas, but the detail analysis will be conducted in future works.

The performance of convergence can be improved by Marquardt's Algorithm<sup>16)</sup>. By this algorithm, the value of  $P^{-1}$  is controlled to smoothly decrease the observation vector residual  $\Delta \mathbf{Z}$  and the state correction vectors  $\Delta \mathbf{X}_0$ ,  $\Delta \mathbf{X}_i$ .

### 5.3 Evaluation of the algorithm by numerical simulations

At first, using the general observation error 15 m, 1 mm/s of range and range rate, the covariance analysis is conducted. Supposing the 5-deg in-plane amplitude librational motion, the observations are obtained with the 5-s interval, and the observation span is 7 days. The results from the range observation only, and the range-rate observation only are compared in **Table 3**. From this result, the case using range only will estimate with large error compared to the range-rate case. When the similar estimate performance is demanded, the observations with 0.1 m RMS should be prepared.

**Table 3** Estimation error from 7-day observation (5-deg in-plane amplitude librational motion, 5-s measurement interval).

			SD of estimated states	
			range only	range-rate only
observation RMS	sigma_rho	m	15	-
	sigma_drho	mm/s	-	1
CM position	r_total	m	152	2
length of tether	lt	m	3534	16
in-plane libration angle (first pass)	theta	deg	9	0.02
out-of-plane libration angle (first pass)	phi	deg	4	0.04
<b>observation span = 7 days</b>				

**Table 4** Results of the orbit determination of a TSS from 3-day observation (5-deg in-plane amplitude librational motion, 15-s measurement interval).

Item	Symbol	Unit	Result					
			true	a priori	estimate	abs. error (estimate - true)	uncertainty (SD)	error / SD
observation RMS (range-rate)	sigma_drho	mm/s	1.000	1.000	0.950	-	-	-
	x	km	6674.137	6674.827	6674.138	0.001	0.000	<b>1.2</b>
center-of-mass inertia position	y	km	0.000	0.868	0.001	0.001	0.001	<b>0.8</b>
	z	km	0.000	1.452	0.001	0.001	0.001	<b>1.4</b>
	dx	m/s	0.000	-1.787	-0.001	0.001	0.001	<b>1.3</b>
center-of-mass inertia velocity	dy	m/s	6794.780	6794.247	6794.778	0.002	0.001	<b>2.2</b>
	dz	m/s	3681.591	3680.900	3681.593	0.002	0.001	<b>3.0</b>
length of tether	lt	km	20.000	19.317	20.004	0.004	0.001	<b>2.8</b>
in-plane angle (first pass)	theta	deg	-1.799	0	-1.797	0.001	0.001	<b>1.2</b>
in-plane angular velocity (first pass)	dtheta	1e-3 deg/s	-9.343	0	-9.345	0.002	0.003	<b>0.7</b>
in-plane angle (last pass)	theta	deg	-2.162	0	-2.160	0.002	0.001	<b>2.3</b>
in-plane angular velocity (last pass)	dtheta	1e-3 deg/s	-9.028	0	-9.035	0.007	0.003	<b>2.0</b>

range-rate observation only, 3-day span, 15-sec interval

In the next simulation, the orbit determination is conducted by the following procedures to improve the stability of convergence.

1. Estimation of the center-of-mass and the length of tether by no librational model (5.1)
2. All the libration angles and angular velocities are set to zero in initial state vector, and state estimation filter of a TSS is adopted (5.2)

If procedure 2 is conducted without the reasonable initial guess of the center-of-mass motion, the process will be difficult to find the optimum estimate states.

In this simulation, 5-deg in-plane amplitude librational motion is supposed, and the observations are obtained with the 15-s interval, and the observation span is 3 days. In addition, the parameters of out-of-plane librational angles and angular velocities are all neglected. The simulation results are shown in **Table 4**. The result shows the estimate of the in-plane librational motion in first pass and final pass.

As a result, the difference of estimate and true is under 3.0 times as the estimate standard deviation. That is, the estimate states are purely reflecting the random measurement error, and this validity of the estimation method is proved.

At this time, some problems of this method are found. At first, when the parameters of the out-of-plane librational motion are included in the state vector, the performance of the convergence becomes worse. Secondly, the calculation speed is very slow. In the future works, this filtering method will be improved to be more effective filter.

## 6. Initial orbit determination of a TSS

### 6.1 Algorithm for the initial orbit determination

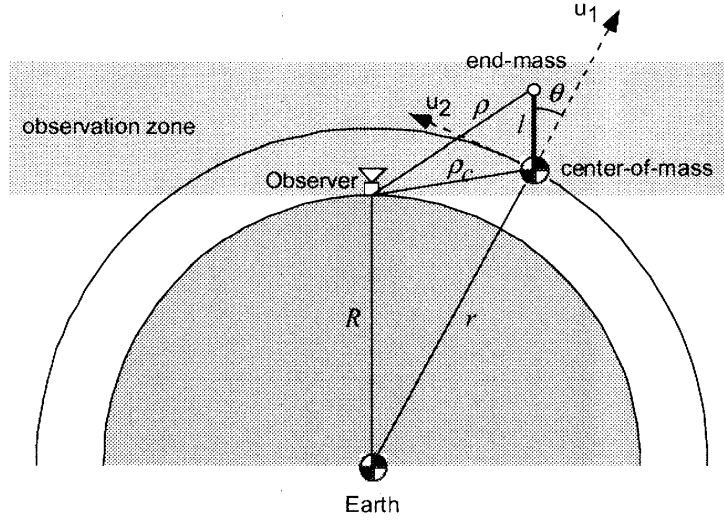
In the preceding chapters, the filtering method for a TSS was discussed. In this chapter, however, the librational motion is not determined from many observations, the method to estimate it from minimum observation data is introduced. This type of method is classified as “initial orbit determination”. For a general single satellites, various techniques of initial orbit determination are proposed. On the other hand, for a TSS, the necessity of initial determination technique are frequently mentioned, but detail considerations are not conducted in the past. In the filtering method, when the librational motion is large, the allowable error to initial guess is very small. If the initial determination technique is available, the time of analysis can be decreased.

The methods proposed in this section will estimate the tether motion only from the difference of end-mass observation and center-of-mass observation. Therefore, the orbit of center-of-mass has to be known before. The observation source is limited in range or range-rate.

## 6.2 Analytical formula of observational deviations

The observation model is shown in **Fig. 5**. Defining the range and range-rate observations of the center-of-mass of a TSS (actual or estimated) by  $\rho_c, \dot{\rho}_c$ , the observational deviations  $\delta\rho, \delta\dot{\rho}$  are defined by the following expressions.

$$\delta\rho = \rho - \rho_c, \quad \delta\dot{\rho} = \dot{\rho} - \dot{\rho}_c \quad (23)$$



**Fig. 5** Observation model.

In the orbit-define coordinate frame  $u_1u_2u_3$ , the position vector of the center-of-mass  $\mathbf{r}_c$ , the position vector of the observer  $\mathbf{R}$ , and the tether length vector  $\mathbf{l}$  are defined as follows, where  $\theta$  is the in-plane librational angle.

$$\mathbf{r}_c = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}_u, \quad \mathbf{R} = \begin{bmatrix} X_u \\ Y_u \\ Z_u \end{bmatrix}_u, \quad \mathbf{l} = \begin{bmatrix} \ell \cos \theta \\ \ell \sin \theta \\ 0 \end{bmatrix}_u \quad (24)$$

Where, the following parameters are adopted,

$$\frac{R}{r} \equiv \gamma_R, \quad \frac{\rho_c}{r} \equiv \gamma_c, \quad \frac{\ell}{r} \equiv x, \quad \frac{X_u}{R} \equiv \alpha, \quad \frac{Y_u}{R} \equiv \beta, \quad \frac{Z_u}{R} \equiv \xi, \quad \ell \ll r \quad (25)$$

The range vector of the center-of-mass  $\rho_c$ , and the range scalar of the end mass  $\rho$  are

$$\rho_c = \mathbf{r}_c - \mathbf{R} = \begin{bmatrix} r - X_u \\ -Y_u \\ -Z_u \end{bmatrix}_u \equiv r \begin{bmatrix} 1 - \gamma_R \alpha \\ -\gamma_R \beta \\ -\gamma_R \xi \end{bmatrix}_u \quad (26)$$

$$\frac{\rho^2}{r^2} = \frac{1}{r^2} \{ \rho_c^2 + \ell^2 + 2(\rho_c \cdot \mathbf{l}) \} = \gamma_c^2 + x^2 + 2x \{ (1 - \gamma_R \alpha) \cos \theta - \gamma_R \beta \sin \theta \}$$

$$\therefore \frac{\rho}{r} \approx \sqrt{\gamma_c^2 + 2x \{ (1 - \gamma_R \alpha) \cos \theta - \gamma_R \beta \sin \theta \}} \quad (27)$$

Defining the formula of  $\rho/r$  as the function of  $x$ , the following expression is obtained,

$$\frac{\rho}{r} = \left(\frac{\rho}{r}\right)_{x=0} + \left(\frac{\partial \rho}{\partial x} \frac{\rho}{r}\right)_{x=0} x + \frac{1}{2} \left(\frac{\partial^2 \rho}{\partial x^2} \frac{\rho}{r}\right)_{x=0} x^2 + \dots \quad (28)$$

where

$$\left(\frac{\partial \rho}{\partial x} \frac{\rho}{r}\right)_{x=0} = \frac{1}{\gamma_\rho} \{(1 - \gamma_R \alpha) \cos \theta - \gamma_R \beta \sin \theta\}, \quad \frac{\delta \rho}{r} \approx \left(\frac{\partial \rho}{\partial x} \frac{\rho}{r}\right)_{x=0} x$$

therefore

$$\therefore \delta \rho_\theta \approx \frac{\ell}{\gamma_\rho} \{(1 - \gamma_R \alpha) \cos \theta - \gamma_R \beta \sin \theta\} \quad \text{or} \quad \delta \rho_\theta \approx -\ell \sqrt{1 - \frac{\gamma_R^2}{\gamma_\rho^2} \xi^2} \sin(\theta - \eta) \quad (29)$$

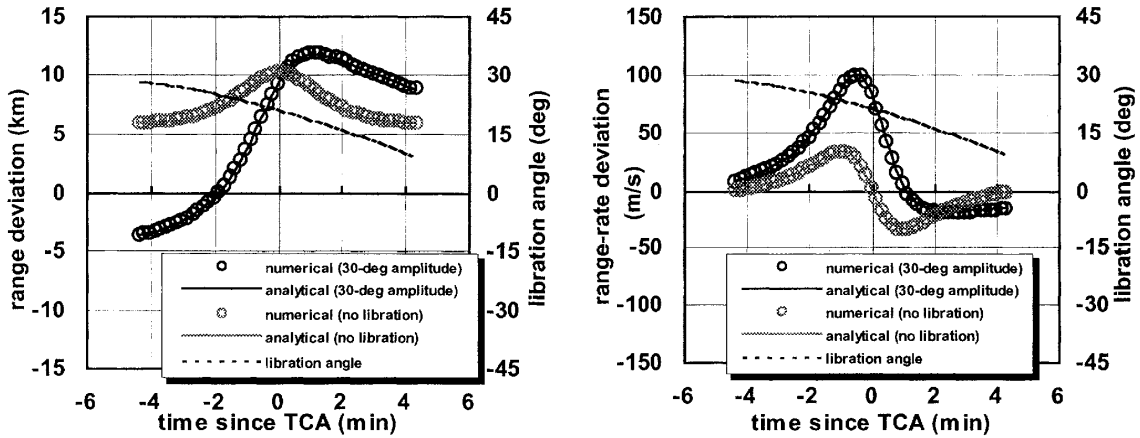
where

$$\cos \eta = \frac{\gamma_R \beta}{\gamma_\rho} \left(1 - \frac{\gamma_R^2}{\gamma_\rho^2} \xi^2\right)^{-\frac{1}{2}}, \quad \sin \eta = \frac{1 - \gamma_R \alpha}{\gamma_\rho} \left(1 - \frac{\gamma_R^2}{\gamma_\rho^2} \xi^2\right)^{-\frac{1}{2}} \quad (30)$$

The range-rate deviation  $\delta \dot{\rho}$  can be derived by the time derivative of range deviation  $\delta \rho$ ,

$$\begin{aligned} \delta \dot{\rho}_\theta \approx & \ell \left\{ \dot{u} \frac{\gamma_R}{\gamma_\rho^2} (\alpha - \gamma_R) - \dot{\theta} \right\} \sqrt{1 - \frac{\gamma_R^2}{\gamma_\rho^2} \xi^2} \cos(\theta - \eta) \\ & + \ell \dot{u} \frac{\gamma_R^2}{\gamma_\rho^3} \xi^2 \sin \theta - \ell \frac{\gamma_R}{\gamma_\rho^2} \frac{\dot{X}_u}{R} \sqrt{1 - \frac{\gamma_R^2}{\gamma_\rho^2} \xi^2} \sin(\theta - \eta) - \ell \frac{\gamma_R}{\gamma_\rho} \left( \frac{\dot{X}_u}{R} \cos \theta + \frac{\dot{Y}_u}{R} \sin \theta \right) \end{aligned} \quad (31)$$

The comparisons of the numerical solution and the analytical solution of range deviation and range-rate deviation are shown in **Fig. 6**, where the results are shown in cases of no librational motion and 30-deg in-plane amplitude librational motion. The observation conditions are already defined in Section 4.1. As a result, the numerical and analytical solutions coincide reasonably.



**Fig. 6** Comparison of numerical and analytical solutions of observational deviation.

### 6.3 Estimation of the librational motion when the length of tether is known

When the length of tether is known, the following three methods to estimate the librational motion are proposed. Every method can estimate in-plane librational angle and angular velocity.

1. Estimation method from the range deviation  $\delta\rho_t$  and the range-rate deviation  $\delta\dot{\rho}_t$  at time  $t$
2. Estimation method from the range deviations  $\delta\rho_t, \delta\rho_{t+\delta t}$  at time  $t, t+\delta t$
3. Estimation method from the range-rate deviations  $\delta\dot{\rho}_t, \delta\dot{\rho}_{t+\delta t}$  at time  $t, t+\delta t$

In the method 1, from the analytical formulas of range and range-rate deviations, libration angle  $\theta_t$  and angular velocity  $\dot{\theta}_t$  can be calculated. In the method 2, from the analytical formula of range deviations, libration angles  $\theta_t, \theta_{t+\delta t}$  at time  $t, t+\delta t$  can be calculated, and the libration angular velocity  $\dot{\theta}_t$  is approximately predicted. In the method 3, the Newton iterative procedure with the parameter of  $\theta_t$  is conducted. At first, the angular velocity  $\dot{\theta}_t$  is calculated from the analytical formula, and then the range-rate deviation  $\delta\bar{\rho}_{t+\delta t}$  is predicted. When the difference of  $\delta\bar{\rho}_{t+\delta t}$  and  $\delta\dot{\rho}_{t+\delta t}$  is converging to zero, the iterative process is finished.

#### 6.4 Estimation of the librational motion when the length of tether is unknown

When the length of tether is unknown, the following four methods to estimate the length of tether and the librational motion are proposed. Every method can estimate the length of tether, in-plane librational angle and angular velocity. Methods 1 and 2 can be used in the ground station that can observe both of a range and a range-rate, and method 3 can be used when a range observation is only available, and method 4 can be used when a range-rate observation is only available.

1. Estimation method from the range and range-rate deviations  $\delta\rho_t, \delta\dot{\rho}_t$  at time  $t$ , and the range deviation  $\delta\rho_{t+\delta t}$  at time  $t+\delta t$
2. Estimation method from the range and range-rate deviations  $\delta\rho_t, \delta\dot{\rho}_t$  at time  $t$ , and the range-rate deviation  $\delta\dot{\rho}_{t+\delta t}$  at time  $t+\delta t$
3. Estimation method from the three range deviations  $\delta\rho_t, \delta\rho_{t-\delta t}, \delta\rho_{t+\delta t}$  at time  $t, t-\delta t, t+\delta t$
4. Estimation method from the three range-rate deviations  $\delta\dot{\rho}_t, \delta\dot{\rho}_{t-\delta t}, \delta\dot{\rho}_{t+\delta t}$  at time  $t, t-\delta t, t+\delta t$

In all the methods, the Newton iterative procedure with the parameter of  $\ell$  should be conducted. In the method 1, the libration angle and angular velocity  $\theta_t, \dot{\theta}_t$  can be calculated from the analytical formula of  $\delta\rho_t$  and  $\delta\dot{\rho}_t$ , and then the range deviation  $\delta\bar{\rho}_{t+\delta t}$  is predicted, and compared with  $\delta\rho_{t+\delta t}$ . Similarly, in the method 2, the predicted  $\delta\bar{\rho}_{t+\delta t}$  and the  $\delta\dot{\rho}_{t+\delta t}$  are compared. In the method 3, from two range deviations of  $\delta\rho_t$  and  $\delta\rho_{t-\delta t}$ , the libration angle and angular velocity  $\theta_t$  and  $\dot{\theta}_t$  are calculated from the method 2 in Section 6.3, and then the range deviation  $\delta\bar{\rho}_{t+\delta t}$  is predicted and compared with  $\delta\rho_{t+\delta t}$ . In the method 4, the libration angle and angular velocity  $\theta_t$  and  $\dot{\theta}_t$  can be calculated from  $\delta\dot{\rho}_t$  and  $\delta\dot{\rho}_{t-\delta t}$  by the method 3 in Section 6.3, and then the range deviation  $\delta\bar{\rho}_t$  is predicted. Similarly, from the  $\delta\dot{\rho}_t$  and  $\delta\dot{\rho}_{t+\delta t}$ ,  $\delta\bar{\rho}_t$  is predicted, and compared to  $\delta\bar{\rho}_t$ .

The performances of four methods are compared here. The estimate errors of length of tether, libration angle, angular velocity are compared, and ranges of initial guess to obtain the converged solutions are summarized. The simulations are conducted at 9 points in a single span. The tendency at TCA (Time of Closest Approach) is different from results at other points. Therefore, average estimation error and TCA estimation error are classified. In all the simulations,  $\delta t = 10$  s, and observation error is neglected. The estimation errors in these simulations are derived from the approximate technique, not the observation error. When the unit of the estimate is deg or deg/s, the equivalent distance or velocity (m or m/s) are also shown in the case of 20-km tether. The results are shown in Fig. 7.

In estimation of the length of tether and librational angle, the performance of method 2 is supe-



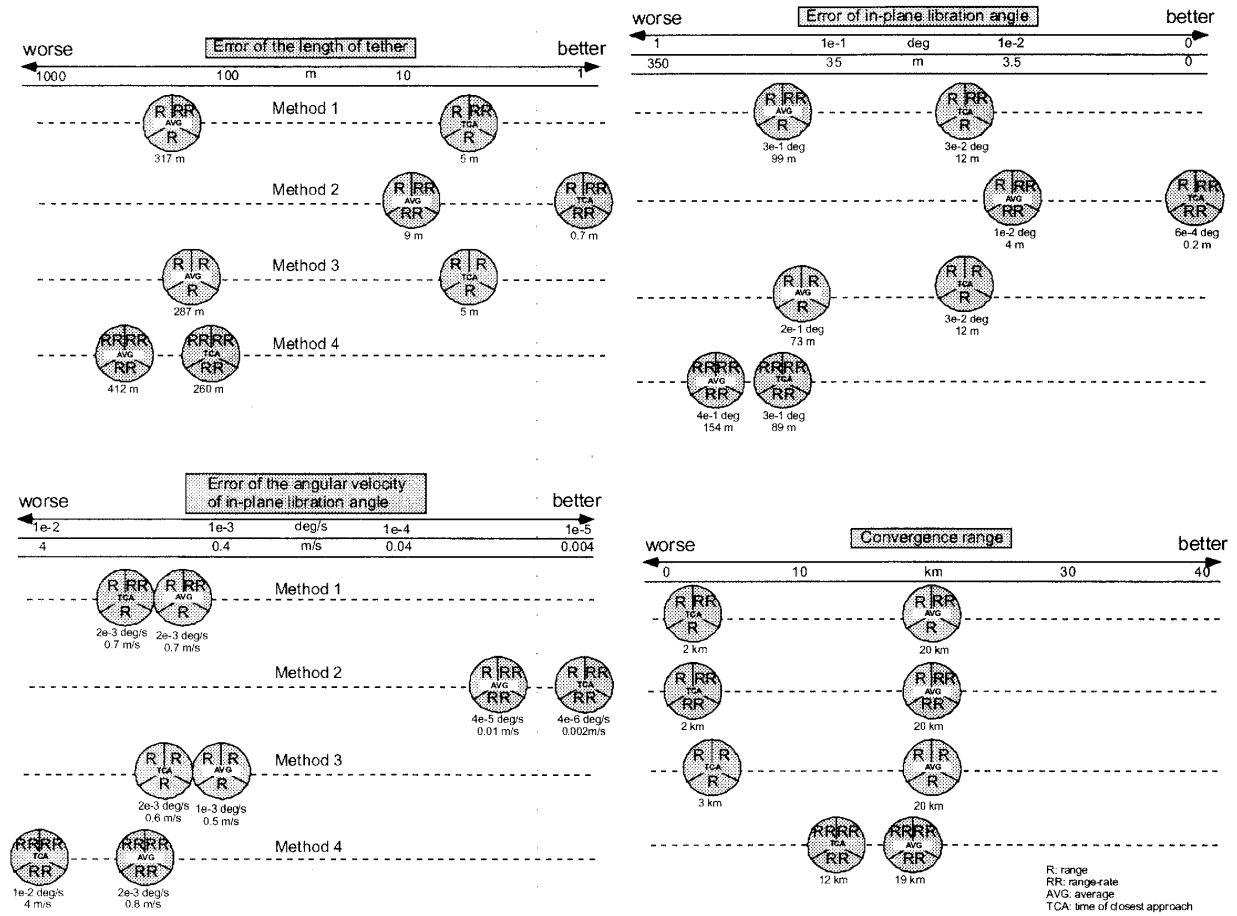


Fig. 7 Comparison of the methods when the length of tether is unknown.

rior to others in both of average and TCA. In other methods, the average performances are worse. In the method 1 and 3, the TCA performances are better. In the estimation of the librational angular velocity, in all methods, the differences of average and TCA are small. The method 2 has the best performance, and the performances of other methods are similar. About the convergence range, the average performance are about 20 km. However, TCA performance except method 4, range is just 2-3 km.

As a result, the following conclusions are obtained. When the observation system can observe both of range and range rate, the method 2 using single range and double range rate is the best choice. When the range observation is only available, the method 3 using triple range can be used. When the range rate observation is only available, the method 4 using triple range rate can be used.

### 7. Conclusions

In general, motion of orbital objects follows the Kepler's law, and an orbit determination system in ground stations has the algorithm to analyze the Kepler's motion. A tethered satellite system (TSS) is the representative future space system that does not follow the conventional space dynamics. Therefore, new methods for the orbit determination of TSS are necessary.

Reviewing the previous researches, the following three problems are considered in this paper. At first, the sensitivity that the length of tether and the librational motion influence observations, which are range, range rate, and direction, is researched. Secondly, the new filtering algorithm is proposed for the orbit determination of a TSS. Thirdly, the initial orbit determination of a TSS is proposed.

In the first subject, the following conclusions are obtained. Comparing the general precision of range, range rate, and direction observations, and influence of a tether motion on observations, it was made clear that observations of range and range rate can reflect the motion of a TSS, on the other hand, observations of direction are not effective for the detection of a TSS motion. Especially, range rate observation is superior to range in detection of the librational motion.

In the second subject, it was proved that the estimation of the center-of-mass of a TSS by supposing no libration is effective in the case of small librational motion. In large librational motion, the estimate of the center-of-mass is still effective, but the estimate of the tether length is not reasonable. Moreover, the new algorithm for the orbit determination of a TSS is proposed. From the numerical simulations, it was confirmed that the excellent estimate purely reflecting the measurement random error could be obtained.

In the third subject, when the motion of the center-of-mass is already known, analytical formulas of range and range rate are introduced. From these formulas, methods of the initial determination of the length of tether, librational angle, and angular velocity were introduced, and the validity was proved by numerical simulations.

At this time, some problems are identified. Firstly, the new state estimation filter cannot sufficiently process the out-of-plane librational motion. In addition, the processing time has to be more shortened. Secondly, the discussed methods of initial orbit determination are not considering the out-of-plane motion and the measurement error. In the future study, the performances of the methods will be compared with the measurement error.

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